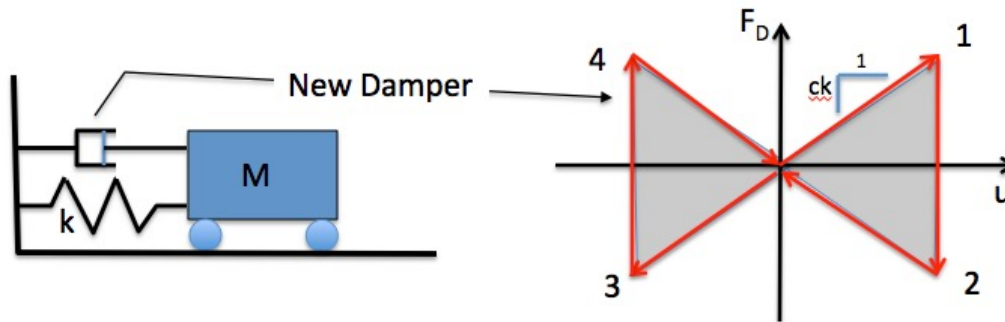


## Preliminary Examination in Dynamics

1. A new type of damper has been developed that has special force ( $F_D$ ) vs. displacement ( $u$ ) characteristics. These are shown in the right hand side figure below.



In this case, the damping force vs. displacement loop under steady state conditions goes from the origin (0) to point 1, then to point 2, then back to the origin (0) and then to point 3, then to point 4, and finally back to the origin (0). This can be mathematically described as:

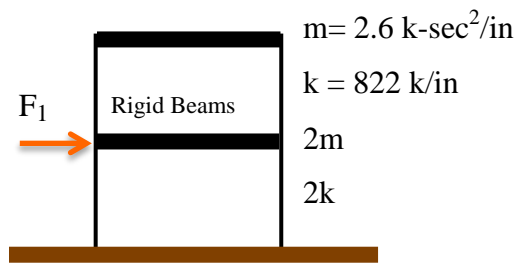
$$F_D = ck|u|\frac{\dot{u}}{|\dot{u}|}$$

Where the parameter  $c$  is a designer assignable property of the damper, and  $k$  and  $M$  are the lateral stiffness and mass of the structure in which the damper is to be installed (see figure above on left). As an approximation in the calculations below, assume that an 'equivalent' linear damper can be used to represent the new type of damper.

The structure shown is subjected to a sinusoidal lateral force ( $p(t) = A \sin \omega_e t$ ) that induces steady state vibration in the system.

1. Develop an equation for the new type of damper expressing the equivalent linear damping ratio as a function of  $c$ ,  $A$ ,  $u$ ,  $k$ ,  $M$  or other basic parameters identified above.
  2. Does the damping ratio depend on  $A$ , the frequency of excitation ( $\omega_e$ ) or the natural frequency ( $k$  and  $M$ ) of the structure?
  3. Estimate the value of ' $c$ ' is needed for the damper to limit the peak steady state displacement of the structure to five times the displacement it would develop if it were loaded statically with constant lateral load  $A$ .
2. Consider the two-degree-of-freedom linear elastic system shown below. Its masses, stiffnesses, mode shapes and frequencies are as given. Note: One of you colleagues put a coffee cup on your exam and one of the terms is now not readable. The structure is subjected to a suddenly imposed, constant lateral force  $F$  at the lower level (instantaneous step function). What is the displacement do you expect at the roof at time  $2\pi/\omega_1$ ?

Name: \_\_\_\_\_



$$\omega_1 = \sqrt{\frac{k}{2m}} \quad \phi_1 = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$$

$$\omega_2 = \sqrt{\frac{2k}{m}} \quad \phi_2 = \begin{bmatrix} -1 \\ \square \end{bmatrix}$$

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Department of Civil and Environmental Engineering

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Structural Engineering, Mechanics and Materials  
Spring Semester 2014

**M.S. Comprehensive Examination**  
**Structural Dynamics**

**Problem 1:** (30 points)

An SDF system is excited by a sinusoidal force. At resonance the amplitude of displacement was measured to be 5 in. At an exciting frequency of one-tenth the natural frequency of the system, the displacement amplitude was measured to be 0.1 in. Estimate the damping ratio of the system.

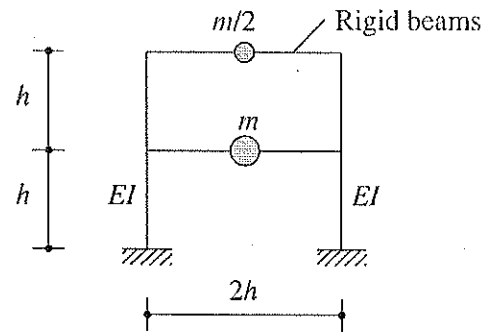
**Problem 2:** (70 points)

The natural vibration frequencies and modes of vibration of the 2-story frame are:

$$\omega_1 = 0.765\sqrt{k/m} \quad \omega_2 = 1.848\sqrt{k/m}$$

$$\phi_1 = \begin{Bmatrix} 1/\sqrt{2} \\ 1 \end{Bmatrix} \quad \phi_2 = \begin{Bmatrix} -1/\sqrt{2} \\ 1 \end{Bmatrix}$$

where  $k = 24EI/h^3$ .



If the frame is excited by horizontal ground motion  $\ddot{u}_g(t)$ , determine (a) the floor displacement response in terms of  $D_n(t)$ , (b) the story shears in terms of  $A_n(t)$ , and (c) the first-floor and base overturning moments in terms of  $A_n(t)$ .

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UNIVERSITY OF CALIFORNIA, BERKELEY  
Fall Semester 2013

Dept. of Civil and Environmental Engineering  
Structural Engineering, Mechanics and Materials

Name: .....

Ph.D. Preliminary Examination  
Structural Dynamics

Note:

1. Write your answers on these sheets.
2. Calculations should be shown in detail with all intermediate steps; it is recommended to manipulate expressions symbolically as far as possible and substitute numbers only at or near the end.

Problem 1 (20%)

An **undamped** SDF system has a natural vibration period of  $T_n = 3$  sec. and stiffness  $k$ . It is subjected to a rectangular pulse force of amplitude  $p_0$  and duration  $t_d$ . **Without using any equation for  $R_d$  or any shock spectrum,** determine the peak deformation  $u_0$  if (a)  $t_d = 2$  sec, and (b)  $t_d = 1$  sec. Do not make any approximations, express results in terms of  $p_0$  and  $k$ .

Problem 2 (30%)

A one-story building is idealized as a massless frame supporting a weight of 40 kips at the beam level;  $I = 80 \text{ in}^4$  and  $40 \text{ in}^4$  for the left and right columns, respectively; and  $E = 30,000 \text{ ksi}$ . Determine the peak response of the structure to ground motion characterized by the given design spectrum scaled to 0.25g peak ground acceleration. The response quantities of interest are the lateral deformation at the top of the frame and the shears in the two columns.

Problem 3 (50%)

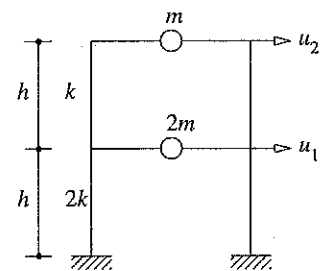
A rigid bar, supported by a weightless column, as shown, is excited by horizontal ground motion  $\ddot{u}_g(t)$ . Determine the bending moment at the base of the column. Express your results in terms of  $A_n(t)$ , the pseudo acceleration response of the  $n$ th mode SDF system.

The natural vibration frequencies and modes of the system are given:

**Doctoral Preliminary Examination**  
**Structural Dynamics**

**Problem 1 (50 points)**

By Rayleigh's method determine the **exact** natural vibration frequencies and modes of the two-story shear frame with floor masses and story stiffnesses as shown

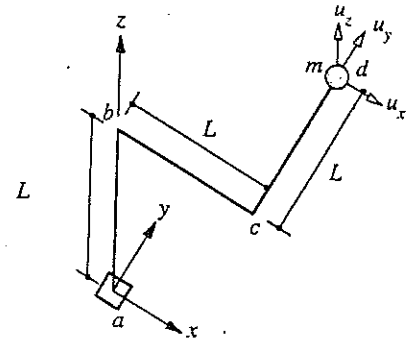


**Problem 2** (50 points)

Shown in the figure is a three-dimensional uniform pipe  $abcd$  clamped at  $a$ , with mass  $m$  at  $d$ . Its natural vibration frequencies and modes are given:

$$\omega_n = \alpha_n \sqrt{\frac{EI}{mL^3}} \quad \alpha_1 = 0.483 \quad \alpha_2 = 0.499 \quad \alpha_3 = 1.483$$

$$\phi_1 = \begin{Bmatrix} -0.777 \\ -0.492 \\ -0.393 \end{Bmatrix} \quad \phi_2 = \begin{Bmatrix} -0.208 \\ 0.388 \\ -0.898 \end{Bmatrix} \quad \phi_3 = \begin{Bmatrix} 0.594 \\ 0.779 \\ 0.198 \end{Bmatrix}$$



The system is excited by ground motion in the  $y$  direction.

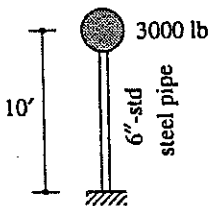
Determine the bending moments about the  $x$  and  $y$  axes and the torque at the clamped end  $a$  in terms of  $A_n(t)$ , the pseudo-acceleration response of the  $n$ th-mode SDF system.

**Suggestion:** Completely determine the response due to the first mode, then for other modes as time permits.

**Doctoral Preliminary Examination**  
**Structural Dynamics**

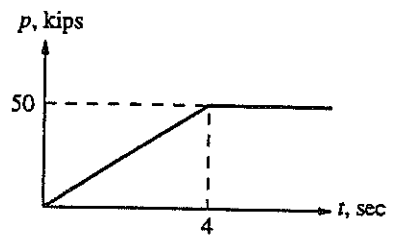
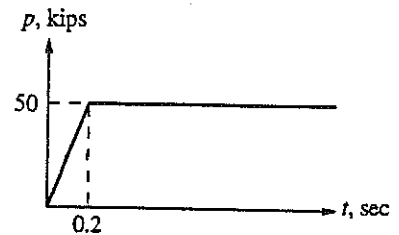
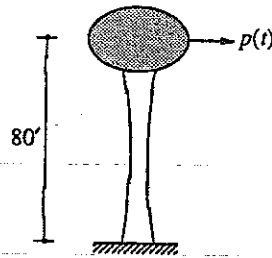
**Problem 1:** (30 percent)

A 10-ft-long vertical cantilever made of a 6-in.-nominal-diameter standard steel pipe supports a 3000-lb weight attached at the tip, as shown. The properties of the pipe are: outside diameter = 6.625 in., inside diameter = 6.065 in., thickness = 0.280 in., second moment of cross-sectional area  $I = 28.1 \text{ in}^4$ , Young's modulus  $E = 29,000 \text{ ksi}$ , and weight = 18.97 lb/ft length. Determine the peak deformation and the peak bending moment diagram due to lateral force equal to a unit impulse:  $p(t) = \delta(t)$ , the Dirac Delta function. Neglect damping.



Problem 2: (20 percent)

The elevated water tank shown weighs 100 kips when full with water. The tower has a lateral stiffness of 8.2 kips/in. Estimate the maximum lateral displacement due to each of the two dynamic forces shown; neglect damping.



**Problem 3: (50 percent)**

For the umbrella structure excited by vertical ground motion  $\ddot{u}_g(t)$ , determine:

- Displacements
- Bending moment at the base of the column and at location  $a$  of the beam

Express displacements in terms of  $D_n(t)$  and forces in terms of  $A_n(t)$ , the deformation and pseudo-acceleration responses of the  $n$ th mode SDF system.

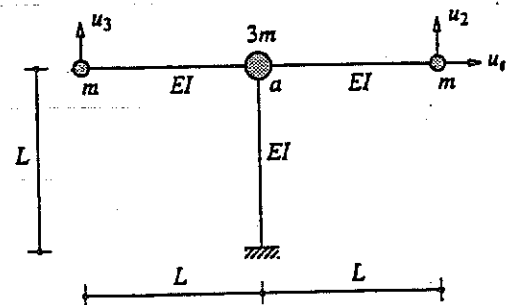
The natural vibration frequencies and modes of the system, assuming axially-rigid members, are given:

$$\omega_n = \alpha_n \sqrt{\frac{EI}{mL^3}} ; \alpha_1 = 0.526, \alpha_2 = 1.614, \alpha_3 = 1.732$$

$$\underline{\phi}_1 = \begin{Bmatrix} 1 \\ -1.949 \\ 1.949 \end{Bmatrix}$$

$$\underline{\phi}_2 = \begin{Bmatrix} 1 \\ 1.283 \\ -1.283 \end{Bmatrix}$$

$$\underline{\phi}_3 = \begin{Bmatrix} 0 \\ 1 \\ 1 \end{Bmatrix}$$



Student name : \_\_\_\_\_

### Doctoral Preliminary Examination in Dynamics

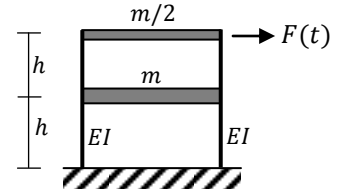
#### Problem 1.

A 2-story building with rigid floors has the mass and stiffness matrices

$$\mathbf{M} = m \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} \quad \mathbf{K} = k \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

where  $k = 24EI/h^3$ . The first modal frequency and mode shape are

$$\omega_1 = 0.765 \sqrt{\frac{k}{m}} \text{ rad/s} \quad \boldsymbol{\phi}_1 = \begin{Bmatrix} 1 \\ \sqrt{2} \end{Bmatrix}$$



- Determine the second mode shape  $\boldsymbol{\phi}_2$  and corresponding modal frequency  $\omega_2$ .
- Suppose the building is subjected to a step forcing function

$$F(t) = F_0 \quad 0 \leq t$$

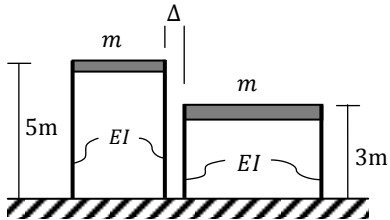
$$= 0 \quad t < 0$$

applied at the roof. Assuming the building is undamped and initially at rest, derive an expression for the displacement at the roof level.

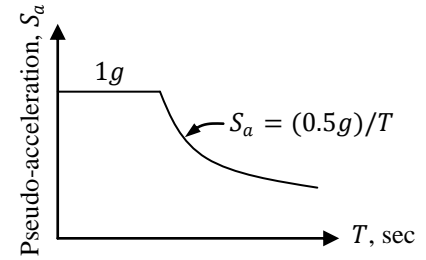
Student name: \_\_\_\_\_

### Problem 2.

The two adjacent single-story buildings with rigid roofs are subject to an earthquake base motion described by the pseudo-acceleration response spectrum for 5% damping shown below. Given the information provided in the figure, estimate the minimum separation  $\Delta$  between the two buildings that is required to avoid their pounding. Assume each building has 5% of critical damping.



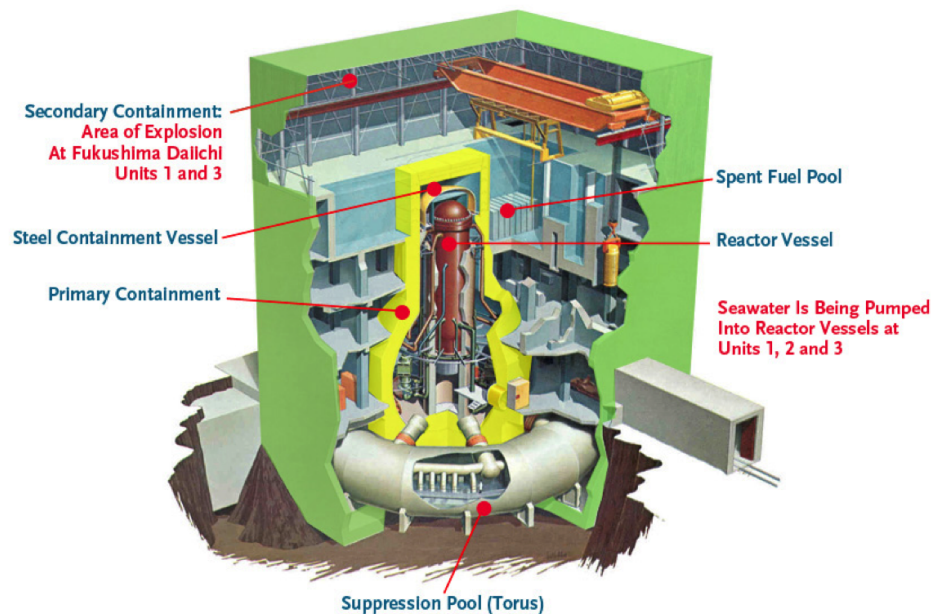
$$\frac{EI}{m} = 500 \text{ m}^3 \text{s}^{-2}$$



Name: \_\_\_\_\_

**Ph.D. PRELIMINARY EXAMINATION:**  
**DYNAMICS**

Let's take a look at a reactor building at the Fukushima Daichi nuclear power plant.

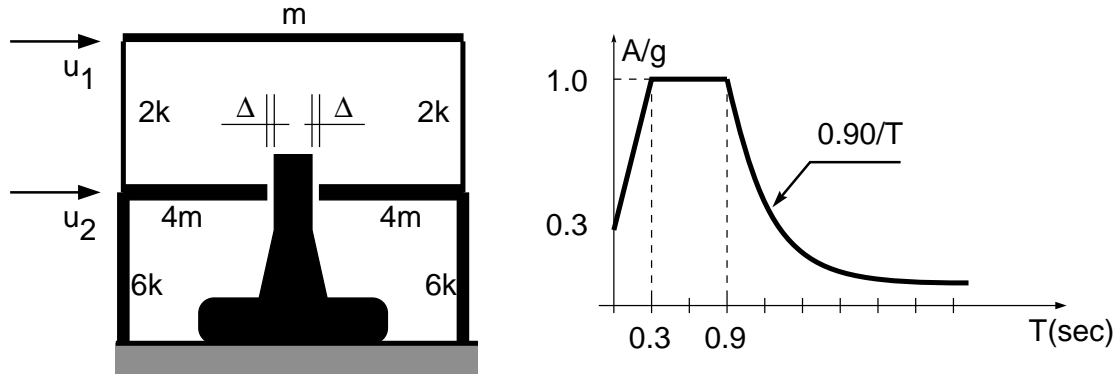


**Problem 1**

A two-story frame is used to model the reactor building. Note the distribution of masses ( $m$  vs.  $8m$ ) that reflect the weight of the spent fuel pool. The building has the following properties:  $m = 5\text{kip} - \text{sec}^2/\text{in}$ ,  $k = 100\text{kips}/\text{in}$ . The reactor containment structure is considered to be rigid at this stage of the analysis. There is a gap sized  $\Delta$  between the containment structure and the spent fuel pool.

A pseudo-acceleration design spectrum given in the figure is derived from the 2011 Great Tohoku earthquake records.

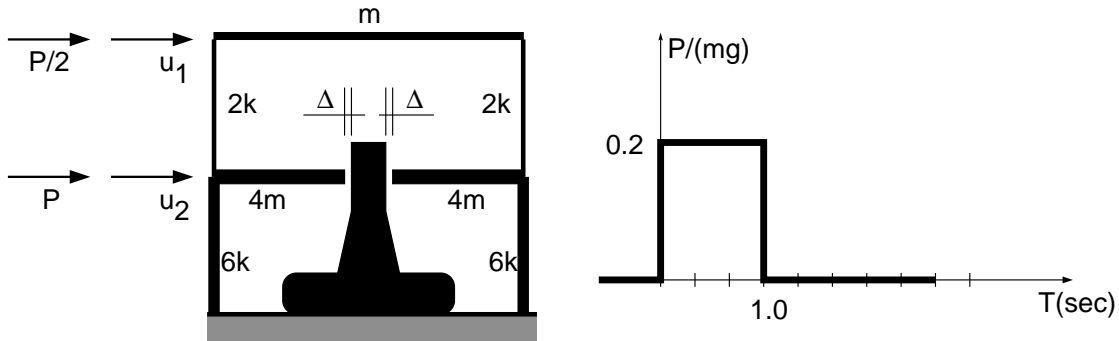
1. Determine the natural frequencies and mode shapes of the reactor building.
2. Using an appropriate modal combination rule, determine the smallest size of the gap  $\Delta_{req}$  such that pounding does not occur given the design spectrum. Justify your choice of the modal combination rule.



## Problem 2

A tsunami wave force is modeled using rectangular pulse function, shown, and two equivalent lateral forces,  $P$  at the spent fuel pool level and  $P/2$  at the roof level.

1. Determine the maximum lateral displacement of the reactor building at the spent fuel pool level.
2. Assuming the containment vessel is unaffected by the tsunami wave, determine if the gap size  $\Delta_{req}$  determined considering earthquake loading (Problem 1) is sufficient to prevent pounding due to tsunami wave loading. Yes or No.



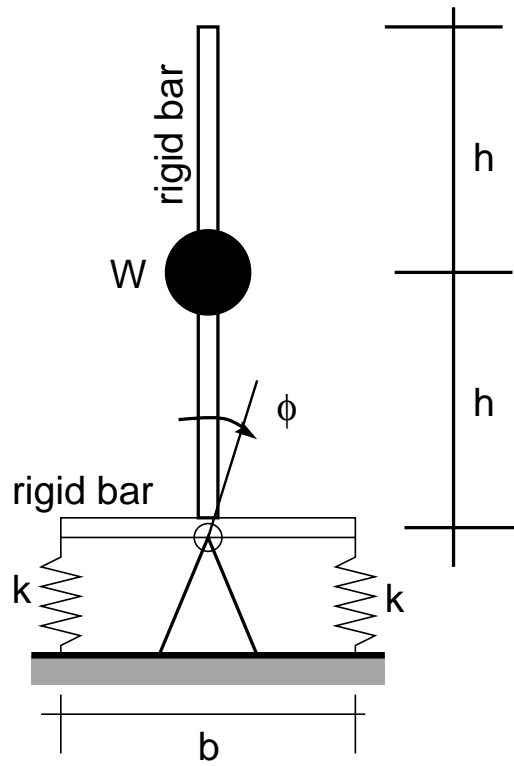
## Problem 3

The containment vessel and the suppression pool structure is sitting on soil. The structure is modeled using a rigid weight  $W$  supported by massless rigid bars. The foundation of the structure is modeled using two symmetrically positioned springs whose axial stiffness is  $k$  and a pin support, as shown. The aspect ratio  $h/b = 1$ . There are no dampers.

Assume deflections are small. Neglect only the rotational inertia of the weight, but account for its translation inertia.

1. Determine the natural frequency  $\omega_n$  of this system. Neglect the action of gravity.
2. Compute the value of the weight  $W_{cr}$  that would cause the system to buckle. By definition,  $W_{cr}$  is the largest possible weight the structure can support in static equilibrium when deflected from its vertical position by a small angle  $\phi$ . Include the effect of gravity.
3. Determine the natural frequency  $\omega_{gn}$  of this system. Include the effect of gravity. Show that:

$$\omega_{gn} = \omega_n \sqrt{1 - \frac{W}{W_{cr}}}$$



UNIVERSITY OF CALIFORNIA AT BERKELEY  
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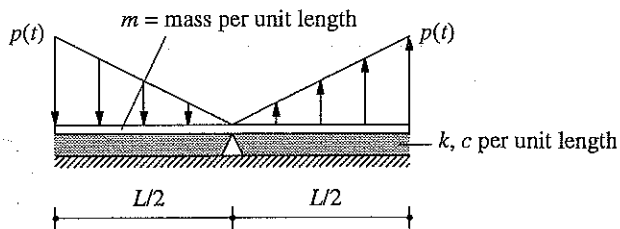
**Doctoral Preliminary Examination**  
**Structural Dynamics**

Problem 1: (20 percent)

An **undamped** SDF system has a natural vibration period of  $T_n = 1$  sec. and stiffness  $k = 20$  kips/in. It is subjected to a rectangular pulse force of amplitude  $p_o = 20$  kips and duration  $t_d$ . Determine the peak deformation  $u_o$  if (a)  $t_d = 0.8$  sec, and (b)  $t_d = 0.4$  sec. Do not make any approximations.

Problem 2: (40 percent)

The rigid bar supported on a pin at the center is bonded to a viscoelastic halfspace foundation, which can be modeled by stiffness  $k$  and damping coefficient  $c$  per unit of length. Determine the vertical displacement at the right end of the bar due to dynamic forces shown, where  $p(t) = p_o \delta(t)$ . No approximations are permitted.

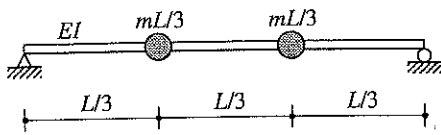


Problem 3: (40 percent)

Shown is a structural steel beam with  $E = 30,000 \text{ ksi}$ ,  $I = 100 \text{ in}^4$ ,  $L = 150 \text{ in.}$ , and  $mL = 0.864 \text{ kip-sec}^2/\text{in.}$  Determine the displacement response of the undamped beam to a rectangular pulse force  $p_o = 100 \text{ kips}$  with duration  $t_d = 0.3 \text{ sec.}$  applied at the right mass. The natural vibration frequencies (in rads/sec) & modes are given

$$\begin{aligned} \omega_1 &= 10 & \omega_2 &= 38.73 \\ \varphi_1 &= \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} & \varphi_2 &= \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} \end{aligned}$$

where the DOFs,  $u_1$  and  $u_2$ , are defined at the left and right mass, respectively.



UNIVERSITY OF CALIFORNIA AT BERKELEY  
Department of Civil and Environmental Engineering

Name: \_\_\_\_\_  
Structural Engineering, Mechanics and Materials  
Spring Semester 2010

**Doctoral Preliminary Examination**  
**Structural Dynamics**

Problem 1: (50 percent)

Determine the pseudo-acceleration response spectrum for undamped systems subjected to ground acceleration defined by a rectangular pulse of acceleration  $\ddot{u}_{g0}$  and duration  $t_d$ . **No approximations are allowed.** Plot the spectrum accurately, label axes and salient values.

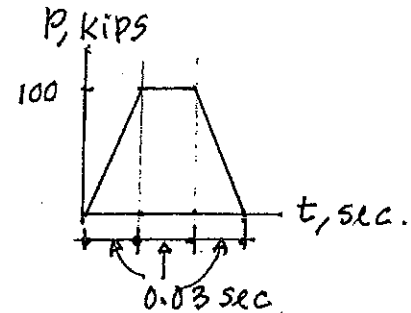
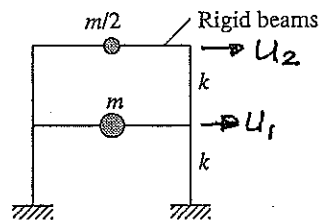
Problem 2: (50 percent)

The undamped shear frame shown is subjected to force  $p(t)$  at the first floor. Determine the lateral floor displacements as functions of time. The natural vibration frequencies and modes are given.

$$\omega_1 = 0.765 \sqrt{k/m} \quad \omega_2 = 1.848 \sqrt{k/m}$$

$$\varphi_1 = \begin{Bmatrix} 1/\sqrt{2} \\ 1 \end{Bmatrix} \quad \varphi_2 = \begin{Bmatrix} -1/\sqrt{2} \\ 1 \end{Bmatrix}$$

For the known values of  $m$  and  $k$  the natural vibration periods are  $T_1=1$  sec and  $T_2=0.41$  sec. Express your result in terms of  $m$ ,  $k$  and  $\omega_n$ . If you choose to make any assumptions or approximations, they must be stated and justified.



**Doctoral Preliminary Examination**  
**Structural Dynamics**

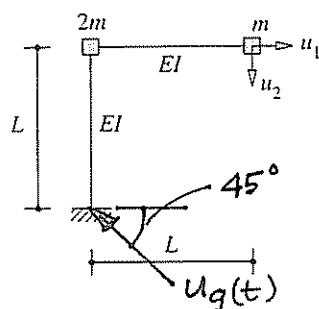
Problem 1 (50 percent)

Derive equations for the deformation, pseudo-velocity, and pseudo-acceleration spectra for ground acceleration  $\ddot{u}_g(t) = \dot{u}_{go}\delta(t)$ , where  $\delta(t)$  is the Dirac delta function and  $\dot{u}_{go}$  is the increment in velocity or the magnitude of the acceleration impulse. Plot the spectra for  $\zeta = 0$  and 10%.

Problem 2 (50 percent)

For the inverted L-shaped frame excited by ground acceleration in the direction shown, determine the shear and bending moment at the base of the column. Express your result in terms of  $A_n(t)$ , the pseudo-acceleration response of the  $n$ th mode SDF system.

The natural vibration frequencies and modes of the system, assuming axially rigid members, are given:



$$\omega_1 = 0.6987 \sqrt{\frac{EI}{mL^3}} \quad \omega_2 = 1.874 \sqrt{\frac{EI}{mL^3}}$$

$$\phi_1 = \begin{Bmatrix} 1 \\ 2.097 \end{Bmatrix} \quad \phi_2 = \begin{Bmatrix} 1 \\ -1.431 \end{Bmatrix}$$

## Doctoral Preliminary Examination

### Dynamics

#### Problem 1

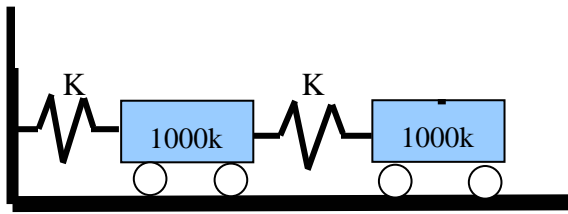
Viscous dampers are often constructed so that the damping force is proportional the velocity raised to a power other than one. Consider a damper where the damping force is given by  $F_D = \text{sgn}(v)a|v|^{0.5}$  where  $\text{sgn}(v)$  is +1 for  $v > 0$  and is -1 for  $v < 0$ ,  $a$  is an arbitrary positive-valued coefficient, and  $v$  is the velocity of the damper. Determine an expression for the equivalent linear viscous damping coefficient  $c_{eq}$  for the forces developed by this damper when it is undergoing harmonic excitation with a non-zero displacement amplitude  $u_0$  and frequency  $\omega$ .

#### Problem 2

Consider the undamped two-degree-of-freedom ELASTIC structure shown below. Each mass block has a mass of  $2 \text{ k-sec}^2/\text{in}$  and each spring has a stiffness of  $315 \text{ k/in}$ . The mass on the left (between the two springs) is subjected to a horizontal harmonic forcing excitation equal to  $F(t) = F_0 \sin \omega t$ . The frequency of the excitation  $\omega$  is  $12.56 \text{ rad/sec}$ .

Determine:

- i) What are the maximum expected displacements of both masses, and
- ii) What are the expected maximum forces in the two springs?



**Doctoral Preliminary Examination**  
**Structural Dynamics**

Problem 1 (25 percent)

A one-story building with roof mass  $m$  and natural frequency  $\omega_n$  is excited by a vibration generator with two counter-rotating masses, each  $m_e/2$ , rotating about a vertical axis at an eccentricity  $e$ . The total force due to the vibration generator is  $p(t) = (m_e e \omega^2) \sin \omega t$ . When the vibration generator turns at  $\omega = \omega_n$ , the amplitude of the roof acceleration is measured to be  $a$ .

Derive an equation for the damping ratio of the structure.

Problem 2 (25 percent)

An undamped SDF system with mass  $m$ , stiffness  $k$ , and natural vibration period of 1 sec is subjected to two pulse forces, both having the same amplitude  $p_o$  but different shapes: one is rectangular, the other is an isosceles (symmetric) triangle.

Estimate the peak deformation of the system due to each pulse for two different pulse durations: (a) 0.2 sec, (b) 5 sec. Express your result in terms of  $p_o$  and  $k$ .

### Problem 3 (50 percent)

#### Given

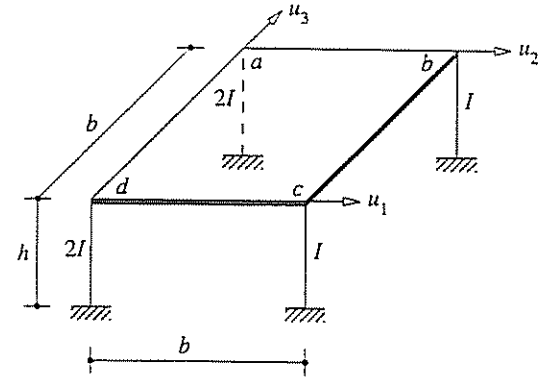
Shown in the figure is a uniform slab supported on four columns rigidly attached to the slab and clamped at the base. The slab has a total mass  $m$  and is rigid in its plane and out of plane. Each column is of circular cross-section and its second moment of cross-sectional area about any diametrical axis is noted. For the DOFs shown, the mass and stiffness matrices are

$$\mathbf{m} = m \begin{bmatrix} 2/3 & -1/6 & 1/2 \\ -1/6 & 2/3 & -1/2 \\ 1/2 & -1/2 & 1 \end{bmatrix} \quad \mathbf{k} = k \begin{bmatrix} 5 & -2 & 2 \\ -2 & 5 & -2 \\ 2 & -2 & 6 \end{bmatrix}$$

where  $k = 12EI/h^3$ . For a system with  $m = 90$  kips/g,  $k = 1.5$  kips/in and  $b = 25$  ft, the natural vibration frequencies and modes are

$$\omega_n = 5.96, 6.21, \text{ and } 10.90 \text{ rads/sec}$$

$$\Phi = \begin{bmatrix} 0.4885 & 2.071 & 2.4889 \\ -0.4885 & 2.071 & -2.4889 \\ 1.5437 & 0 & -2.8878 \end{bmatrix}$$



The structure is subjected to ground motion in the x-direction (direction of DOFs  $u_1$  and  $u_2$ ).

#### Determine

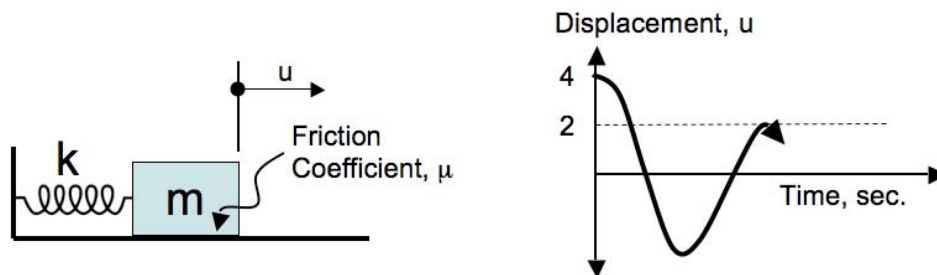
The x- and y-components of base shear and base torque in terms of  $A_n(t)$ , the pseudo-acceleration response of the nth-mode SDF system; y direction is along DOF  $u_3$ .

#### Suggestion

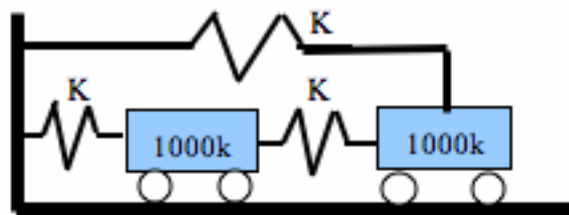
Completely determine the response due to the first mode then for other modes as time permits.

**Preliminary Examination in Dynamics**

1. Consider a simple single degree-of-freedom system shown below. The rigid block with mass **m** slides on the horizontal surface. The block is attached to a rigid support by a linear elastic spring with stiffness **k**. Movement of the block is also resisted by friction between the block and the sliding surface. The horizontal force needed to overcome the friction equals  $F_f = \mu W$ , where  $\mu$  is an unknown friction coefficient and  $W$  is the weight of the mass block.  $W$  equals 100 kips and **k** equals 50 kips/inch. The mass is moved to the right 4 inches and then released. What friction coefficient is needed so that after one cycle of oscillation, the amplitude of horizontal oscillation is reduced to 2 inches?



2. Consider the two-degree of freedom system shown below. It is subjected to a horizontal earthquake excitation represented by a simplified response spectrum  $D_n = 3T$ , where the spectral displacement  $D_n$  is measured in inches, and period is specified in seconds. It is desired that the structure be proportioned so that the expected peak horizontal response of the mass on the right is 8 inches. Modal contributions to response can be estimated using SRSS methods. It may be assumed that peak responses in each mode can be represented by the specified spectrum.
- What is the stiffness required of the springs (all equal) to achieve the stipulated peak displacement?
  - What is the expected maximum force in the spring located between the two masses?



**Doctoral Preliminary Examination**  
**Structural Dynamics**

Problem 1: (50 percent)

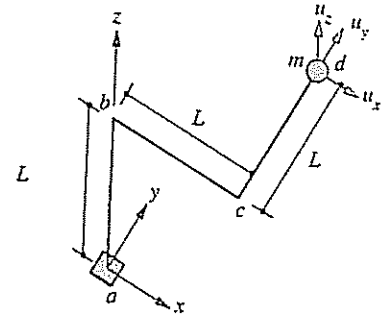
- (a) A uniform cantilever tower of length  $L$  has mass per unit length  $= m$  and flexural rigidity  $= EI$ . Assuming the shape of the fundamental vibration mode as  $\psi(x) = x^2/L^2$ , estimate the vibration period in terms of  $m$ ,  $EI$ , and  $L$ .
- (b) Determine the response of the tower to lateral forces that vary linearly over height from zero at the base to  $p(t)$  at the top, where  $p(t)$  is a rectangular pulse with amplitude 4 kips/ft and duration 1 sec. Properties of the tower are:  $L = 600$  ft,  $m = 1.738$  kip  $\cdot$  sec<sup>2</sup>/ft<sup>2</sup>, and  $EI = 5.469 \times 10^{10}$  kip-ft<sup>2</sup>.

Problem 2: (50 percent)

Shown in the figure is a three-dimensional uniform pipe  $abcd$  clamped at  $a$ , with mass  $m$  at  $d$ . Its natural vibration frequencies and modes are given:

$$\omega_n = \alpha_n \sqrt{\frac{EI}{mL^3}} \quad \alpha_1 = 0.483 \quad \alpha_2 = 0.499 \quad \alpha_3 = 1.483$$

$$\phi_1 = \begin{Bmatrix} -0.777 \\ -0.492 \\ -0.393 \end{Bmatrix} \quad \phi_2 = \begin{Bmatrix} -0.208 \\ 0.388 \\ -0.898 \end{Bmatrix} \quad \phi_3 = \begin{Bmatrix} 0.594 \\ 0.779 \\ 0.198 \end{Bmatrix}$$



The system is excited by ground motion in the  $xy$  plane in the direction  $45^\circ$  between the positive  $x$  and  $y$  axes.

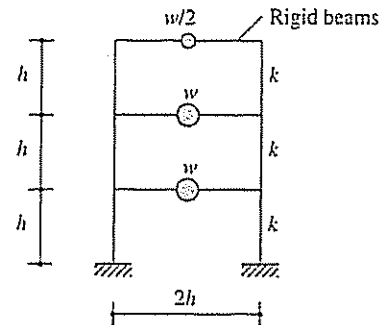
Determine the bending moments about the  $x$  and  $y$  axes and the torque at the clamped end  $a$  in terms of  $A_n(t)$ , the pseudo-acceleration response of the  $n$ th-mode SDF system.

**Suggestion:** Completely determine the response due to the first mode, then for other modes as time permits.

**Doctoral Preliminary Examination**  
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Problem 1: (40 percent)

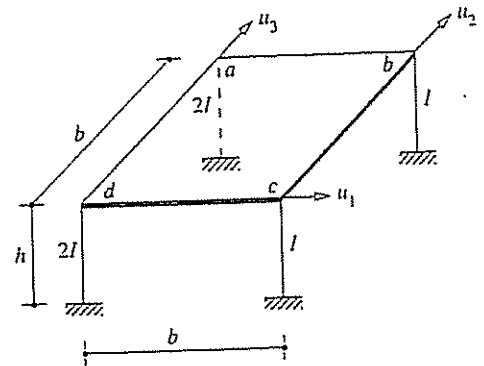
A three-story shear frame with lumped weights is shown. By solving the  $3 \times 3$  eigen problem, the natural frequencies and modes of vibration had been determined. In the first mode, the displacements at the first and third floors are 0.5 and 1.0, respectively; however, a part of the results -- the displacement at the second floor and the natural frequency -- have been lost. Using the properties of Rayleigh's quotient, determine the **exact** values of the missing data. Express the natural frequency in terms of  $w$  and story stiffness  $k$ .



Problem 2: (40 percent)

The figure shows a uniform slab supported on four columns rigidly attached to the slab and clamped at the base. The slab has a total mass  $m$  and is rigid in plane and out of plane. Each column is of circular cross section, and its second moment of cross-sectional area about any diametrical axis is as noted. With the DOFs selected as shown, and using influence coefficients:

- Formulate the mass and stiffness matrices in terms of  $m$  and the lateral stiffness  $k = 12EI/h^3$  of the smaller columns;  $h$  is the height.
- Formulate the equations of motion for ground motion in the direction  $d-b$ .



Problem 3:

The undamped two-story shear frame is subjected to an impulsive force at the first floor mass:  $p_1(t) = p_0 \delta(t)$ . Determine the floor displacements as a function of time in terms of  $p_0$ ,  $m$ ,  $\omega_1$  and  $\omega_2$ . The natural vibration frequencies and modes are given:

$$\omega_1 = 0.765 \sqrt{\frac{k}{m}}$$

$$\omega_2 = 1.848 \sqrt{\frac{k}{m}}$$

$$\underline{\phi}_1 = \begin{Bmatrix} 1/\sqrt{2} \\ 1 \end{Bmatrix}$$

$$\underline{\phi}_2 = \begin{Bmatrix} -1/\sqrt{2} \\ 1 \end{Bmatrix}$$

