UNIT 1: DC CIRCUITS

(Lecture 1 to 7 + Tutorial 1 to 3)

Prepared By:

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Outcome: Understand the fundamental behaviour of circuit elements and solve dc networks by different circuit reduction techniques

Fundamentals of D.C. circuits : resistance, inductance, capacitance, voltage, current, power and energy concepts, ohm's law, Kirchhoff's laws, basic method of circuit analysis, intuitive method of circuit analysis- series and parallel simplification, voltage division rule, current division rule, star–delta transformation, mesh and nodal analysis, introduction to dependent and independent sources, network theorems- superposition theorem, Thevenin's theorem, Norton's theorem, maximum power transfer theorem

UNIT 1: DC CIRCUITS

Lecture 1

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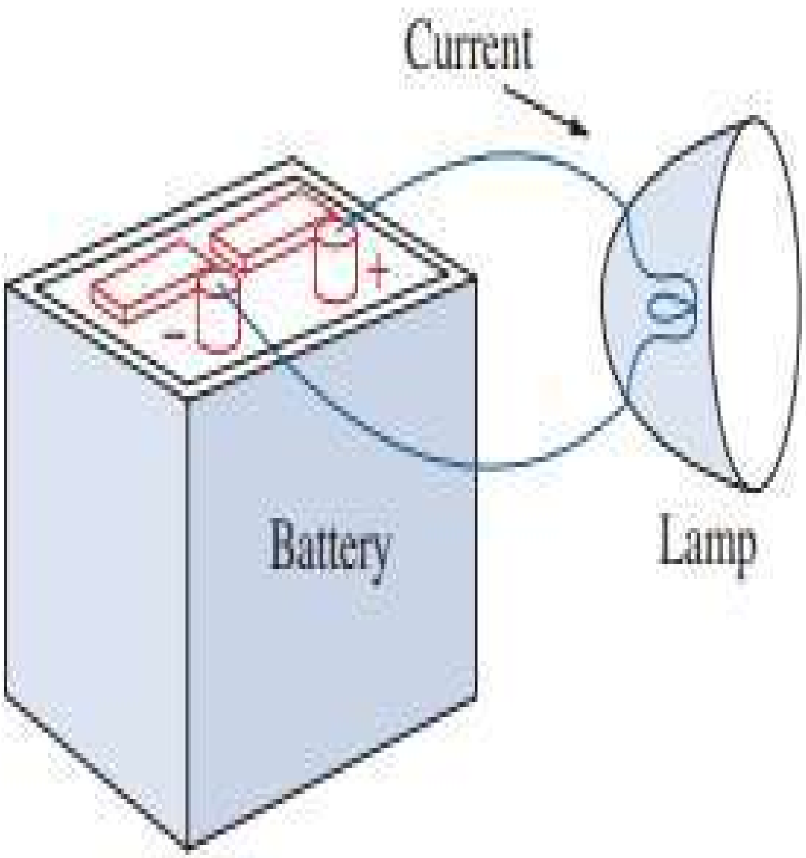
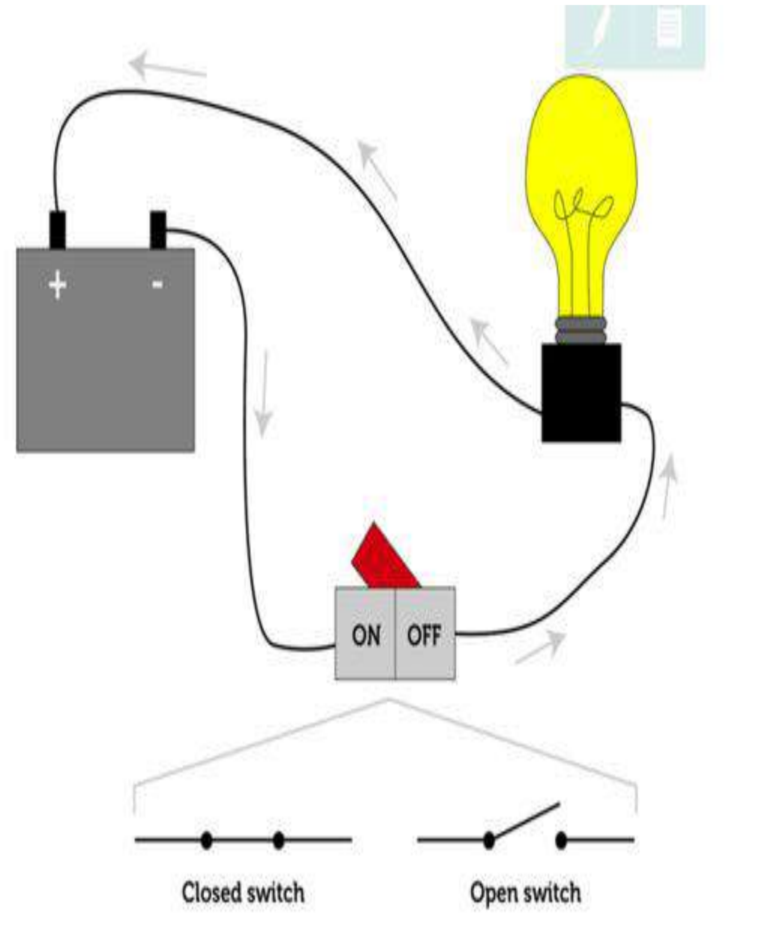
Electrical Circuit





Charge and Current

- Charge: Charge is an electrical property of the atomic particles of a matter.

  S.I Unit: Coulomb (C)

  Symbol: Q

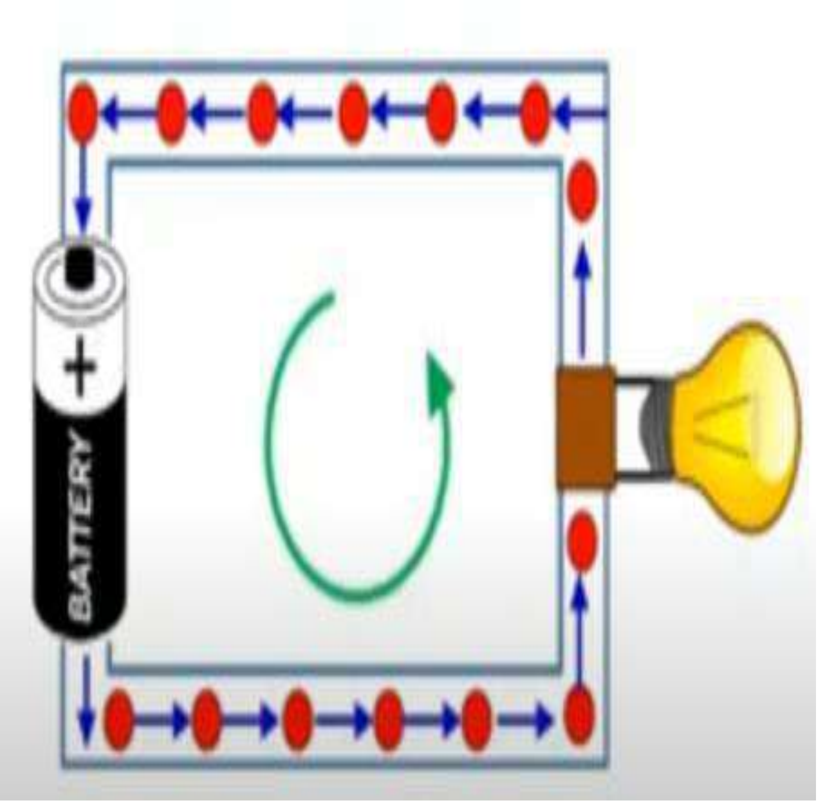
- Current: Rate of change of charge.

  OR

  Continuous flow of electrons in an electrical circuit.

  S.I Unit: Ampere (A)

  Symbol: I



Charge and Current

- Mathematically,

$$I = \frac{dQ}{dt} \; or \; Q = \int_{t_0}^{t} I.dt$$

Or, in simple terms:

$$I = \frac{Q}{T}$$

So, 1 Ampere = 1 coulomb/ 1 second.

QUICK QUIZ (Poll 1)

1 Coulomb is same as:

A. Watt /sec
B. Ampere/sec
C. Joule-sec
D. Ampere-sec

QUICK QUIZ (Poll 2)

Example: The total charge entering a terminal is given by $q(t)=5t\sin 4\pi t$ mC. Calculate the current at t=0.5s

A. 31.42A
B. 31.42mA
C. 62.8mA
D. 62.8A

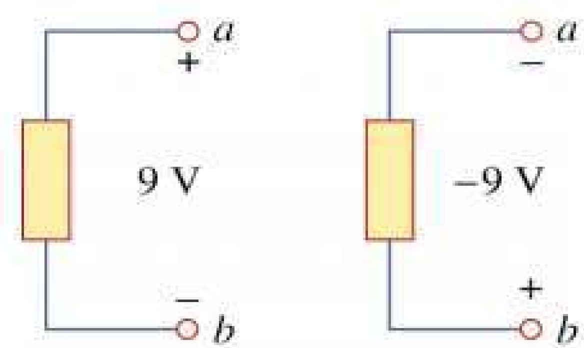
Explanation

$$i(t) = \frac{dq(t)}{dt} = \frac{d}{dt}(5t \sin 4\pi t) = (5 \sin 4\pi t + 20\pi t \cos 4\pi t)\text{mA}$$

$$i(0.5) = 5 \sin 2\pi + 10\pi \cos 2\pi = 31.42\text{mA}$$

Voltage

- To move an electron from point *a* to point *b*, external electromotive force (emf), typically a battery, is needed
- The voltage v_{ab} between two points *a* and *b* is the energy needed to move a unit charge from *a* to *b*
- Mathematically, $v_{ab} \triangleq \frac{dw}{dq}$
- 1 volt = 1 joule / coulomb
- Two equivalent representations:
  - Point *a* is v_{ab}=+9V above point *b*
  - Point *b* is v_{ba}=-9*V* above point *a*
  - In general, $v_{ab}= -v_{ba}$



(a) (b)

Power and Energy

- Power: Rate at which the work is done.

  OR

  Time rate of absorbing or supplying energy

  S.I Unit: Watts (W)

  Symbol: P

Mathematically,

$$P = \frac{dW}{dt} = \frac{dW}{dq} . \frac{dq}{dt} = V . I$$

Implies, $\boxed{P = V . I}$

Power and Energy

- Energy: Capacity of doing work.

  S.I Unit: Joules(J)

  Symbol: E

QUICK QUIZ (Poll 3)

Calculate the current ratings of 100 Watt incandescent bulb and 15 Watt LED lamp operated with the domestic supply of 220 Volt?

A. Bulb = 0.068 A and LED = 0.45 A
B. Bulb = 0.45 A and LED = 0.068 A
C. Bulb =0.50 A and LED = 0.068 A
D. Bulb = 0.50 and LED = 0.68 A

QUICK QUIZ (Poll 4)

From the previous question, it can be inferred that:

A. LED consumes 5 times more current than Bulb.
B. Bulb consumes 5 times more current than LED..
C. LED consumes 6.6 times more current than Bulb.
D. Bulb consumes 6.6 times more current than LED.

Network Components

- Passive elements: not capable of generating energy
  - e.g., resistors, capacitors, and inductors
- Active elements: capable of generating energy
  - e.g., generators, batteries, and operational amplifiers

Active

Battery

Transistor, Op-amp, etc

Passive

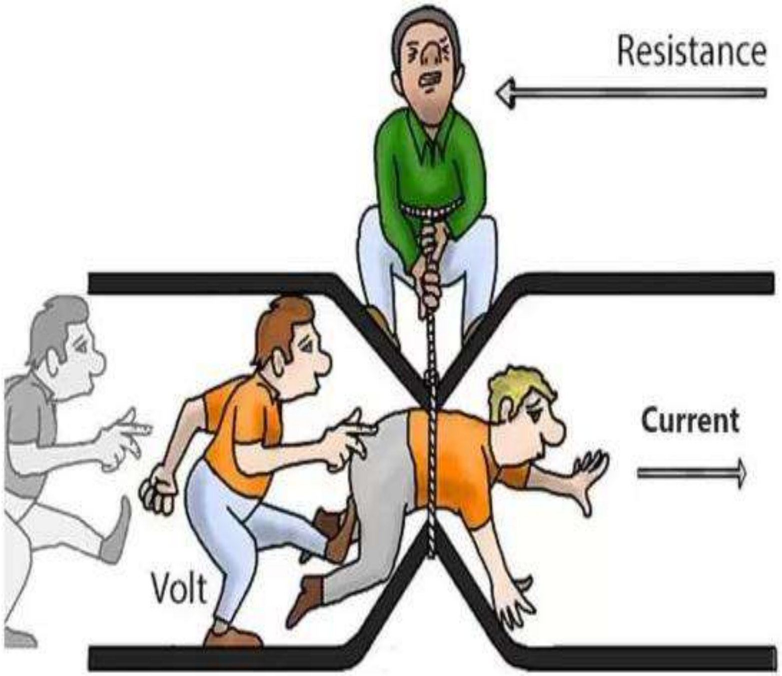
Resistance (R)
Capacitance (C)
Inductance (L)

Resistance

- Resistance: It is an opposition to the flow of current.

  S.I Unit: Ohm (Ω)

  Symbol: R



Capacitance

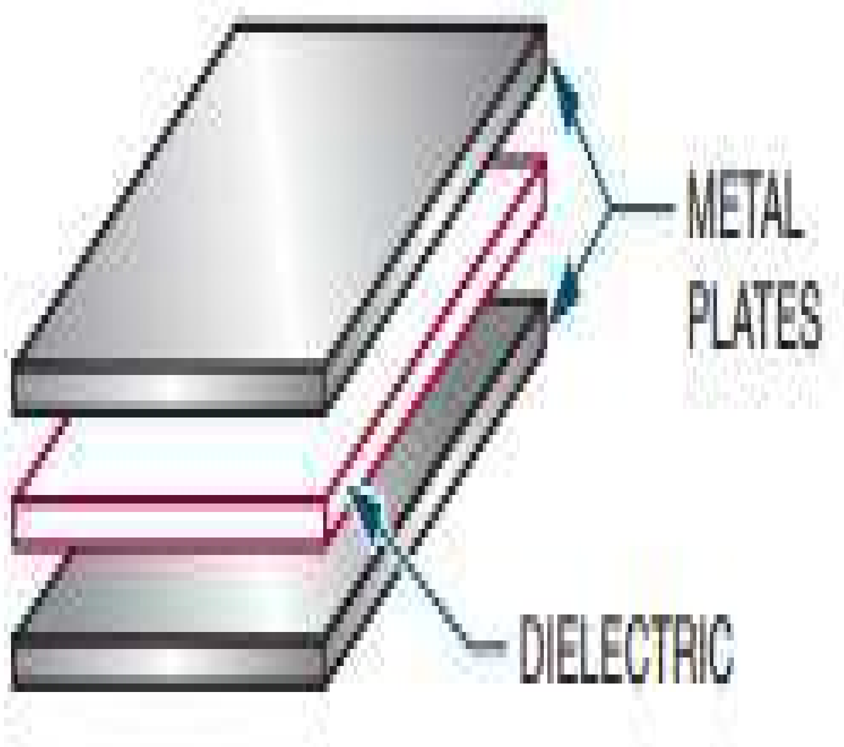
- Capacitance is the ability of a device to store electrical energy in an electrostatic field.
- A capacitor is a device that stores energy in the form of an electrical field..
- A capacitor is made of two conductors separated by a dielectric.

  S.I Unit: Farad (F)

  Symbol: C

Two important Properties:

1. No current flows through the capacitor, if the voltage remains constant.
2. Voltage across a capacitor cannot change instantaneously.



Inductance

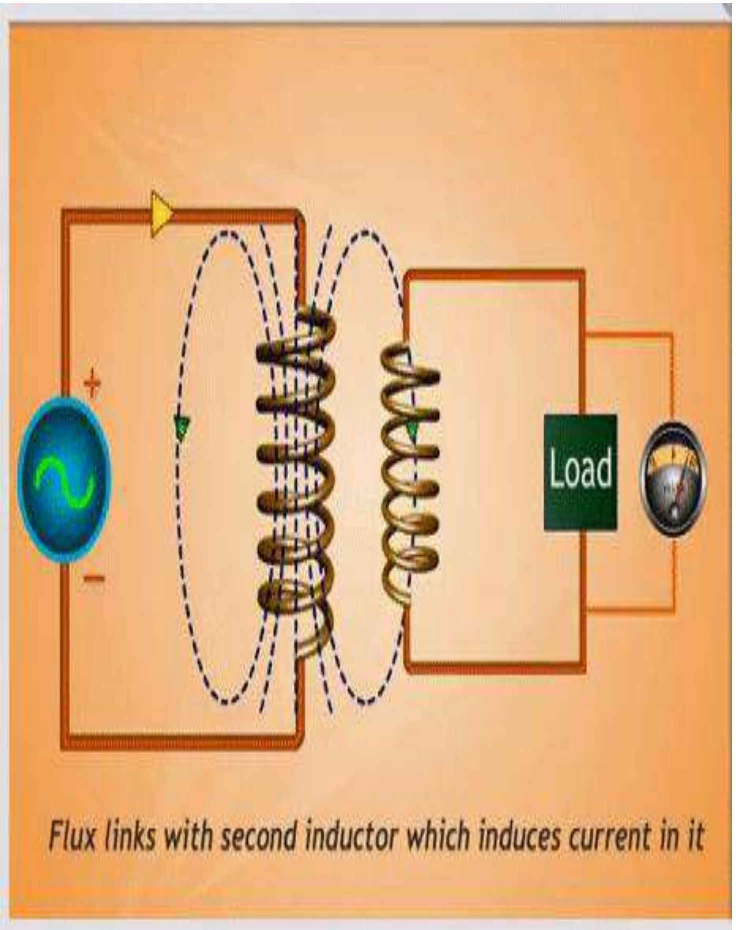
- Inductance is the characteristic of an electrical conductor that opposes a change in current flow.
- An inductor is a device that stores energy in a magnetic field.
- When a current flows through a conductor, magnetic field builds up around the conductor. This field contains energy and is the foundation for inductance

  S.I Unit: Henry (H)

  Symbol: L

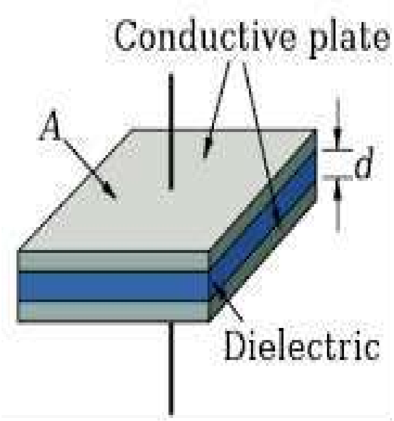
Two important Properties:

1. No voltage appears across an inductor, if the current through it remains constant.
2. The current through an inductor cannot change instantaneously.

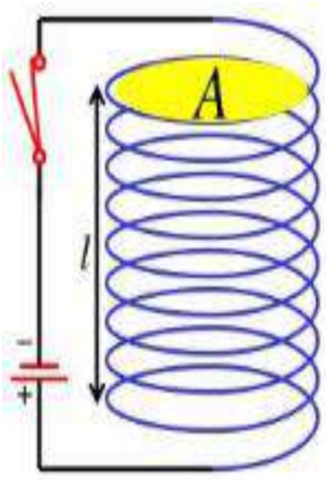


Capacitance and Inductance

- $Q = CV$
- $I = \frac{dQ}{dt} = \frac{d\ CV}{dt} = C\frac{dV}{dt}$
- $E = \frac{1}{2}CV^2$
- $C = \frac{A\epsilon}{d}$



- $V = L\frac{dI}{dt}$
- $E = \frac{1}{2}LI^2$
- $L = \frac{\mu N^2 A}{L}$



QUICK QUIZ (Poll 5)

Identify the passive element

A. Battery
B. Transformer
C. Transistor
D. OP-amp
E. None of these

QUICK QUIZ (Poll 6)

Find the value of capacitance if the value of voltage increases linearly from 0 to 100 V in 0.1 s causing a current flow of 5 mA?

A. 10 μF

B. 5 F

C. 10 F

D. 5 μF

Ohm's Law

- Ohm's law states that:

"the current in an electric circuit is directly proportional to the voltage across its terminals, provided that the physical parameters like temperature, etc. remain constant"

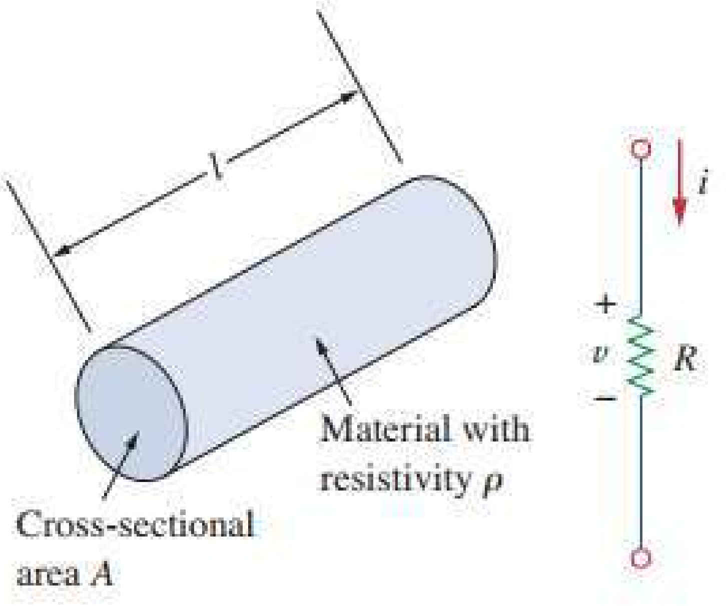
Mathematically,

$$I \propto V$$

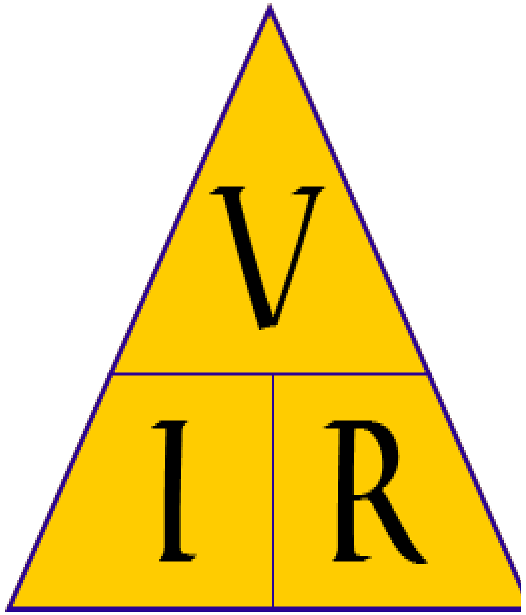
Or,

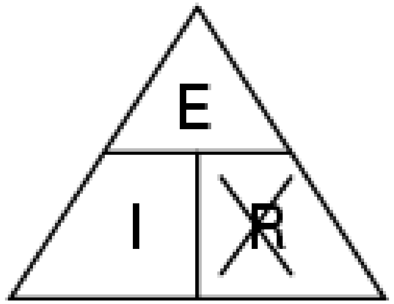
$$I = \frac{V}{R}$$

Where, Resistance $R = \frac{\rho l}{A}$

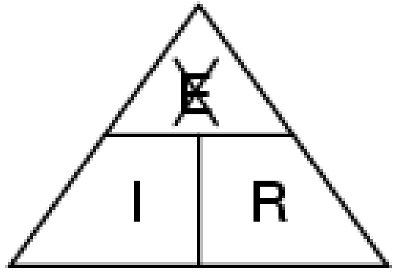


Ohm's law magic triangle

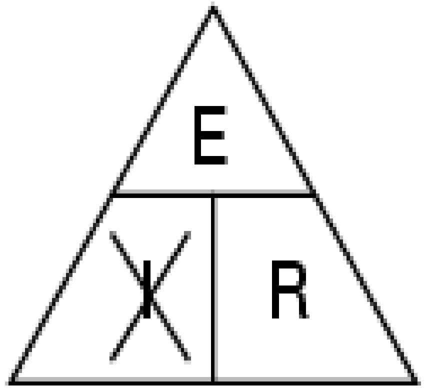




$$R = \frac{E}{I}$$



$$E = I\,R$$



$$I = \frac{E}{R}$$

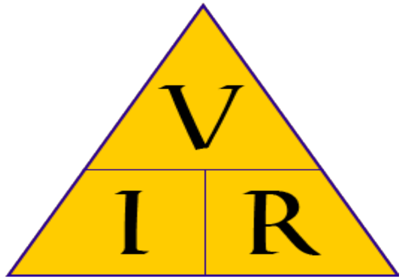
Voltage measured in *volts*, symbolized by the letters "E" or "V".

Current measured in *amps*, symbolized by the letter "I".

Resistance measured in *ohms*, symbolized by the letter "R".

Resistivity Table

Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator



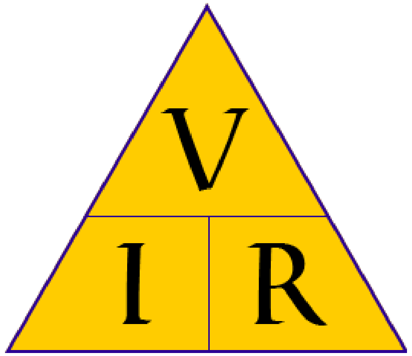
Practice problems

In a circuit, 0.5 A is flowing through the bulb. The voltage across the bulb is 4.0 V. What is the bulbs resistance?

1. Write the equation → $R = \frac{V}{I}$
2. Replace the known values → $R = \frac{4.0}{0.5}$
3. Solve → $R = 8$
4. Label → $R = 8\ \Omega$

Practice problem

- You light a light bulb with a 1.5 volt battery. If the bulb has a resistance of 10 ohms, how much current is flowing?



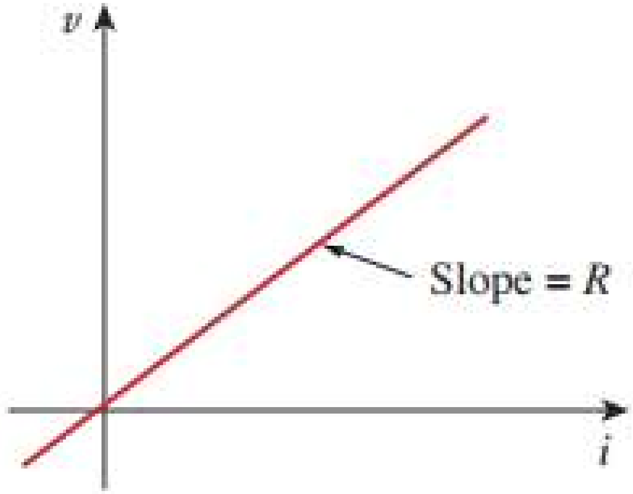
1. Write the equation → $I = \frac{V}{R}$
2. Replace the known values → $I = \frac{1.5}{10}$
3. Solve → $I = 0.15$

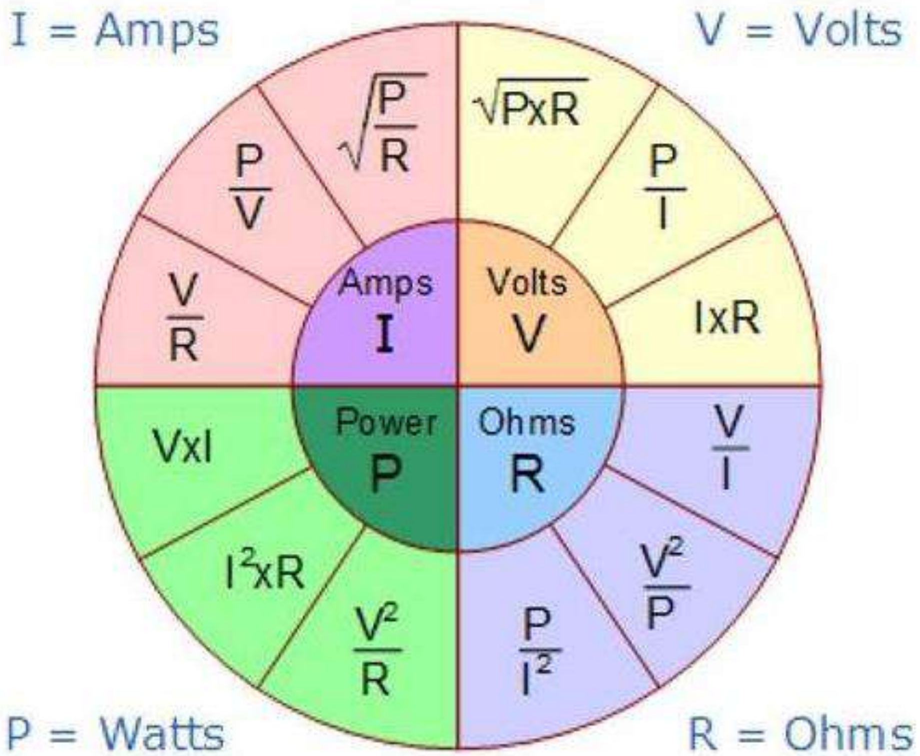
Conductance

- A useful quantity in circuit analysis is the reciprocal of resistance R, known as conductance and denoted by G
- $G = \frac{1}{R} = \frac{I}{V}$
- S.I Unit: mho (ohm spelled backwards) or Siemens
- Symbol: ℧, the inverted omega.

$$1\,\text{S} = 1\,℧ = 1\,\text{A/V}$$

- Power dissipated in the resistor can be expressed as:
- $P = VI = I^2R = \frac{V^2}{R}$

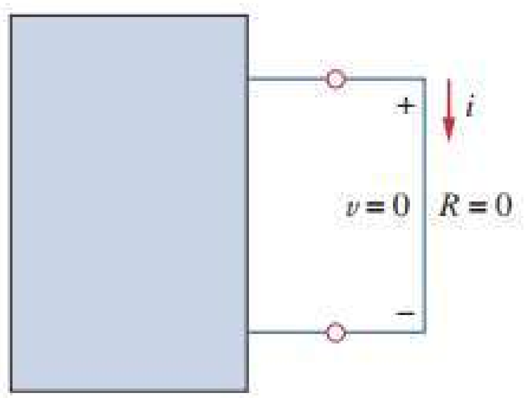




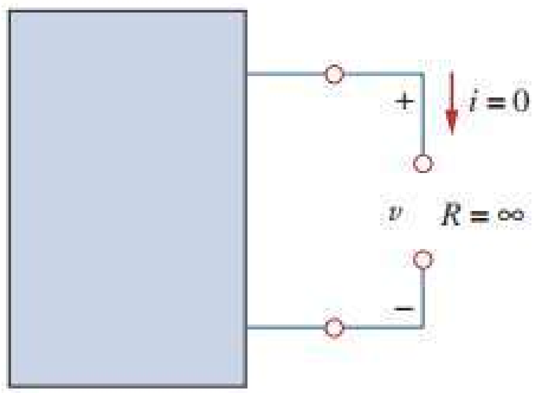
Ohm's Law Pie Chart (Source: Electronics-Tutorials.ws)

Short-circuit and Open-circuit

- For a short circuit, R = 0 Ω
- Therefore, V = I.R = 0 V
- NOTE: (current, I can be of any value)



- For an open circuit, R = ∞Ω
- Therefore, I = V/R = 0 V
- NOTE: (voltage,V can be of any value)



Applications of Ohm's Law

1. To find unknown Voltage (V)
2. To Find unknown Resistance (R)
3. To Find unknown Current (I)
4. Can be used to find Unknown Conductance (G)=1/R
5. Can be used to find unknown Power (P)=VI
6. Can be used to find unknown conductivity or Resistivity

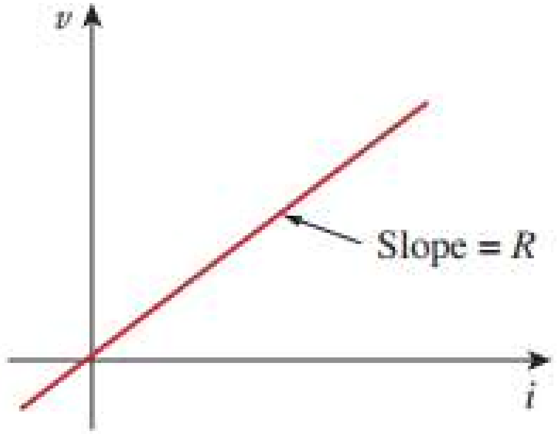
$v = iR$ $R = \frac{v}{i}$ $I = V/R$ $R = \rho \frac{\ell}{A}$

Applications of Ohm's Law

1. It is widely used in circuit analysis.
2. It is used in ammeter, multimeter, etc.
3. It is used to design resistors.
4. It is used to get the desired circuit drop in circuit design (Example, Domestic Fan Regulator).
5. Advanced laws such as Kirchhoff's Norton's law, Thevenin's law are based on ohm's law.
6. Electric heaters, kettles and other types of equipment working principle follow ohm's law.
7. A laptop and mobile charger using DC power supply in operation and working principle of DC power supply depend on ohm's law.

Limitations of Ohm’s Law

- Ohm’s law holds true only for a conductor at a constant temperature. Resistivity changes with temperature.
- Ohm’s law by itself is not sufficient to analyze circuits.
- It is NOT applicable to non linear elements, For example, Diodes, Transistors, Thyristors, etc.
- This law cannot be applied to unilateral networks.



QUICK QUIZ (Poll 7)

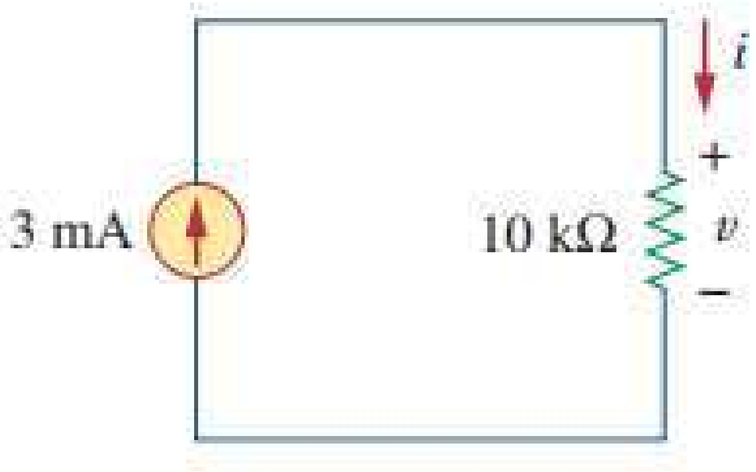
The voltage and the conductance of the given circuit is:

A. 30 V, 10 µS

B. 30 mV, 100 µS

C. 30 V, 100 µS

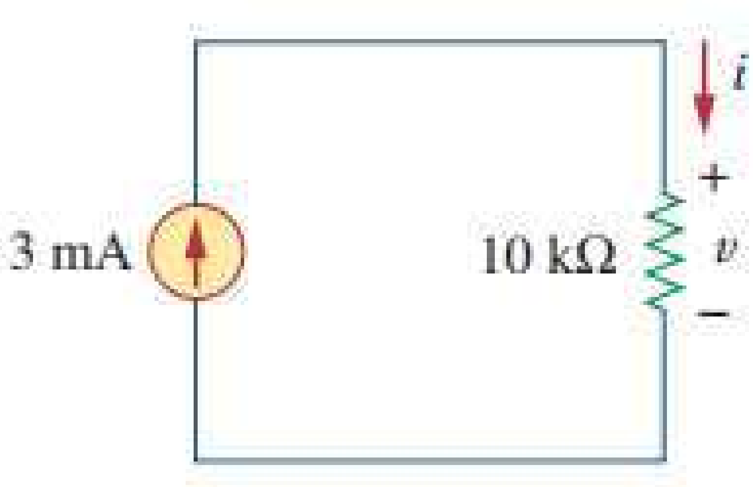
D. 30 mV, 10 µS



QUICK QUIZ (Poll 8)

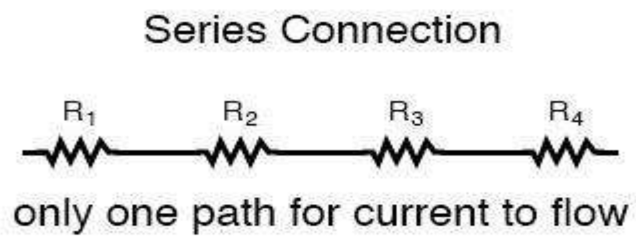
The power of the given circuit is:

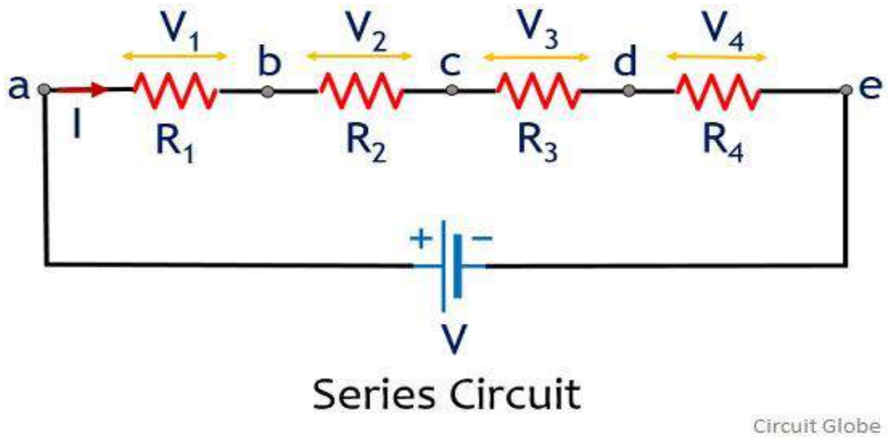
A. 60 mW
B. 70 mW
C. 80 mW
D. 90 mW

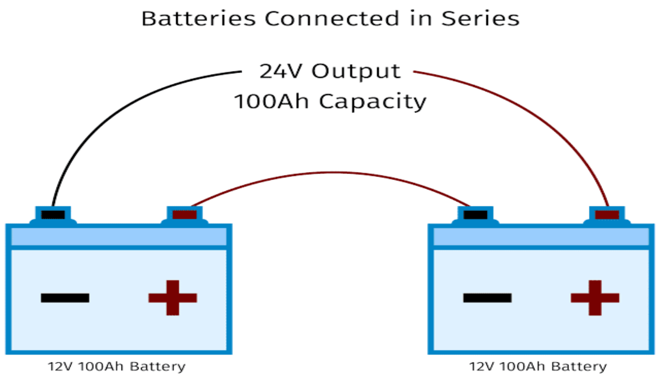


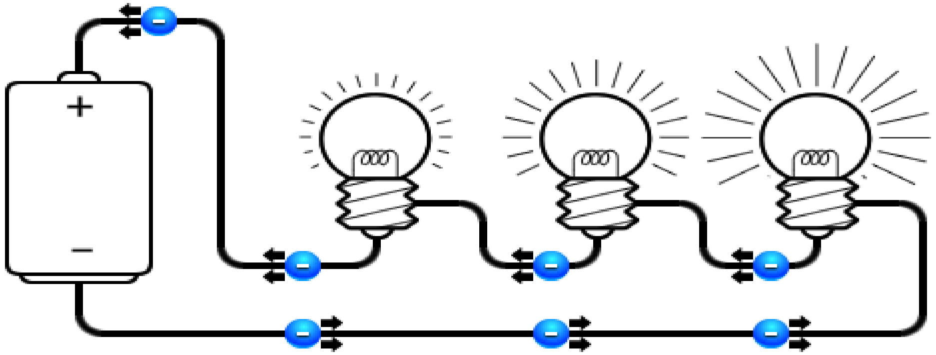
Series Connection

- **SERIES CONNECTION:** Two or more elements are in series if they exclusively share a single node and consequently carry the same current.

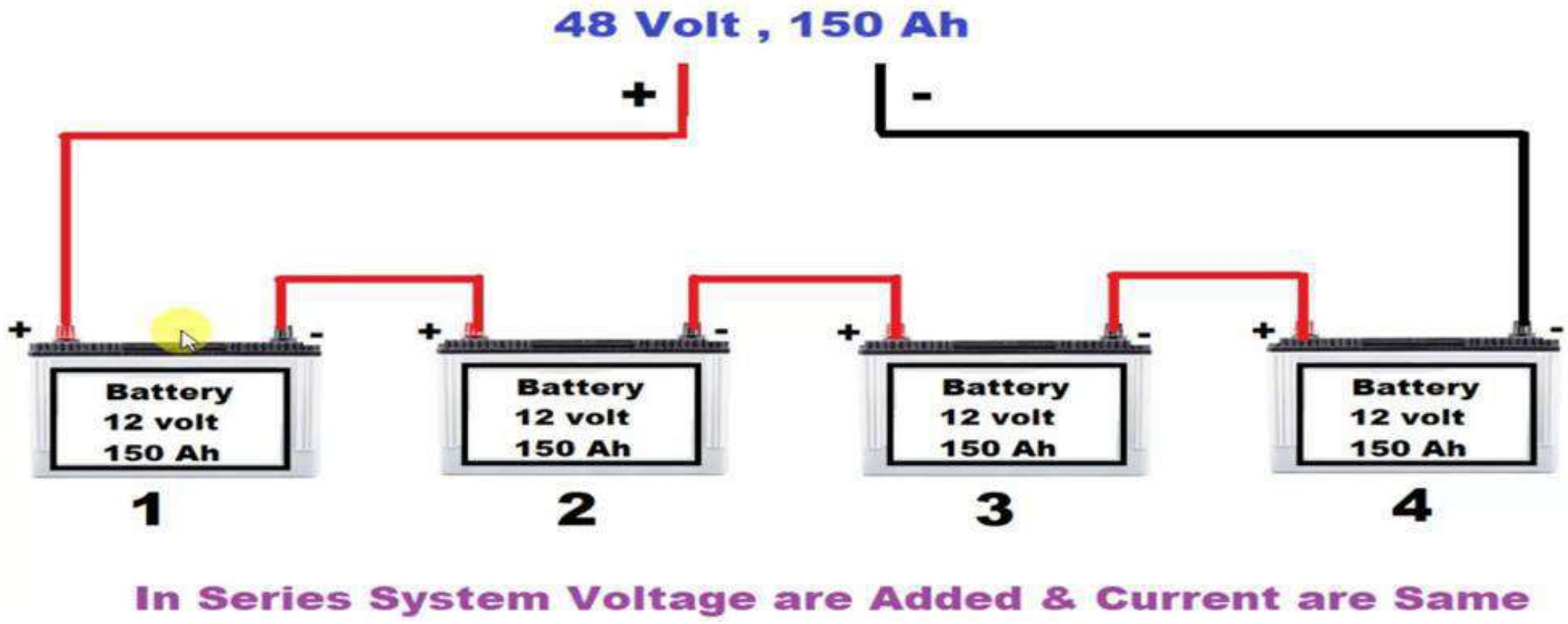






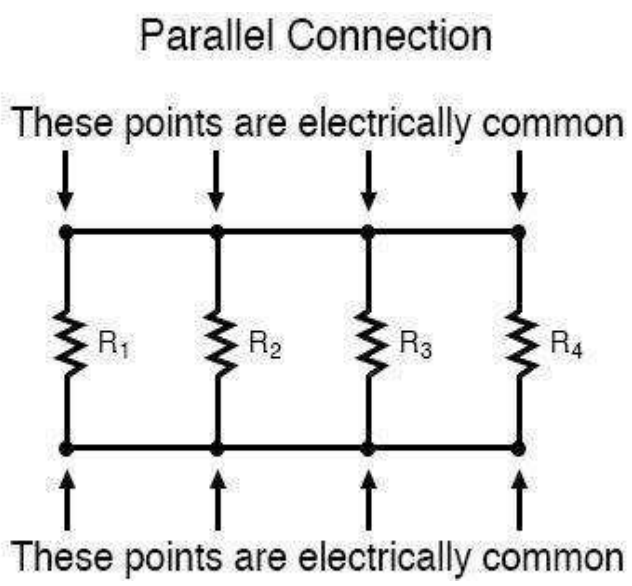


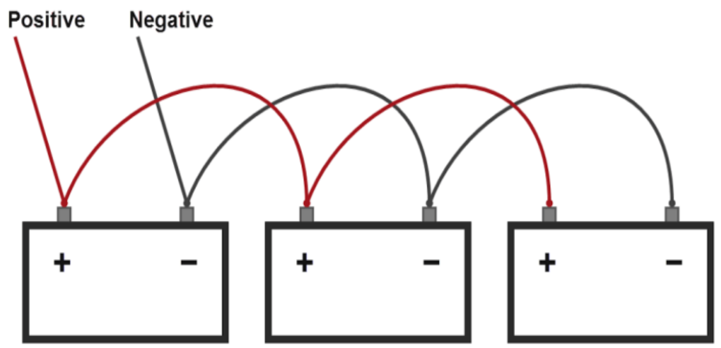
Point to Remember for Series Circuits

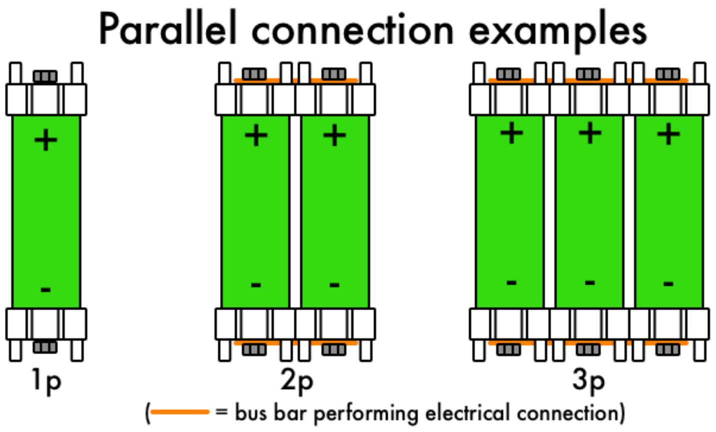


Parallel Connection

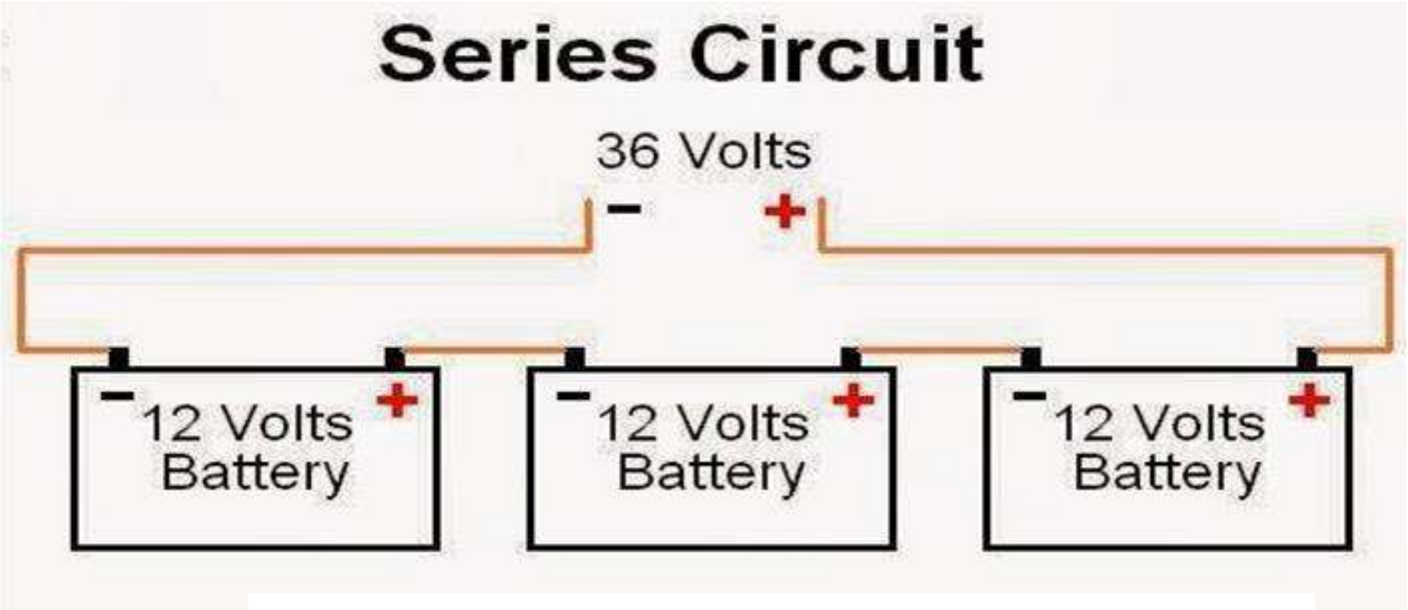
- PARALLEL CONNECTION: Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them

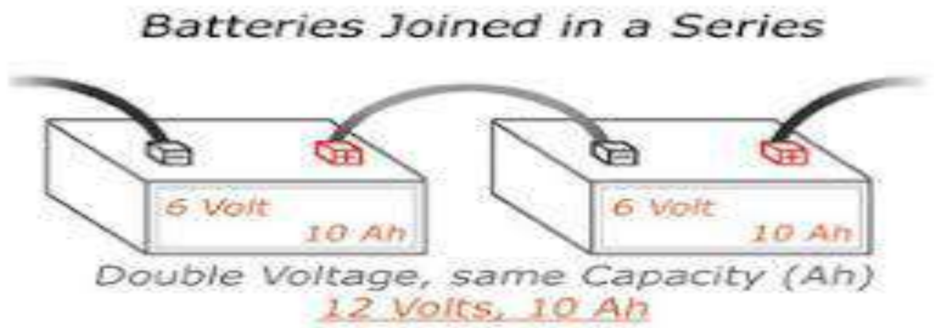


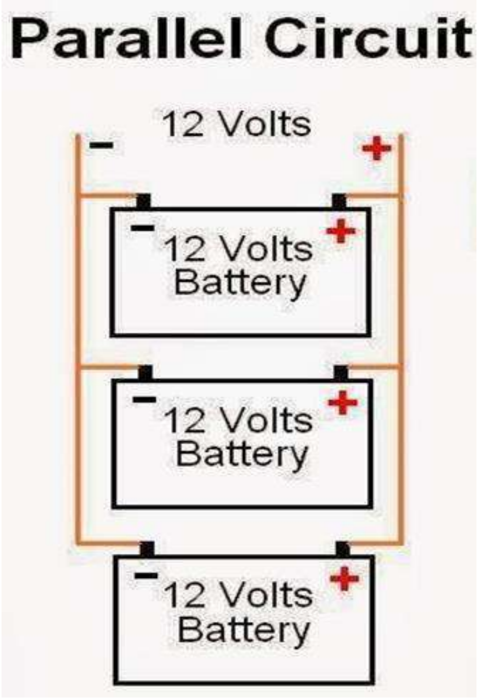


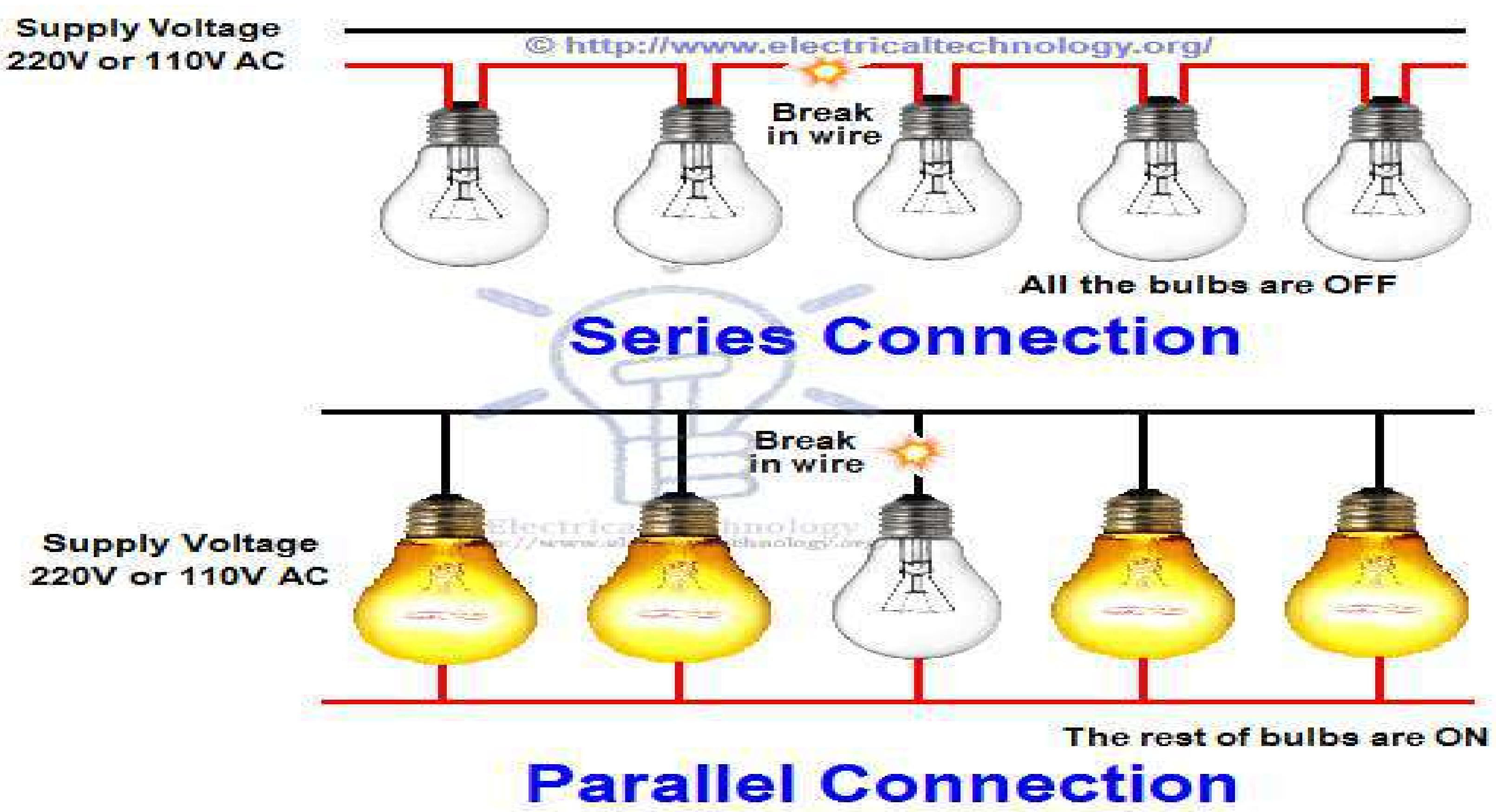


Battery Voltage In Series And Parallel









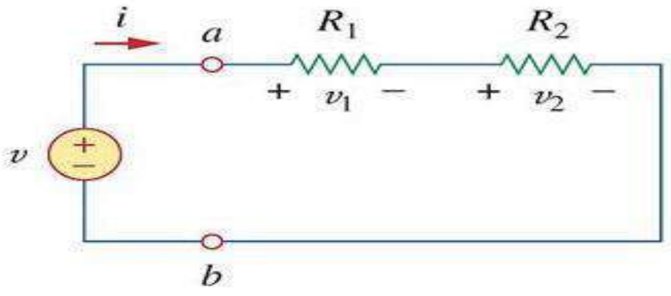
Why Parallel Connection is Preferred over Series Connection?

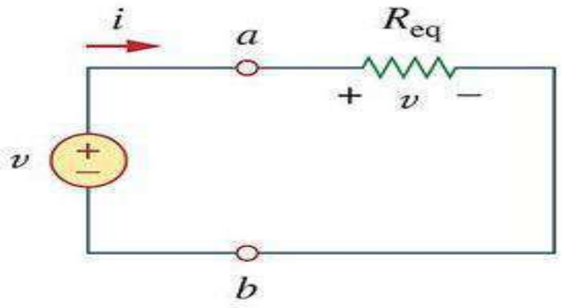
RESISTORS IN SERIES

Series: Two or more elements are in series if they are cascaded or connected sequentially and consequently carry the same current.

The equivalent resistance of any number of resistors connected in a series is the sum of the individual resistances

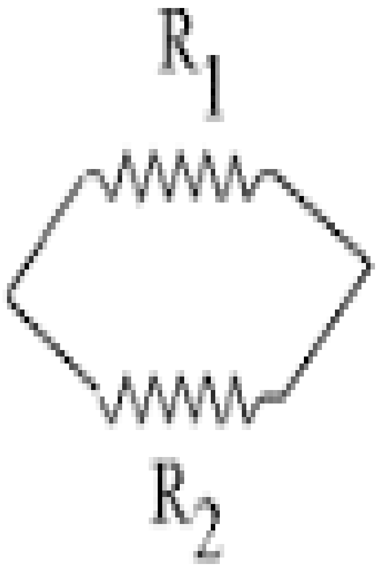
$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$





Note: Resistors in series behave as a single resistor whose resistance is equal to the sum of the resistances of the individual resistors.

Resistors in Parallel



$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2}$$

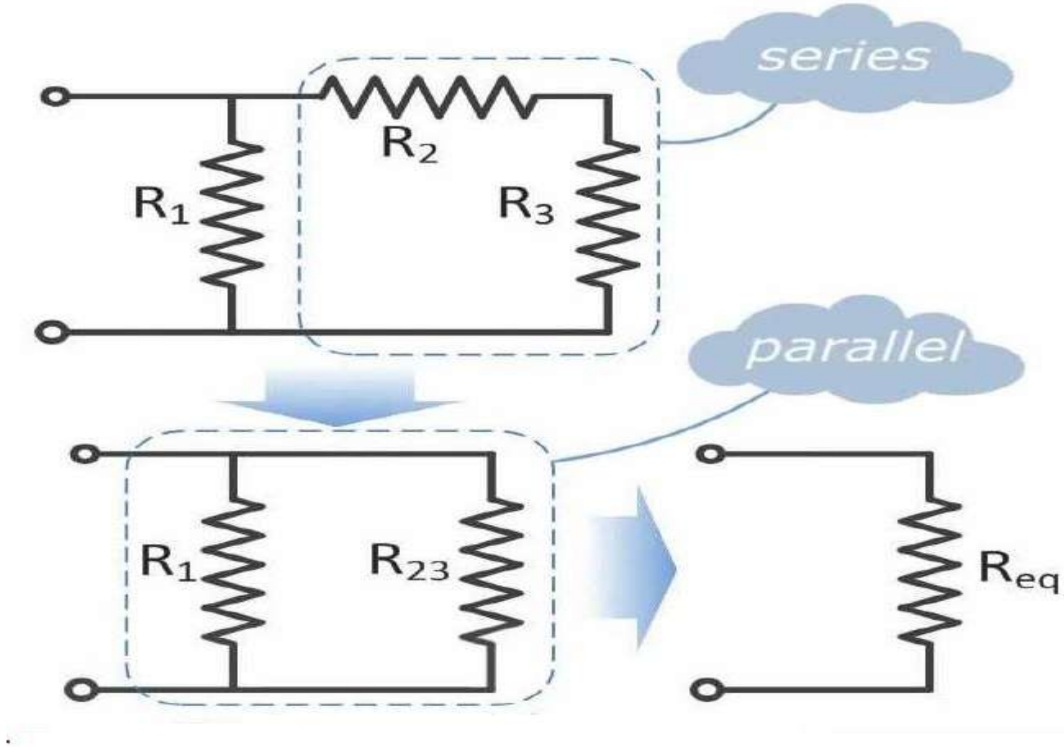
$$\frac{1}{R_t} = \frac{R_2 + R_1}{R_1 R_2}$$

$$R_t = \frac{R_1 R_2}{R_2 + R_1}$$

The equivalent of two parallel resistor is equal to their product divided by their sum .

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

How to find Equivalent Resistance for Series-Parallel Combinations



$$R_{23} = R_2 + R_3$$

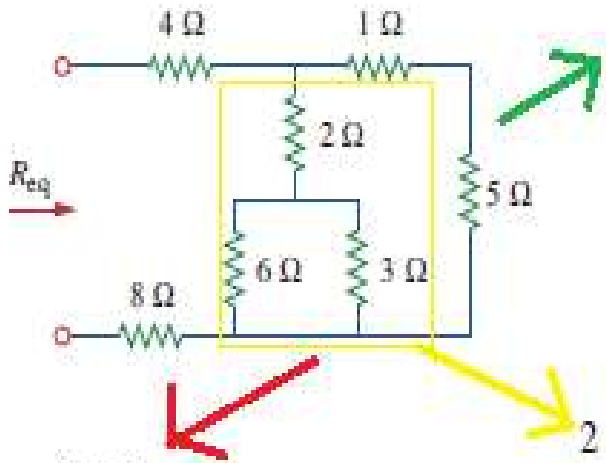
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_{23}}$$

$$R_{eq} = \frac{R_1 \cdot R_{23}}{R_1 + R_{23}}$$

$$R_{eq} = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3}$$

Example: To find R_{eq}

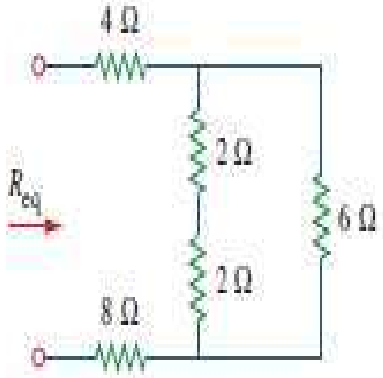
Find R_{eq} for the circuit shown in Fig.

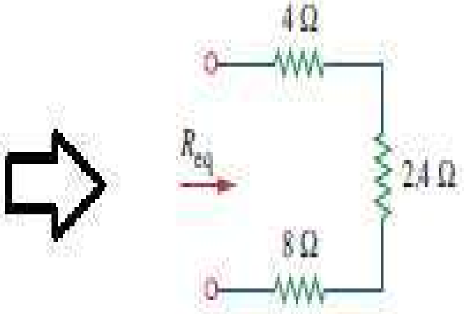


$1\,\Omega + 5\,\Omega = 6\,\Omega$

$2\,\Omega + 2\,\Omega = 4\,\Omega$

$$6\,\Omega \parallel 3\Omega = \frac{6 \times 3}{6+3} = 2\,\Omega$$





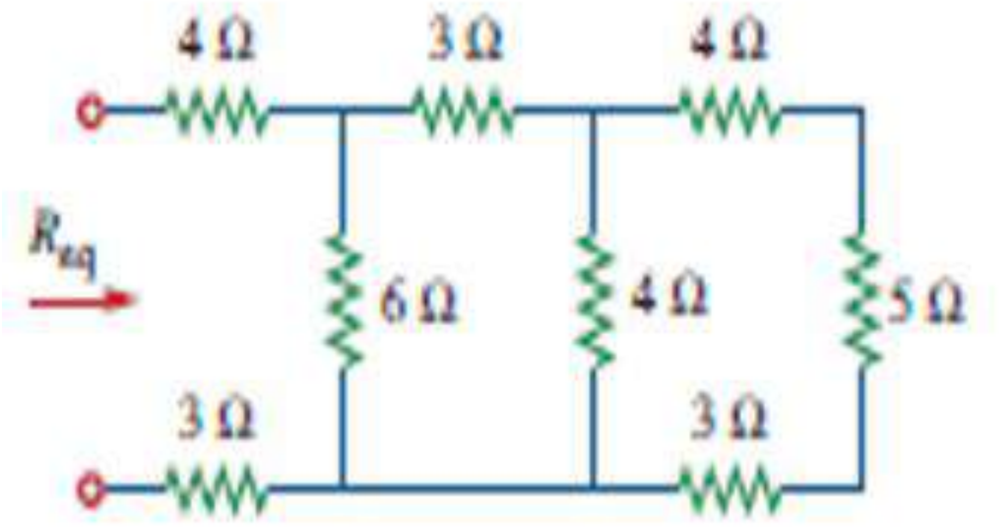
$$4\,\Omega \parallel 6\,\Omega = \frac{4 \times 6}{4+6} = 2.4\,\Omega$$

$$R_{eq} = 4\,\Omega + 2.4\,\Omega + 8\,\Omega = 14.4\,\Omega$$

QUICK QUIZ (Poll 9)

Find Equivalent Resistance in Ohms?

A. 5
B. 10
C. 15
D. 20

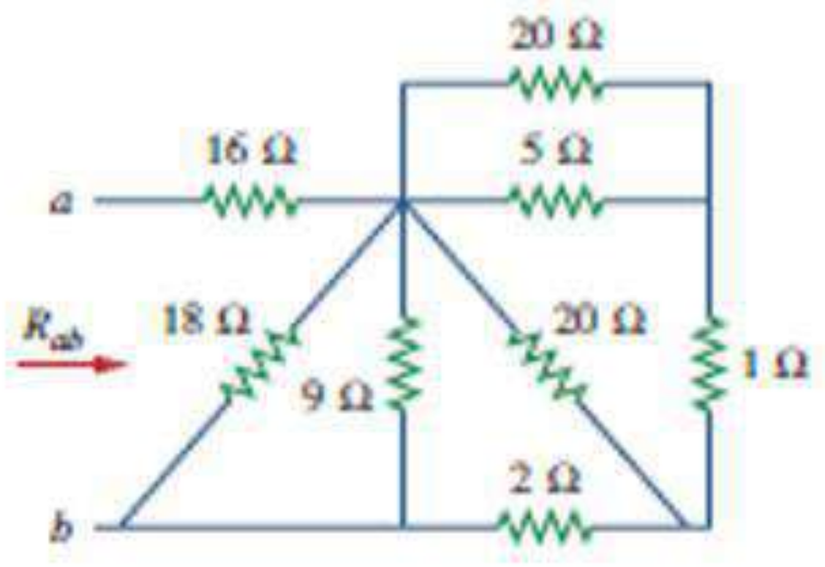


QUICK QUIZ (Poll 10)

LOVELY PROFESSIONAL UNIVERSITY

Find Equivalent Resistance in Ohms?

A. 12
B. 17
C. 19
D. 29



Useful Links

- http://www.dynamicscience.com.au/tester/solutions1/electric/voltage.htm
- https://gfycat.com/directhauntinglamb
- https://www.youtube.com/watch?v=NfcgA1axPLo

UNIT 1: DC CIRCUITS

Lecture 3

Prepared By:

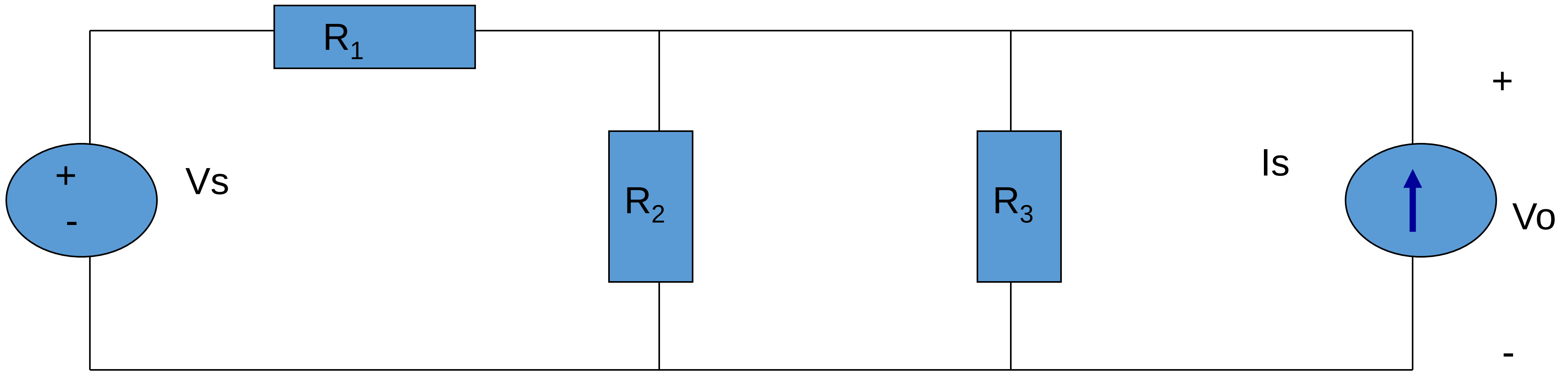
Krishan Arora
Assistant Professor and Head

Circuit Definitions

- Node – any point where 2 or more circuit elements are connected together
  - Wires usually have negligible resistance
  - Each node has one voltage (w.r.t. ground)
- Branch – a circuit element between two nodes
- Loop – a collection of branches that form a closed path returning to the same node without going through any other nodes or branches twice

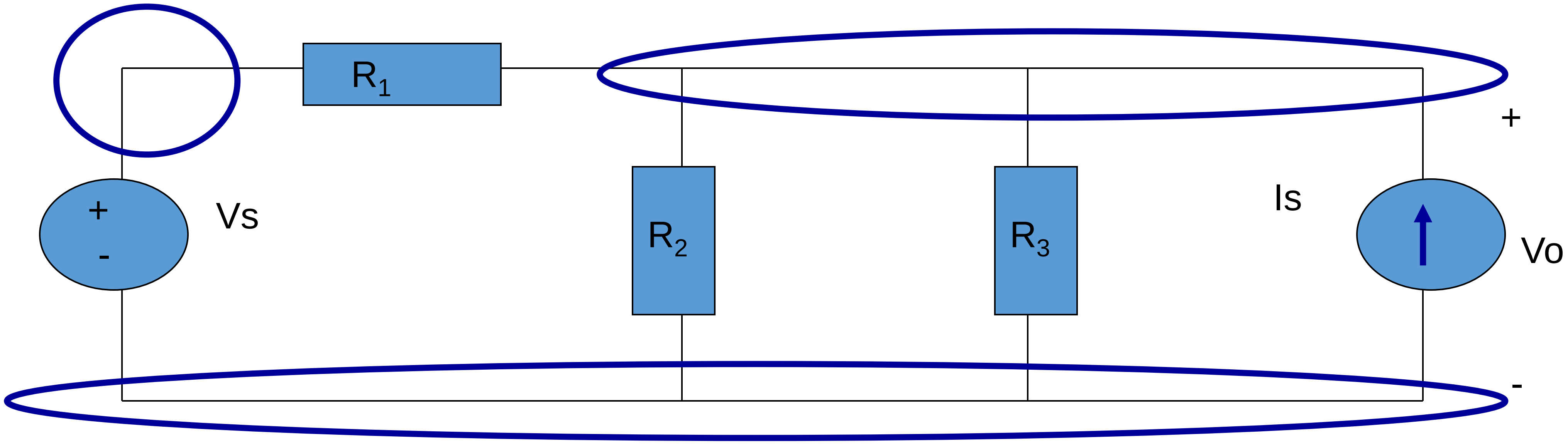
Example

- How many nodes, branches & loops?



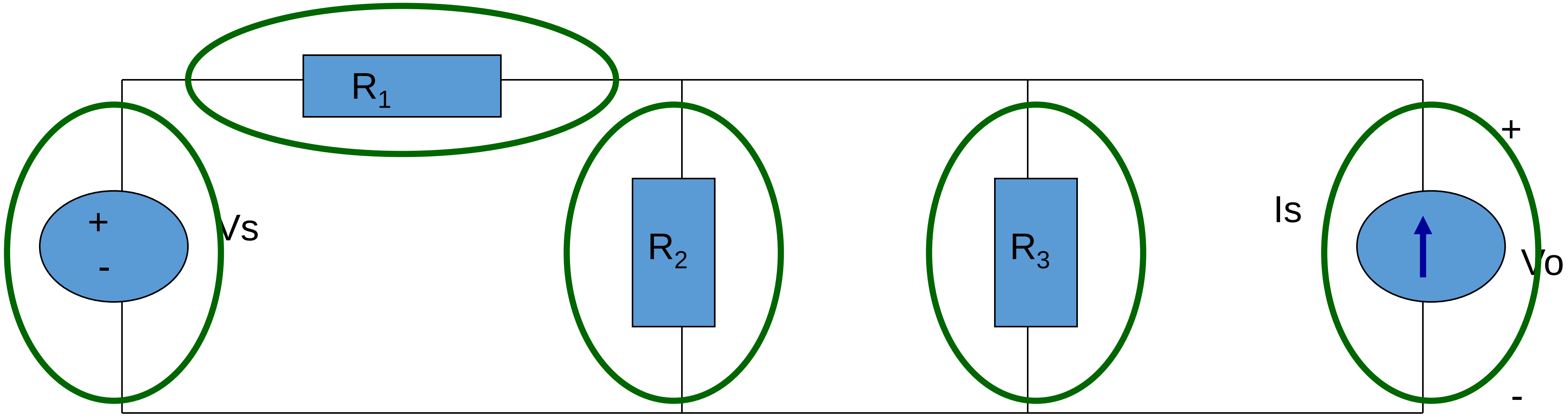
Example

- Three nodes



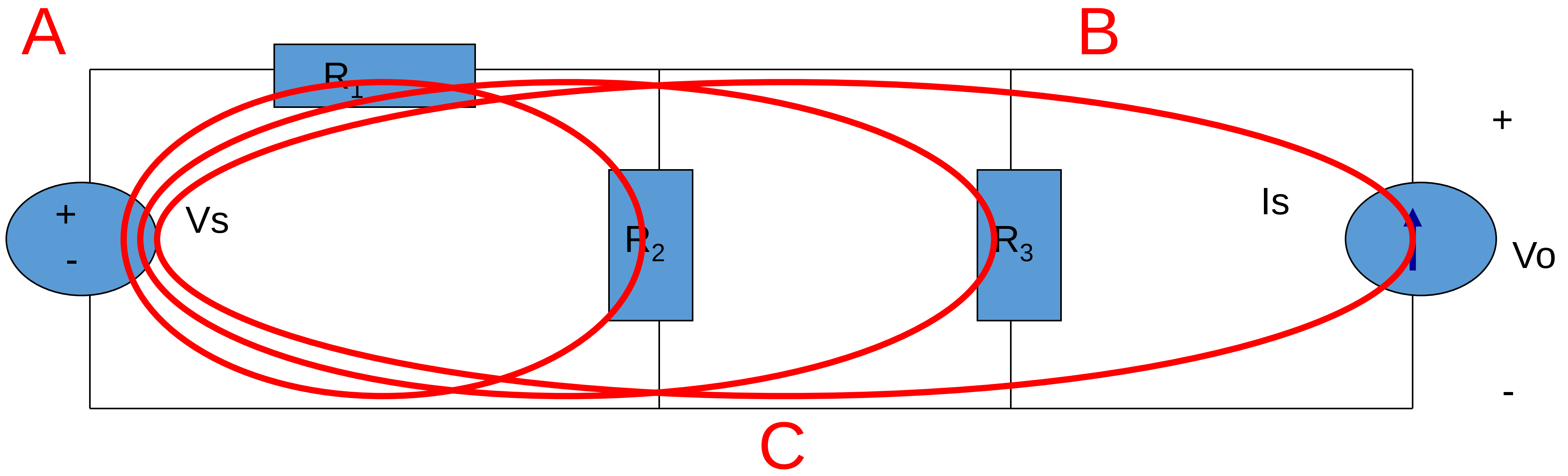
Example

- 5 Branches



Example

- Three Loops, if starting at node A



QUICK QUIZ (Poll 1)

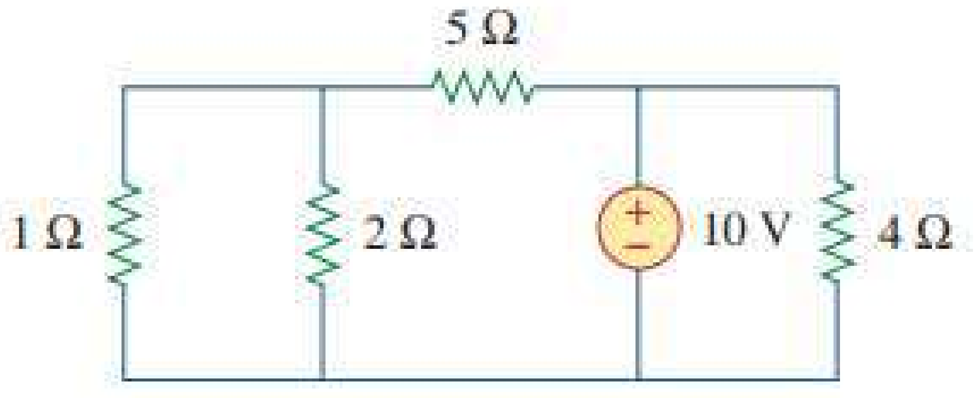
How many branches, nodes and independent loops are present in the given circuit?

A. b=3, n=5, l=6

B. b=5, n=3, l=6

C. b=5, n=3, l=3

D. b=3, n=5, l=3



Kirchhoff’s Law

- Ohm’s law by itself is not sufficient to analyze circuits.
- However, when it is coupled with Kirchhoff’s two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits.
- These laws are:

1. Kirchhoff’s Voltage Law (KVL)
2. Kirchhoff’s Current Law (KCL)

Kirchhoff's Current Law (KCL)

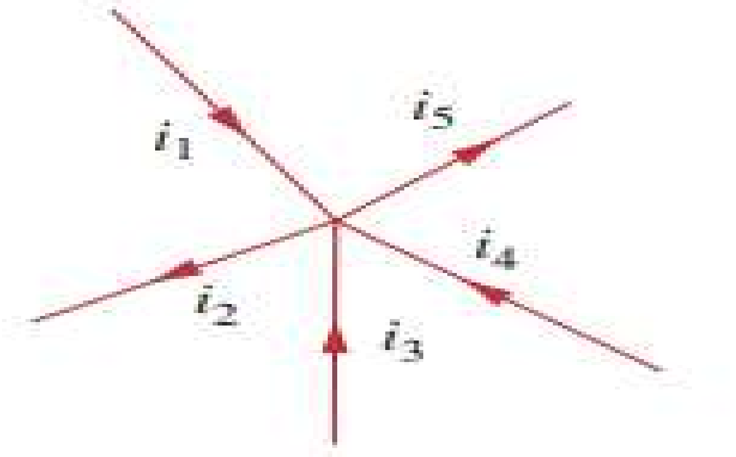
- It states that:

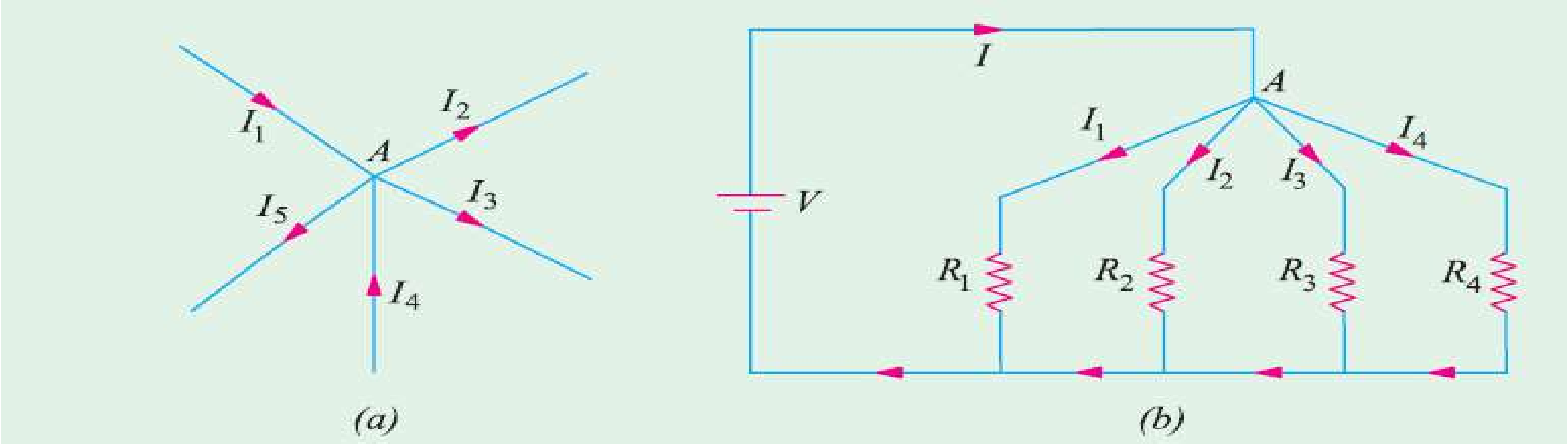
"the algebraic sum of currents entering a node is zero".

OR

" Sum of currents entering a node = Sum of currents leaving a node "

- Based on Law of Conservation of Charge.
- Mathematically, $\sum I = 0$





$I_1 + (-I_2) + (-I_3) + (+I_4) + (-I_5) = 0$

$I_1 + I_4 - I_2 - I_3 - I_5 = 0$ or $I_1 + I_4 = I_2 + I_3 + I_5$

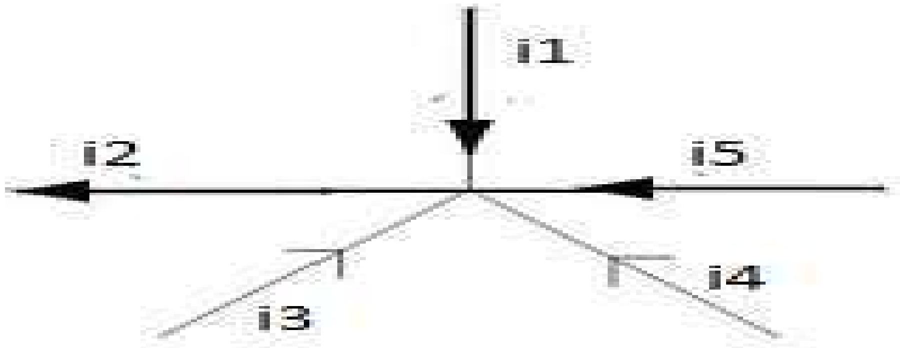
incoming currents = outgoing currents

$+I + (-I_1) + (-I_2) + (-I_3) + (-I_4) = 0$ or $I = I_1 + I_2 + I_3 + I_4$

We can express the above conclusion thus : $\Sigma I = 0$

QUICK QUIZ (Poll 2)

What is the relation between currents in above figure



a. i2 = i1 + i3 + i4 +i5
b. i2 − i1 = i3 − i4 +i5
c. i3 +i4 =i1+i2+i5
d. I1+i5 =i2+i3+i4

Kirchhoff's Voltage Law (KVL)

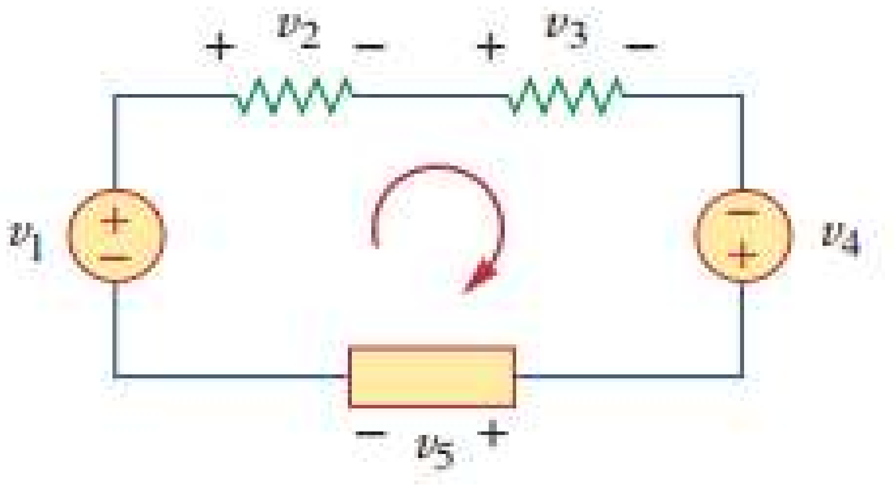
- It states that:

"algebraic sum of all voltages around a closed path (or loop) is zero."

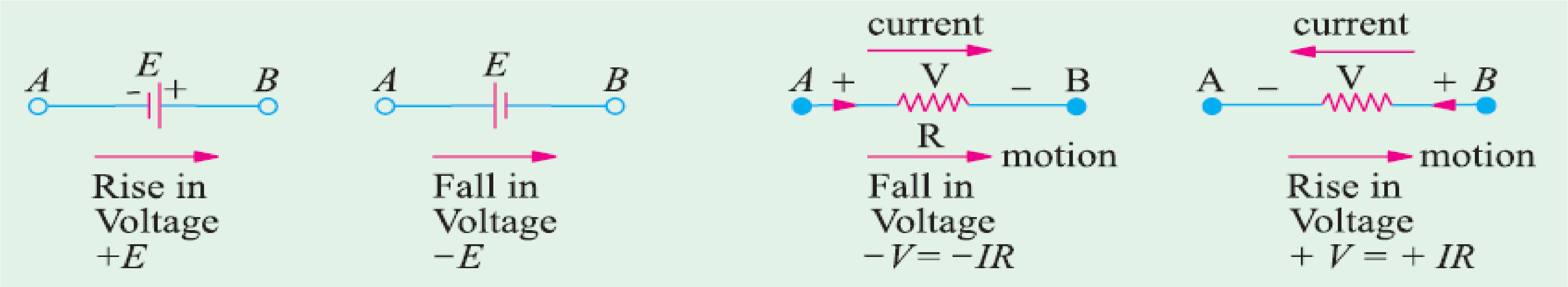
OR

" Sum of voltage drops = Sum of voltage rises."

- Based on Law of Conservation of Energy
- Mathematically, $\sum V = 0$



Sign Convention for KVL

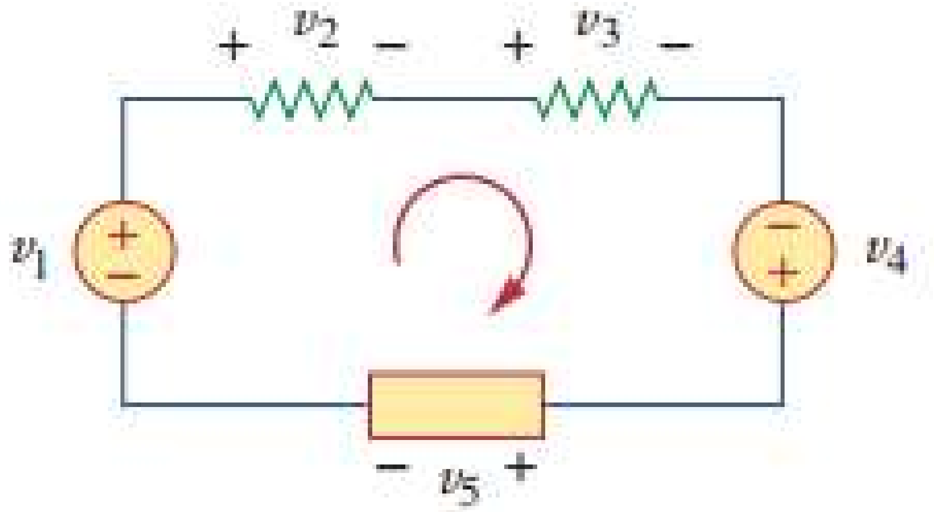


A *rise in voltage should be given a + ve sign and a fall in voltage a −ve sign. Keeping this in* mind, it is clear that as we go from the −ve terminal of a battery to its +ve terminal, there is a *rise in potential, hence this voltage should be given a + ve sign. If, on the other hand, we go from* +ve terminal to −ve terminal, then there is a *fall in potential, hence this voltage should be preceded* by a −ve sign.

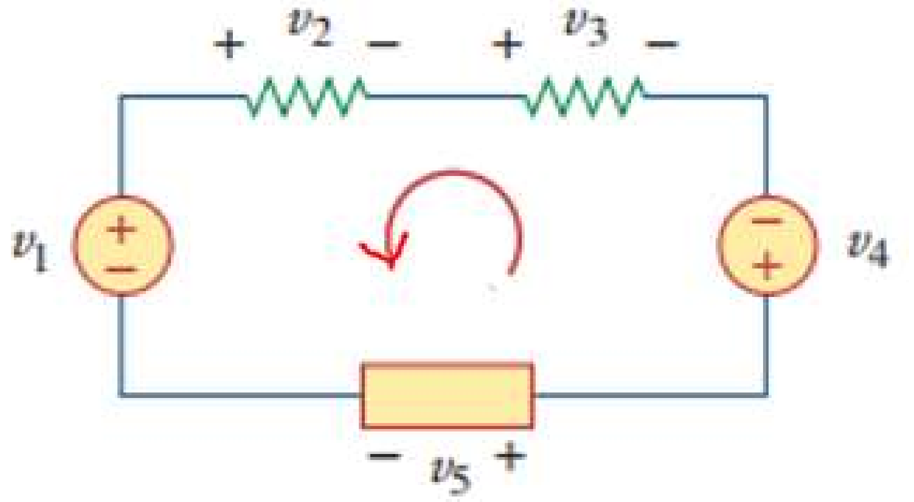
Now, take the case of a resistor. If we go through a resistor in the *same direction as the* current, then there is a fall in potential because current flows from a higher to a lower potential. Hence, this voltage fall should be taken −ve. However, if we go in a direction opposite to that of the current, then there is a *rise in voltage. Hence, this voltage rise should be given a positive sign.*

Let us Recall!

- Taking Clockwise direction (Def. 1):
  $$+V_1 - V_2 - V_3 + V_4 - V_5 = 0$$



- Taking Anti-clockwise direction(Def. 1):
  $$-V_4 + V_3 + V_2 - V_1 + V_5 = 0$$



- Voltage rise = Voltage drop
  $$+V_1 + V_4 = V_2 + V_3 + V_5$$

Example

I_1R_2 is $-$ve (fall in potential)

I_2R_2 is $-$ve (fall in potential)

I_3R_3 is $+$ve (rise in potential)

I_4R_4 is $-$ve (fall in potential)

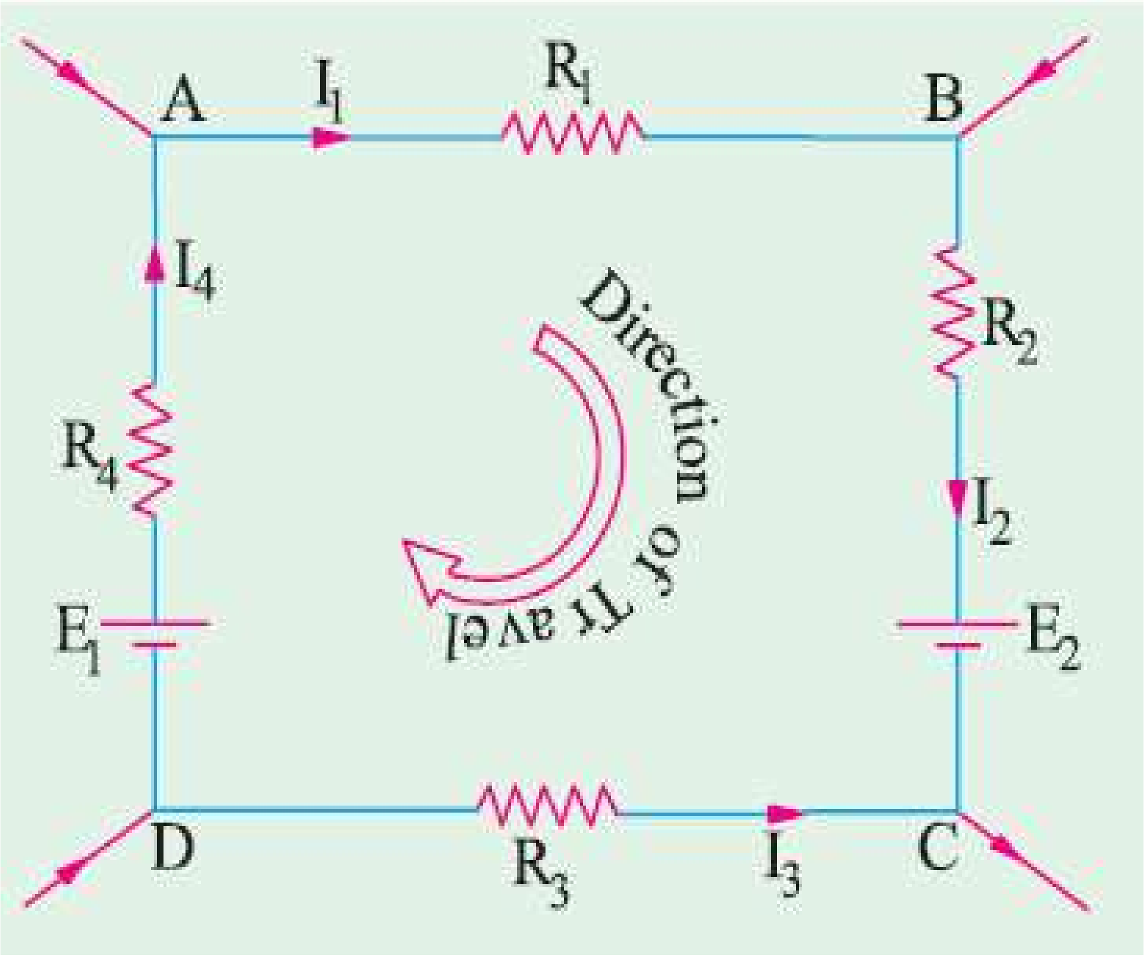
E_2 is $-$ve (fall in potential)

E_1 is $+$ve (rise in potential)

Using Kirchhoff's voltage law, we get

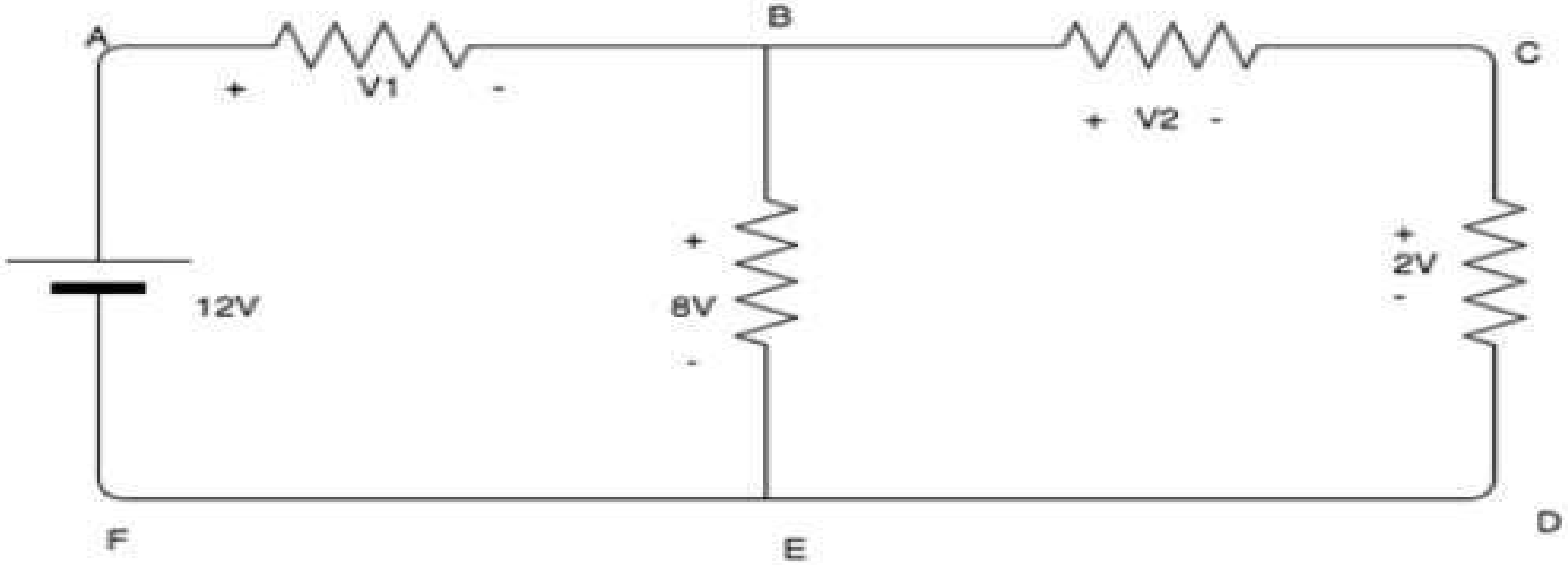
$$-I_1R_1 - I_2R_2 - I_3R_3 - I_4R_4 - E_2 + E_1 = 0$$

or $$I_1R_1 + I_2R_2 - I_3R_3 + I_4R_4 = E_1 - E_2$$



QUICK QUIZ (Poll 3)

Calculate the value of V1 and V2



a) 4V, 6V
b) 5V, 6V
c) 6V, 7V
d) 7V, 8V

Explanation

- Answer: a

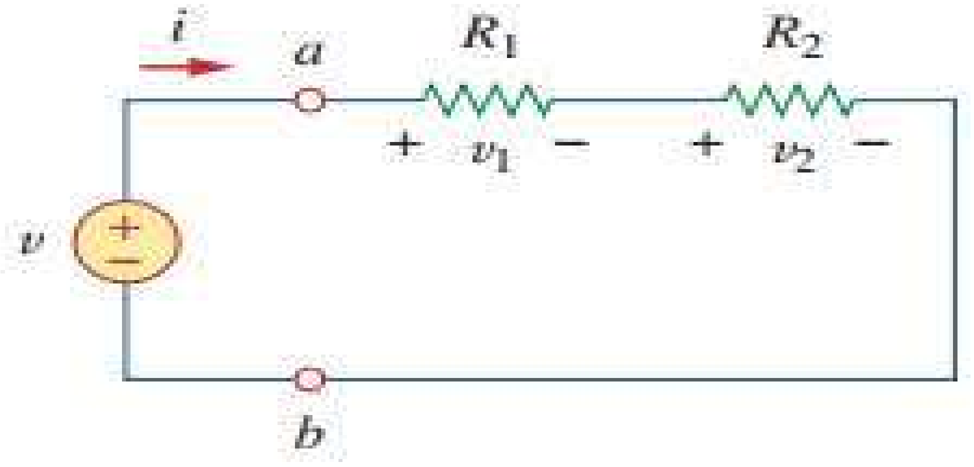
  Explanation: Using KVL, 12-V1-8=0. V1= 4V.

  8-V2-2=0. V2=6V.

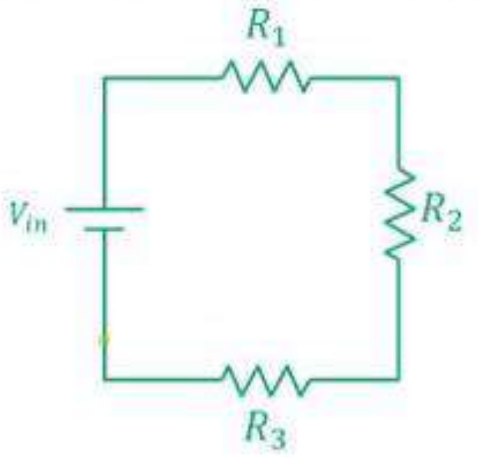
Voltage Division Rule

- The important relations are:

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$



VOLTAGE DIVISION RULE FOR 3- RESISTORS



$$V_1 = \frac{R_1}{R_1 + R_2 + R_3} \cdot V_{in}$$

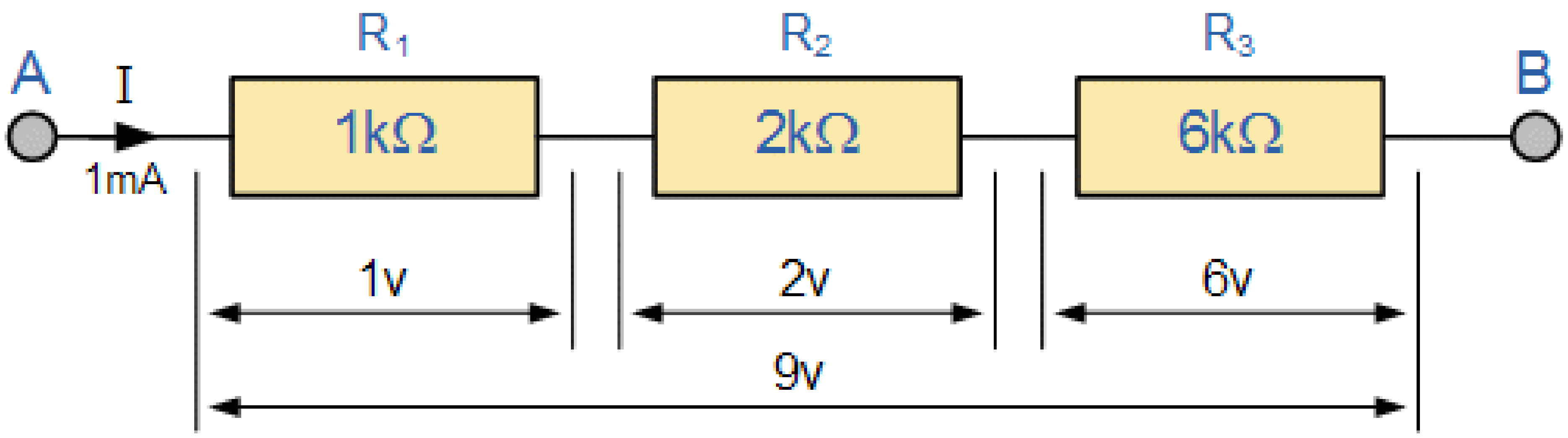
$$V_2 = \frac{R_2}{R_1 + R_2 + R_3} \cdot V_{in}$$

$$V_3 = \frac{R_3}{R_1 + R_2 + R_3} \cdot V_{in}$$

Voltage Division Rule for N-Resistors

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

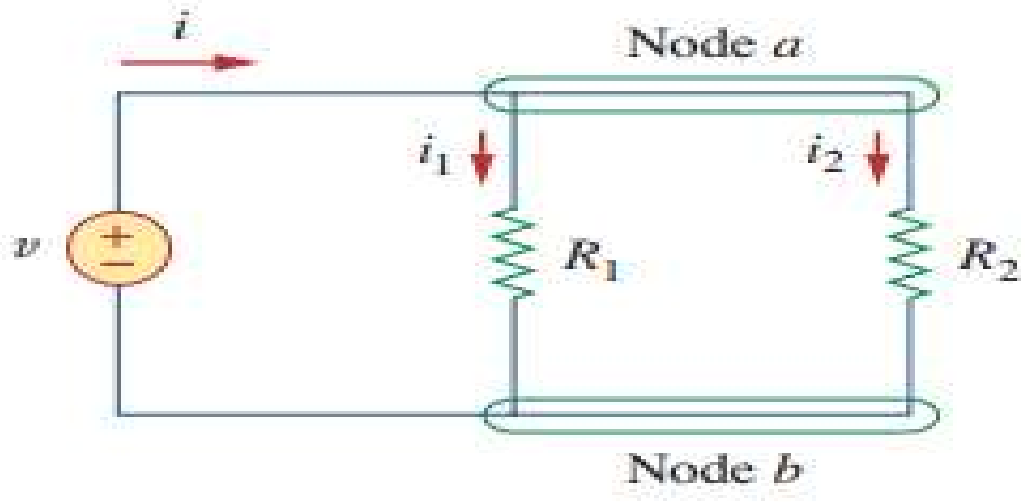
Example for Voltage Division Rule



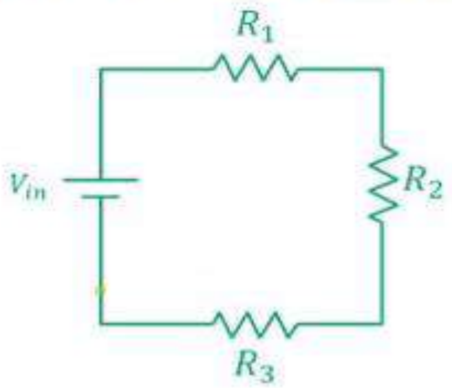
Current Division Rule

- The important relations are:

$$i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}$$



VOLTAGE DIVISION RULE FOR 3- RESISTORS



$$V_1 = \frac{R_1}{R_1 + R_2 + R_3} \cdot V_{in}$$

$$V_2 = \frac{R_2}{R_1 + R_2 + R_3} \cdot V_{in}$$

$$V_3 = \frac{R_3}{R_1 + R_2 + R_3} \cdot V_{in}$$

Voltage Division Rule for N-Resistors

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

QUICK QUIZ (Poll 4)

3. If there are 3 Resistors R_1, R_2 and R_3 in series and V is total voltage and I is total current then Voltage across R_2 is

a) $V R_3/ R_1 + R_2 + R_3$

b) $V R_2/ R_1 + R_2 + R_3$

c) $V R_1/R_1 + R_2 + R_3$

d) V

Applications of Kirchhoff's Laws

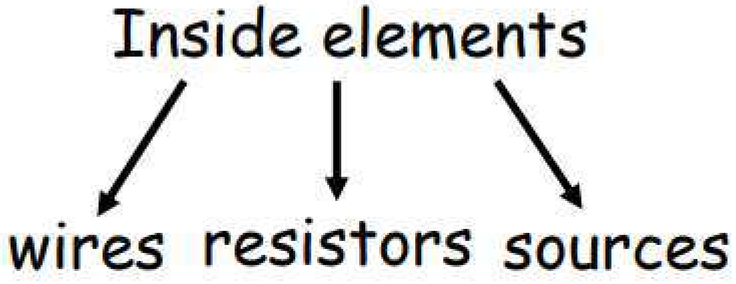
- They can be used to analyze any electrical circuit.
- Computation of current and voltage of complex circuits.

Limitations of Kirchhoff's Laws

- The limitation of Kirchhoff's both laws is that it works under the assumption that there is no fluctuating magnetic field in the closed loop and the current flows only through conductors and wires.

$$\frac{\partial \phi_B}{\partial t} = 0$$ Outside elements

$$\frac{\partial q}{\partial t} = 0$$



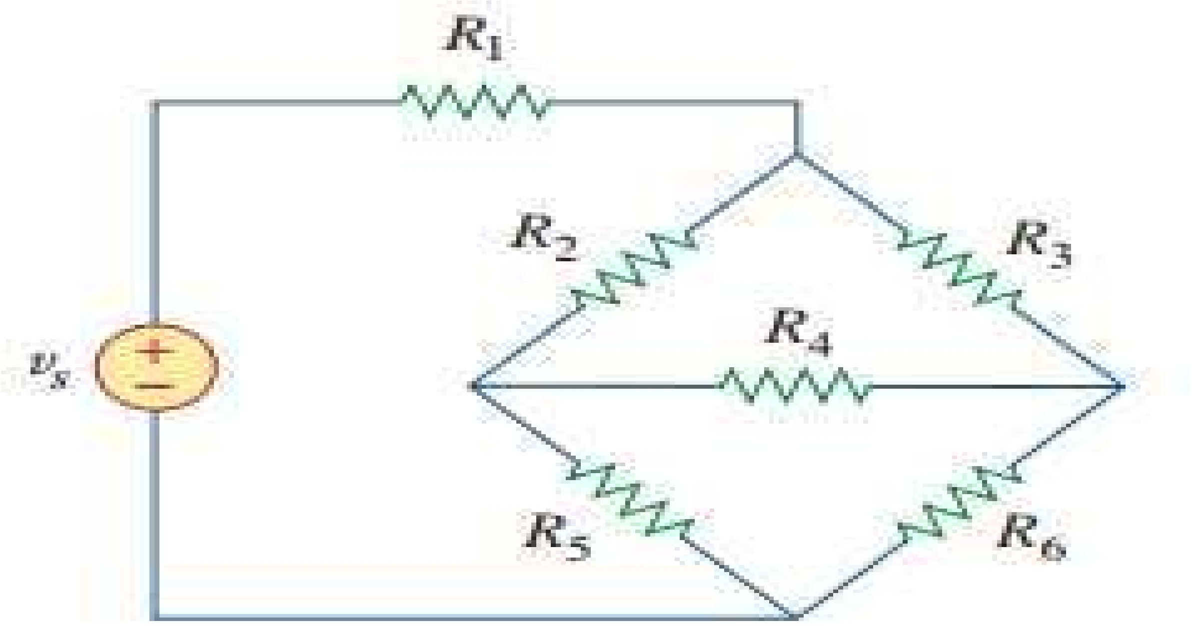
UNIT 1: DC CIRCUITS

Lecture 4

Prepared By:
Krishan Arora
Assistant Professor and Head

Star Delta Transformation

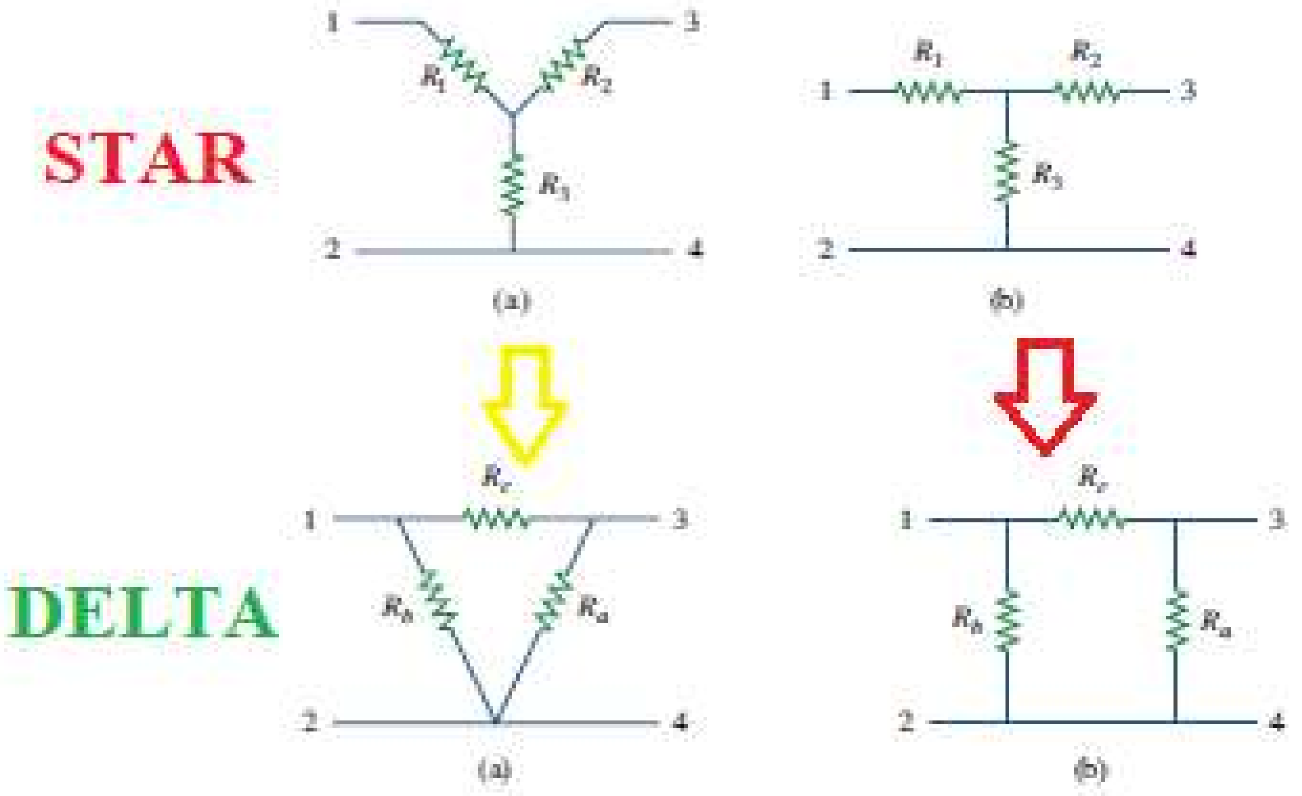
- Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge shown in the figure.



Star Delta Transformation

There are two types of such circuits

1. Star Connection
2. Delta Connection

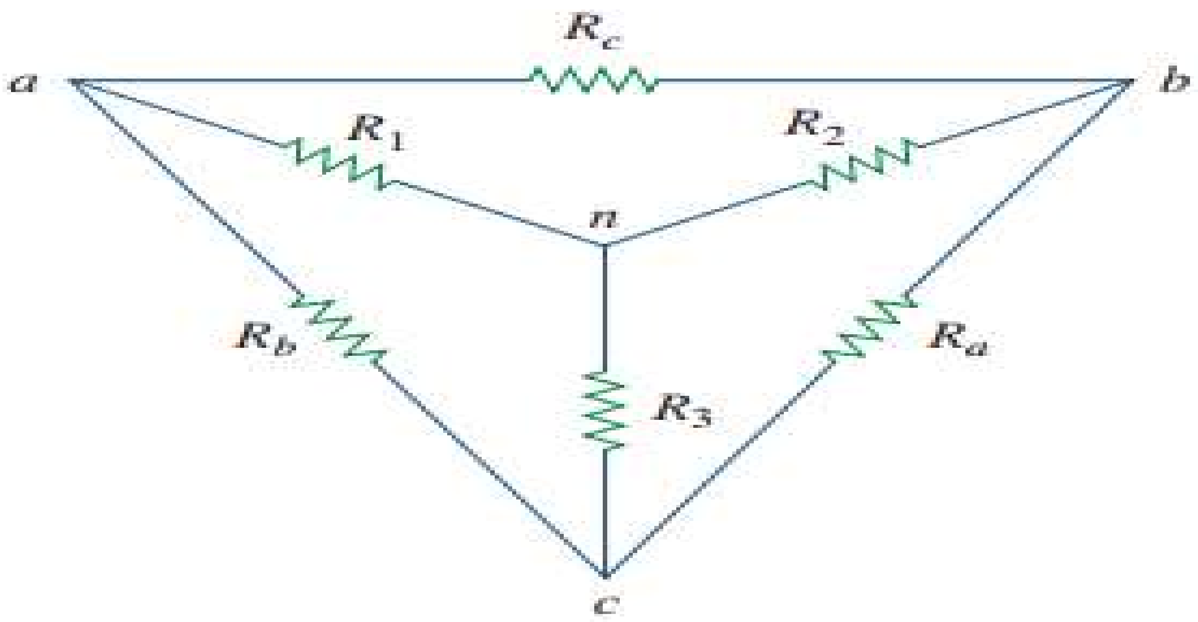


Delta to Star Conversion

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



How to Remember ?

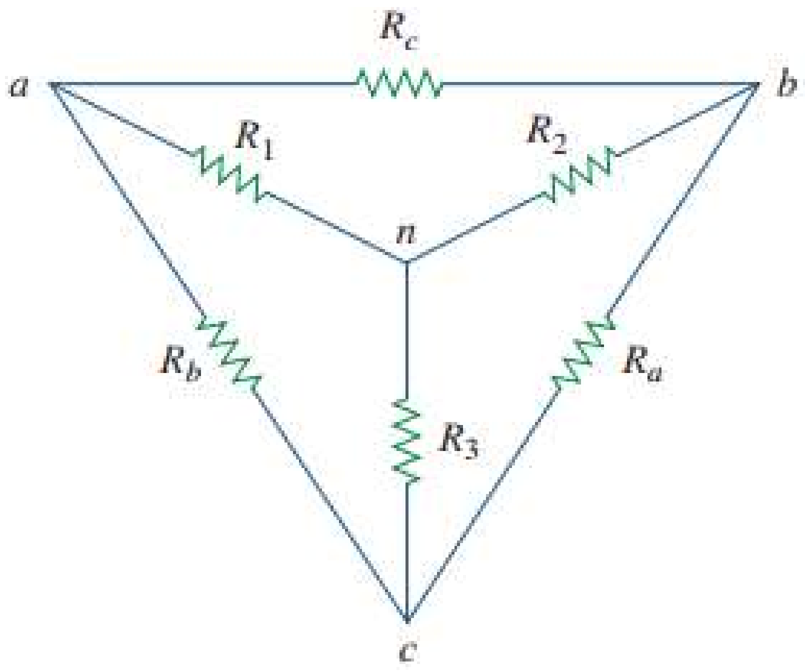
It is seen from above that each numerator is the product of the two sides of the delta which meet at the point in star. Hence, it should be remembered that : *resistance of each arm of the star is given by the product of the resistances of the two delta sides that meet at its end divided by the sum of the three delta resistances.*

Star to Delta Conversion

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

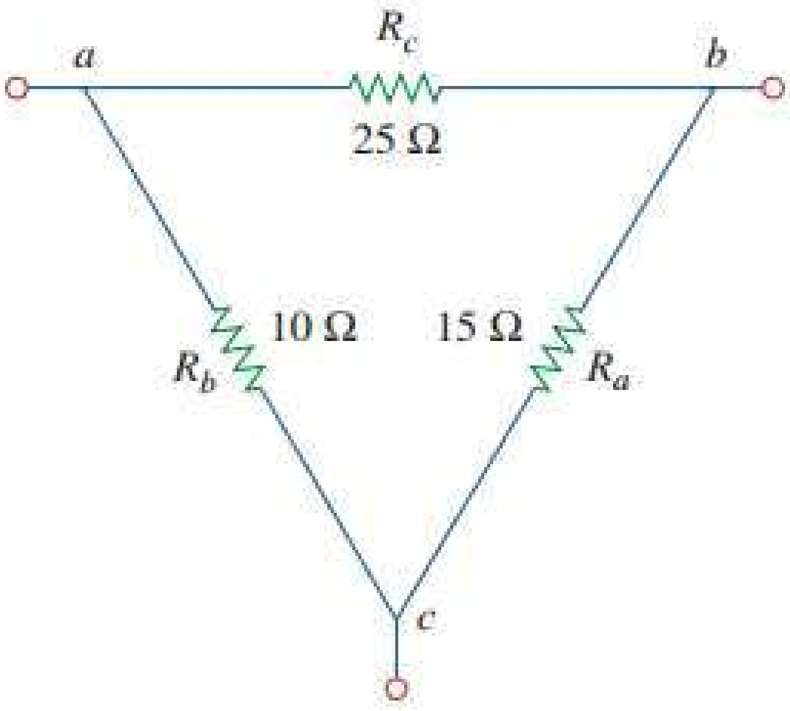


How to Remember ?

The equivalent delta resistance between any two terminals is given by the sum of star resistances between those terminals plus the product of these two star resistances divide by the third star resistances.

Practice Problem

Q: Convert Δ network into a Y network?



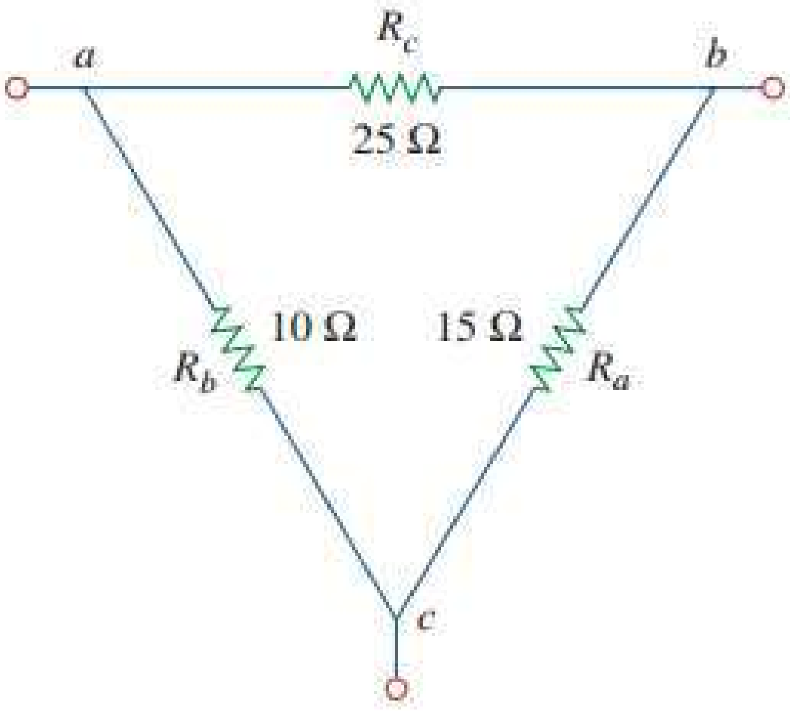
Practice Problem

Q: Convert Δ network into a Y network?

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$



QUICK QUIZ (Poll 1)

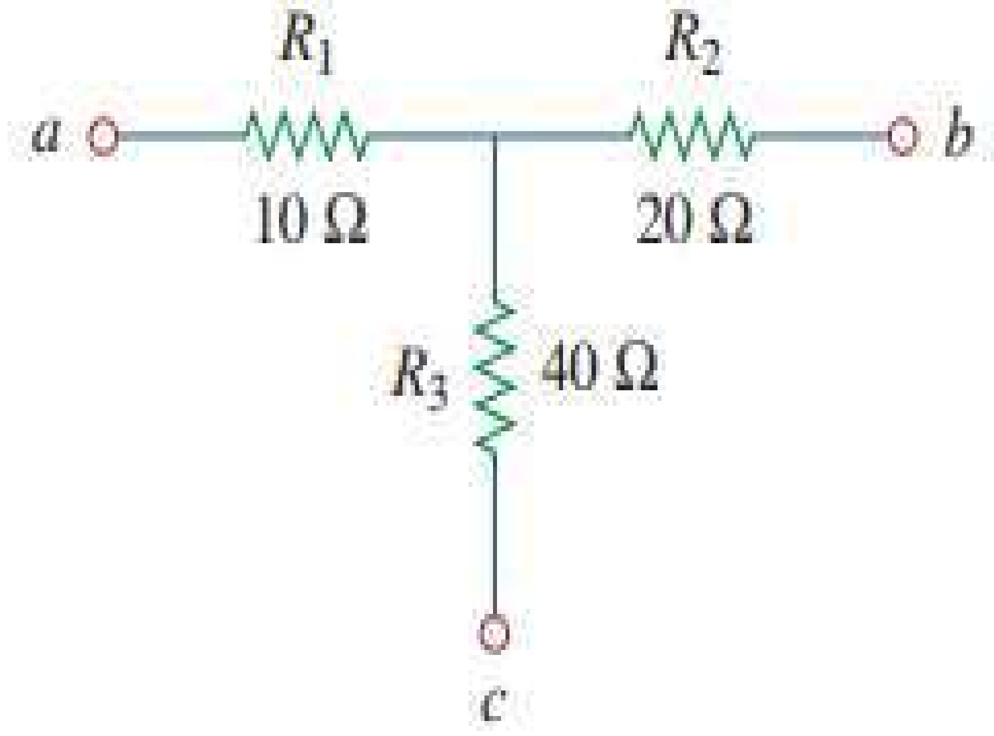
Resistance R_{bc} for the Δ network of the corresponding Figure is:

A. 140

B. 70

C. 35

D. 100



QUICK QUIZ (Poll 2)

4. Delta connection is also known as________

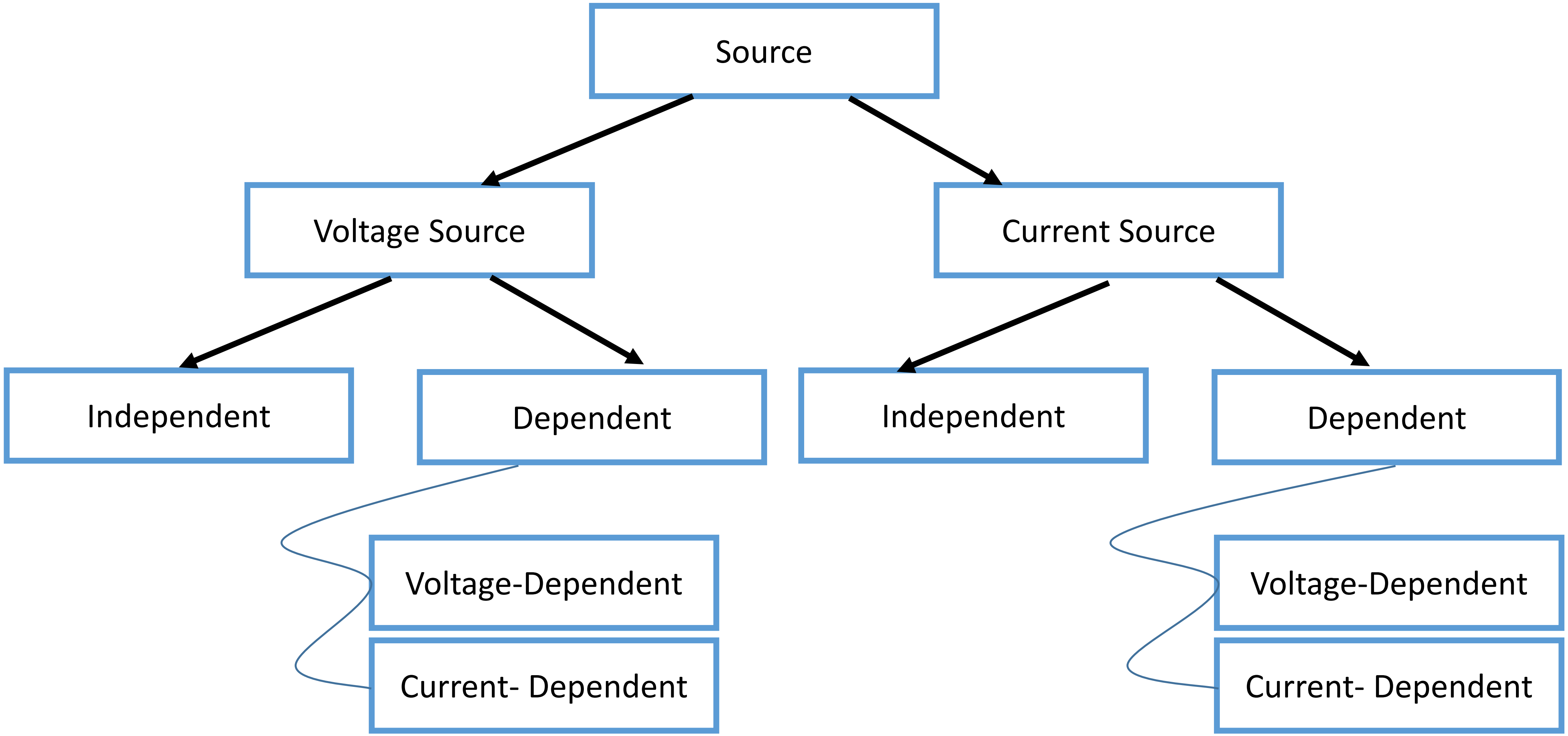
a) Y-connection

b) Mesh connection

c) Either Y-connection or mesh connection

d) Neither Y-connection nor mesh connection

Energy Sources



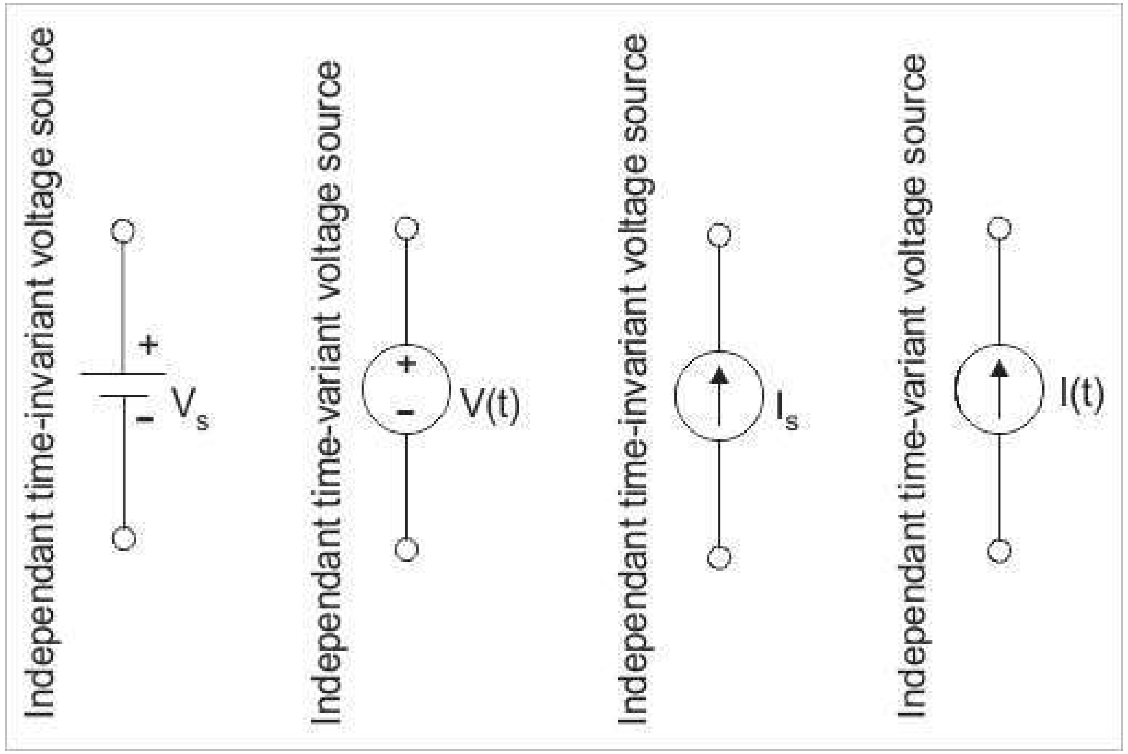
Independent and Dependent Sources

- Independent sources are those which does not depend on any other quantity in the circuit. They are two terminal devices and has a constant value, i.e. the voltage across the two terminals remains constant irrespective of all circuit conditions. The Independent sources are represented by a circular shape.

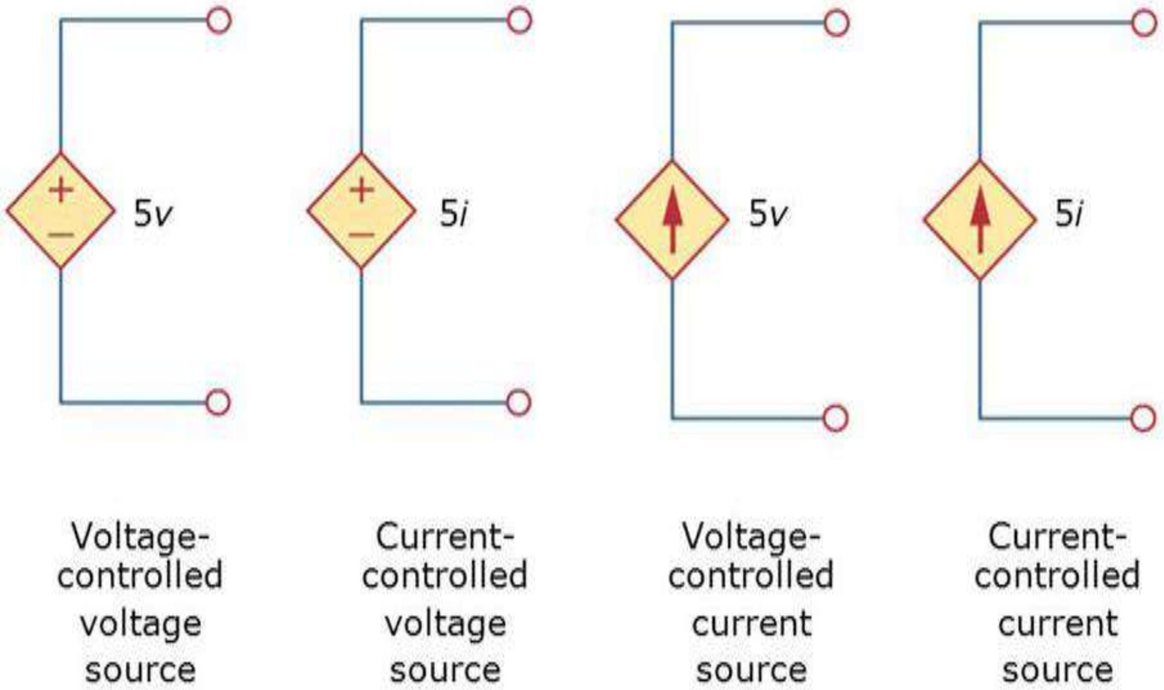
- Dependent or Controlled sources are those whose output voltage or current is NOT fixed but depends on the voltage or current in another part of the circuit is called. They are four terminal devices. When the strength of voltage or current changes in the source for any change in the connected network, they are called dependent sources. The dependent sources are represented by a diamond shape.

Independent and Dependent Sources

- Independent



- Dependent



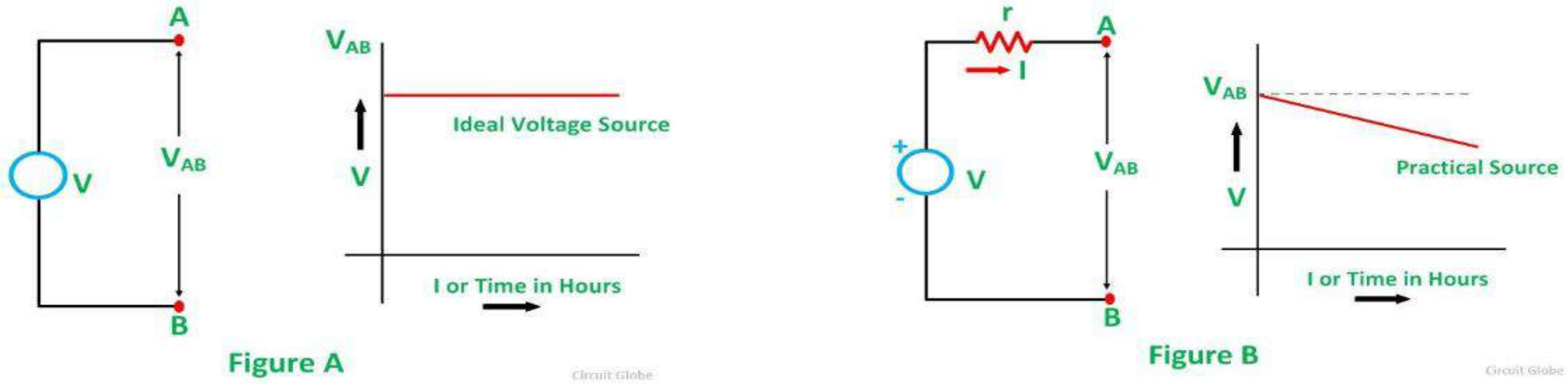
Ideal and Practical Voltage Source

- Ideal is one where internal resistance does NOT exist.

NOTE:

1. For a voltage source, internal resistance must be ZERO.
2. For a current source, internal resistance must be INFINITY.

- Practical is one where internal resistance is present.



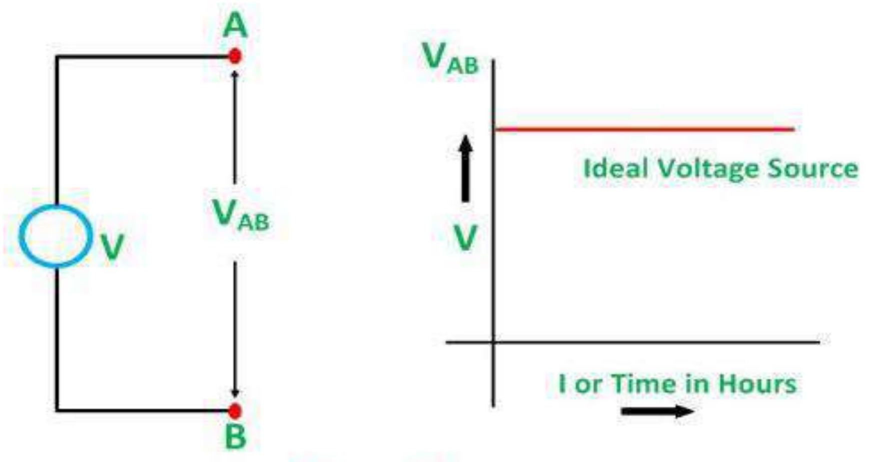


Figure A

The figure B shown below gives the circuit diagram and characteristics of Practical Voltage Source

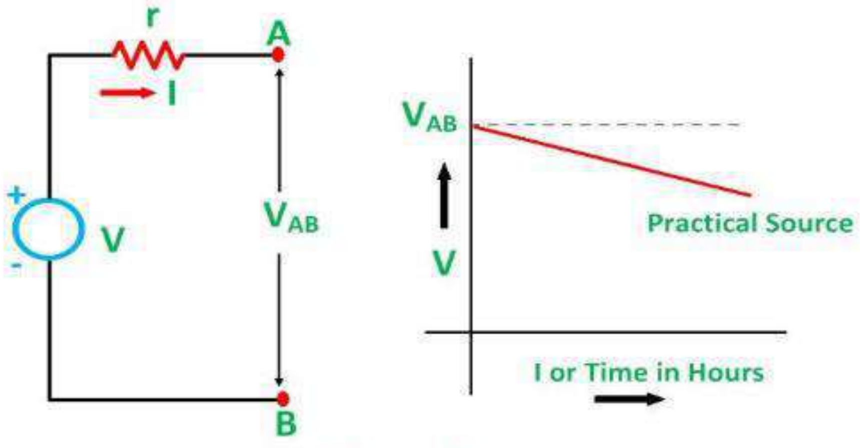


Figure B

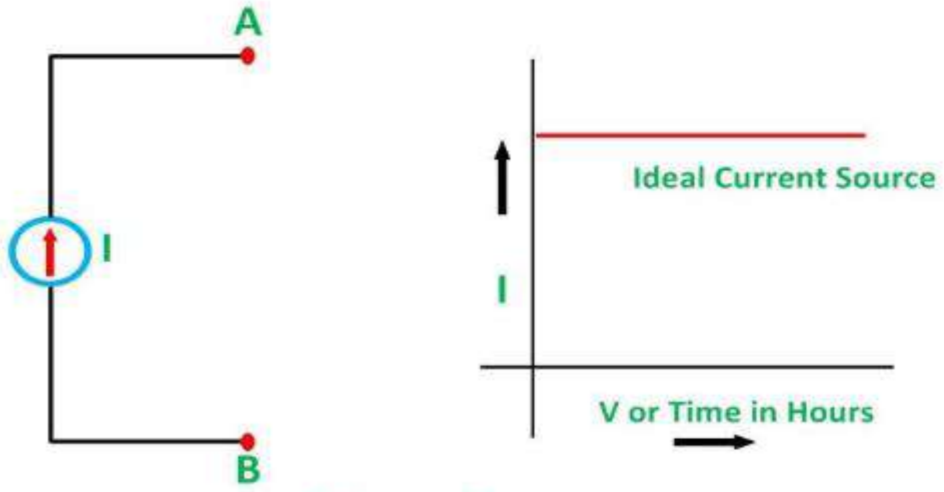


Figure C

Figure D shown below shows the characteristics of Practical Current Source.

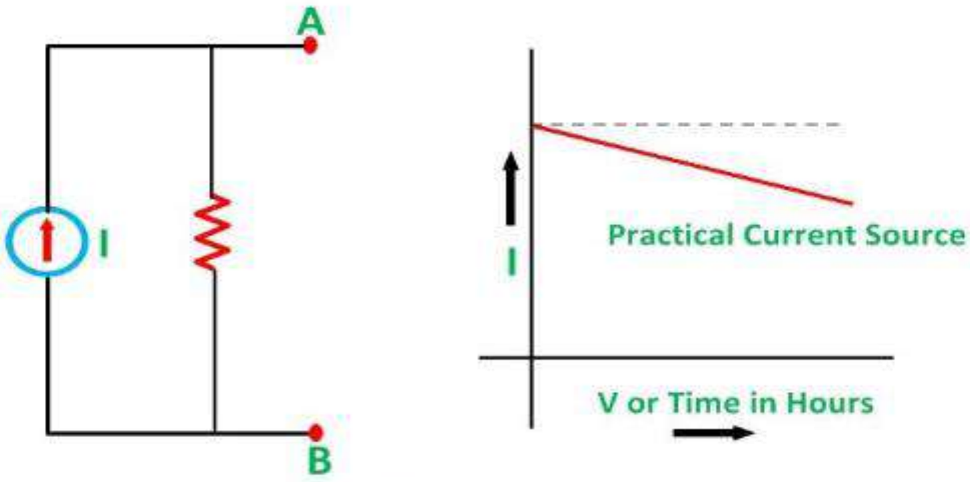


Figure D

QUICK QUIZ (Poll 3)

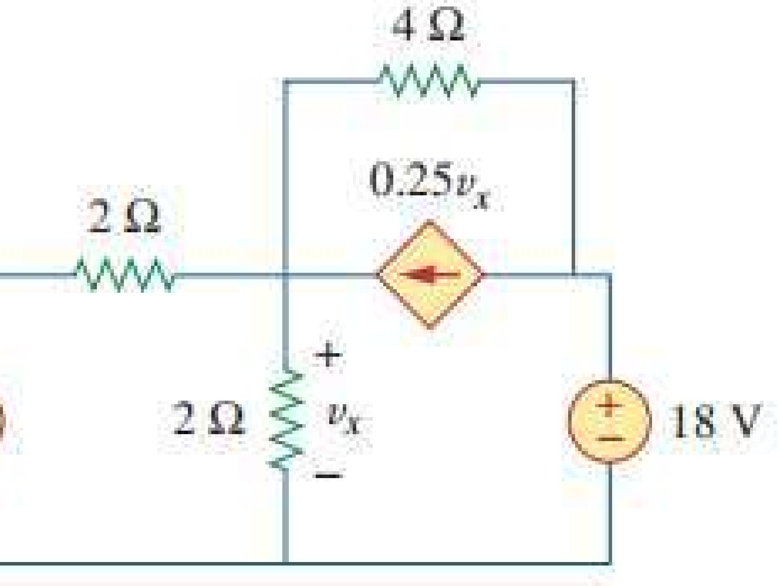
Identify the type of dependent source used in the network:

A. VCVS

B. CCCS

C. VCCS

D. CCVS



QUICK QUIZ (Poll 4)

1) The symbol used for representing Independent sources

a) Diamond

b) Square

c) Circle

d) Triangle

QUICK QUIZ (Poll 5)

2. Controlled sources are also known as

a) Independent sources

b) Dependent sources

c) Ideal sources

d) Voltage sources

QUICK QUIZ (Poll 6)

The analysis of a circuit containing dependent sources can be done using nodal and mesh analysis.

a) True

b) False

QUICK QUIZ (Poll 7)

- In case of a dependent voltage/current source, the value of this voltage/current source depends on __________

  a) Voltage/current sources of an external circuit
  b) Voltage/current source present somewhere in the circuit
  c) Only on voltage sources
  d) Only on current sources

UNIT 1: DC CIRCUITS

Lecture 5

Prepared By:
Krishan Arora
Assistant Professor and Head

Nodal Analysis

- Nodal analysis provides a general procedure for analyzing circuits using node voltages as the circuit variables.
- Choosing node voltages instead of element voltages as circuit variables is convenient and reduces the number of equations one must solve simultaneously.
- Applicable to nodes only.
- It is used to find the unknown node voltages.
- This Method is Application of KCL+Ohm's Law Only

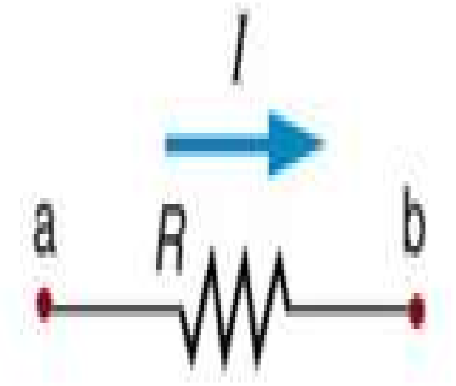
Steps to Determine Node Voltages

1. Select one nodes out of 'n' node as the reference node. Assign voltages to the remaining nodes. The voltages are referenced with respect to the reference node.

2. Apply KCL to each of the non-reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.

3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

- The number of non-reference nodes is equal to the number of independent equations that we have to derive.
- Current flows from a higher potential to a lower potential in a resistor



$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

QUICK QUIZ (Poll 6)

For “N” number of nodes, the number of non-reference nodes is equal to:

A. $N + 1$

B. $N - 1$

C. 2N

D. 2N – 1

QUICK QUIZ (Poll 7)

- In nodal analysis how many nodes are taken as reference nodes?

a) 1
b) 2
c) 3
d) 4

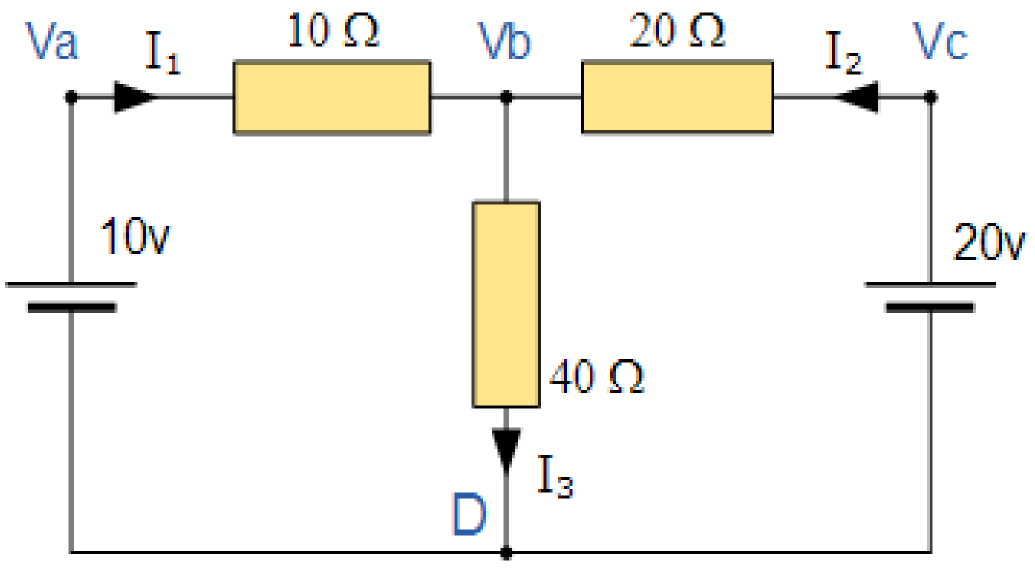
QUICK QUIZ (Poll 8)

- If there are 8 nodes in network, we can get _____ number of equations in the nodal analysis.

  a) 9
  b) 8
  c) 7
  d) 6

Example 1

- Obtain the node voltages in the given circuit?



In the above circuit, node D is chosen as the reference node and the other three nodes are assumed to have voltages, Va, Vb and Vc with respect to node D. For example;

$$\frac{(V_a - V_b)}{10} + \frac{(V_c - V_b)}{20} = \frac{V_b}{40}$$

As Va = 10v and Vc = 20v , Vb can be easily found by:

$$\left(1-\frac{Vb}{10}\right)+\left(1-\frac{Vb}{20}\right)=\frac{Vb}{40}$$

$$2=Vb\left(\frac{1}{40}+\frac{1}{20}+\frac{1}{10}\right)$$

$$Vb=\frac{80}{7}V$$

$$\therefore I_3=\frac{2}{7}\ or\ 0.286Amps$$

Mesh Analysis

- Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables.
- It is based on KVL.

RECALL!

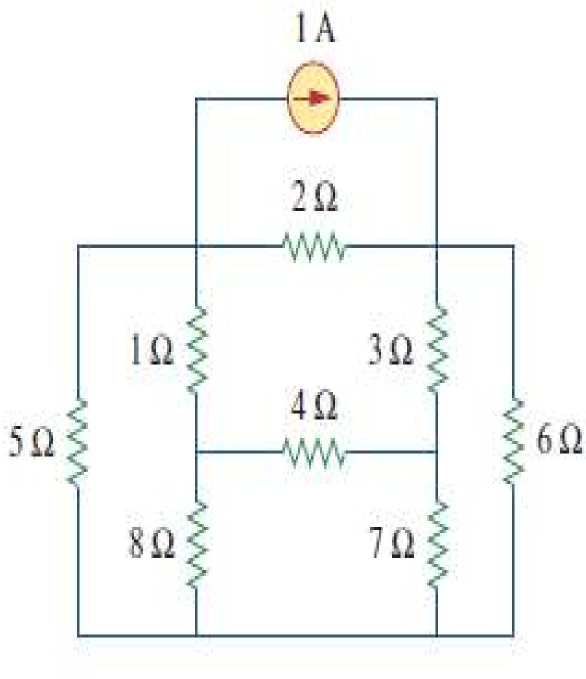
- LOOP: A loop is a closed path with no node passed more than once.
- MESH: A mesh is a loop that does not contain any other loop within it.
- Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is planar.
- **PLANAR CIRCUIT:** A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is nonplanar.

Steps to Determine Mesh Currents

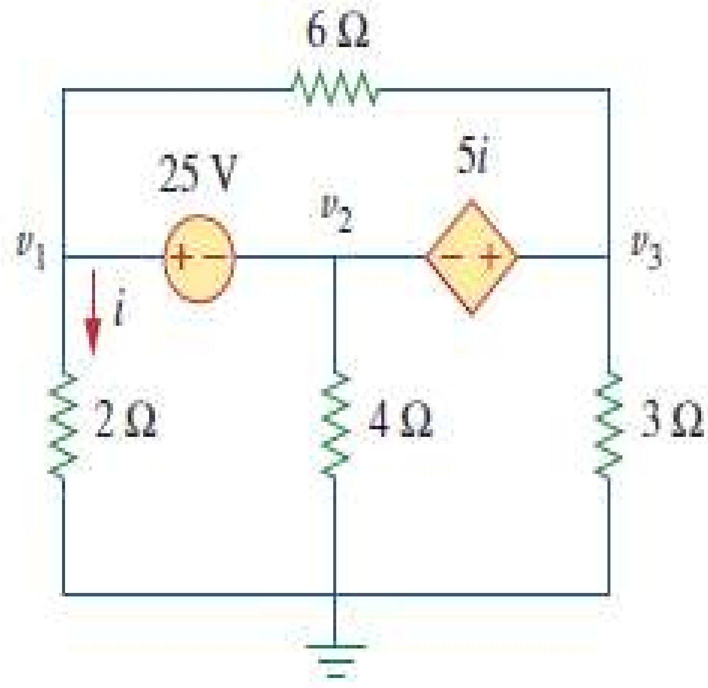
1. Assign mesh currents to ‘n’ meshes
2. Apply KVL to each of the ‘n’ meshes.
3. Solve the resulting ‘n’ simultaneous equations to obtain the unknown mesh currents.

Examples of Planar Circuits

Planar circuit is circuit which can be represented on plane without crossing any other branch.

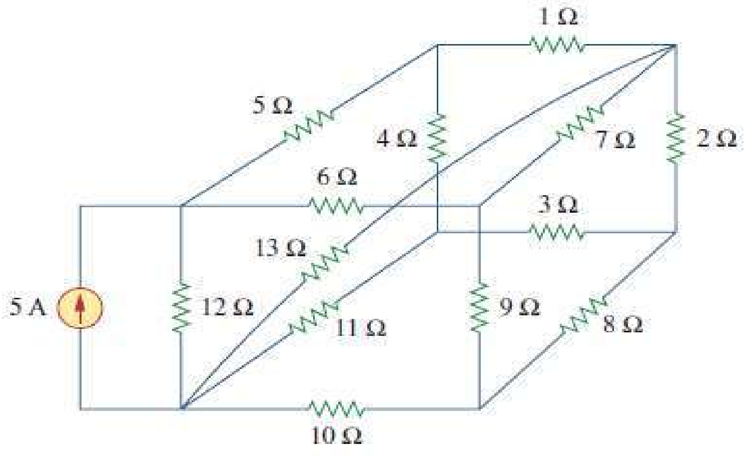


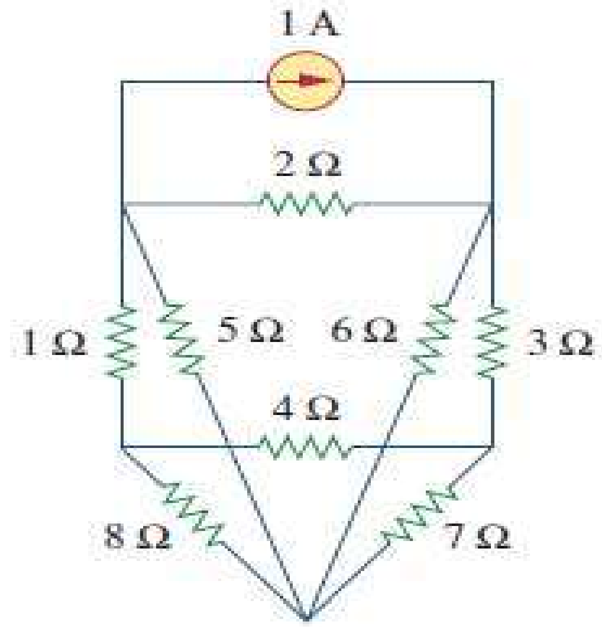
(b)



Examples of Non-Planar Circuits

Non planar circuit is one which cannot be represented on paper without crossing other branch.





(a)

QUICK QUIZ (Poll 8)

Mesh Analysis to applicable to______ type networks.:

A. Planar and Loop
B. Non planar and mesh
C. Planar and mesh
D. Non planar and Loop

QUICK QUIZ (Poll 9)

Mesh analysis, which is based on KVL is used to find unknown:

A. *current*

B. *voltage*

QUICK QUIZ (Poll 10)

- Nodal analysis, which is based on KCL, is used to find unknown value of

A) Current

B) Voltage

UNIT 1: DC CIRCUITS

Lecture 6

Prepared By:
Krishan Arora
Assistant Professor and Head

Superposition Theorem

- The superposition theorem extends the use of Ohm's Law to circuits with multiple sources.
- **Definition** :- *The current through, or voltage across, an element in a linear bilateral network equal to the algebraic sum of the currents or voltages produced independently by each source.*
- The Superposition theorem is very helpful in determining the voltage across an element or current through a branch when the circuit contains multiple number of voltage or current sources.

Superposition Theorem

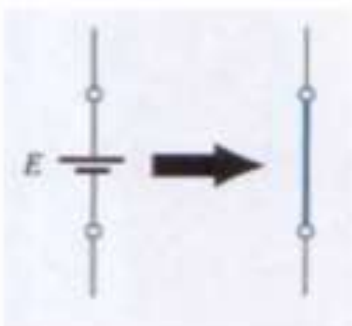
- In order to apply the superposition theorem to a network, certain conditions must be met :

1. All the components must be linear, for e.g.- the current is proportional to the applied voltage (for resistors), flux linkage is proportional to current (in inductors), etc.
2. All the components must be bilateral, meaning that the current is the same amount for opposite polarities of the source voltage.
3. Passive components may be used. These are components such as resistors, capacitors, and inductors, that do not amplify or rectify.
4. Active components may not be used. Active components include transistors, semiconductor diodes, and electron tubes. Such components are never bilateral and seldom linear.

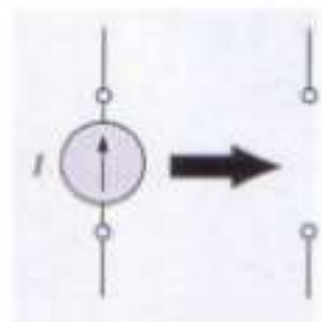
Procedure for applying Superposition Theorem

➢ Circuits Containing **Only Independent Sources**

- Consider only one source to be active at a time.
- Remove all other **IDEAL VOLTAGE SOURCES** by **SHORT CIRCUIT** & all other **IDEAL CURRENT SOURCES** by **OPEN CIRCUIT.**



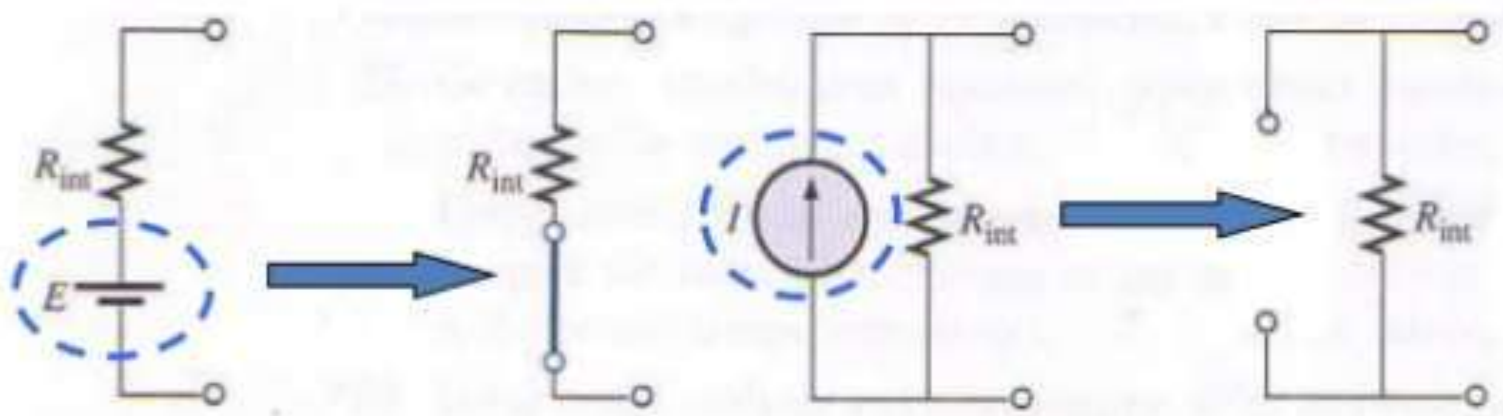
Voltage source is replaced by a Short Circuit



Current source is replaced by a Open Circuit

Procedure for applying Superposition Theorem

- If there are practical sources, replace them by the combination of **ideal source** and an **internal resistances** (as shown in figure).
- After that, **short circuit** the **ideal voltage source** & **open circuit** the **ideal current source** (as shown in figure).



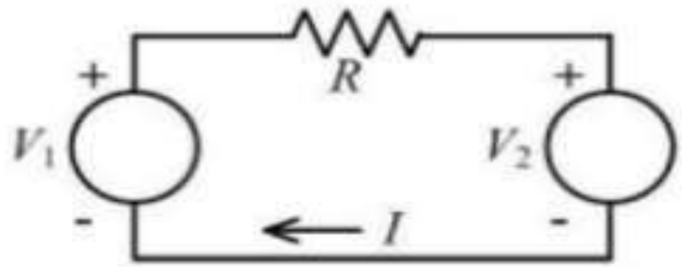
QUICK QUIZ (Poll 1)

- In superposition theorem, when we consider the effect of one voltage source, all the other voltage sources are _______________
  a) Shorted
  b) Opened
  c) Removed
  d) Undisturbed

QUICK QUIZ (Poll 2)

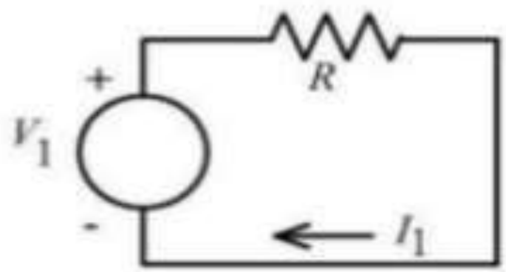
- In superposition theorem, when we consider the effect of one voltage source, all the other current sources are ______________
  a) Shorted
  b) Opened
  c) Removed
  d) Undisturbed

Example : 1



Find the current flowing through R.

1.

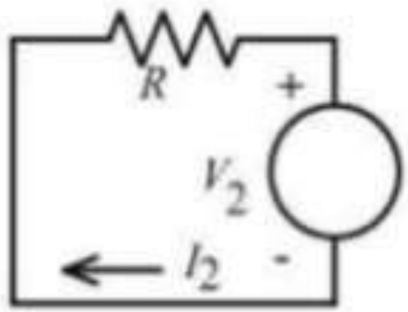


Short Circuiting Voltage source V_2 & finding the current I_1

$$I_1 = \frac{V_1}{R}$$

Example : 1

2.



Short Circuiting Voltage source V_1 & finding the current I_2

$$I_2 = -\frac{V_2}{R}$$

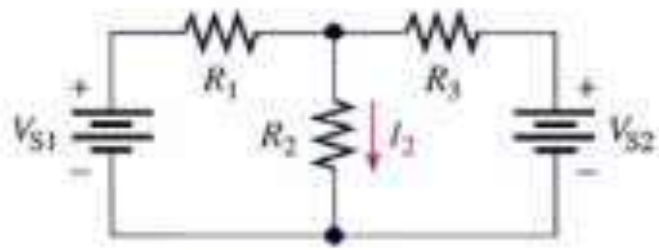
The net current is :- $I = I_1 + I_2 = \frac{V_1}{R} - \frac{V_2}{R}$

Same answer is obtained by another method (shown below) which would turn out to be tedious when applied to bigger circuits as in next example….

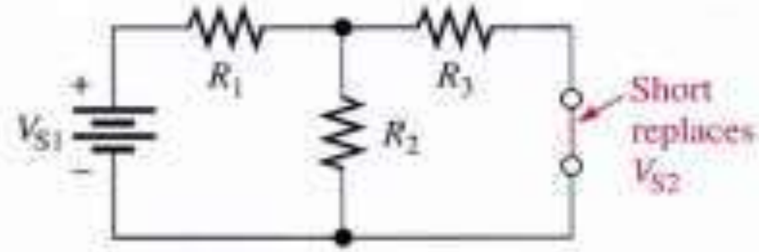
$$V_R = V_1 - V_2 \ (1),\ \ I = \frac{V_R}{R} = \frac{V_1 - V_2}{R} \ (2),\ I = \frac{V_1}{R} - \frac{V_2}{R} \ (3)$$

Superposition Theorem

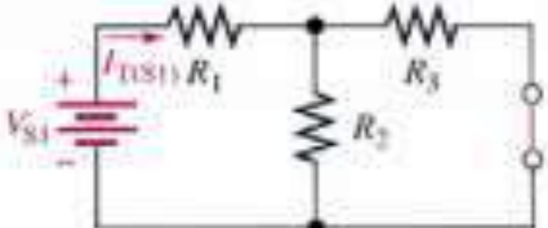
The approach to superposition is demonstrated in the figure for a series-parallel circuit with two ideal voltage sources.



(a) Problem: Find I_2.



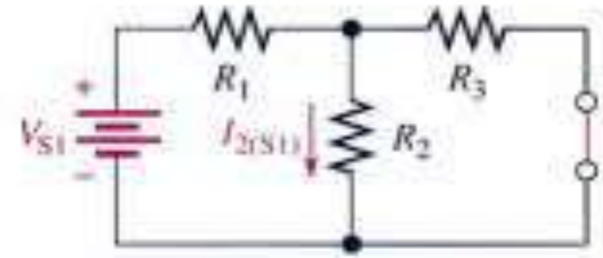
(b) Replace V_{S2} with zero resistance (short).



(c) Find R_T and I_T looking from V_{S1}:

$R_{T(S1)} = R_1 + R_2 \parallel R_3$

$I_{T(S1)} = V_{S1}/R_{T(S1)}$

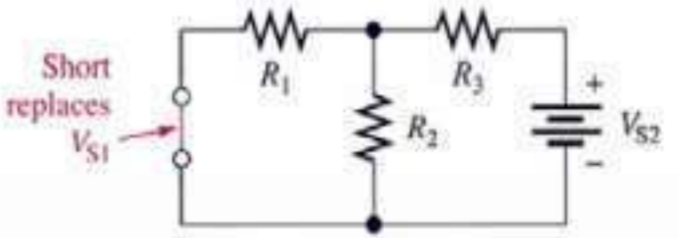


(d) Find I_2 due to V_{S1} (current divider):

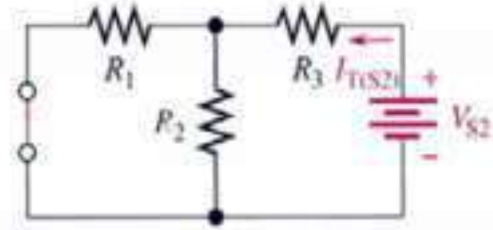
$$I_{2(S1)} = \left(\frac{R_3}{R_2 + R_3}\right) I_{T(S1)}$$

Superposition Theorem

The approach to superposition is demonstrated in the figure for a series-parallel circuit with two ideal voltage sources.



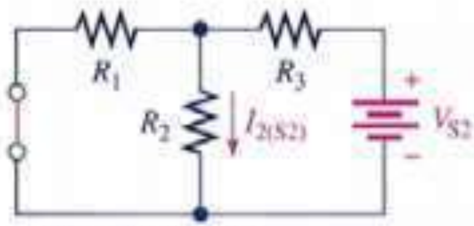
(e) Replace V_{S1} with zero resistance (short).



(f) Find R_T and I_T looking from V_{S2}:

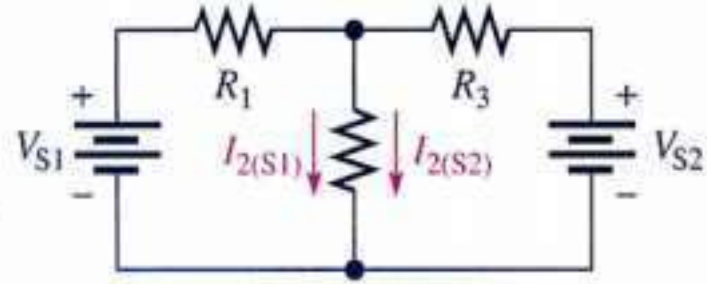
$$R_{T(S2)} = R_3 + R_1 \parallel R_2$$

$$I_{T(S2)} = V_{S2}/R_{T(S2)}$$



(g) Find I_2 due to V_{S2}:

$$I_{2(S2)} = \left(\frac{R_1}{R_1 + R_2}\right) I_{T(S2)}$$

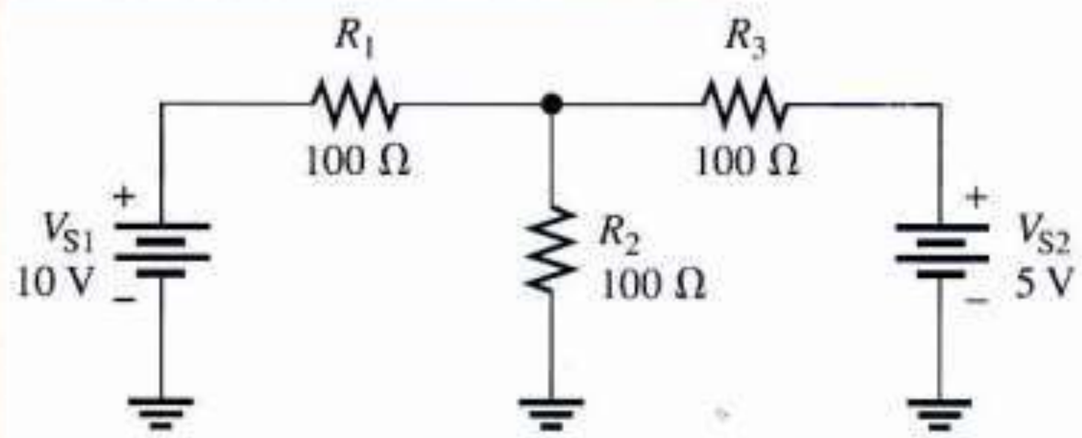


(h) Restore the original sources. Add $I_{2(S1)}$ and $I_{2(S2)}$ to get the actual I_2 (they are in same direction):

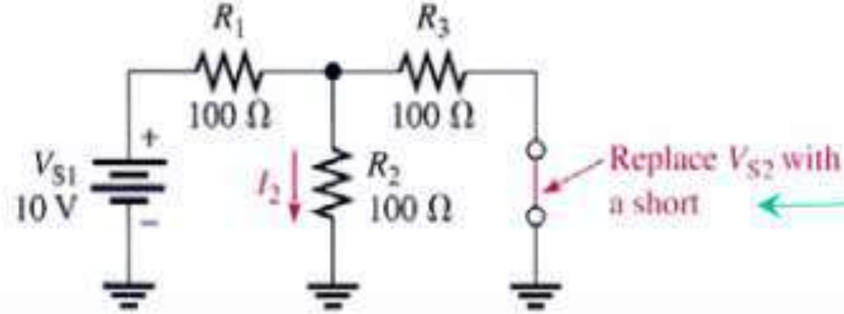
$$I_2 = I_{2(S1)} + I_{2(S2)}$$

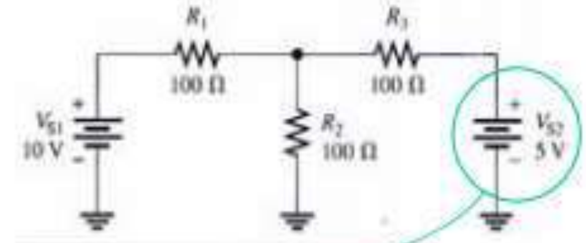
Superposition Theorem

EXAMPLE Use the superposition theorem to find the current through R_2.



Superposition Theorem





Solution **Step 1:** Replace V_{S2} with a short and find the current through R_2 due to voltage source V_{S1}, as shown in Figure 8–18. To find I_2, use the current-divider formula (Equation 6–6). Looking from V_{S1},

$$R_{T(S1)} = R_1 + \frac{R_3}{2} = 100\,\Omega + 50\,\Omega = 150\,\Omega$$

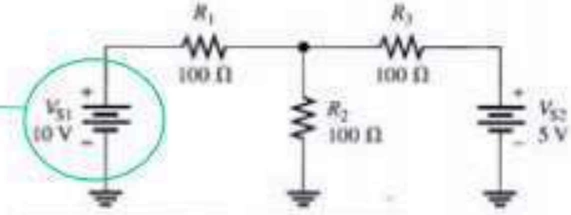
$$I_{T(S1)} = \frac{V_{S1}}{R_{T(S1)}} = \frac{10\text{ V}}{150\,\Omega} = 66.7\text{ mA}$$

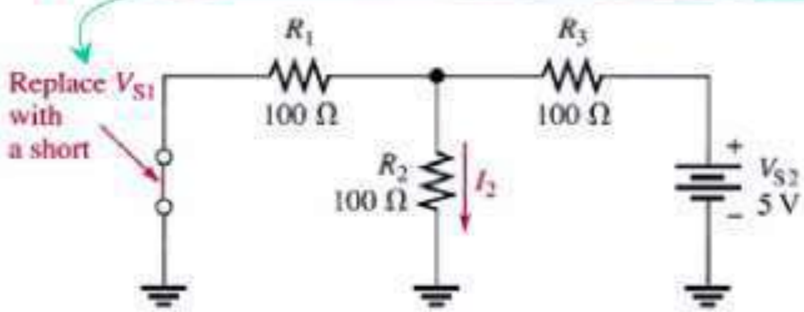
The current through R_2 due to V_{S1} is

$$I_{2(S1)} = \left(\frac{R_3}{R_2 + R_3}\right)I_{T(S1)} = \left(\frac{100\,\Omega}{200\,\Omega}\right)66.7\text{ mA} = 33.3\text{ mA}$$

Note that this current is downward through R_2.

Superposition Theorem





Step 2: Find the current through R_2 due to voltage source V_{S2} by replacing V_{S1} with a short, as shown in Figure 8–19. Looking from V_{S2},

$$R_{T(S2)} = R_3 + \frac{R_1}{2} = 100\ \Omega + 50\ \Omega = 150\ \Omega$$

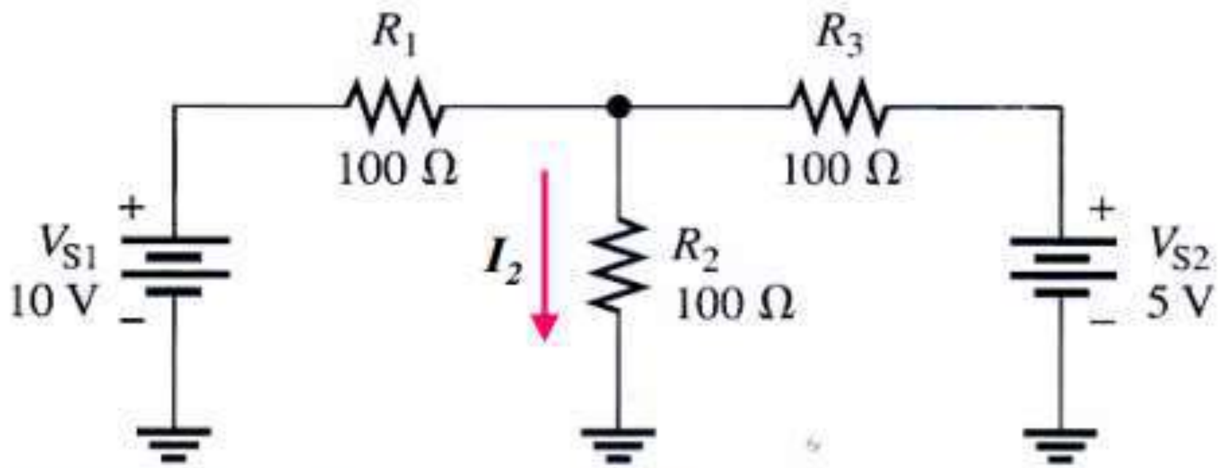
$$I_{T(S2)} = \frac{V_{S2}}{R_{T(S2)}} = \frac{5\ \text{V}}{150\ \Omega} = 33.3\ \text{mA}$$

The current through R_2 due to V_{S2} is

$$I_{2(S2)} = \left(\frac{R_1}{R_1 + R_2}\right) I_{T(S2)} = \left(\frac{100\ \Omega}{200\ \Omega}\right) 33.3\ \text{mA} = 16.7\ \text{mA}$$

Note that this current is downward through R_2.

Superposition Theorem

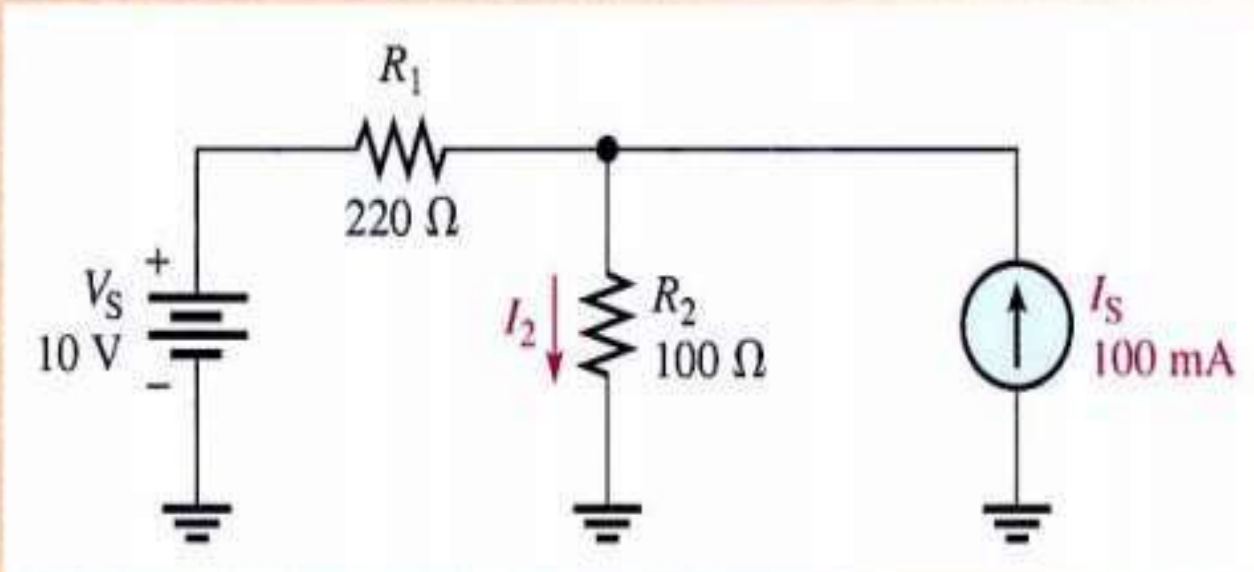


Step 3: Both component currents are downward through R_2, so they have the same algebraic sign. Therefore, add the values to get the total current through R_2.

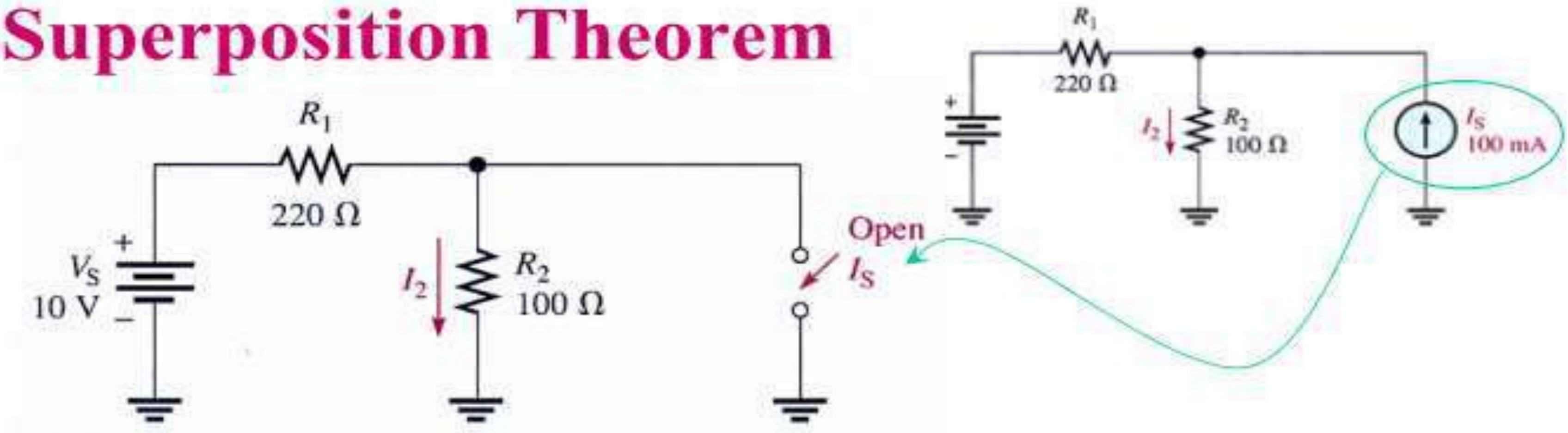
$$I_{2(tot)} = I_{2(S1)} + I_{2(S2)} = 33.3\,\text{mA} + 16.7\text{mA} = \mathbf{50\ mA}$$

Superposition Theorem

EXAMPLE Find the current through R_2 in the circuit.



Superposition Theorem



Solution

Step 1: Find the current through R_2 due to V_S by replacing I_S with an open.

Notice that all of the current produced by V_S is through R_2. Looking from V_S,

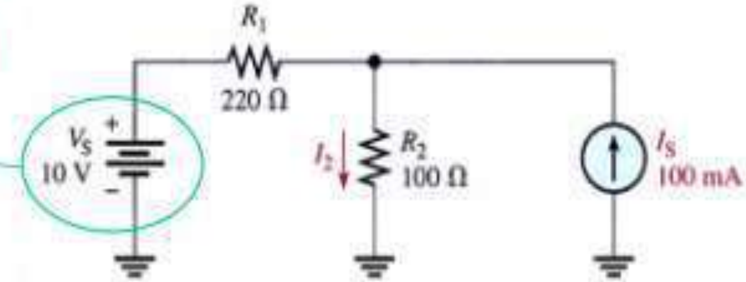
$$R_T = R_1 + R_2 = 320\ \Omega$$

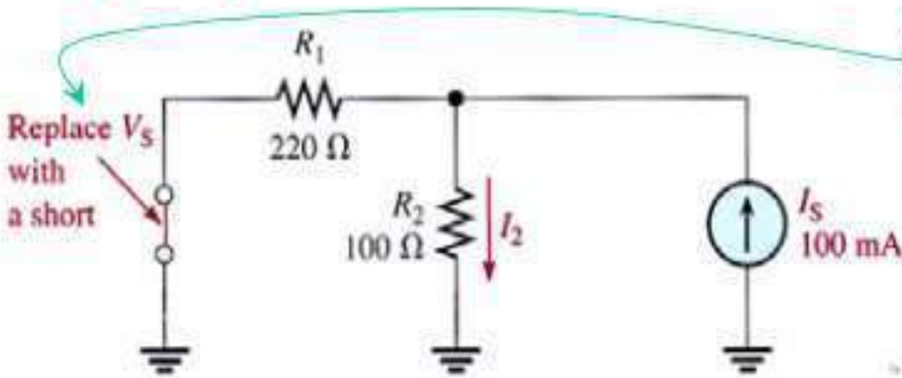
The current through R_2 due to V_S is

$$I_{2(V_S)} = \frac{V_S}{R_T} = \frac{10\text{ V}}{320\ \Omega} = 31.2\text{ mA}$$

Note that this current is downward through R_2.

Superposition Theorem



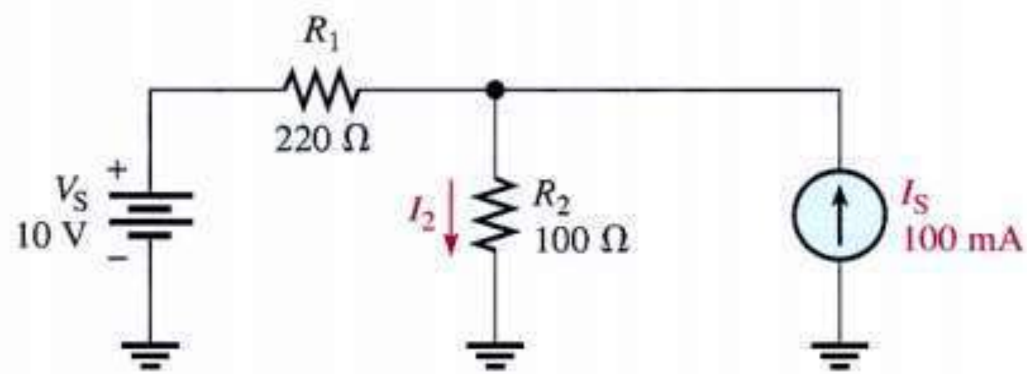


Step 2: Find the current through R_2 due to I_S by replacing V_S with a short.
Use the current-divider formula to determine the current through R_2 due to I_S.

$$I_{2(I_S)} = \left(\frac{R_1}{R_1 + R_2}\right)I_S = \left(\frac{220\ \Omega}{320\ \Omega}\right)100\ \text{mA} = 68.8\ \text{mA}$$

Note that this current also is downward through R_2.

Superposition Theorem



Step 3: Both currents are in the same direction through R_2, so add them to get the total.

$$I_{2(\text{tot})} = I_{2(V_S)} + I_{2(I_S)} = 31.2\,\text{mA} + 68.8\,\text{mA} = \mathbf{100\,mA}$$

UNIT 1: DC CIRCUITS

Prepared By:
Krishan Arora
Assistant Professor and Head

Thevenin’s Theorem

- Thevenin’s theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

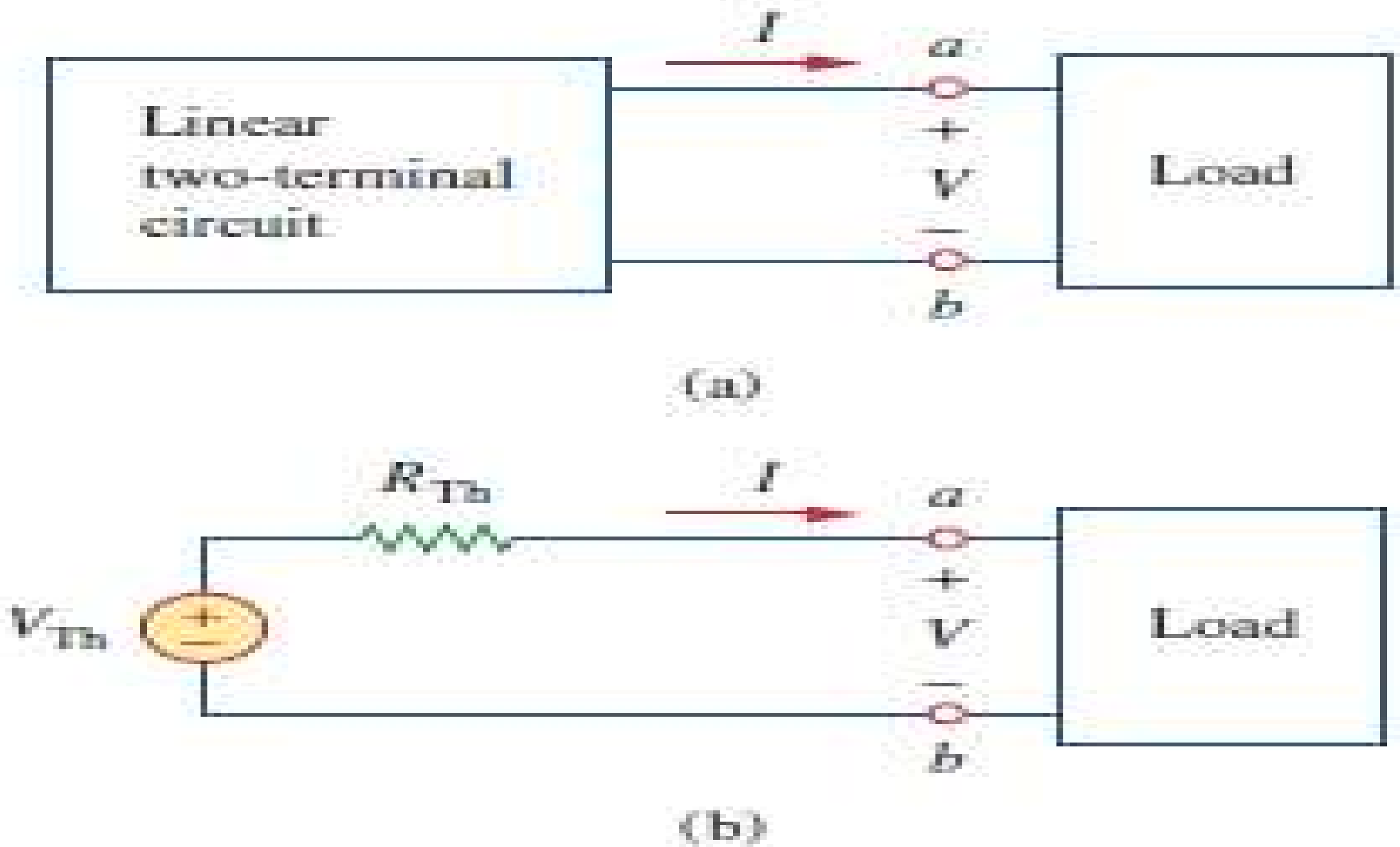
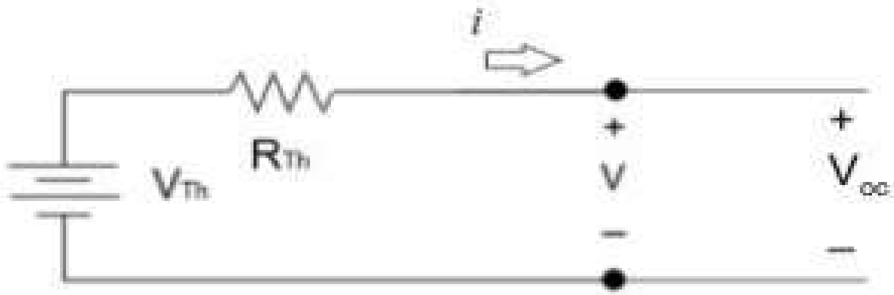


Figure 4.23
Replacing a linear two-terminal circuit by its Thevenin equivalent: (a) original circuit, (b) the Thevenin equivalent circuit.

Definitions for Thévenin's Theorem

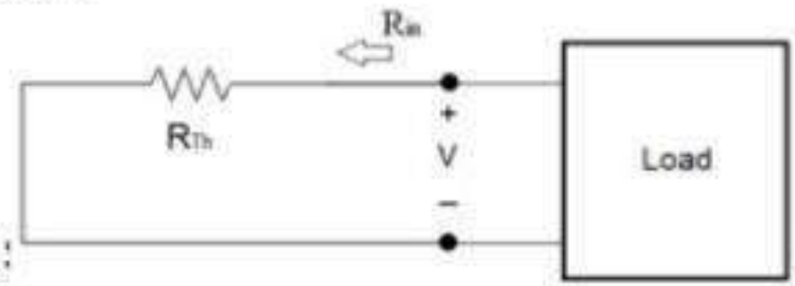


Open-circuit voltage V_{oc} is the voltage, V, when the load is an open circuit (i.e., $R_L = \infty\Omega$).

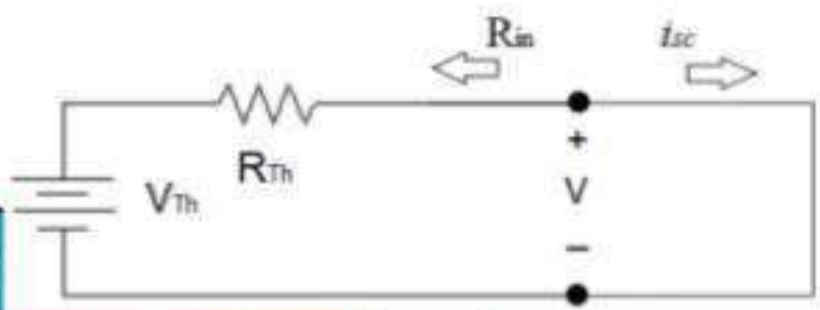
$$V_{OC} = V_{Th}$$

Definitions for Thévenin's Theorem

- Input resistance is the resistance seen by the load when $V_{Th} = 0V$.



- It is also the resi: when the load is a short circuit ($R_L = 0\Omega$).



$$R_{in} = R_{Th} = V_{Th}/i_{SC}$$

Summary of Thevenin's Theorem

Step 1. Open the two terminals (remove any load) between which you want to find the Thevenin equivalent circuit.

Step 2. Determine the voltage (V_{TH}) across the two open terminals.

Step 3. Determine the resistance (R_{TH}) between the two open terminals with all sources replaced with their internal resistances (ideal voltage sources shorted and ideal current sources opened).

Step 4. Connect V_{TH} and R_{TH} in series to produce the complete Thevenin equivalent for the original circuit.

Step 5. Replace the load removed in Step 1 across the terminals of the Thevenin equivalent circuit. You can now calculate the load current and load voltage using only Ohm's law. They have the same value as the load current and load voltage in the original circuit.

QUICK QUIZ (Poll 1)

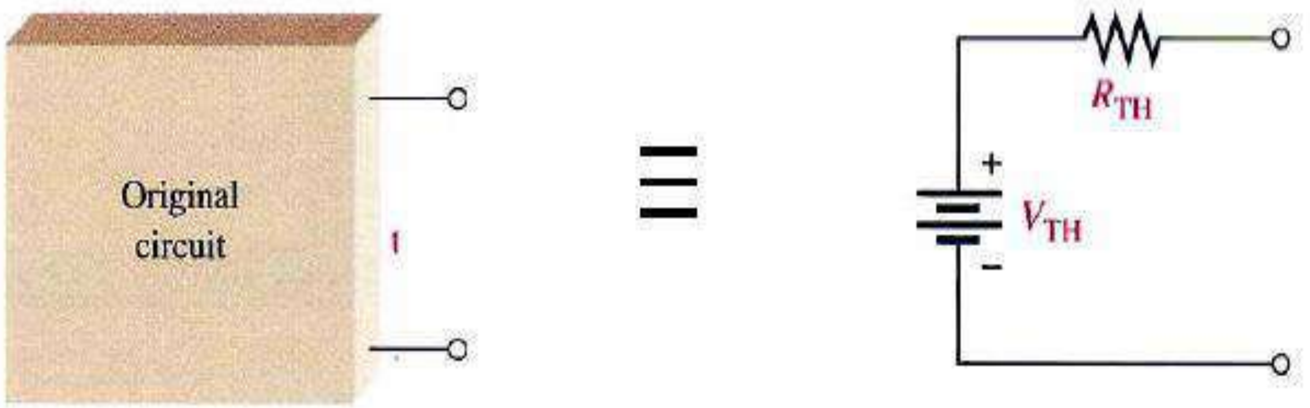
- The Thevenin voltage is the___________
  a) Open circuit voltage
  b) Short circuit voltage
  c) Open circuit and short circuit voltage
  d) Neither open circuit nor short circuit voltage

QUICK QUIZ (Poll 2)

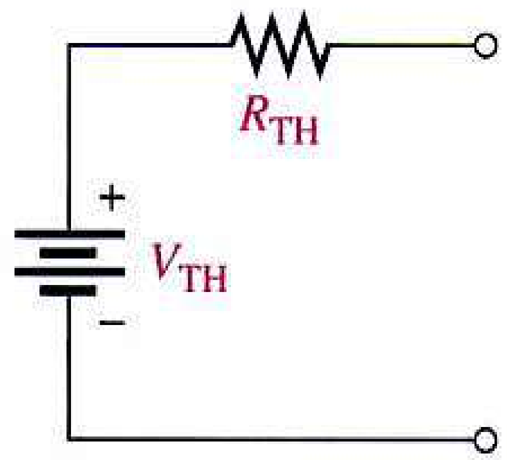
- Thevenin’s theorem is true for ___________
  a) Linear networks
  b) Non-Linear networks
  c) Both linear networks and nonlinear networks
  d) Neither linear networks nor non-linear networks

Thevenin's Theorem

The Thevenin equivalent form of any two-terminal resistive circuit consists of an equivalent voltage source (V_{TH}) and an equivalent resistance (R_{TH}).



Thevenin's Theorem

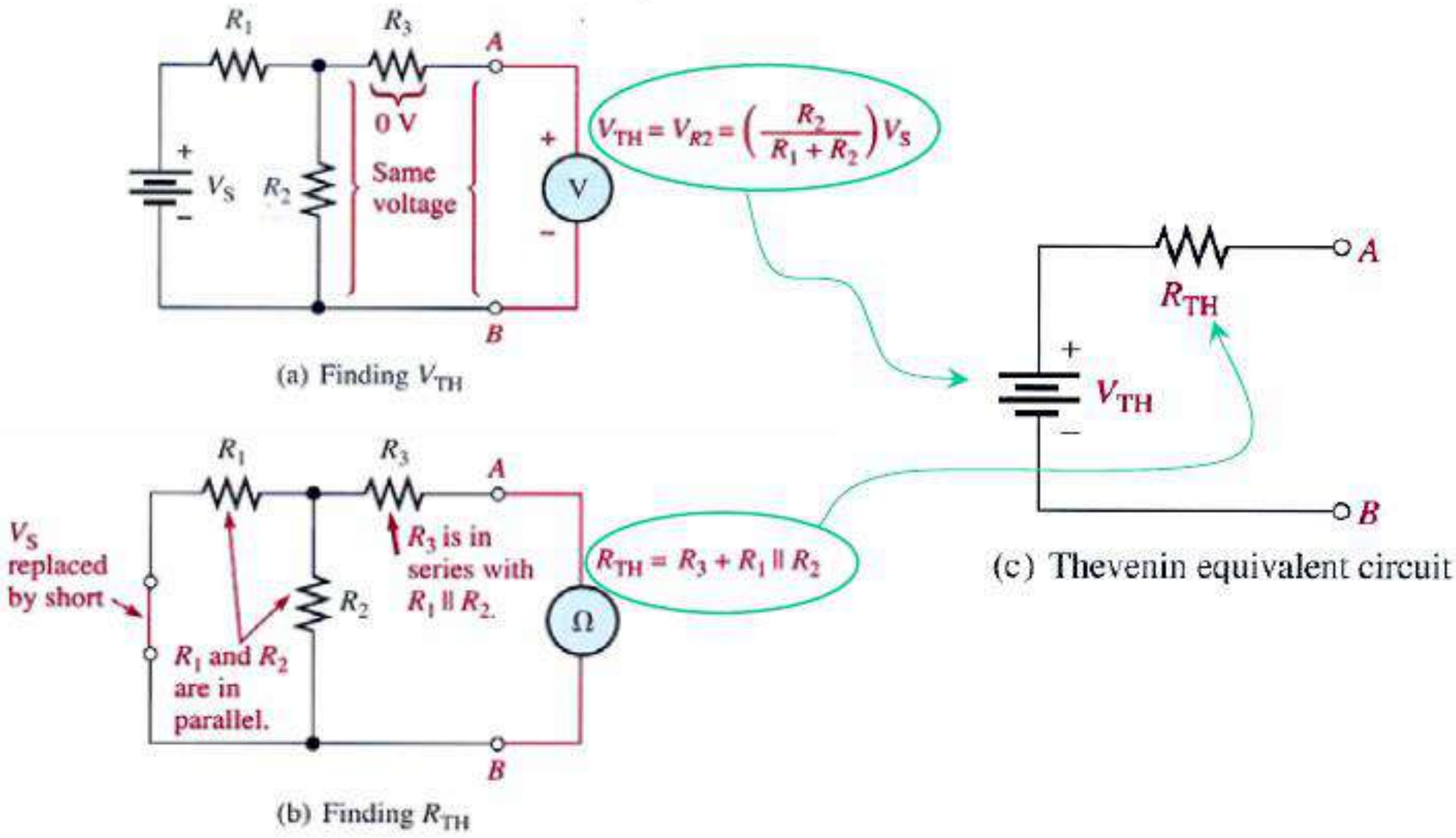


The Thevenin equivalent voltage (V_{TH}) is the open circuit (no-load) voltage between two output terminals in a circuit.

The Thevenin equivalent resistance (R_{TH}) is the total resistance appearing between two terminals in a given circuit with all sources replaced by their internal resistances.

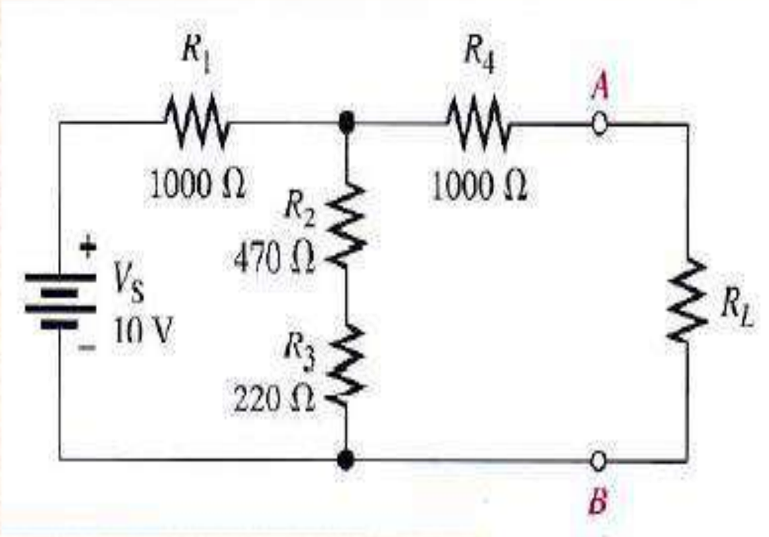
Thevenin's Theorem

Example of the simplification of a circuit by Thevenin's theorem.

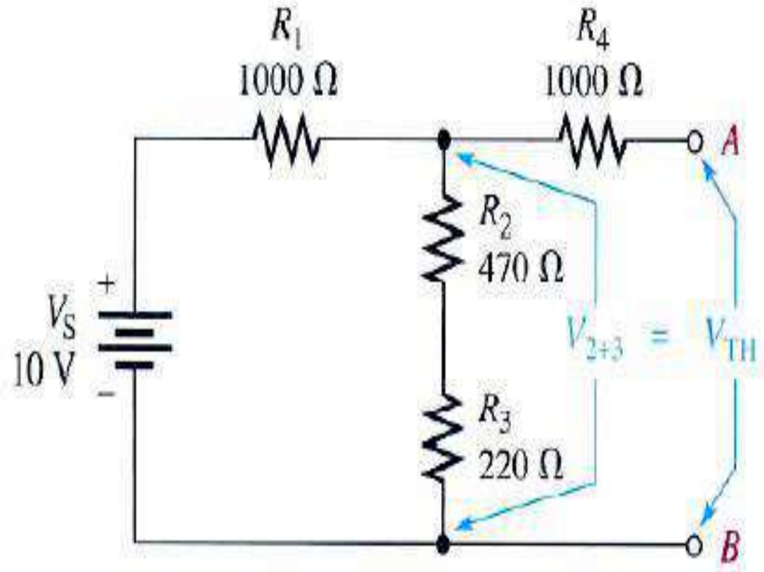


Thevenin's Theorem

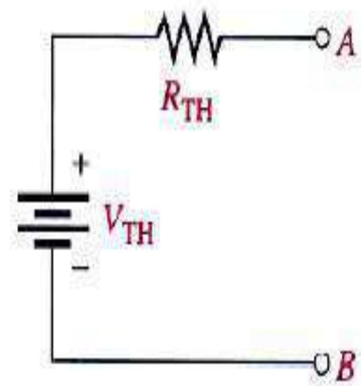
EXAMPLE Find the Thevenin equivalent circuit between *A* and *B* of the circuit.



Thevenin's Theorem



The voltage from *A* to *B* is V_{TH} and equals V_{2+3}.

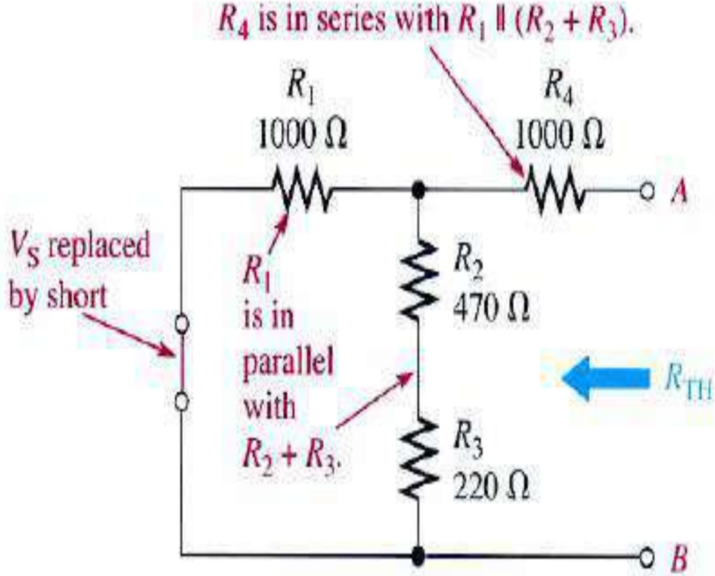


(c) Thevenin equivalent circuit

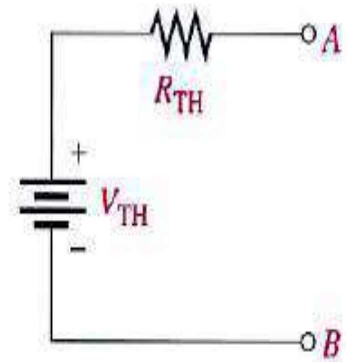
Solution First, remove R_L. Then V_{TH} equals the voltage across $R_2 + R_3$, because $V_4 = 0$ V since there is no current through it.

$$V_{TH} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3}\right)V_S = \left(\frac{690\ \Omega}{1690\ \Omega}\right)10\text{ V} = \mathbf{4.08\ V}$$

Thevenin's Theorem



Looking from terminals A and B, R_4 appears in series with the combination of R_1 in parallel with $(R_2 + R_3)$.

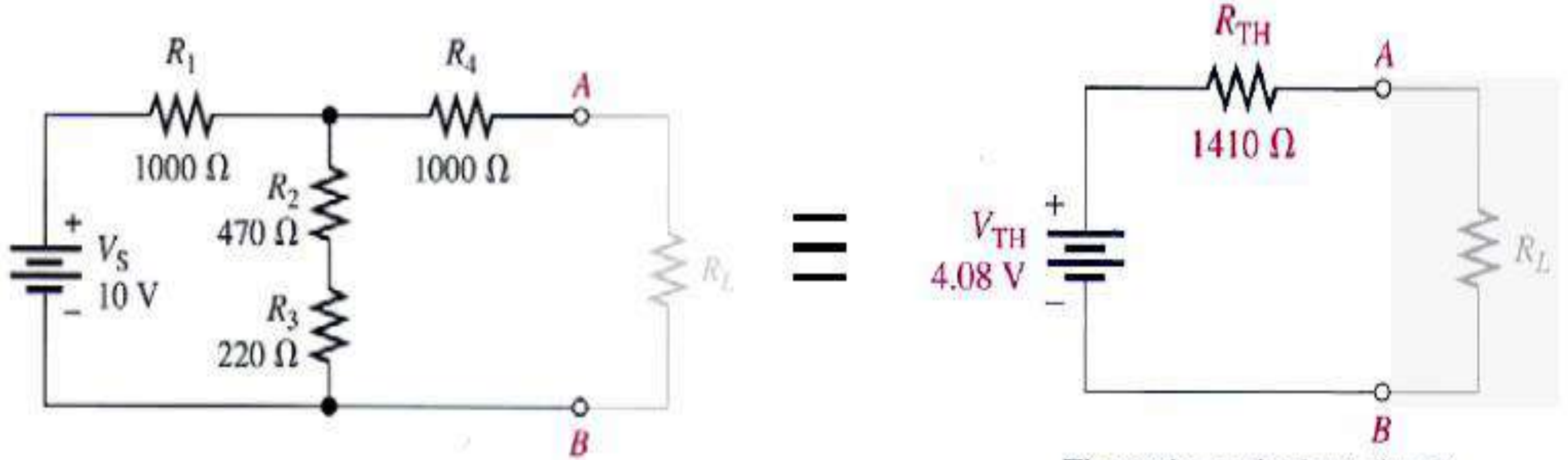


(c) Thevenin equivalent circuit

To find R_{TH}, first replace the source with a short to simulate a zero internal resistance. Then R_1 appears in parallel with $R_2 + R_3$, and R_4 is in series with the series-parallel combination of R_1, R_2, and R_3,

$$R_{TH} = R_4 + \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} = 1000\ \Omega + \frac{(1000\ \Omega)(690\ \Omega)}{1690\ \Omega} = \mathbf{1410\ \Omega}$$

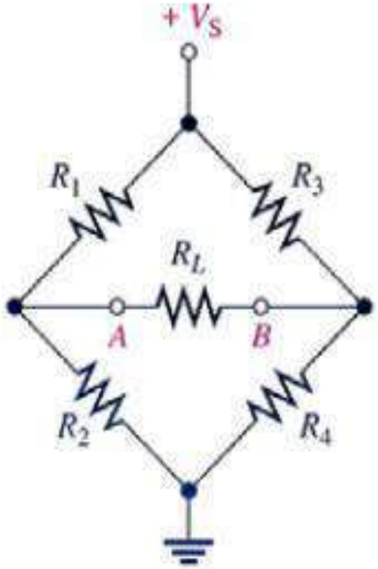
Thevenin's Theorem



Thevenin equivalent circuit

Thevenin's Theorem

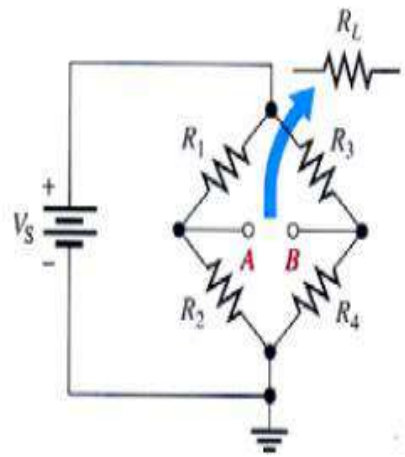
Thevenizing a Bridge Circuit



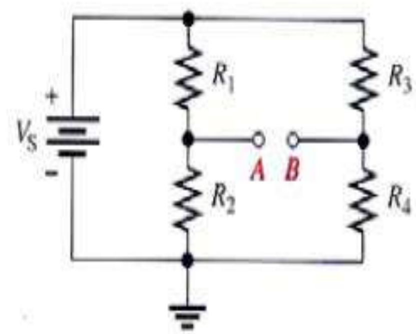
Using Thevenin's theorem, you can simplify the bridge circuit to an equivalent circuit viewed from the load resistor.

Thevenin's Theorem

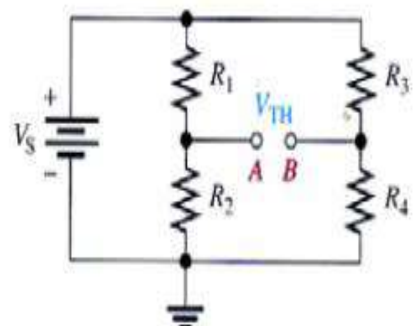
Thevenizing a Bridge Circuit



(a) Remove R_L.



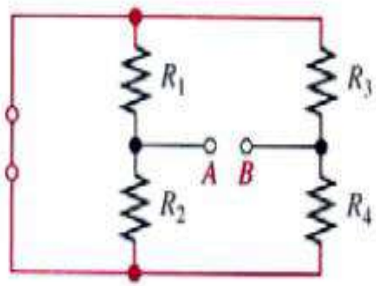
(b) Redraw to find V_{TH}.



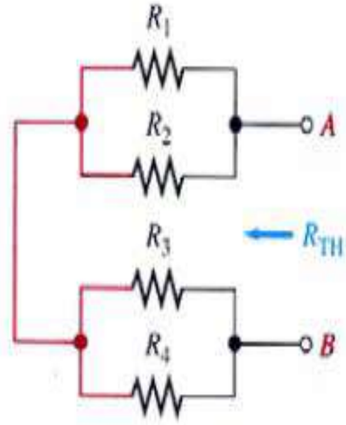
(c) $V_{TH} = V_A - V_B = \left(\dfrac{R_2}{R_1 + R_2}\right)V_S - \left(\dfrac{R_4}{R_3 + R_4}\right)V_S$

Thevenin's Theorem

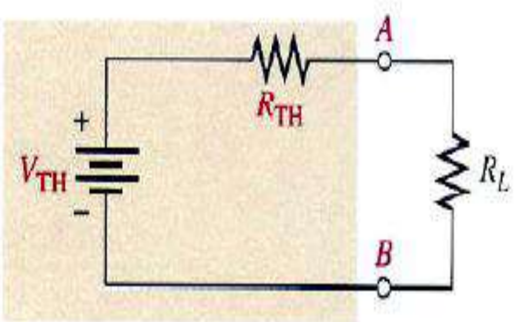
Thevenizing a Bridge Circuit



(d) Replace V_S with a short.



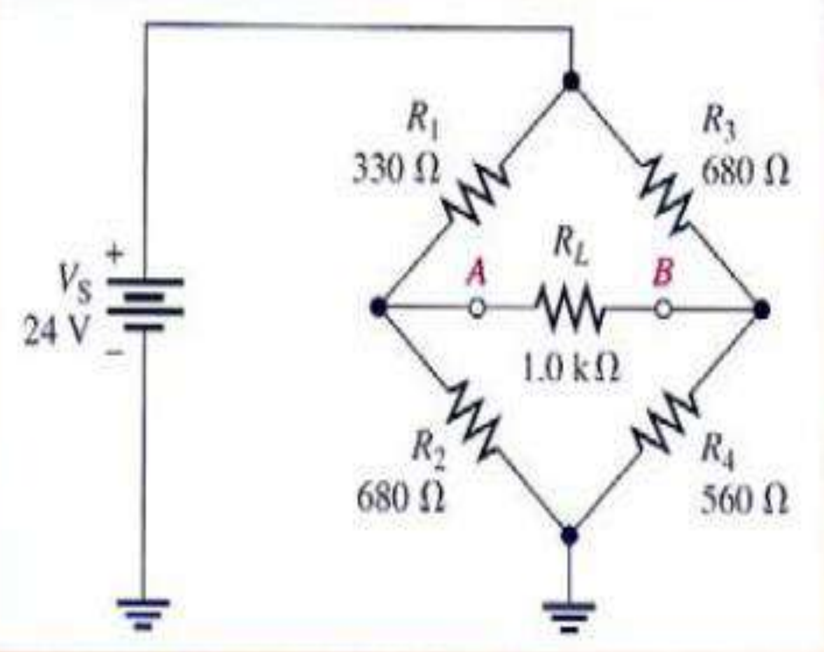
(e) Redraw to find R_{TH}:
$R_{TH} = R_1 \parallel R_2 + R_3 \parallel R_4$



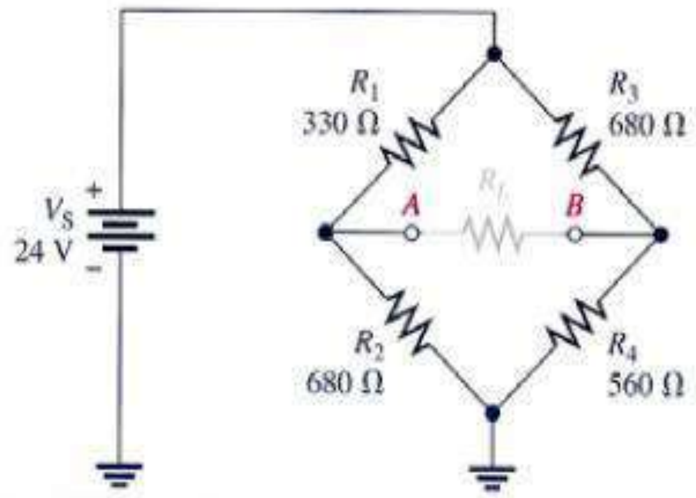
(f) Thevenin's equivalent
with R_L reconnected

Thevenizing a Bridge Circuit

EXAMPLE Determine the voltage and current for the load resistor, R_L, in the bridge



Thevenizing a Bridge Circuit



Solution

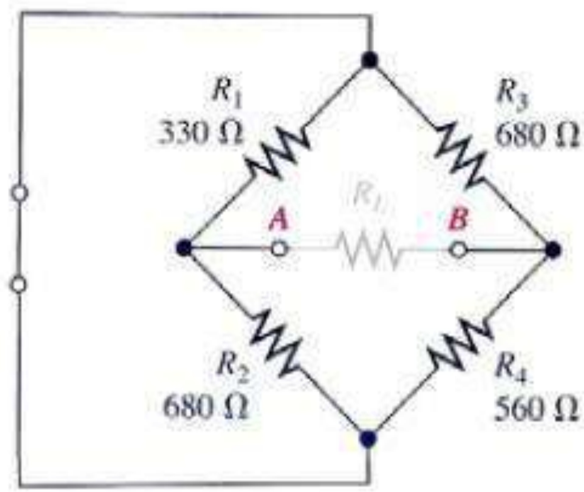
Step 1: Remove R_L.

Step 2: To thevenize the bridge as viewed from between terminals A and B, first determine V_{TH}.

$$V_{TH} = V_A - V_B = \left(\frac{R_2}{R_1 + R_2}\right)V_S - \left(\frac{R_4}{R_3 + R_4}\right)V_S$$

$$= \left(\frac{680\ \Omega}{1010\ \Omega}\right)24\text{ V} - \left(\frac{560\ \Omega}{1240\ \Omega}\right)24\text{ V} = 16.16\text{ V} - 10.84\text{ V} = 5.32\text{ V}$$

Thevenizing a Bridge Circuit

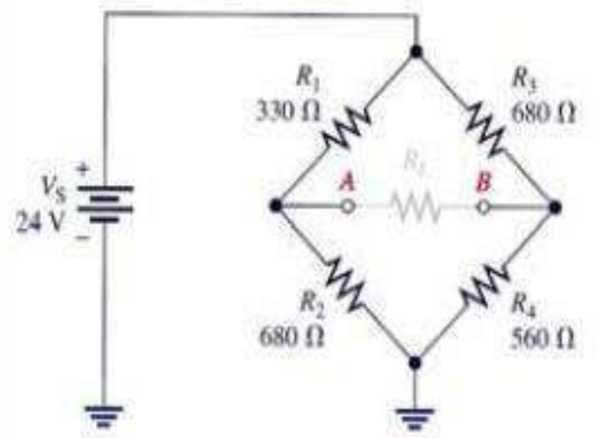


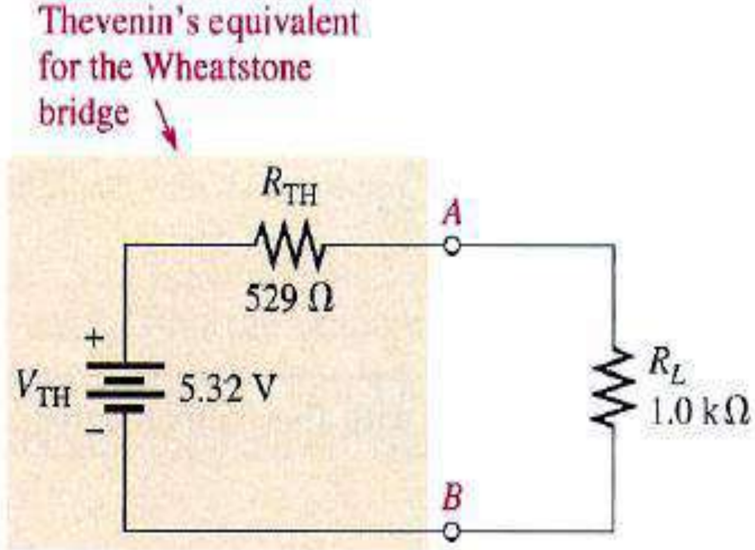
Step 3: Determine R_{TH}.

$$R_{TH} = \frac{R_1R_2}{R_1 + R_2} + \frac{R_3R_4}{R_3 + R_4}$$

$$= \frac{(330\ \Omega)(680\ \Omega)}{1010\ \Omega} + \frac{(680\ \Omega)(560\ \Omega)}{1240\ \Omega} = 222\ \Omega + 307\ \Omega = 529\ \Omega$$

Thevenizing a Bridge Circuit





Step 4: Place V_{TH} and R_{TH} in series to form the Thevenin equivalent circuit.

Step 5: Connect the load resistor between terminals A and B of the equivalent circuit, and determine the load voltage and current .

$$V_L = \left(\frac{R_L}{R_L + R_{TH}}\right)V_{TH} = \left(\frac{1.0\,\text{k}\Omega}{1.529\,\text{k}\Omega}\right)5.32\text{ V} = \mathbf{3.48\ V}$$

$$I_L = \frac{V_L}{R_L} = \frac{3.48\text{ V}}{1.0\,\text{k}\Omega} = \mathbf{3.48\ mA}$$

QUICK QUIZ (Poll 3)

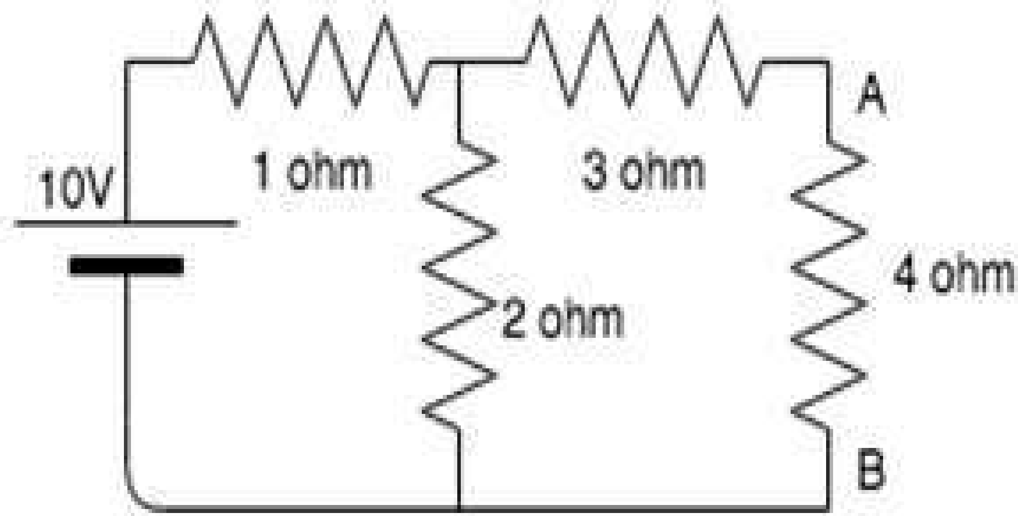
Thevenin resistance is found by

____

a) Shorting all voltage sources
b) Opening all current sources
c) Shorting all voltage sources and opening all current sources
d) Opening all voltage sources and shorting all current sources

QUICK QUIZ (Poll 4)

- Calculate the current across the 4 ohm resistor.



a) 0.86A
b) 1.23A
c) 2.22A
d) 0.67A

Answer: a

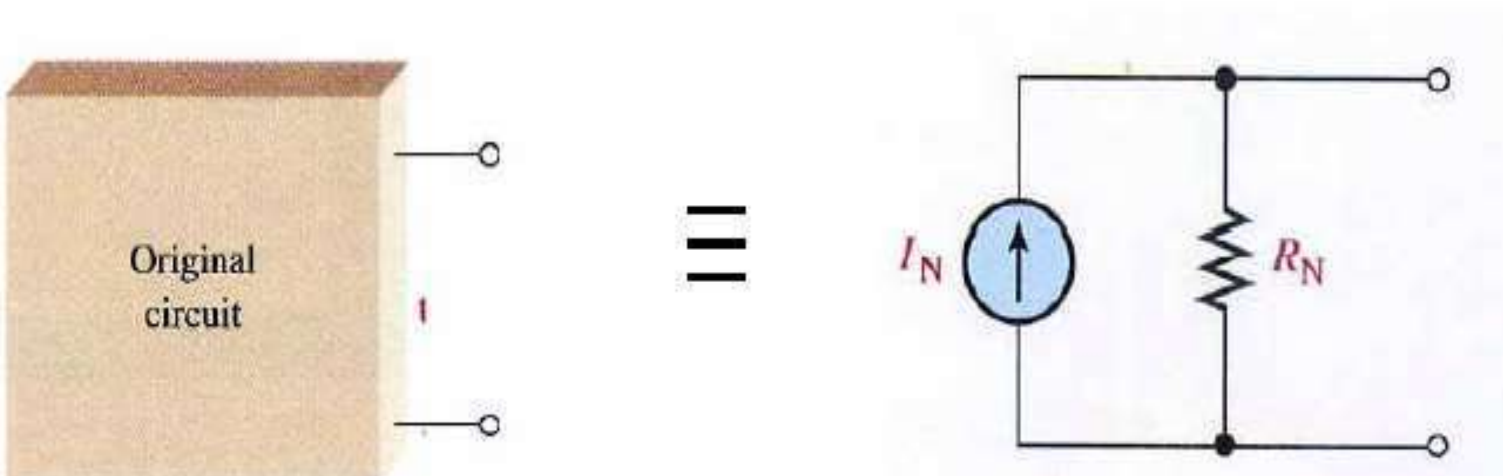
Explanation: Thevenin resistance is found by opening the circuit between the specified terminal and shorting all voltage sources.
When the 10V source is shorted, we get:
Rth=(1||2)+3=3.67 ohm.
Vth is calculated by opening the specified terminal.
Using voltage divider, Vth= 2*10/(2+1)=6.67V.
On drawing the Thevenin equivalent circuit, we get Rth, 4 ohm and Vth in series.
Applying Ohm’s law, I=Vth/(4+Rth) = 0.86A.

UNIT 1: DC CIRCUITS

Prepared By:
Krishan Arora
Assistant Professor and Head

Norton's Theorem

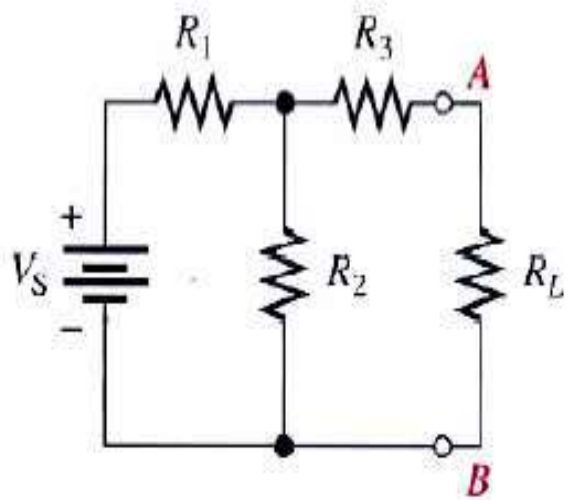
Like Thevenin's theorem, Norton's theorem provides a method of reducing a more complex circuit to a simpler equivalent form.



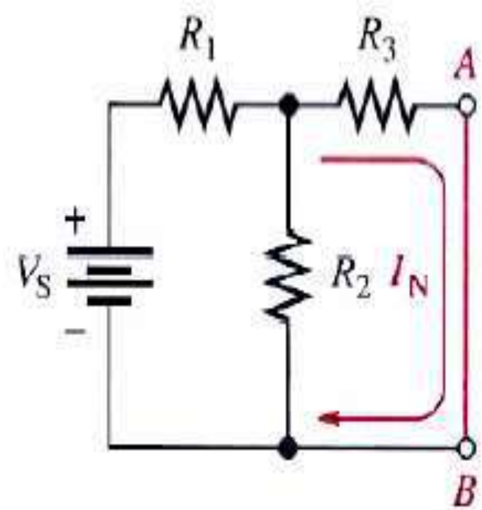
Norton's Theorem

Norton's Equivalent Current (I_N)

Norton's equivalent current (I_N) is the short-circuit current between two output terminals in a circuit.



(a) Original circuit



(b) Short the terminals to get I_N.

QUICK QUIZ (Poll 1)

- The Norton current is the________
  a) Short circuit current
  b) Open circuit current
  c) Open circuit and short circuit current
  d) Neither open circuit nor short circuit current

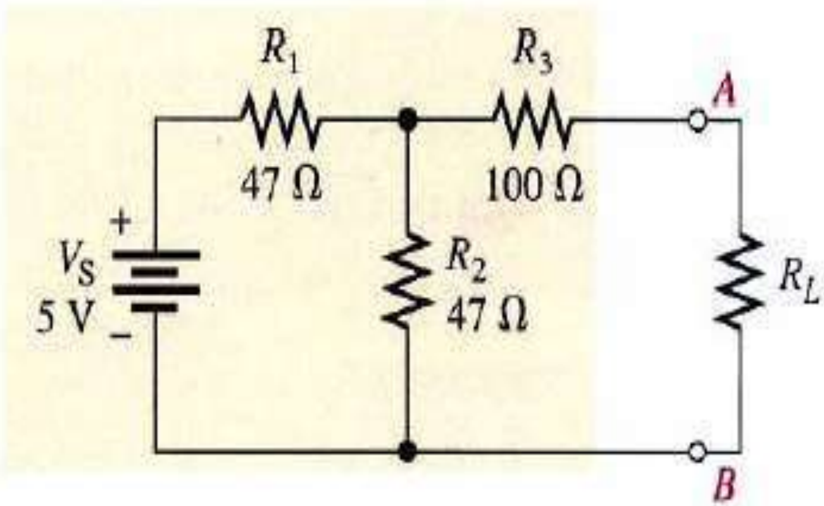
QUICK QUIZ (Poll 2)

- Norton resistance is found by?
  a) Shorting all voltage sources
  b) Opening all current sources
  c) Shorting all voltage sources and opening all current sources
  d) Opening all voltage sources and shorting all current sources

Norton's Theorem

Norton's Equivalent Current (I_N)

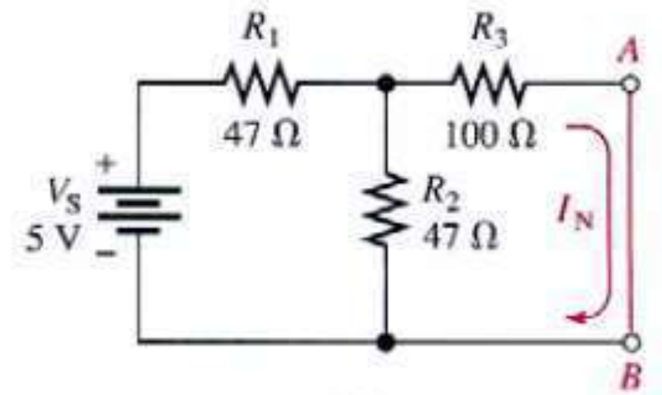
EXAMPLE Determine I_N for the circuit within the beige area.



(a)

Norton's Theorem

Norton's Equivalent Current (I_N)



(b)

Solution

Short terminals *A* and *B*. I_N is the current through the short. First, the total resistance seen by the voltage source is

$$R_T = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 47\ \Omega + \frac{(47\ \Omega)(100\ \Omega)}{147\ \Omega} = 79\ \Omega$$

The total current from the source is

$$I_T = \frac{V_S}{R_T} = \frac{5\ \text{V}}{79\ \Omega} = 63.3\ \text{mA}$$

Now apply the current-divider formula to find I_N (the current through the short).

$$I_N = \left(\frac{R_2}{R_2 + R_3}\right) I_T = \left(\frac{47\ \Omega}{147\ \Omega}\right) 63.3\ \text{mA} = \mathbf{20.2\ mA}$$

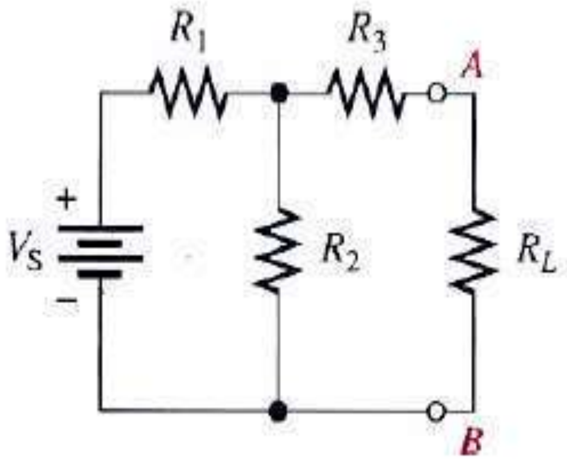
This is the value for the equivalent Norton current source.

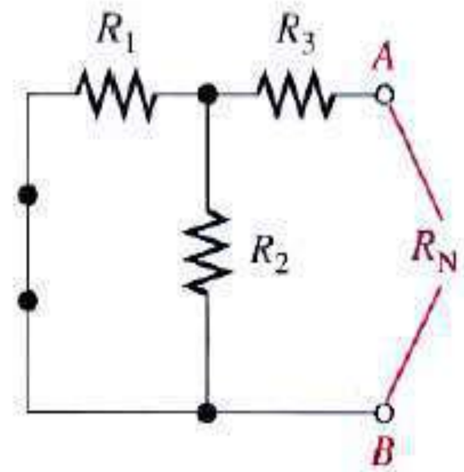
Norton's Theorem

Norton's Equivalent Resistance (R_N)

Norton's equivalent resistance (R_N) is defined in the same way as R_{TH}.

The Norton equivalent resistance, R_N, is the total resistance appearing between two output terminals in a given circuit with all sources replaced by their internal resistances.

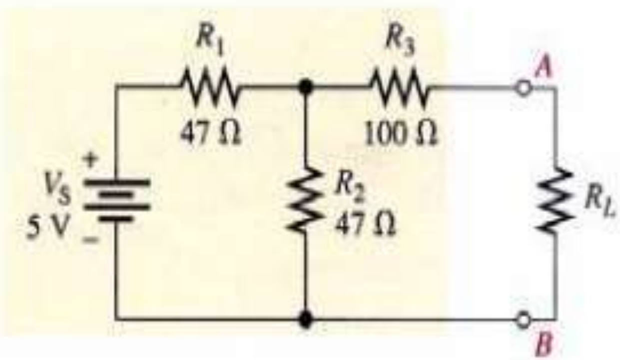


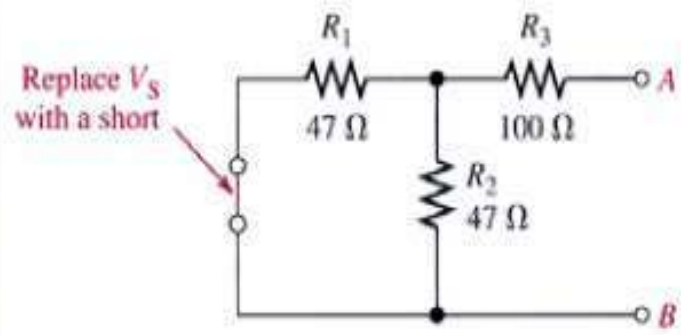


Norton's Theorem

Norton's Equivalent Resistance (R_N)

EXAMPLE Find R_N for the circuit within the beige area





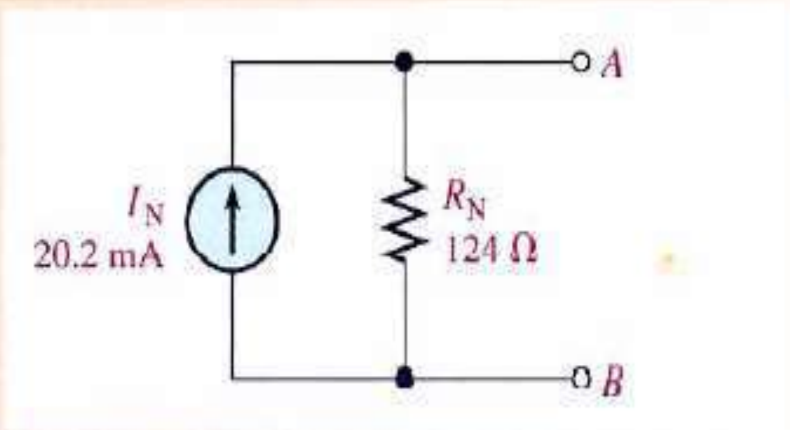
Solution

$$R_N = R_3 + \frac{R_1}{2} = 100\ \Omega + \frac{47\ \Omega}{2} = \mathbf{124\ \Omega}$$

Norton's Theorem

EXAMPLE Draw the complete Norton equivalent circuit for the original circuit that $I_N = 20.2$ mA and $R_N = 124\ \Omega$.

Solution



Norton's Theorem

Summary of Norton's Theorem

Step 1. Short the two terminals between which you want to find the Norton equivalent circuit.

Step 2. Determine the current (I_N) through the shorted terminals.

Step 3. Determine the resistance (R_N) between the two open terminals with all sources replaced with their internal resistances (ideal voltage sources shorted and ideal current sources opened). $R_N = R_{TH}$.

Step 4. Connect I_N and R_N in parallel to produce the complete Norton equivalent for the original circuit.

QUICK QUIZ (Poll 3)

- Norton's theorem is true for ___________
  a) Linear networks
  b) Non-Linear networks
  c) Both linear networks and nonlinear networks
  d) Neither linear networks nor non-linear networks

QUICK QUIZ (Poll 4)

Short circuit current Isc is found across the _______________ terminals of the network.

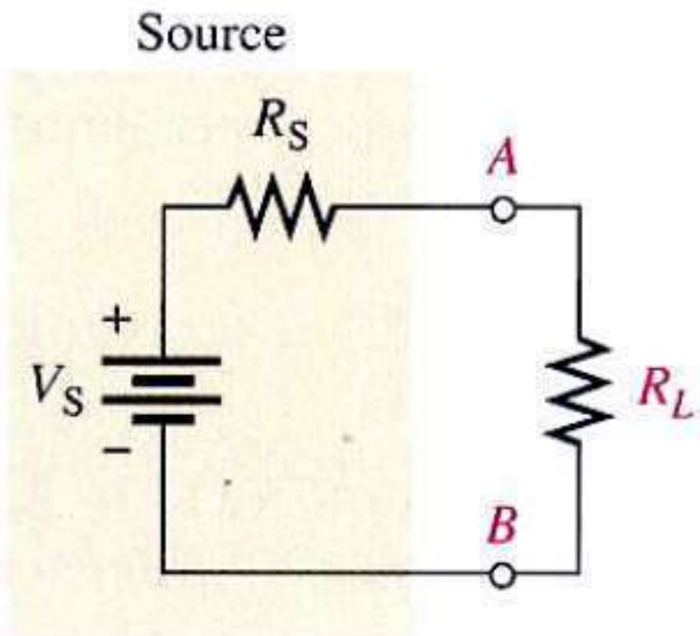
a) Input
b) Output
c) Neither input nor output
d) Either input or output

Maximum Power Transfer Theorem

The maximum power transfer theorem is important when you need to know the value of the load at which the most power is delivered from the source.

The **maximum power transfer** theorem is stated as follows:

For a given source voltage, maximum power is transferred from a source to a load when the load resistance is equal to the internal source resistance.



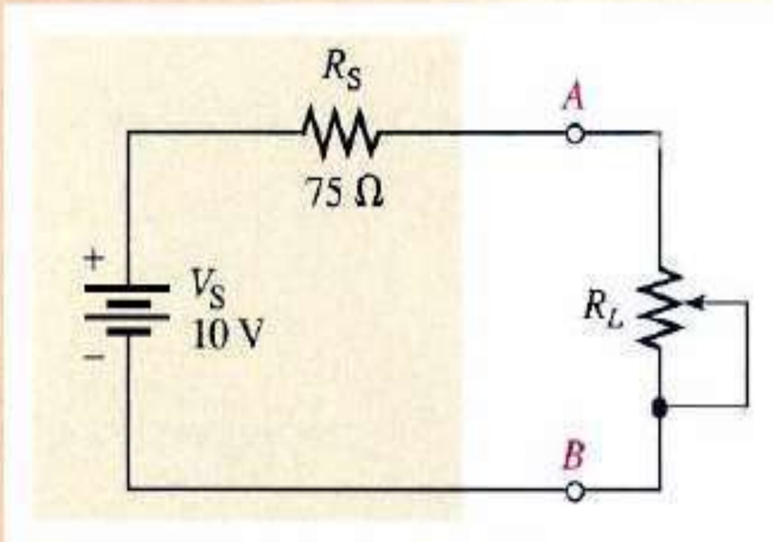
Maximum power is transferred to the load when $R_L = R_S$.

Maximum Power Transfer Theorem

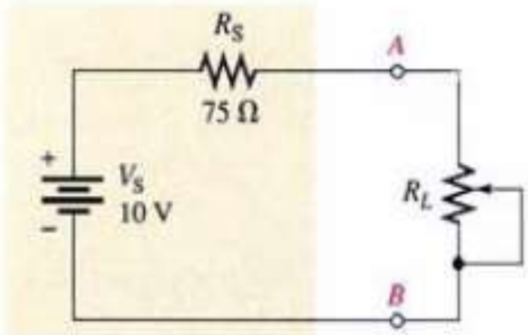
EXAMPLE The source has an internal source resistance of 75 Ω. Determine the load power for each of the following values of load resistance:

(a) 0 Ω **(b)** 25 Ω **(c)** 50 Ω **(d)** 75 Ω **(e)** 100 Ω **(f)** 125 Ω

Draw a graph showing the load power versus the load resistance.



Maximum Power Transfer Theorem



Solution

Use Ohm's law ($I = V/R$) and the power formula ($P = I^2R$) to find the load power, P_L, for each value of load resistance.

(a) For $R_L = 0\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\text{ V}}{75\ \Omega + 0\ \Omega} = 133\text{ mA}$$

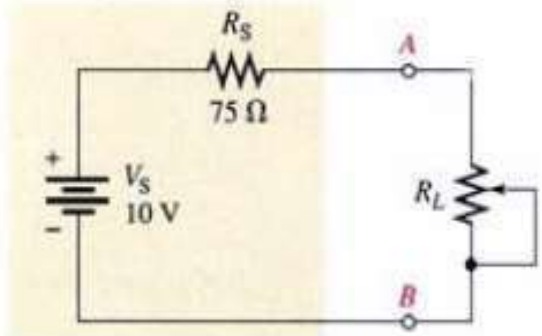
$$P_L = I^2R_L = (133\text{ mA})^2(0\ \Omega) = \mathbf{0\ mW}$$

(b) For $R_L = 25\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\text{ V}}{75\ \Omega + 25\ \Omega} = 100\text{ mA}$$

$$P_L = I^2R_L = (100\text{ mA})^2(25\ \Omega) = \mathbf{250\ mW}$$

Maximum Power Transfer Theorem



(c) For $R_L = 50\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\text{ V}}{125\ \Omega} = 80\text{ mA}$$

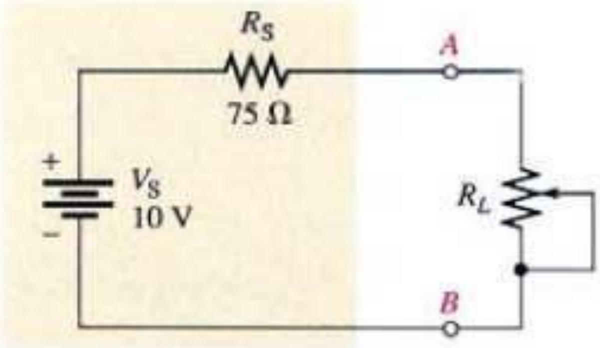
$$P_L = I^2R_L = (80\text{ mA})^2(50\ \Omega) = \mathbf{320\text{ mW}}$$

(d) For $R_L = 75\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\text{ V}}{150\ \Omega} = 66.7\text{ mA}$$

$$P_L = I^2R_L = (66.7\text{ mA})^2(75\ \Omega) = \mathbf{334\text{ mW}}$$

Maximum Power Transfer Theorem



(e) For $R_L = 100\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\text{ V}}{175\ \Omega} = 57.1\text{ mA}$$

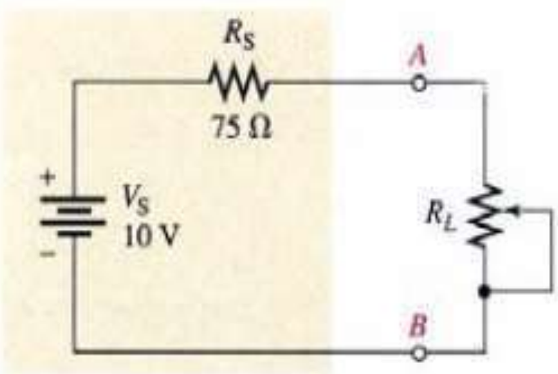
$$P_L = I^2 R_L = (57.1\text{ mA})^2(100\ \Omega) = \mathbf{326\ mW}$$

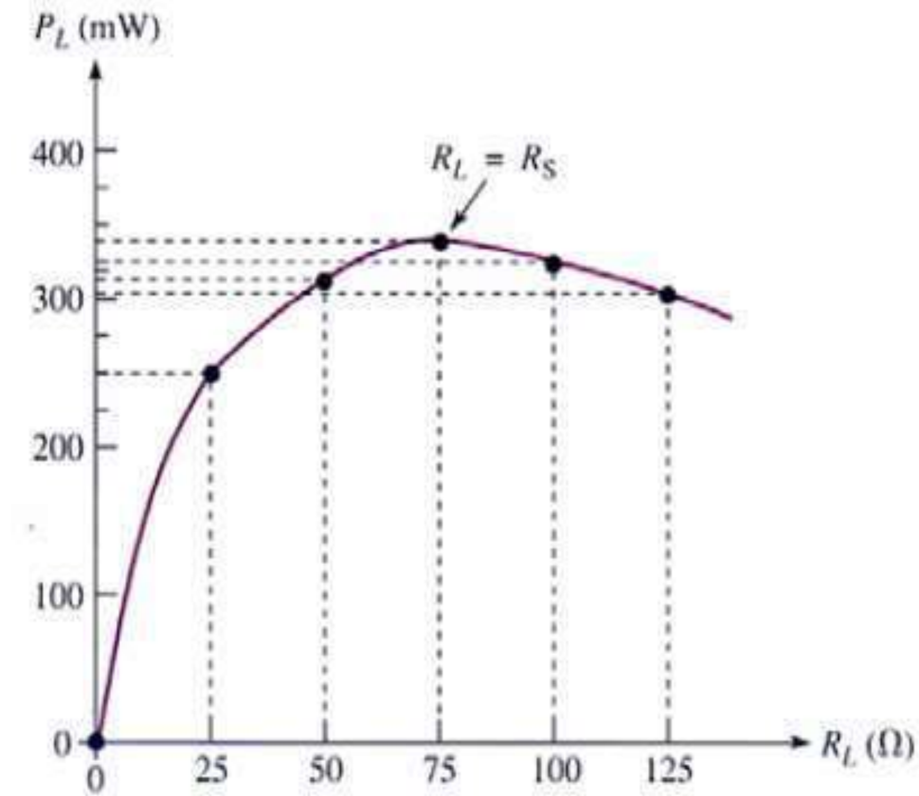
(f) For $R_L = 125\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\text{ V}}{200\ \Omega} = 50\text{ mA}$$

$$P_L = I^2 R_L = (50\text{ mA})^2(125\ \Omega) = \mathbf{313\ mW}$$

Maximum Power Transfer Theorem





The load power is greatest when $R_L = 75\ \Omega$, which is the same as the internal source resistance.

QUICK QUIZ (Poll 5)

- For maximum transfer of power, internal resistance of the source should be

A. Equal to the load resistance

B. Less than the load resistance

C. Greater than the load resistance

D. None of the above

Tutorial 1

Question 1

Example 1.3. *A coil consists of 2000 turns of copper wire having a cross-sectional area of 0.8 mm^2. The mean length per turn is 80 cm and the resistivity of copper is 0.02 μΩ–m. Find the resistance of the coil and power absorbed by the coil when connected across 110 V d.c. supply.*

Solution

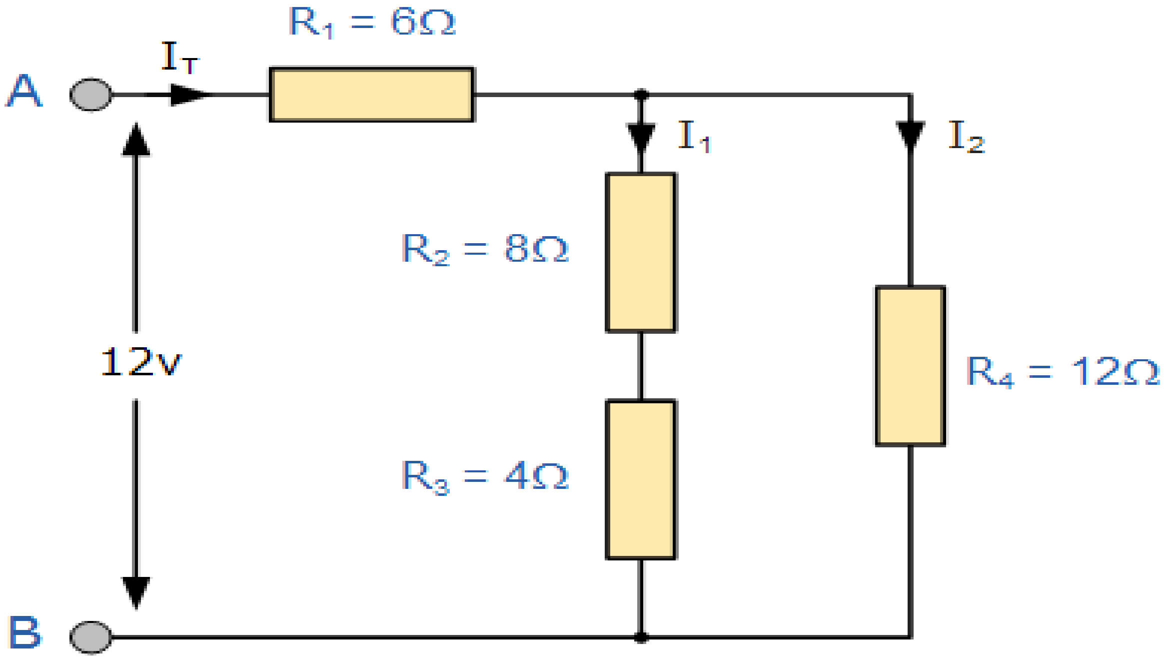
Solution. Length of the coil, $l = 0.8 \times 2000 = 1600$ m ;
$A = 0.8\ \text{mm}^2 = 0.8 \times 10^{-6}\ \text{m}^2$.

$$R = \rho \frac{l}{A} = 0.02 \times 10^{-6} \times 1600/0.8 \times 10^{-6} = \mathbf{40\ \Omega}$$

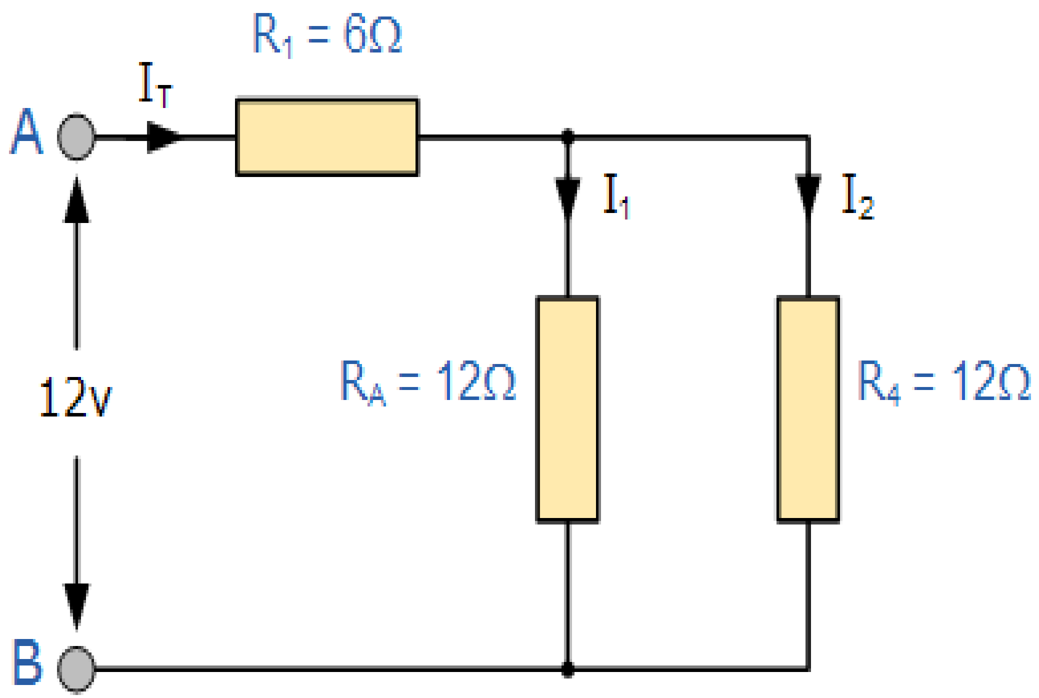
Power absorbed $= V^2 / R = 110^2/40 =$ **302.5 W**

Question 2

- Calculate the total current (I_T) taken from the 12v supply.



- At first glance this may seem a difficult task, but if we look a little closer we can see that the two resistors, R_2 and R_3 are actually both connected together in a “SERIES” combination so we can add them together to produce an equivalent resistance the same as we did in the series resistor tutorial. The resultant resistance for this combination would therefore be:
- $R_2 + R_3 = 8\Omega + 4\Omega = 12\Omega$
- So we can replace both resistor R_2 and R_3 above with a single resistor of resistance value 12Ω



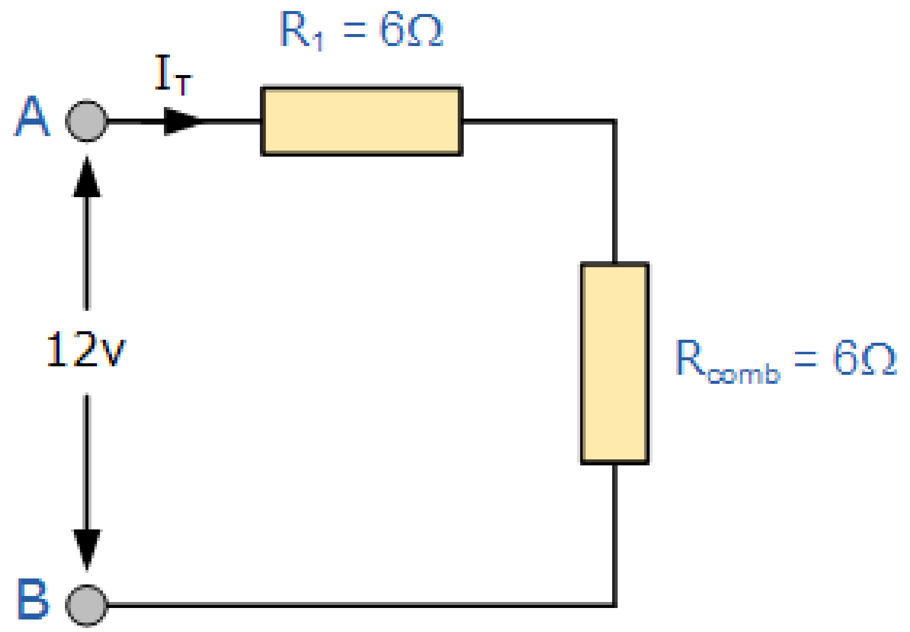
R1 = 6Ω
IT
A
I1
I2
RA = 12Ω
R4 = 12Ω
12v
B

So our circuit now has a single resistor R_A in "PARALLEL" with the resistor R_4. Using our resistors in parallel equation we can reduce this parallel combination to a single equivalent resistor value of $R_{(combination)}$ using the formula for two parallel connected resistors as follows.

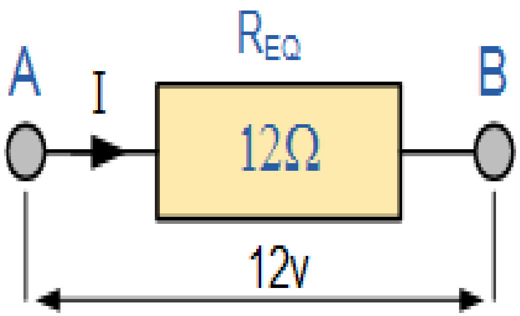
$$R_{(eq)} = \frac{1}{R_A} + \frac{1}{R_4} = \frac{1}{12} + \frac{1}{12} = 0.1667$$

$$R_{(combination)} = \frac{1}{R_{(eq)}} = \frac{1}{0.1667} = 6\Omega$$

The resultant resistive circuit now looks something like this:



$$R_{(ab)} = R_{comb} + R_1 = 6\Omega + 6\Omega = 12\Omega$$



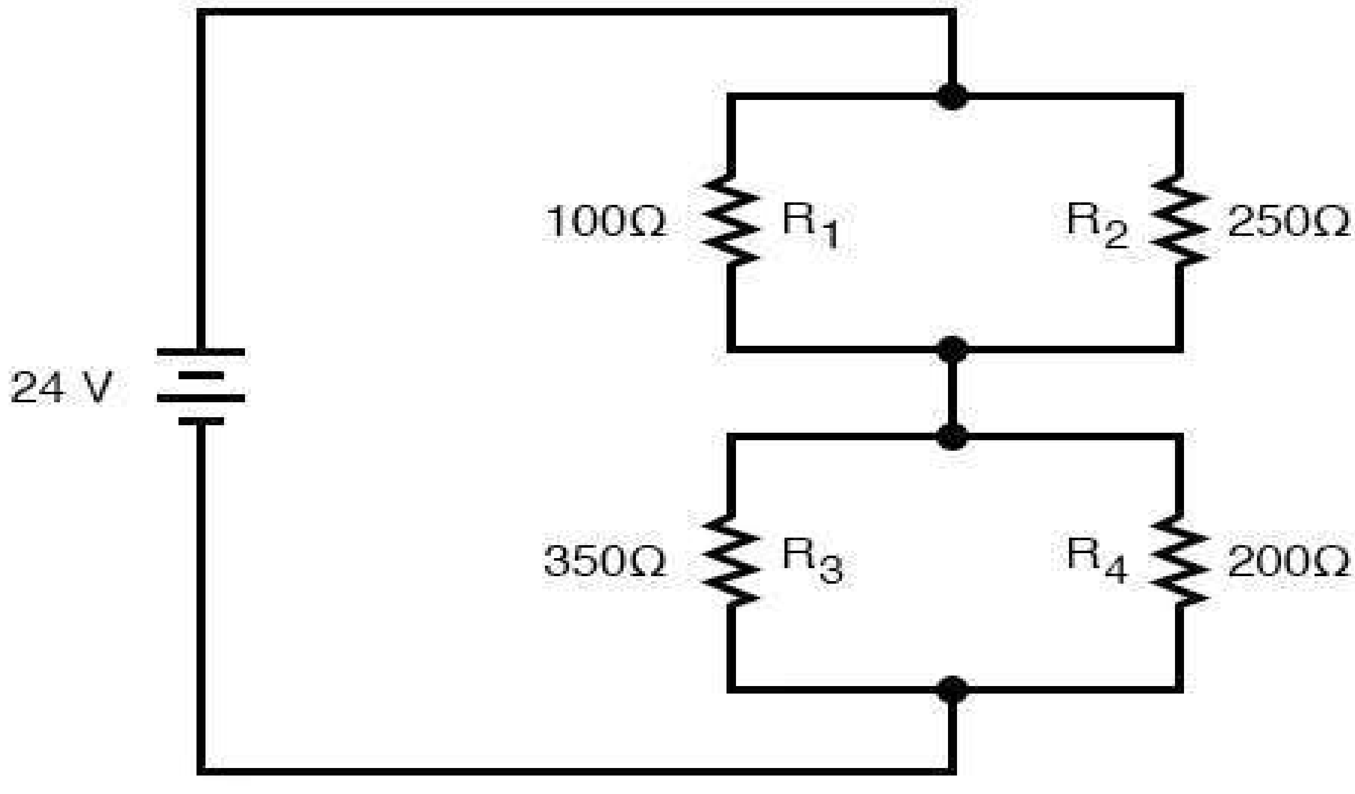
Thus a single resistor of just 12Ω can be used to replace the original four resistors connected together in the original circuit above.

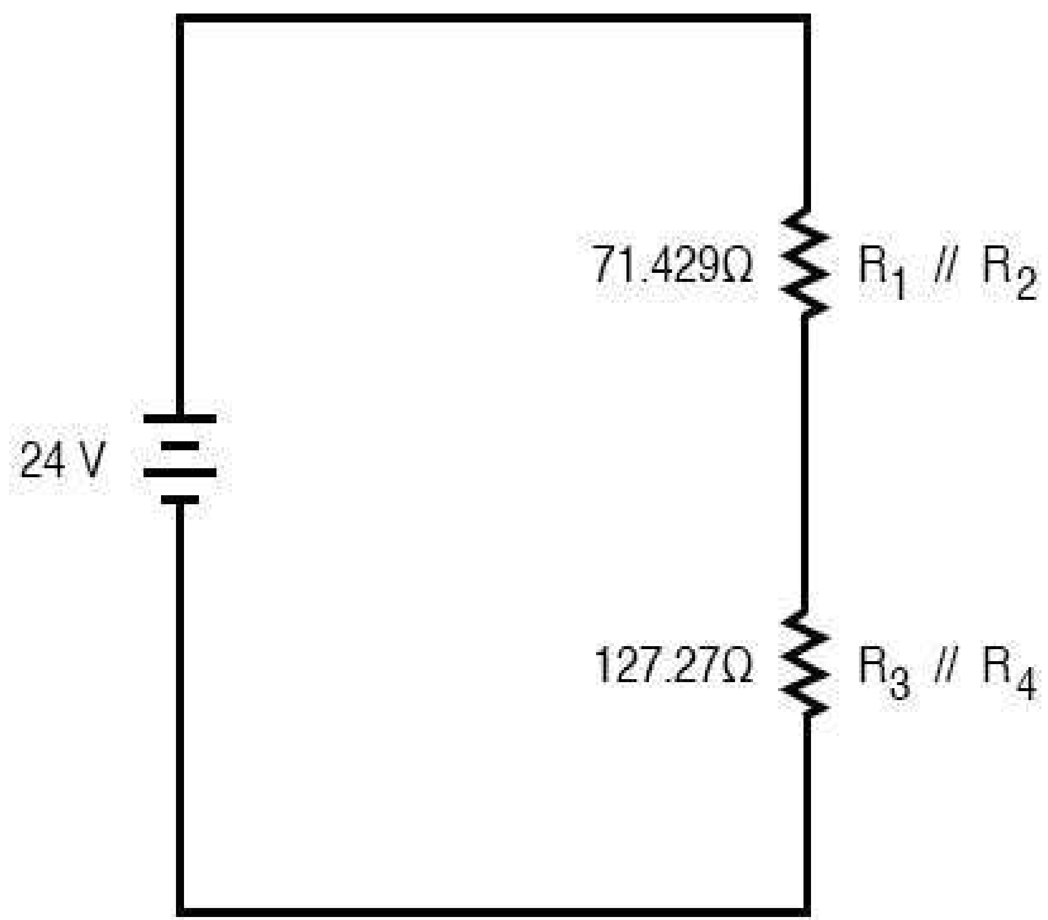
By using Ohm's Law, the value of the current (I) flowing around the circuit is calculated as:

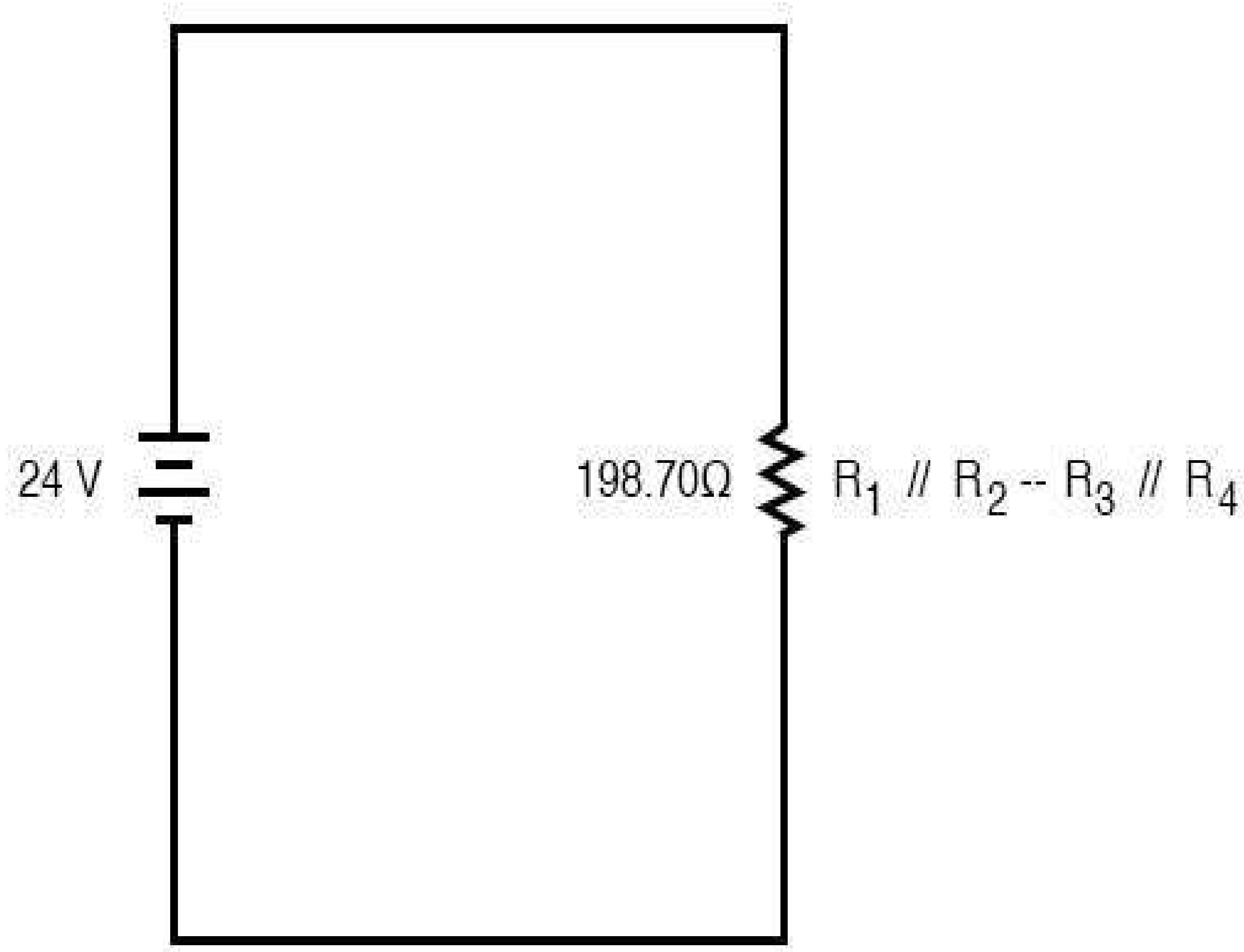
$$\text{Circuit Current (I)} = \frac{V}{R} = \frac{12}{12} = 1\,\text{Ampere}$$

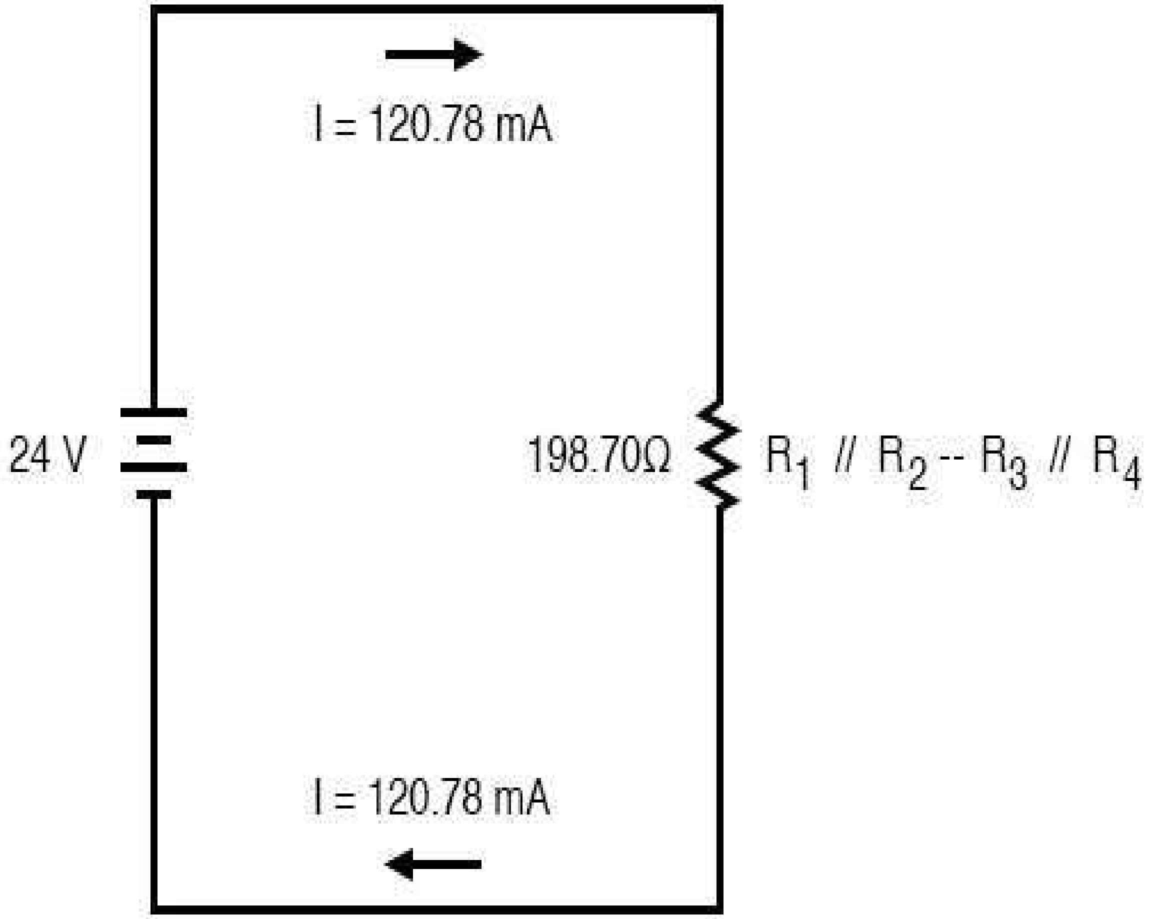
Question 3

A series-parallel combination circuit



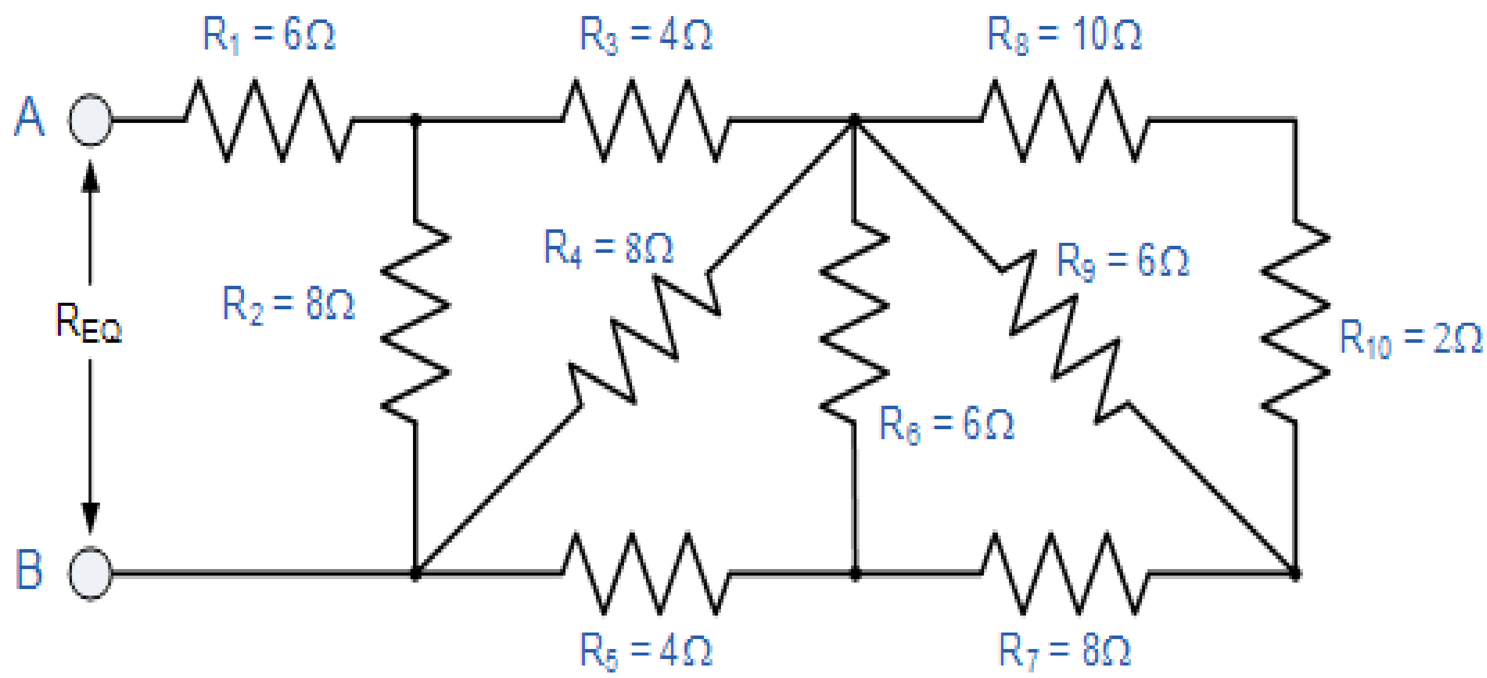

24 V
71.429Ω
R1 // R2
127.27Ω
R3 // R4

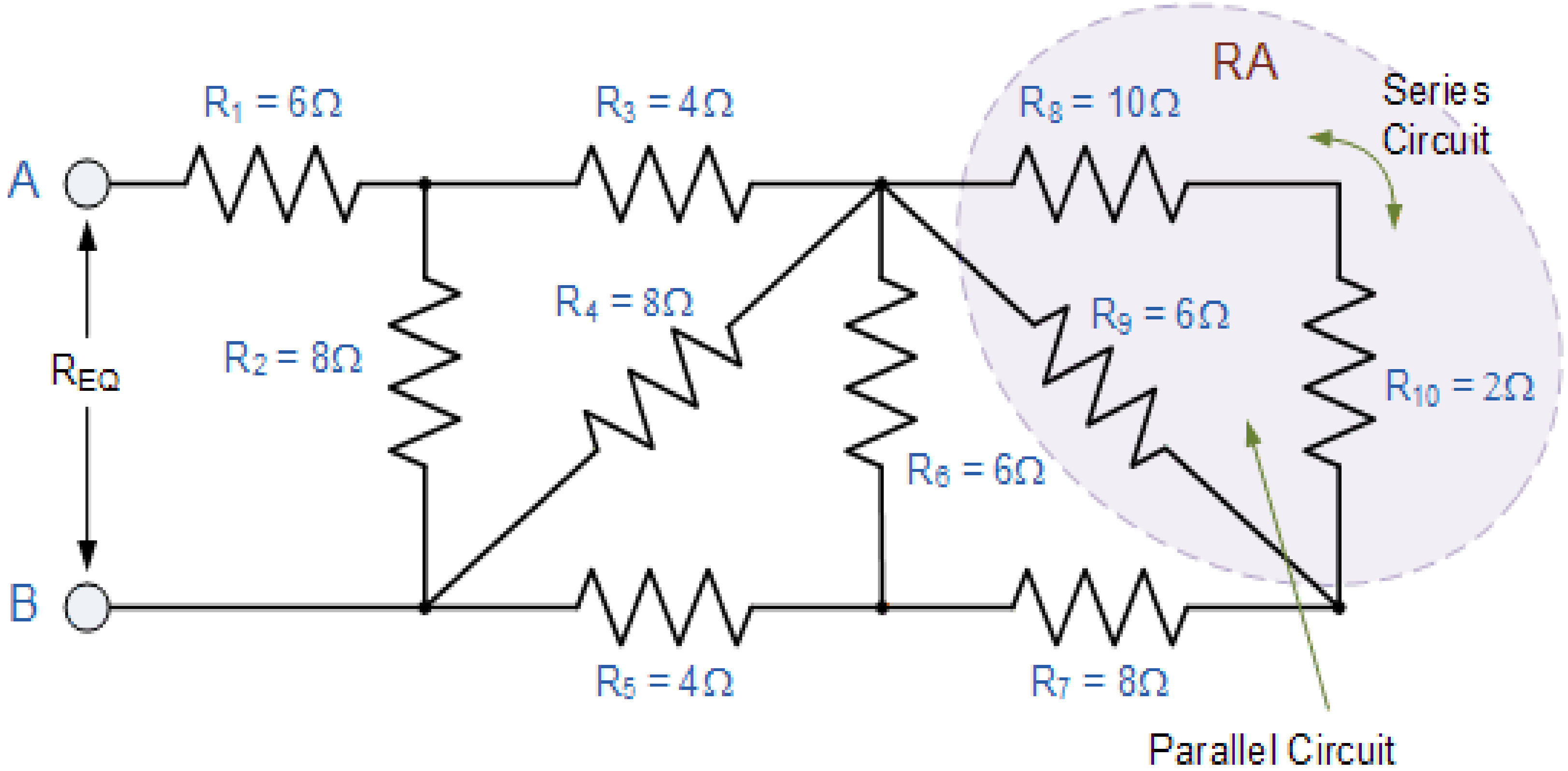

24 V
198.70Ω
R1 // R2 -- R3 // R4


I = 120.78 mA
24 V
198.70Ω
R_1 // R_2 -- R_3 // R_4
I = 120.78 mA

Question 4

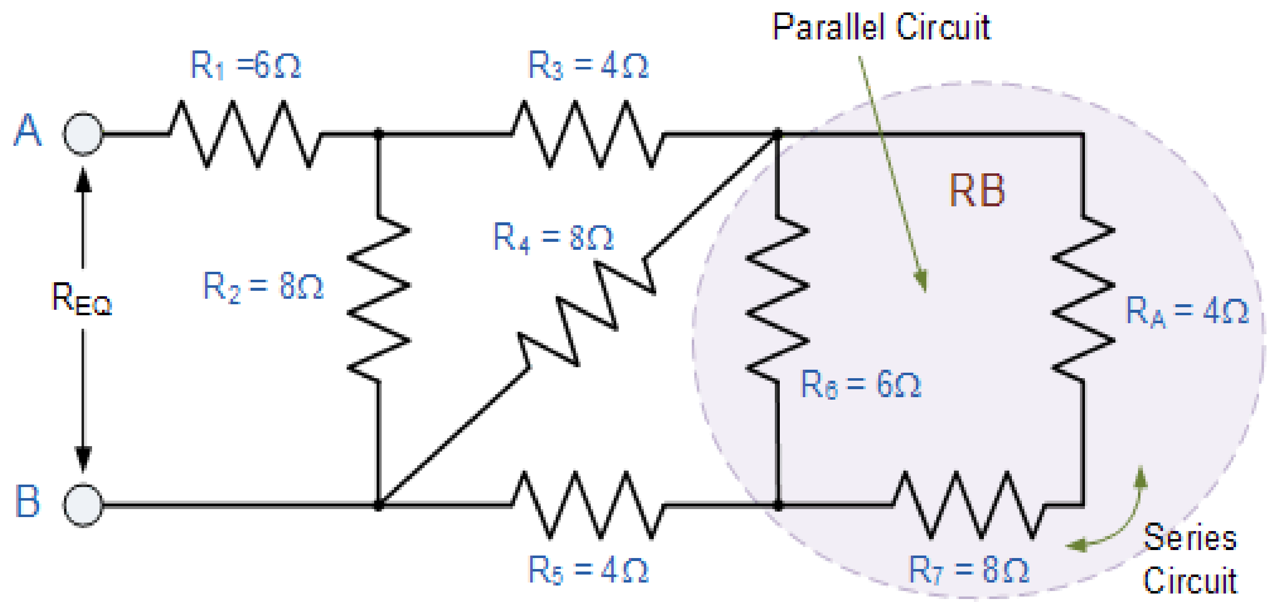
Find the equivalent resistance, R_{EQ} for the following resistor combination circuit.





$$R_A = \frac{R_9 \times (R_8 + R_{10})}{R_9 + R_8 + R_{10}} = \frac{6 \times (10 + 2)}{6 + 10 + 2} = 4\Omega$$

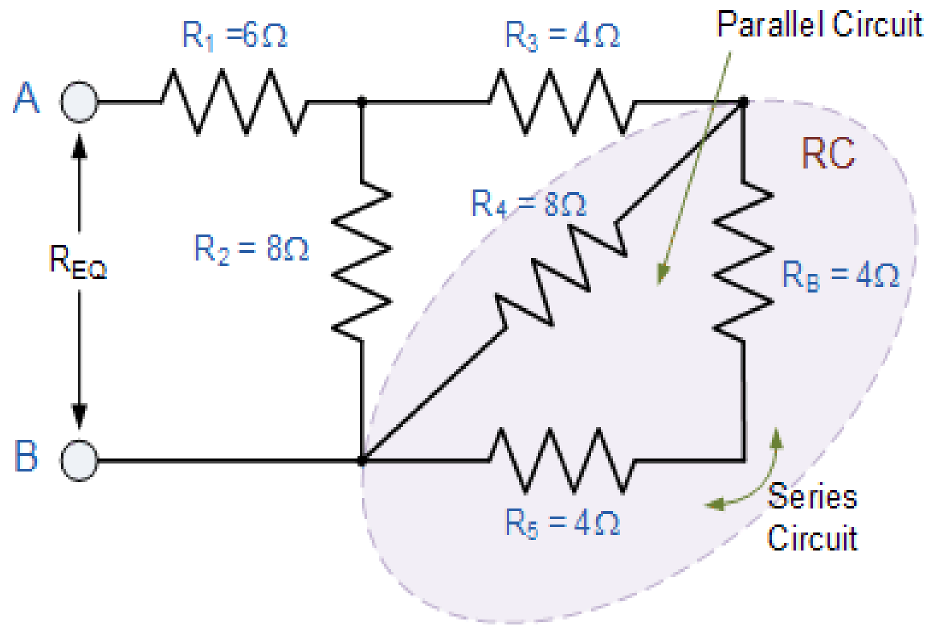
R_A is in series with R_7 therefore the total resistance will be $R_A + R_7 = 4 + 8 = 12\Omega$ as shown.



This resistive value of 12Ω is now in parallel with R_6 and can be calculated as R_B.

$$R_B = \frac{R_6 \times (R_A + R_7)}{R_6 + R_A + R_7} = \frac{6 \times (4 + 8)}{6 + 4 + 8} = 4\Omega$$

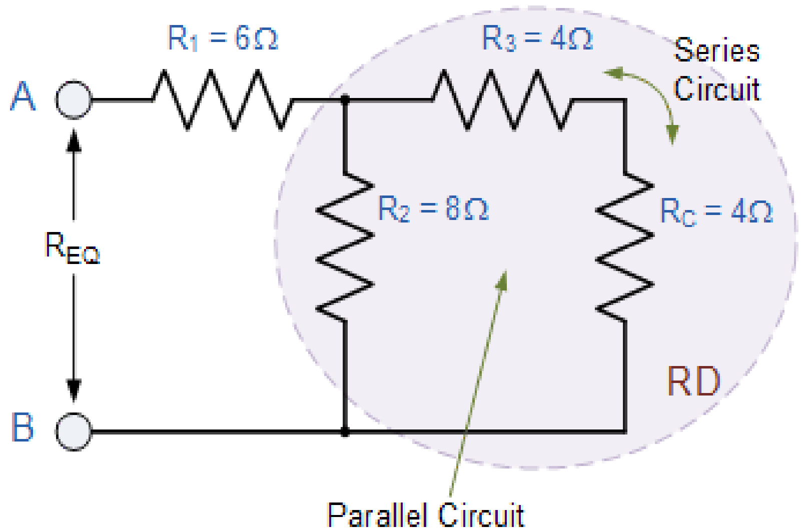
R_B is in series with R_5 therefore the total resistance will be $R_B + R_5 = 4 + 4 = 8\Omega$ as shown.



This resistive value of 8Ω is now in parallel with R_4 and can be calculated as R_C as shown.

$$R_C = \frac{R_4 \times (R_B + R_5)}{R_4 + R_B + R_5} = \frac{8 \times (4 + 4)}{8 + 4 + 4} = 4\Omega$$

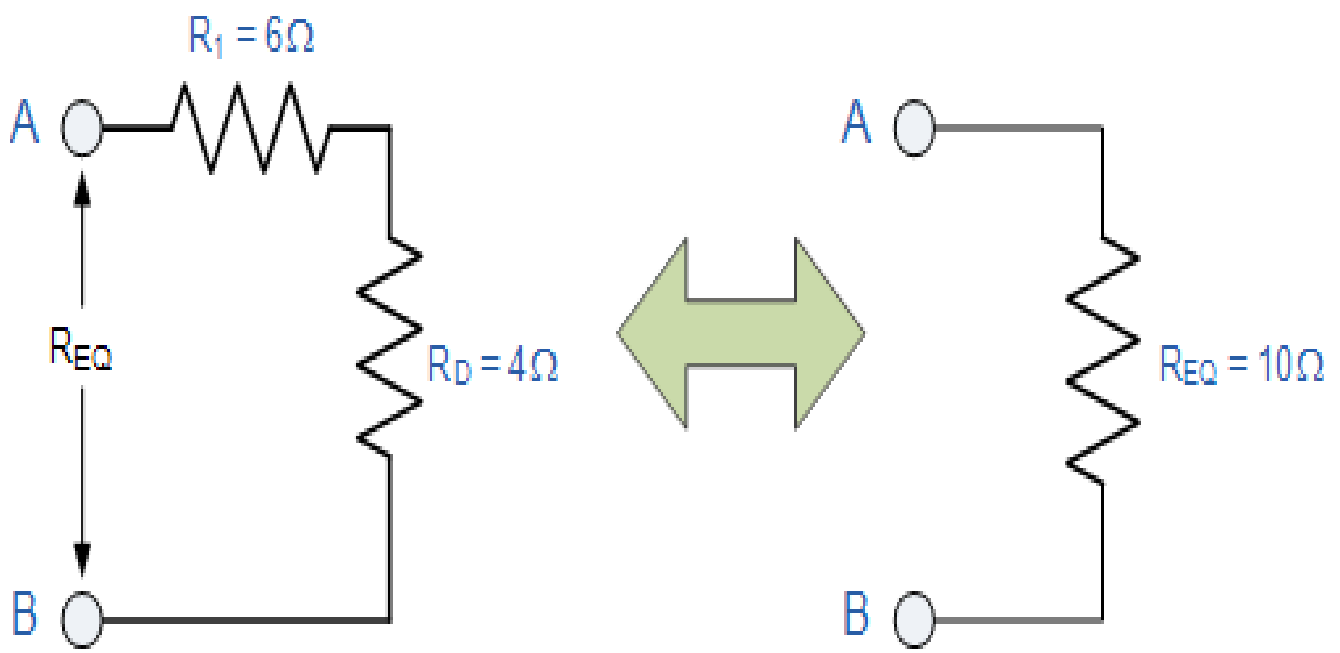
R_C is in series with R_3 therefore the total resistance will be $R_C + R_3 = 8\Omega$ as shown.



This resistive value of 8Ω is now in parallel with R_2 from which we can calculated R_D as:

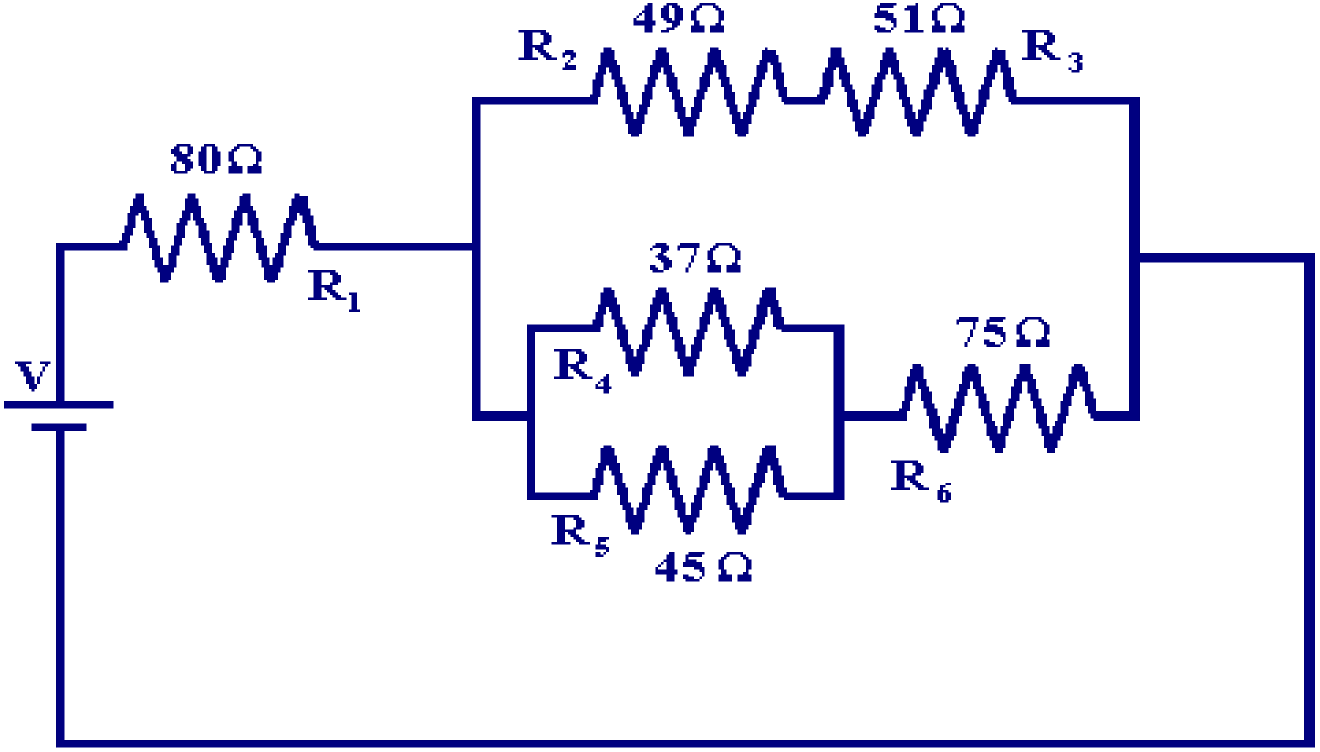
$$R_D = \frac{R_2 \times (R_C + R_3)}{R_2 + R_C + R_3} = \frac{8 \times (4 + 4)}{8 + 4 + 4} = 4\Omega$$

R_D is in series with R_1 therefore the total resistance will be $R_D + R_1 = 4 + 6 = 10\Omega$ as shown.



Question 5

Calculate the value of current if V= 10V in above circuit?



Tutorial 2

Question 1

Example 1.37. *A resistance of 10 Ω is connected in series with two resistances each of 15 Ω arranged in parallel. What resistance must be shunted across this parallel combination so that the total current taken shall be 1.5 A with 20 V applied ?*

(Elements of Elect. Engg.-1; Banglore Univ.)

Solution. The circuit connections are shown in Fig. 1.46.

Drop across 10-Ω resistor = 1.5 × 10 = 15 V

Drop across parallel combination, $V_{AB} = 20 - 15 = 5$ V

Hence, voltage across each parallel resistance is 5 V.

$$I_1 = 5/15 = 1/3 \text{ A}, I_2 = 5/15 = 1/3 \text{ A}$$

$$I_3 = 1.5 - (1/3 + 1/3) = 5/6 \text{ A}$$

$$\therefore \quad I_3 R = 5 \quad \text{or} \quad (5/6) R = 5 \quad \text{or} \quad R = 6\ \Omega$$

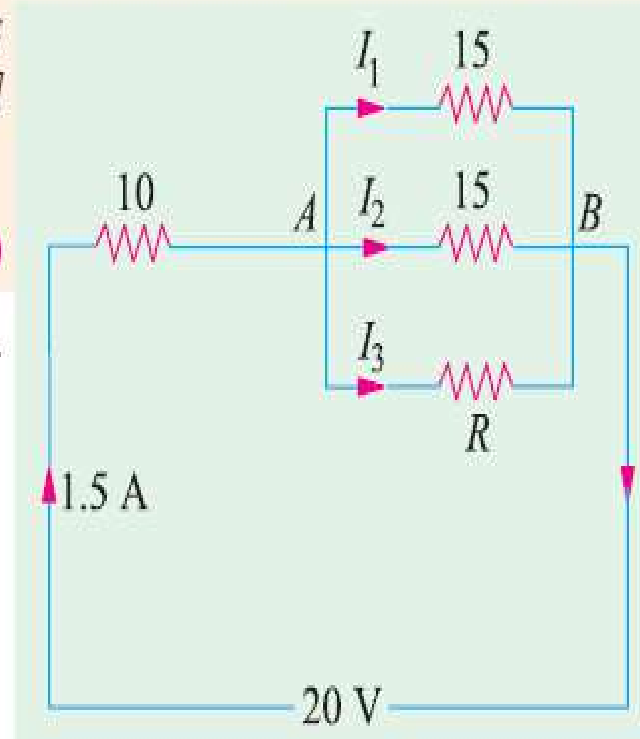
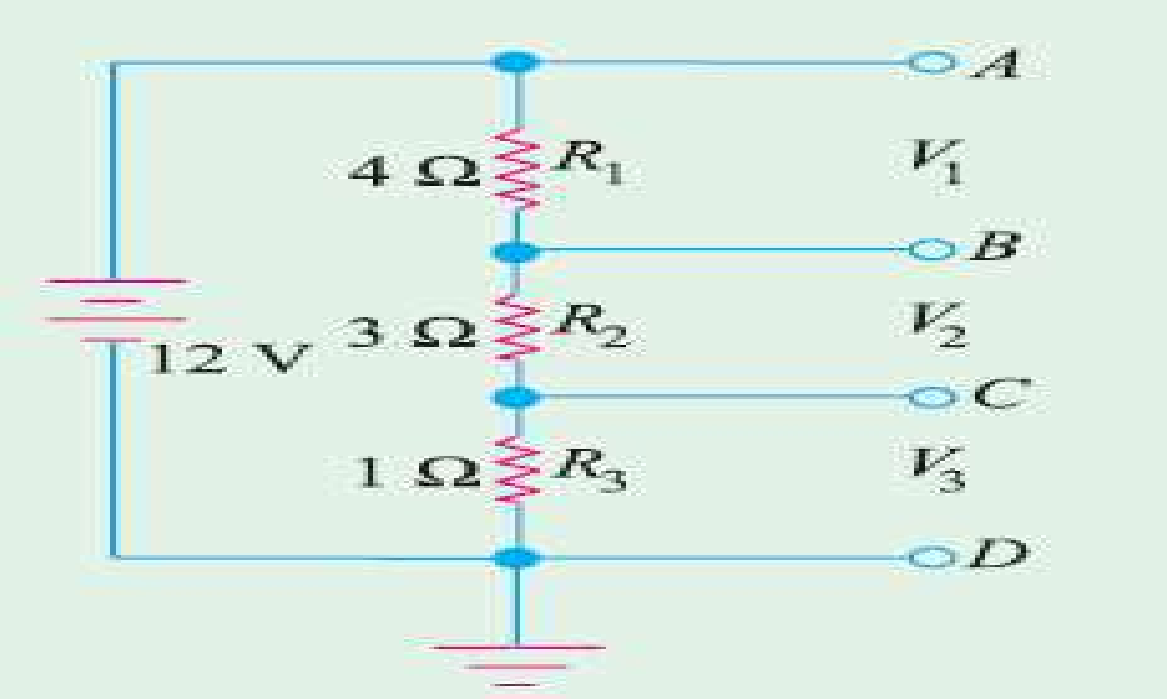


Fig. 1.46

Question 2

- *Find the values of different voltages that can be obtained from a 12-V battery with the help of voltage divider circuit of Fig*



Solution. $R = R_1 + R_2 + R_3 = 4 + 3 + 1 = 8\ \Omega$

Drop across $R_1 = 12 \times 4/8 = 6\text{ V}$

$\therefore$ $V_B = 12 - 6 = 6\text{ V}$ above ground

Drop across $R_2 = 12 \times 3/8 = 4.5\text{ V}$

$\therefore$ $V_C = V_B - 4.5 = 6 - 4.5 = 1.5$

Drop across $R_3 = 12 \times 1/8 = 1.5\text{ V}$

Different available load voltages are :

(*i*) $V_{AB} = V_A - V_B = 12 - 6 =$ **6 V**

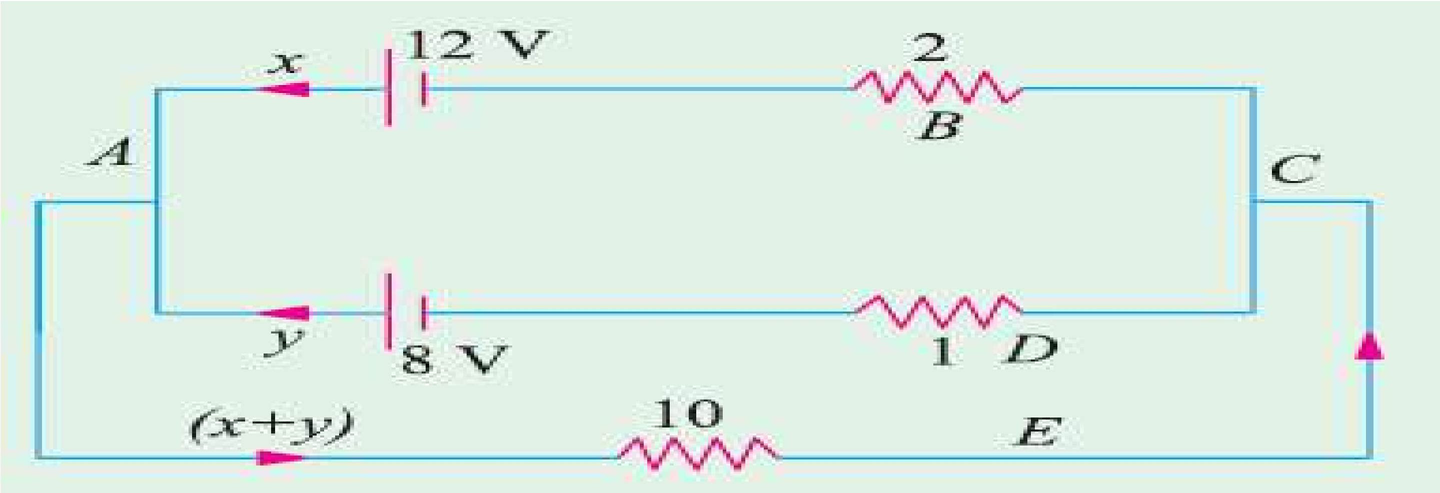
(*ii*) $V_{AC} = 12 - 1.5 =$ **10.5 V**

(*iii*) $V_{AD} =$ **12 V**

(*iv*) $V_{BC} = 6 - 1.5 =$ **4.5 V**

(*v*) $V_{CD} =$ **1.5 V**

Two batteries A and B are connected in parallel and load of 10 Ω is connected across their terminals. A has an e.m.f. of 12 V and an internal resistance of 2 Ω ; B has an e.m.f. of 8 V and an internal resistance of 1 Ω. Use Kirchhoff's laws to determine the values and directions of the currents flowing in each of the batteries and in the external resistance. Also determine the potential difference across the external resistance.



Explanation

Applying KVL to the closed circuit *ABCDA of Fig., we get*

$-12 + 2x - 1y + 8 = 0$ or $2x - y = 4$ ...***(i)***

Similarly, from the closed circuit *ADCEA, we get*

$-8 + 1y + 10(x + y) = 0$ or $10x + 11y = 8$ ...***(ii)***

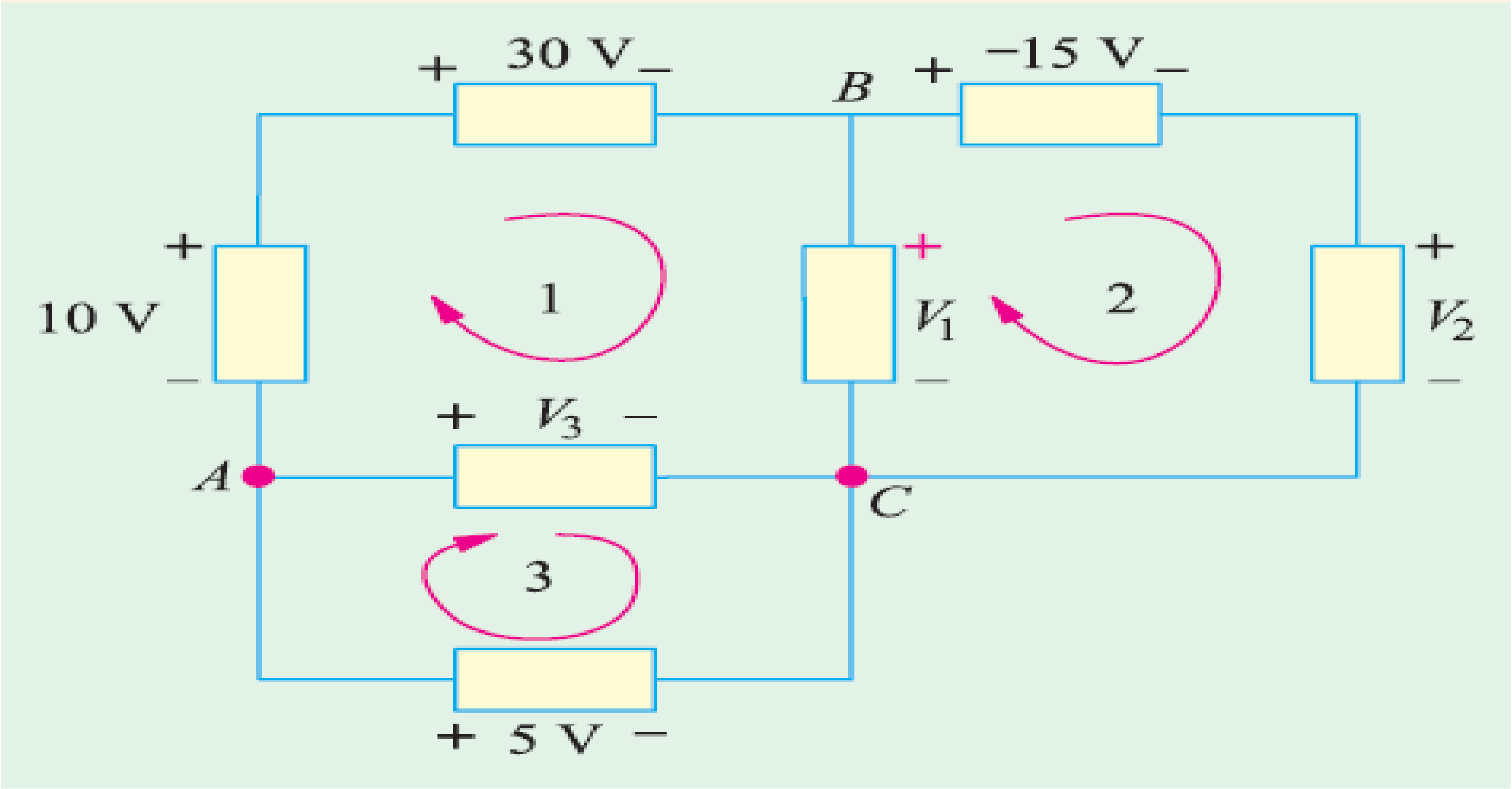
- From Eq. **(*i) and (ii), we get***
- $x = \mathbf{1.625\ A}$ ***and*** $\mathbf{y = -0.75\ A}$

The negative sign of *y shows that the current is flowing into the 8-V battery and not out of it. In* other words, it is a charging current and not a discharging current.

- Current flowing in the external resistance = $x + y = 1.625 - 0.75 = \mathbf{0.875\ A}$
- P.D. across the external resistance = 10 × 0.875 = **8.75 V**

Question 4

- *Applying Kirchhoff's laws to different loops in Fig., find the values of V1 and V2.*



Explanation

Starting from point *A and* applying Kirchhoff's voltage law to loop No.3, we get

- *−V3 + 5 = 0 or V3 = 5 V*

Starting from point *A and applying* Kirchhoff's voltage law to loop No. 1, we

get

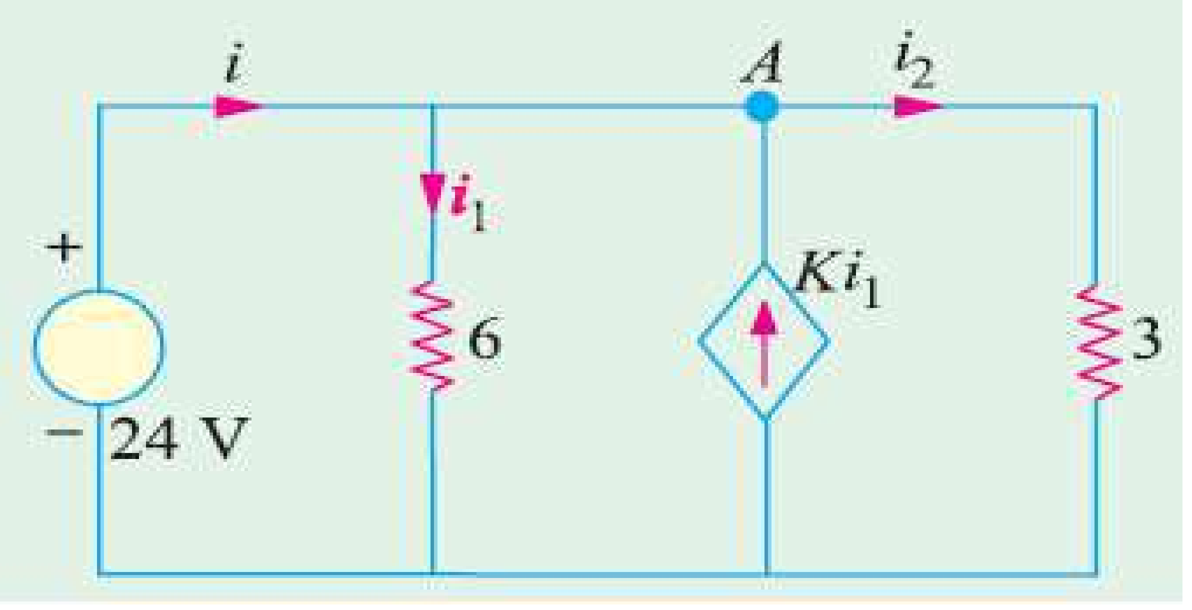
- 10 − 30 *−V1 +5 = 0 or V1 = −**15 V***
- The negative sign of *V1 denotes that its* polarity is opposite to that shown in the figure.

Starting from point *B in loop No. 2, we* get

- − (− 15) − V2 + (− 15) = 0 or *V2 = **0***

Question 5

- *In the circuit of Fig. 2.26, apply KCL to find the value of current i when* ***(a) K = 2 (b) K = 3 and (c) K = 4. Both resistances are in ohms.***



Explanation

- Since 6 Ω and 3 Ω resistors are connected in parallel across the 24-V battery,
- *$i1 = 24/6 = 4$ A. and $i2 = 24/3 = 8$ A*

Applying *KCL to node A,*

- *we get $i - 4 + 4K - 8 = 0$ or*
- *$i = 12 - 4K$.*

(*a*) *When $K = 2$, $i = 12 - 4 \times 2 = 4$ A*

(*b*) *When $K = 3$, $i = 12 - 4 \times 3 = 0$ A*

(*c*) *When $K = 4$, $i = 12 - 4 \times 4 = -4$ A*

It means that current *i flows in the opposite direciton.*

Unit 1: DC Circuits

TUTORIAL : Week 3

Prepared By: Irfan Ahmad Pindoo

Recap: POLL

Nodal Analysis is applicable to_______ network.

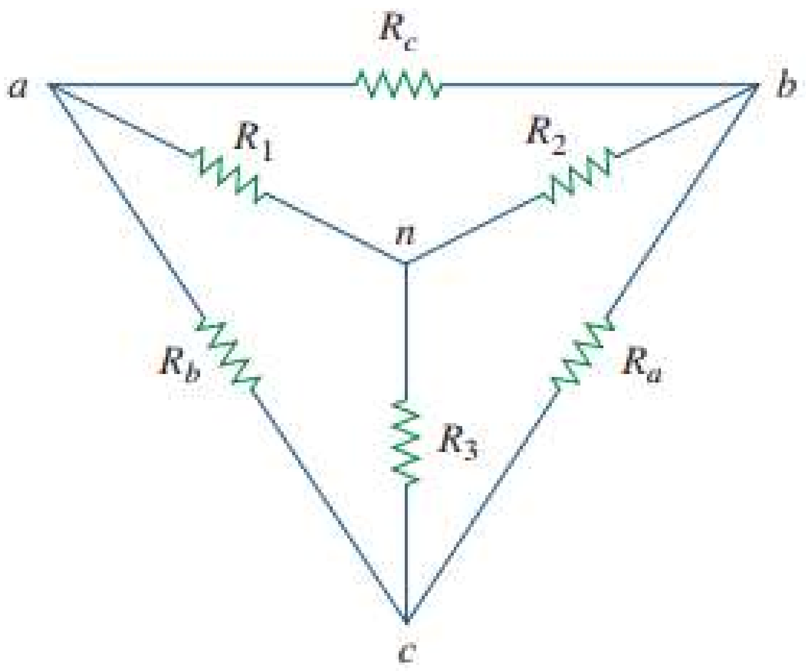
A. Planar only
B. Non Planar
C. Both planar and non planar
D. Only meshes

Delta to Star Conversion

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

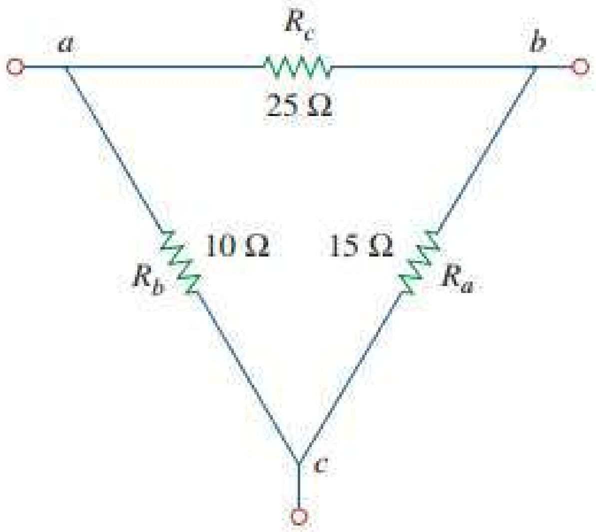
$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



Problem 1

Q: Convert Δ network into a Y network?



POLL

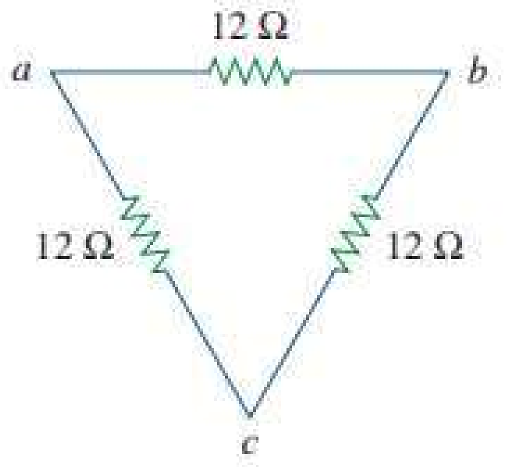
While converting Δ network into a Y , the equivalent values of the resistor would be?

A. 12

B. 4

C. 36

D. 40

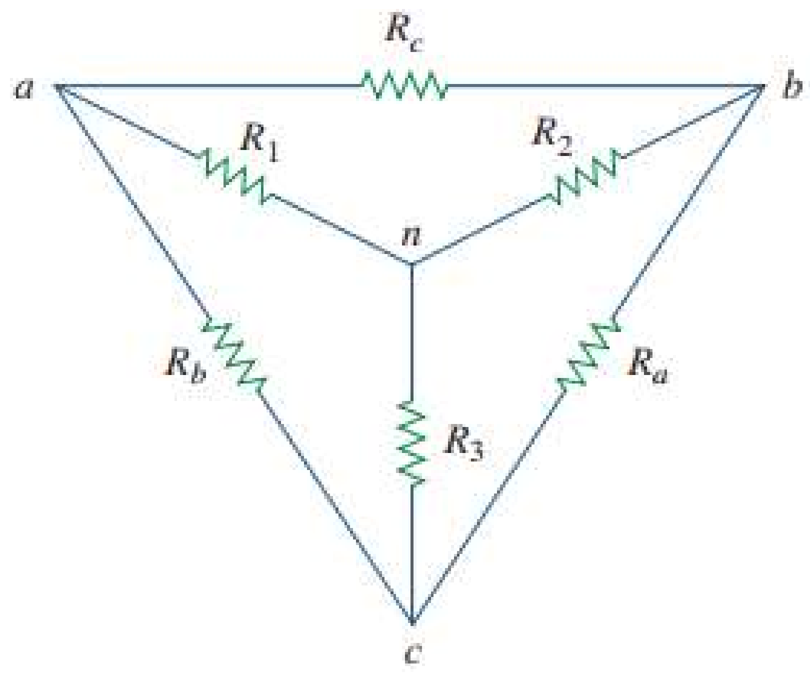


Star to Delta Conversion

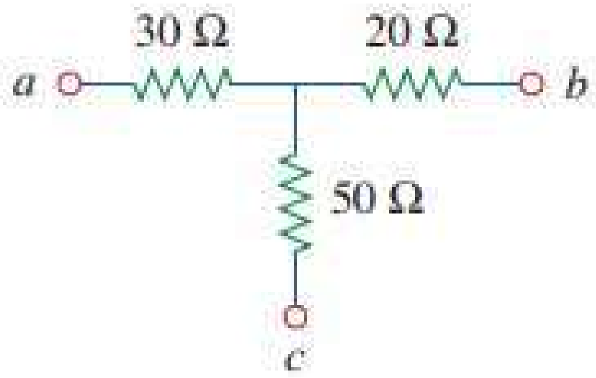
$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$



Example



QUICK QUIZ (Poll)

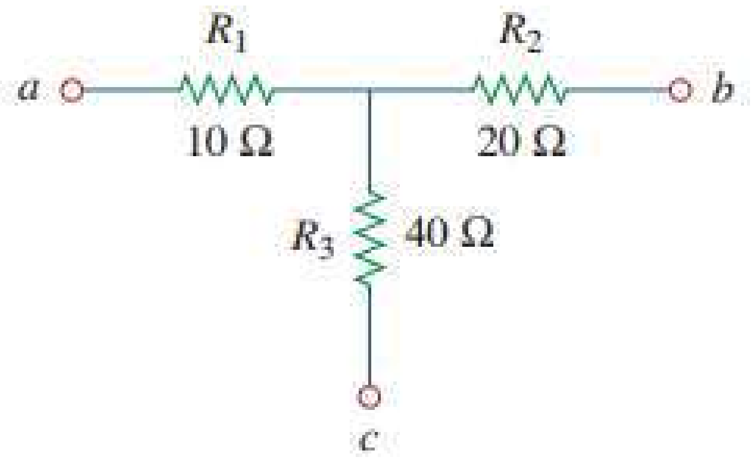
Resistance R_{bc} for the Δ network of the corresponding Figure is:

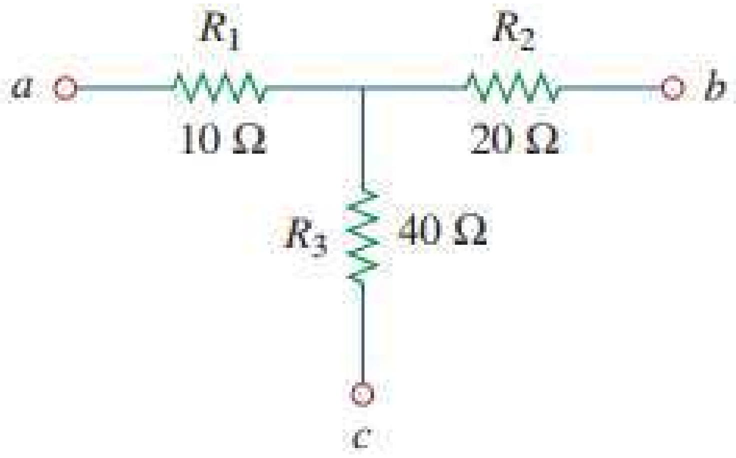
A. 140

B. 70

C. 35

D. 100

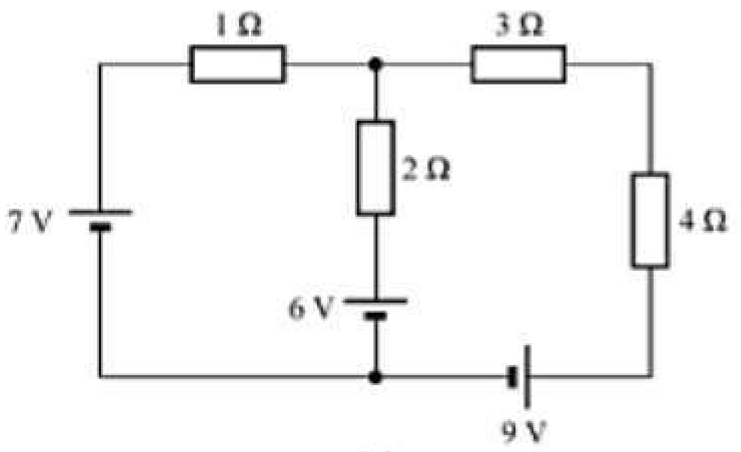




R_1
R_2
a
b
10 Ω
20 Ω
R_3
40 Ω
c

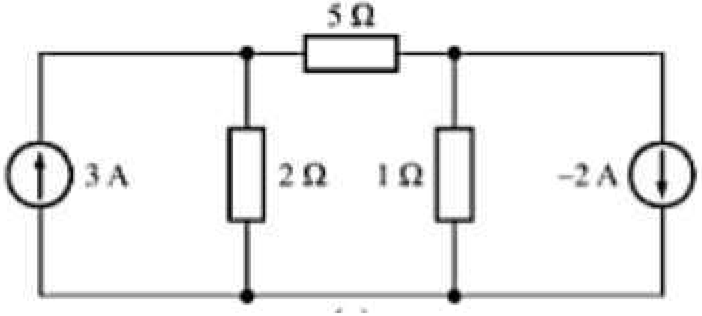
Problem 2

Find Mesh Currents?



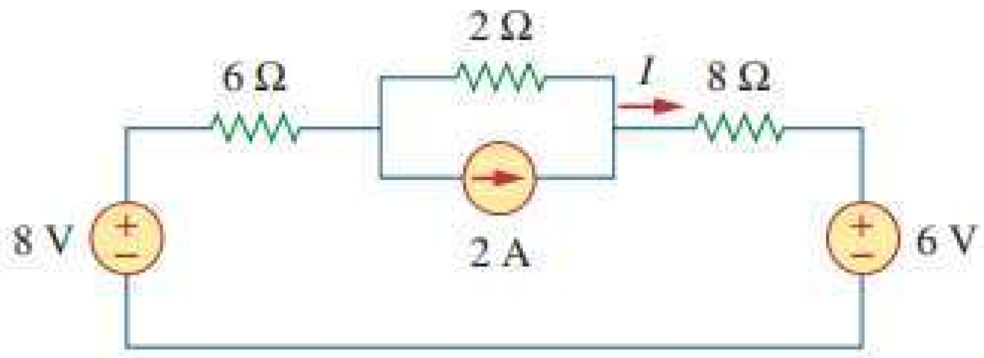
Problem 3

Find current through 5ohm using Nodal Analysis?



Problem 4

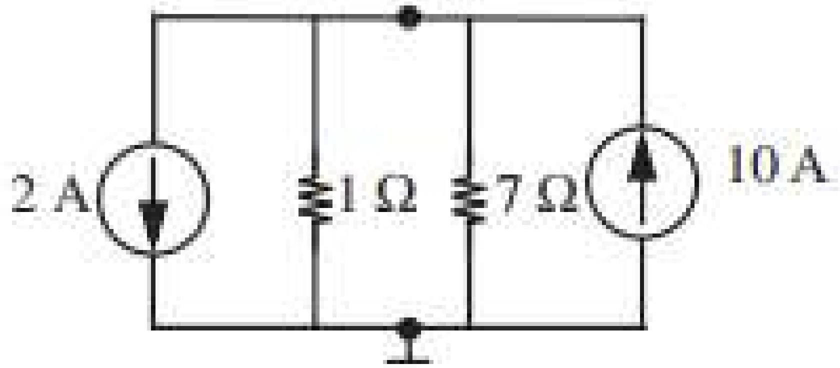
Find I using Superposition?



QUICK QUIZ (POLL)

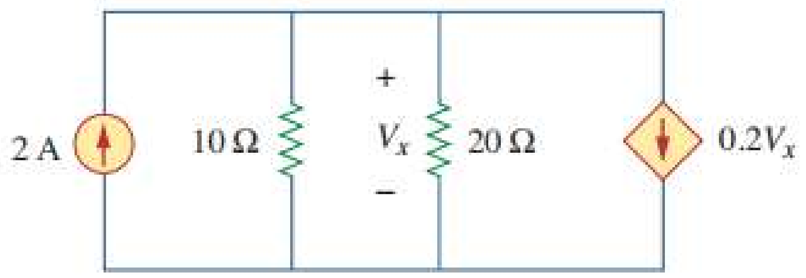
Find node voltages?

A. 6V
B. 7V
C. 8V
D. 9V



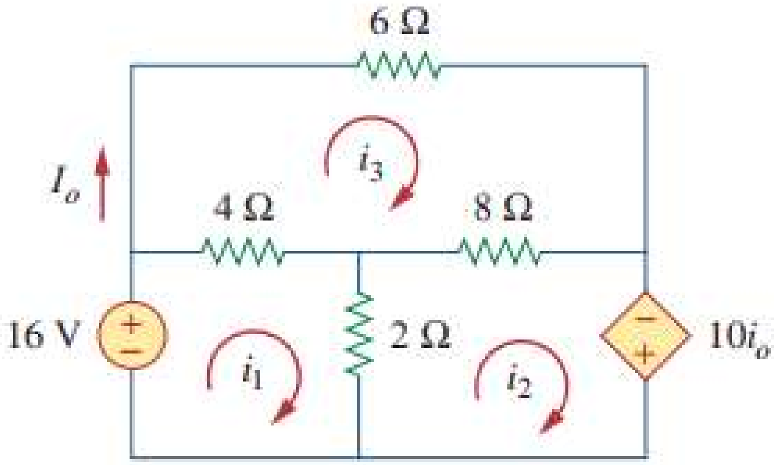
Dependent Sources:
Problem 5

Find Vx in the circuit using Nodal Analysis?



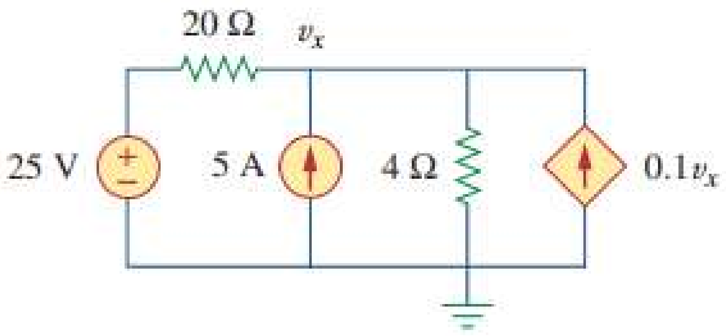
Problem 6

Use Mesh Analysis to find Io?



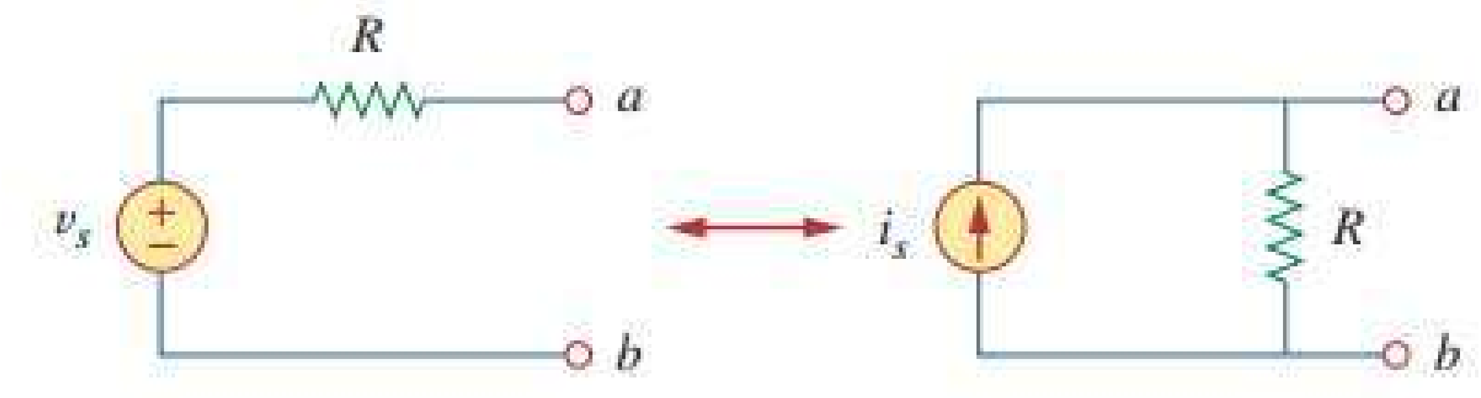
Problem 7

Find Vx using Superposition?



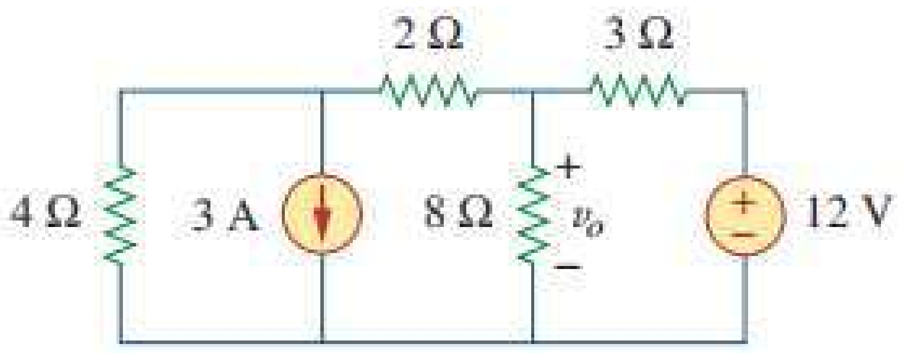
Source Transformation

- We have noticed that series-parallel combination and wye-delta transformation help simplify circuits.
- *Source transformation* is another tool for simplifying circuits. Basic to these tools is the concept of *equivalence*.



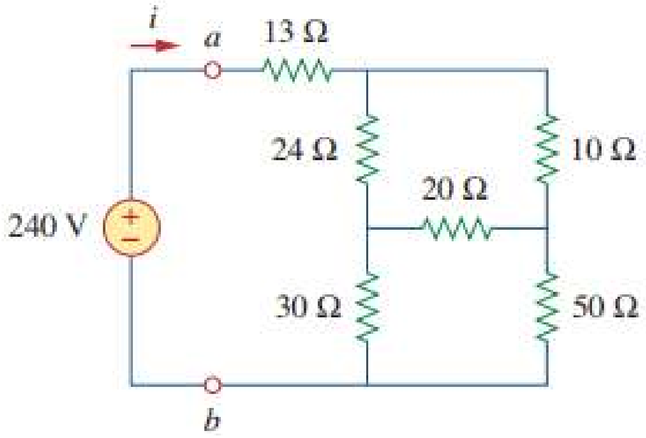
PRACTICE PROBLEM

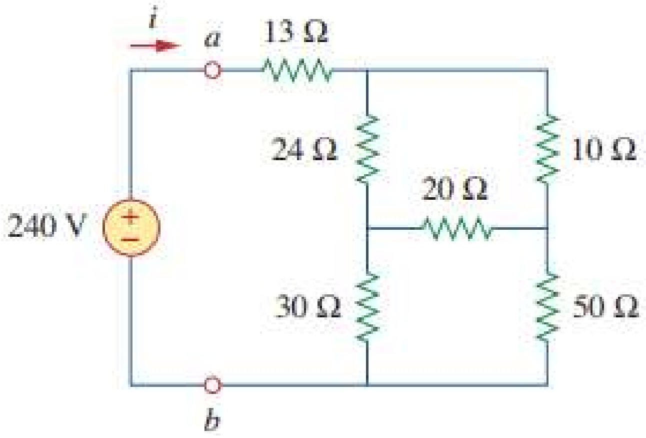
Using Source Transformation, find Vo?



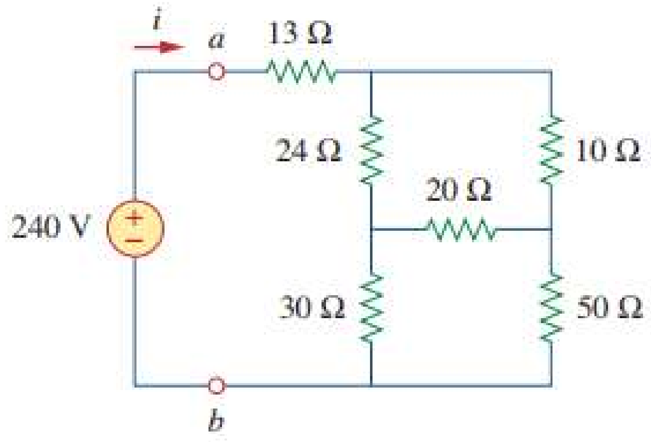
Problem 8

- Find R_{ab} and i in the given circuit:





i
a
13 Ω
24 Ω
10 Ω
20 Ω
240 V
30 Ω
50 Ω
b



i
a
13 Ω
24 Ω
10 Ω
20 Ω
240 V
30 Ω
50 Ω
b

Problem 9

Find io in the circuit?