

# PREPARING A LETTER/STATEMENT OF PURPOSE/INTENT

## 1. Math Highlights

Remember some concrete moments and activities in your life that turned you towards mathematics, ignited some passion about math, or expanded your view on mathematics. Make a (chronological) list of them. Include some key words how and why these events influenced you. Decide on one or two that you may want to describe in some more detail.

## 2. Look Ahead

Think what intrigues you about mathematics that you still want to learn. The idea is not to come up with a program of study for future years – which is likely to appear very odd anyway not having had extensive exposure to graduate mathematics already. Rather give specific examples of things you read about or things you learned the beginnings of and wanted to understand more about. Those should be personal and specific and help underscore your eagerness to move further into math. At all times be yourself and honest with what you can understand. It is OK to say you are undecided and are excited about a broad range of mathematics.

## 3. Specific Mathematics

Choose a specific piece of mathematics that you can talk about that fascinates you in particular. Try to describe the math fact in a short paragraph (you may assume a mathematician is reading this). Try to figure out what is fascinating you about this math fact.

## 4. Evolution of Interest

Think about how these key events can be put together to tell a story about yourself and how your relation about mathematics evolved. Keep early things (childhood etc) very brief – emphasis should be things that happen after high school.

## 5. Career Clarifications

Is your career path standard and obvious from your CV? The answer is probably YES if you went straight from high school to college and stayed at the same college/university.

For more complicated career paths that may not be clear from the CV you can summarize in a narrative and explain some choices (worked for a while between schools, transferred schools due to recommendations and interests, etc)

Be careful with explaining poor and trying to excuse performance during some time, which can come across as whining. You might state basic facts (medical or family issues, over-enrollment, work) but then leave longer explanations to a professor writing a letter for you. Make sure this is explained as a temporary issue and emphasize that a lot of positive things came later.

## 6. Strengths

What are instances, you think, that particularly demonstrate your mathematics strengths and talent – what are you proud of having accomplished.

Think how you can turn the focus of the reader on these points without outright bragging about them – that is, thread them in the stories above, explain that those things gave you confidence, talks about something you were persistent at (instead of saying you are persistent), etc.

## **7. Research Graduate Programs**

Look on the web and talk to people about graduate programs, their specific mathematical strengths and general qualities. Get an idea what type of program would be a good fit for you interests and personality.

Find out what the type of research is done in general. Look up individual professors, the research groups, publications, and seminars. Browse other resources to find out in which area(s) some of the mathematical keyword that you come across come up and what they *very* roughly mean.

## **8. Expressing Specific Interest in Programs**

Can you use what you found to express a more specific interest in the program or in particular faculty. This can be very effective but also has its pitfalls:

Make sure you do not get ahead of yourself but stay honest with what you understand. It will look very strange if you say that you want to work with Professor X on sheaf cohomology when it is highly unlikely from your course background that you have understanding of what that is. You can say that you are fascinated by the idea of using higher algebraic objects to describe geometric spaces and it seems to you that Professor X is doing great things in this direction. As with anything else be honest, be yourself, and show enthusiasm about mathematics.

Make sure professor or area you refer to are accurate and active in the program you write to. Look whether someone had published in recent years, if there are seminars or other activities in the area, and your interests really relate to what is done. Ask a professor from your program to help if you have doubts.

This application is very important to me because completion of a PhD degree in Mathematics will be the best chance for a unifying career development with my labor of love - mathematics. I am confident that when I become a serious and mature researcher my desire to make a contribution to mathematics and to our understanding of this world will remain my main driving force.

I don't remember when I became keen on mathematics. At first it was only entrainment as I found it exciting to solve intricate problems. Later it became something more than a mere hobby. I particularly enjoyed the ineffable feeling of triumph when you realize that the problem is solved; that you have got the idea. I think it is the profundity of this feeling that made mathematics my chief enthusiasm. I have taken part in Olympiads and Conferences. The most valuable contribution of these competitions was a possibility to meet the same enthusiasts in mathematics. I was particularly impressed when I was invited to participate in an International Summer School and Conference in the ancient town of Pereslavl-Zalessky. There I had an opportunity not only to work on appealing research problems but also to interact closely with working mathematicians such as X and Y. It is difficult to describe the feelings that overwhelmed me but they did incite me to further progress.

Being a high school student I was doing a course on inequalities at Kiev State University where I conducted my first research work. It was essentially proof of Karamat inequality that utilized properties of convex functions and Murhead inequality. It was unforgettable, how the main idea of my central proof dawned upon me. That evening I went to sleep the happiest boy in the world. Although, relatively simple, I did something really innovative, something that no one had ever done before with elementary methods. That was my first moderate contribution to mathematics.

Later, while being a freshman at the Moscow Institute of Physics and Technology (MIPT), I refined the proof and presented it at the 52nd MIPT Scientific Conference where it was honored the first prize.

In my fourth year when students of our University get involved in research activity, Professor X suggested to be my supervisor. Under his guidance I wrote my thesis  $\diamond$ Integration of Multivalued Mappings $\diamond$  and defended it with Honors. Essentially it consisted in a study of necessary and sufficient conditions under which there exists Riemann integral of multivalued maps. My task was to find some classes of sets of attainability and to investigate relation between Lebesgue and Riemann integral for Multivalued Maps. In this work I analyzed properties of spaces of compacts with Housdorff metric and properties of support functions of convex compacts and applied these theoretical findings to several problems of theoretical mechanics. During this work I acquired a broad range of research experience and background necessary for further research in convex analysis.

My current research work is devoted to differentiating of multivalued mappings and differential inclusions. The main task is to become familiar with ideas and approaches introduced in the Sci. D. thesis of Professor X and to improve the results that are obtained in it. One of the most challenging tasks in the project is to obtain Pontryagin maximum principle in Hamiltonian form from Lagrange form (in terms of tangent cones).

After graduation I plan to continue my scientific career in mathematics. Differential games, convex analysis and Optimization theory are of particular interest to me. I have the strongest incentive to advance as far as I can in this captivating science and feel confident that application to the University of Chicago is the best possible step to accomplish it. I would regard my admission to your University not only as a great honor but also as a great responsibility and an obligation to work hard.

My interest in math was sparked as an undergraduate student at Rutgers University when I decided to take a course in pure math at the suggestion of a friend. The course covered naïve set theory and logic, some elementary number theory, and concluded with a discussion of cardinalities. Having previously transferred from the Eastman School of Music, where I studied jazz saxophone performance, I was beginning my studies at Rutgers as a physics major. Although I had taken some calculus courses to supplement the physics I was learning, I had never seen math from a rigorous, formal perspective before. I was blown away by Cantor's diagonal argument, showing that the real numbers and the natural numbers have different cardinalities. Through a beautiful and abstract, yet intuitively correct formalization, Cantor had shown that it really made sense to say there were different sizes of infinity! I became fascinated by pure mathematics. A fourth year undergraduate at the time, I decided to switch my major to math, and after taking as many mathematics courses as I could during my fifth year, I graduated from Rutgers University with a B.A. in mathematics.

Upon graduation I knew that mathematics was right for me, and I wanted to pursue a Ph.D. in math, but because I had a limited exposure to pure math I decided to broaden my background by obtaining a Master's degree before applying to Ph.D. programs. Although I applied to their Master's program, Temple University offered me admission to their Ph.D. program, and I accepted.

While attending Temple University I've had the opportunity to study many different areas of mathematics such as measure theory, complex analysis, abstract algebra, topology, and differential geometry. Throughout my studies the interplay between algebra and geometry has stood out to me as one of the most beautiful relationships in mathematics. On one hand, solutions to algebraic equations can be visualized geometrically, allowing the use of geometric intuition to aid in their understanding. On the other hand, once a correspondence between algebra and geometry has been established, geometric results can be derived solely through techniques of algebra. As I have advanced in my studies I have developed an interest in algebraic geometry, an area where this relationship is fundamental.

Although Temple has strong researchers in their algebra department, the department is small and no professors are working in the areas that interest me most. In particular, no one studies algebraic geometry and courses in algebraic geometry are not offered. Although I have been making good progress in the Ph.D. program at Temple University and have passed my qualifying exams, I have decided to pursue graduate study at a program with active research in algebraic geometry.

I am excited by the many ways that study at Ohio State University would provide for me to advance in the field of algebraic geometry. The numerous faculty working in the area, seminar, and advanced coursework, would provide the ideal environment for me to immerse myself in the study of algebraic geometry. In addition to algebra and algebraic geometry, I also have an interest in logic, so the presence of a strong logic department at Ohio State is another attraction. As my eventual goal is to become a professor of mathematics at a research university it is crucial for me to attend a program with an emphasis compatible with my interests. Applying to Ohio State University is a natural choice.

Thank you for your consideration of my application. I look forward to your response.

# Statement of Purpose

██████████ Math Ph.D.

My reasons for wanting to do a Ph.D. are (in my eyes) the obvious ones. I am interested in Mathematics and I enjoy doing it, I hope this is reason enough. Ideally, I would like a career in academia as I have enjoyed studying mathematics thus far and want to continue.

I am now in my final year of a MMath degree at the University of Warwick, UK. In previous years I achieved high grades, high enough to alleviate the worry concerning graduating with a first class classification (equivalent to a GPA of 4.0). This grade buffer from my previous years' examination success has enabled me to take the courses which interest me most, regardless of their difficulty; this is evidenced by the fact that I am only taking graduate level courses this year. By far my favourite courses this term are Algebraic Geometry, Modular Forms and Elliptic Curves; it is somehow rewarding to use the tools and knowledge I gained from technical, purely abstract subjects such as Group Theory, Commutative Algebra, and Complex Analysis in a much more tangible fashion. In particular I find Modular Forms very interesting: I am continually surprised by the subject's order and 'niceness'. I hope to learn more about how modular forms relate to algebraic geometry and number theory over the coming academic year. Being in the final and master's year of my degree I am required to write a dissertation. Under the supervision of Professor Miles Reid FRS, I am writing a dissertation in the area of algebraic geometry and modular forms. The exact title is yet to be decided, but the project will most probably be concerning Hilbert Modular Forms and their associated surfaces with emphasis on resolving their singularities and toric varieties. I aim to give examples of some of the more abstract concepts using toric varieties. Currently, I am reading about cyclic quotient singularities and their resolutions using Herzbruch's continued fractions method.

I have written projects in both my second and third year; an exposition on "Waring's problem" and the Hardy-Littlewood Circle Method, and last year a paper titled 'Primes of the Form  $x^2 + ny^2$ ', which was a basic introduction to Quadratic Forms and Hilbert Class Field Theory. I particularly enjoyed my project last year, the member of faculty who supervised me, Dr Johan Bosman, really encouraged and motivated me; my time spent working with him has inspired me to study algebraic number theory further. During my second year I was awarded the Ron Lockhart scholarship. It is awarded once every three years to an undergraduate mathematician who has shown potential during their first year. Each year since starting university, I have taken the maximum number of extra mathematics courses permitted by my department. Some of my favourite courses from previous years include Galois Theory, Rings and Modules, Groups and Representations, Algebraic Number Theory and Manifolds. During my Galois Theory course I enjoyed seeing applications to Algebraic Number Theory and Classical Number Theory; for example, seeing a proof of quadratic reciprocity using cyclotomic fields. Although I have never taken a course in topology, I have read the first two chapters of Hatcher's "Algebraic Topology" while on an REU last summer in Minnesota. Thus, I do have a working knowledge of point set topology, homotopy, the fundamental group and simplicial homology. I plan to take a course in algebraic topology next term. I hope to learn more algebraic topology during graduate school; it is a subject in which I am becoming very interested. During my time at the University Of Warwick I have given many seminars to fellow undergraduates, the last of which was on quadratic forms and genus theory. In general, I am a big supporter of discussing mathematics: I spend a good portion of my time in the mathematics common room arguing and discussing mathematics with friends. I hope to find a similar dynamic at graduate school.

I have a good amount of teaching experience, both at university level and below. During my second year of university I took part in a government-funded teaching scheme, which entailed

learning basic teaching theory during weekly seminars juxtaposed with teaching Mathematics in a local secondary school (high school) for half a day a week. I was then selected to take part in a charitable programme organised by the university, teaching Mathematics in underprivileged schools in Der es Salaam, Tanzania, for six weeks of the summer vacation. At university level, I worked as a teaching assistant for a first year Analysis course for two consecutive years. I am currently working as a supervisor; a group of five first year mathematics students are assigned to me, I am responsible for marking all their assignments as well as organising twice weekly meetings during which I cover topics including analysis, linear algebra, abstract algebra and differential equations.

During the course of my degree, I have become close with Dr. John Moody, my academic tutor. Over the years we have had many conversations during which he exposed me to new and cool mathematics. In my opinion the most influential lesson he has taught me is to forget about the course syllabus; to learn mathematics instead of a mathematics course. This is a philosophy I plan to implement further at graduate school.

This year I have taken advantage of the many seminars and talks given by my department. The department has a strong number theory/algebra group, hosting regular talks on current research areas. The talks are motivating and inspirational, perhaps best of all they give me a sense of perspective and direction. It is nice to get away from the rigidity of formal lecture courses, and get away from the Definition, Theorem, Proof style of learning, to get a taste of the big picture. I recently attended a talk by Professor Roger Heath-Brown titled "Diophantine Equations: Algebra, Geometry, Analysis and Logic", during which he exhibited some of the many ways to study Diophantine equations. I left the talk motivated and wanting to know more; in general, the more mathematics I learn and see, the more I am amazed by its 'niceness' and the more I want to understand it.

I am applying to Ohio State because of its strong algebraic geometry and number theory groups, more importantly the current research topics of some department members immediately interest me. For example, I see that Professor Christian Friesen has done work on the class number of quadratic extensions. I briefly touch this subject during my algebraic number theory course and also during an essay I wrote on primes of the form  $x^2 + ny^2$ , and I would love the opportunity to learn more. Also, I am currently learning about the resolutions of singularities in relation to the dissertation that I am doing under Professor Miles Reid. I would be interested in continuing this work, possibly under the advisement of Professor Mirel Caibar, who I see is doing research in related areas. In general I hope to understand more about algebraic geometry, elliptic curves, modular forms and homological algebra at graduate school, and I believe Ohio State is a good place to achieve this.

## Statement of Research Interests

This is a question I hear often about mathematics: «When will I ever use this?» This question has probably been asked at some time to every mathematics teacher by students of various levels. While practical examples can easily be found for elementary mathematics, which is taught early in the students' careers, more advanced math principles shown during high school and college are rarely, if at all, justified by teachers, except in related domains, like physics and engineering. Being in this latter category early in my university career at the University of Ottawa, I was not asking myself that question, as calculus and algebra was an important part of my program. Moreover, I was enjoying the challenge of mathematical problems since high school, so there were never any complains from me.

However, if he were following the same courses as I was during the first two years, a social sciences student, for example, would not change his usual presumption about mathematics. Indeed, these courses were for the most parts very abstract and mostly used examples from fields traditionally associated with mathematics. Everything changed when I read a paper from students of the University of Ottawa and professor Robert Smith (2009) about a model for an infection that transform people into zombies. Aside from the obvious novelty of the study, I was intrigued by the application of mathematics to immunology and biology, where an advanced math background is not the norm. The next fall, I took a course in dynamical systems, given by prof. Smith, and I discovered the versatility of applied mathematics. A given set of equations could be used to build a model which could be used in physics, but also in disciplines often overlooked by mathematicians, like biology or social sciences. I was then that the seed of applied mathematics was implanted in me, as the question which starts this letter was finally answered.

I then undertook a research project funded by the NSERC Undergraduate Student Research Awards Program in the fall of 2010 on dynamical systems, supervised by professor Victor G. LeBlanc and this encouraged me to pursue the project for a Master's degree with the same supervisor. The research was about spiral waves, a particular pattern of reaction-diffusion systems, described by a partial differential equation. Spiral waves patterns arises in biology as well as in chemistry, so the interdisciplinary aspect of applied mathematics is present in the project. We used a dynamical system method on eigenvalues to formulate general terms for a grid perturbation and described relative equilibria for simple spiral wave solutions.

For possible graduate studies at a doctorate level, I am looking for research in applied mathematics using dynamical systems, bifurcation theory or evolution partial differential equations (PDEs) with applications in physics, hopefully with other applications in biology or social sciences. I am always amazed at the sheer number of unresolved problems in seemingly simple systems that deal with these mathematical tools, like with spiral waves. The Ohio State University is a good match for my interest. For my doctoral studies, I would like to further my knowledge on dynamics with symmetries, which I started studying during my master's program. Most of my research on the subject have been about rigidly rotating waves, which is a very simple specimen of spiral waves, under symmetry-breaking perturbations. The work done by the Mathematical Biosciences Institute, especially from Professor Martin Golubitsky would be a great complement to this. In addition to its reference text in bifurcation theory, *Singularities and Groups in Bifurcation Theory*, the research of prof. Golubitsky deals with dynamical systems and bifurcation theory with symmetries and has multiple applications in biology such as in the visual cortex. Indeed potential patterns such as spiral waves occur in the visual cortex during migraines and these exhibit several types of symmetry. This potential research subject, combined with an elite establishment in applied mathematics make for the Ohio State University a motivating working environment and a prime potential destination for me.

Thank you for considering my application.



I'm a math major with a physics minor at Arizona State University and my research interests are algebraic geometry and algebraic number theory. Some of the professors I'd potentially like to work with at Ohio State University are Emanuele Macri and Wenzhi Luo.

Over the last three years I've pushed myself by attending Pennsylvania State University's Mathematics Advanced Study Semesters (MASS) program, participating in three research projects, and taking classes that were beyond my level at the time: twelve graduate courses and seminars on real and complex analysis, algebra, differential geometry, p-adic analysis, general relativity, formal groups, and functional analysis. The topics that have excited me the most throughout these research experiences and classes have been those that interact with many different areas of mathematics like exact sequences, the special linear group, projective space, and Galois theory. As a concrete example, I learned about universal properties in my functional analysis class. My professor assigned problem 5.19 from Folland's Real Analysis, and it seemed like just another exercise involving products of topological spaces—show there exists a unique map that makes some diagrams commute. But as I was thinking about the problem, I suddenly understood that what was I was trying to prove was not just a property of Cartesian products but that it was actually a characterization of products. It was a precise way of defining what a product was and I was thrilled because I knew direct sums, tensor products, inverse limits, and many other things could be defined this way too and that one could prove many things with such an exact definition. Later I found out that what I had been excited about is called a universal property and since then I've been very interested in category theory. My coursework refined my mathematical interests and increased my mathematical ability, but it was my research experiences that made me want to attend graduate school in mathematics.

Last summer I participated in an REU at the University of Connecticut. Over the course of the ten-week program, I worked on difficult problems, learned algebraic geometry, and tasted the mathematician's life. Our mentors began by teaching us some fundamentals about toric varieties and we quickly solved one of the questions about a class of toric varieties associated to trivalent trees. I grew confident that the next problems could be unraveled just as easily, but found instead that as we continued to work, the problems became more intractable and complicated. By the end of the next seven or eight weeks, our tactics failed again and again to penetrate the heart of the problems, and at the end of the full ten weeks, though we had solved some particular cases, a complete solution still remained elusive. Though we didn't finish answering all the questions, I loved working on these problems and discovered that what I find most rewarding is obtaining a deeper understanding of the context of these problems and making even a little progress—we are now preparing our paper, *Frobenius Splitting of Projective Toric Varieties*, for publication. I've learned to enjoy this type of extended struggle and I know that I'll be able to achieve much more if I keep working and learning about mathematics.

This semester, instead of taking another four math classes on things like graph theory or probability, I decided to take one math class, one seminar, and read on my own. Recently, I've been studying Rotman's Introduction to Homological Algebra and Miyake's Modular Forms, and working problems from past textbooks so that I can pass the qualifiers and jump into research as soon as possible when I go to graduate school. I also took the math GRE this semester, but didn't perform as well as I would have liked because I was distracted by my studies didn't practice enough problems beforehand. However, I have passed ASU's qualifying exams in real analysis I, real analysis II, and algebra II. After obtaining my PhD, I want to become a professor at a university that emphasizes research and teaching equally. I'd like to collaborate with people in other mathematical areas or physicists and biologists to solve interesting mathematical and real-world problems, but besides doing significant research, one of my passions is to become a great teacher. I enjoy teaching and explaining



math (I was the TA for a physics class and have tutored throughout my undergraduate years) and I want to inspire students to strive for excellence and help them achieve their dreams as I've been supported and inspired by my professors and mentors.

## Statement of Purpose:

Upon graduation from high school in 2003, I chose not to go straight to college. At that time I was fortunate enough to get a good job at a local plywood mill, where, through multiple promotions, I eventually earned a position of maintenance electrician apprentice. Working on industrial electrical systems brought to light my passion for diagnostics, analysis and problem solving. Using deductive logic in daily work prepared me excellently for a career choice in mathematics, and for the first time I truly wanted to become a successful student. Moreover, working hard and earning a living on my own taught me invaluable life lessons that I would not have otherwise learned. This experience showed me the importance of assuming my responsibilities to both myself and others. As a result, I had a renewed and burgeoned appreciation for education and because of this I credit my academic success significantly to everything I learned during that period of my life.

Returning to school has been the best decision ever; I look forward to school work and find every single lecture intellectually stimulating. As a result, I have discovered great fervor for learning and intellectual advancement, making these recent years the most enjoyable, productive and enlightening time of my life. With clarity, I see myself having a career in academia. Even more so I look forward to furthering my education, scholarship, and wisdom by drawing from the wealth of opportunity at The Ohio State University. Considering the measure and quality of research being done at OSU, I anticipate myriad opportunities to match my ambition. My level of experience, passion for studying mathematics, responsibility, and clear determination make me a prime candidate for your world-class graduate program in mathematics.

Among the several subjects one studies as an undergraduate and beginning graduate student, I take reservation in choosing a favorite. Every subject enthralls me, and the notable diversity of research being done at Ohio State is a reason for my wish to study with your renowned faculty. I must say that algebra in particular stands out slightly because of several exceedingly inspirational instructors, mentors, and peers. Dr. Michael Tsatsomeros exemplifies the level of scholarship I hope to achieve in my lifetime; he was an immediate inspiration. Moreover, Dr. Judith McDonald, now my graduate advisor, introduced me to abstract algebra, and has been my most supportive and meaningful mentor during my education. To me, the people are one of the most essential parts to academia, which is why I look forward to working under the first-rate faculty at The Ohio State University.

As I mentioned, peers have been an inspiration for me as well. A doctoral student at Washington State, Timothy Melvin, with whom I began my friendship as an undergraduate, was a notable contributor to why I chose my particular master's research project. I have been researching what is described as qualitative linear algebra. Essentially, given a matrix, how much information can we deduce about it (possible eigenvalues, invertibility, rank, etc.) with as little known about the matrix as possible. In particular, I am studying what are called 'zero-nonzero patterns' and the allowable spectra of these patterns. It is a very interesting, highly non-trivial problem that will be studied for years to come. At OSU, I look forward to the immense experience and diverse research fields of the faculty to continue my study in algebra, and all its intriguing subfields.

Thank you very much for your time and consideration. I greatly appreciate the opportunity to become a part of the outstanding mathematics department at The Ohio State University.

# Statement of Purpose



December 12, 2012

Meeting with the problem solving group every Wednesday is one of my best experiences at Carleton, not only because I enjoy problem solving, but also because I get to work on new problems with my professors. We work on problems that none of us has seen before, share ideas, and sometimes even get stuck together. Although I do not learn much knowledge from them directly, I benefit immensely from learning how they approach problems as experienced mathematicians. Furthermore, whenever I throw questions about some random concepts, it seems that the mathematics is just at their fingertips. I secretly hope that someday I can be a mathematician like them.

My SMALL REU experience at Williams enabled me to taste the joy of thinking like a mathematician. I did a project in phylogenetics in a group of three people, advised by Professor Devadoss. We read about the classical space of phylogenetic metric trees introduced by Billera, Holmes, and Vogtmann, and came up ways to describe the link at the origin of the space by the combinatorial polytopes such as associahedra and permutohedra. The space has a CAT(0) structure, enabling the computation of geodesics and centroids. Then we moved on to another space of phylogenetic trees, introduced by Kim, and studied mathematically first by Moulton and Steel. Our main goal was to figure out the connection between these two spaces. But how, in a concrete way, should we describe the connection? That was the hard part that was different from pure problem solving we needed to create the problems first. Generally speaking, what makes the new space different is that it allows edge lengths of the trees it parameterizes to go to infinity, which introduces unwieldy gluings of the original space that mess up the nice geometric properties. We tried similar approach via associahedra and permutohedra first, but that did not work. A different approach would be to describe the mapping between these two spaces that tells the story about how one space transforms to the other. Prof. Devadoss had a very vague idea of breaking the map into a discrete folding part that does not kill any dimensions, and a collapsing part that does. Nobody knew if it was even possible, but I thought the idea was illuminating. After hours of careful pondering, I finally came up with a set of combinatorial moves on trees induced by the infinite edges that perfectly capture the discrete folds, and further offer straightforward descriptions of the collapsing map. This became one of the key ideas of our research.

As an undergraduate, the knowledge I wished to know about my research was much more than what I actually knew. In this nine-week research, I found my previous experience in reading challenging math books helped me pick out what was worth learning in a short

period of time, and my experience in problem solving helped me tell what problems were worth attacking. However, the next project I did in combinatorics made me realize in order to push my limit further, there is so much more I need to learn.

I worked with another student on an independent study project in symmetric functions combinatorics advised by Prof. Egge for two terms. During the first term I learned the basics of symmetric functions and Young Tableaux such as Schur functions, Pieri rules, the RSK correspondence, and the Littlewood-Richardson rules. I also studied on my own some related representation theory and Specht modules. For the second term Prof. Egge suggested us to study the connection between Schur functions and the six-vertex ice model in statistical mechanics. We started off by studying the work done by Brubaker, Bump, and Friedberg, trying to understand how the partition function of a vertex model can be expressed in terms of a product that contains Schur function, with aid of the Yang-Baxter equation in proving symmetry. Applying this they gave a short proof of Tokuyama's deformation of the Weyl character formula. Although I understood the proof without much trouble, I had a hard time trying to figure out motivations behind their ideas. For example, the Weyl character formula and the denominator formula kept popping up in the papers we read. I had no clue why this concept in Lie groups has natural connection with this kind of combinatorics. But I could not wait to connect the missing links. A term was really short for a project like this, especially since I was not able to dedicate enough time for it. When I revisit this problem after taking more courses systematically in graduate school, the questions I have now will probably become clear.

That's said, I could not wait to learn more Lie algebra, algebraic geometry, and representation theory. For graduate studies I hope to work on problems in representation theory, geometry, and combinatorics. Coming from a background in computer science, I am also interested in problems related to graph theory, algorithms, and optimization.

Ohio State has a strong and diverse department that will allow me to learn and explore extensively in my areas of interest. Coming from a liberal arts college, I was sometimes frustrated that our math department does not offer enough advanced level math classes in the areas I am interested in. However, this forced me to become a dedicated self-studier. Studying independently has become a major way for me to acquire new math knowledge since the second half of junior year. However, although the texts I read were all written by masters, I could not communicate with them in a single word. I truly hope that in graduate school, I can work with peers and professors who can inspire me. Even sparkles of ideas would make a big difference for me. It is almost guaranteed that I will be able to find these people at Ohio State. I am confident that after college I will be prepared for graduate studies at Ohio State, and I am looking forward to all the mathematics that lies ahead of me.

My name is [REDACTED] and I am an aspiring number theorist. Motivated by a desire to see the big picture of mathematics, I have pursued mathematics seriously since the fall of 2007. As a leader in research and education, Ohio State University offers the ideal environment for me to continue my studies as a graduate student. Abundant coursework, research experience, and years of tutoring give me confidence that I will thrive in Ohio State's rigorous academics.

At Amherst College, I took 19 courses in mathematics, participated in 4 research projects, and tutored for 3 years in most math subjects. Professor Benjamin Hutz advised me on the first research project, in arithmetic dynamics, during the summer after my first year of college. I computed a uniform bound on the rational, iterated pre-images of  $-1$  under the family of quadratic polynomials over  $\mathbb{Q}$  using heights on elliptic curves. My result was published in [4]. The success of this project led us to a generalized uniform bound for the rational, iterated pre-images of  $\mathbb{Q}$  under quadratic polynomials over  $\mathbb{Q}$ . We published the results in [3].

In the summer of 2011, I participated in an NSF funded REU at Amherst College under the guidance of Professor Robert Benedetto in the field of arithmetic dynamics. We collected dynamical data [1] for quadratic polynomials over  $\mathbb{Q}$ . I resumed work with Prof. Benedetto in the Fall of 2012 to implement our computational technique for quadratic rational functions defined over  $\mathbb{Q}$ —the work is ongoing.

Professor David A. Cox advised me on a senior honors thesis in which I investigated the Galois theory of the lemniscate.<sup>1</sup> One may construct so-called lemnatomic extensions of  $\mathbb{Q}(i)$  by adjoining division points of the lemniscate much in the same way one constructs cyclotomic extensions of  $\mathbb{Q}$  by adjoining division points of the circle. The Galois groups of such extensions had been studied by Abel and others in their relation to ruler and compass constructions, but the group had not been fully determined. By developing a theory of lemnatomic polynomials akin to their cyclotomic cousins, I determined, with proof, the Galois groups of lemnatomic extensions.

I presented my findings at the SUMS, GSUMC, and HRUMC undergraduate math conferences in the spring of 2012. For my work, Amherst College awarded me the Robert Breusch Prize<sup>2</sup>, a Post-Baccalaureate Fellowship to continue research the following summer, and Summa Cum Laude. Together, Professor Cox and I found another proof of my main result as well as other original discoveries. We collected our work in a paper and submitted for publication; a preprint may be found at [2].

My interest in number theory began in 2007 while reading a book entitled *Excursions in Number Theory*. Upon matriculation at Amherst College, I registered for the introductory number theory course. Quadratic reciprocity, Gaussian integers, and the classification of primes expressible as the sum of squares inspired me to learn more. Prof. Hutz introduced me to elliptic curves and heights during my summer project in 2009. Subsequent courses in algebra, algebraic geometry, and complex analysis prepared me for a more advanced study of arithmetic. When Professor Djordje Milićević joined the Amherst faculty for the 2010-11

<sup>1</sup>The lemniscate is a plane curve defined by the polar equation  $r^2 = \cos(2\theta)$ .

<sup>2</sup>The Breusch Prize is awarded annually at Amherst College for the best senior thesis in mathematics.

academic year, I studied analytic number theory with him in a special topics course. David Cox taught me Galois theory, which led to the senior honors project discussed above. Senior year I devoted most of my time to the study of algebraic number theory from Marcus's *Number Fields* and Cox's *Primes of the Form  $x^2 + ny^2$* . I concluded my time at Amherst with a course on group representation theory.

Following graduation I continued my studies in number theory and geometry. The initial curiosity in the Gaussian integers developed into a fascination with the structure of number fields, and in particular the insights obtained via class field theory. Thus I have been working on Iwasawa's *Local Class Field Theory* and Silverman's *Arithmetic of Elliptic Curves*. Other projects include digesting all the wonderful expository pieces available on Keith Conrad's web page and pursuing further studies of the lemnatomic extensions. One nice result of this research is a generalized Wallis product related to the family of clover curves—I plan to submit the paper for publication once finished with graduate applications.

If admitted to Ohio State University, I would like to study algebraic number theory and the arithmetic of function fields with Professor David Goss. After consulting my advisers and reviewing his publications, I feel that our interests are well-aligned. In addition, the large and diverse department will offer opportunity to explore a variety of fields and expand my breadth. The active research seminar in number theory indicate a large and vibrant mathematical community of which I would very much like to participate. On a personal note, after my time on the East coast as an undergraduate I would be delighted to return to the midwest in closer proximity to my family.

I began work as a full-time math, physics, and computer science tutor in the fall of 2012 at Amherst College's Moss Quantitative Center. My studies and research are ongoing; I continue to work closely with the faculty at Amherst as well as regularly attending the weekly Pioneer Valley Number Theory Seminar to hear about recent developments in the field. I decided to not proceed directly to graduate school from Amherst for two reasons: one, between coursework and a thesis, I did not feel that I had enough time to give the application process the thought and effort it required; two, the Moss Quantitative Fellowship offered a great opportunity to simultaneously develop my teaching skills, master the undergraduate curriculum, and provide the time and resources for self-study. Having just completed the first semester of my life since age 5 not in school, I feel rejuvenated and excited to begin a doctoral program. Come fall, I will have an extra year of mathematics and maturity under my belt for the challenges to come.

Thus, considering my background, experience, and motivation, I feel prepared and qualified to join the Ohio State University community.

## References

- [1] R. Benedetto, R. Chen, T. Hyde, D. Kovacheva, and C. White, *Data from the small heights for quadratic polynomials 2011 REU*, <http://www.cs.amherst.edu/~rlb/quadpolydata/>.
- [2] D. Cox and ██████████, *The Galois theory of the lemniscate*, <http://arxiv.org/abs/1208.2653>.
- [3] B. Hutz, ██████████, and B. Krause, Pre-images of quadratic dynamical systems, *Involve*, 4(4): 343-363, 2011.

- [4] On the number of rational iterated pre-images of -1 under quadratic dynamical systems, *American Journal of Undergraduate Research*, 9(1):19-26, 2010.



My love for math began in first grade with the math games that we played every day after lunch. Obviously, the math that was a part of those games was quite different than the math I am currently studying. However, those games, combined with my desire to win, jump-started my passion for math. In third grade, I tested into the advanced math track that began in the fourth grade and lasted through high school. As time went on, I began to stand out in these math classes and my enjoyment of math continued to grow. In seventh, I started competing at math competitions and continued to enjoy the challenge they presented me as well as the success I had in competitions. At that point in time, there was no doubt in my mind that I wanted to study mathematics at the collegiate level and eventually become a professor. Because I was good at math and enjoyed it so much, majoring in math seemed like the logical thing to do. I had never really considered anything else.

Entering college, I began as math major in the honors program with no additional majors. During my freshman year, I had an outstanding physics professor, Dr. Robert Perry, whose interest in me led to my decision to double major in math and physics. Thanks to Dr. Perry, I was able to get physics research opportunities after both my freshman and sophomore years. Without his influence and guidance, I never would have thought about doing research that early in my college career. The two summers of physics research taught me many things, but one more important than the rest—that more advanced study in physics and research in that area has less appeal to me than continuing my study of math. Through my work in physics, I realized that it is math's elegance, beauty, and preciseness that draw me to it. In both physics research and physics classes, we have made many approximations—working in frictionless environments, looking only at dipole effects in electric fields, leaving off interactions when quantizing the hydrogen atom. I dislike these approximations. It is the exact nature of pure math that I love and the reason I want to study math at the doctoral level.

During the first quarter of my junior year at The Ohio State University, the Abstract Algebra course taught by Dr. Ronald Solomon piqued my interest more than any other course had up until that point due to its dealing with mathematical objects. To me, this was a new, interesting, unique way of looking at math. This topic helped to show me how diverse math is as a field of study which in turn began to solidify the idea in my mind that math was something I could enjoy spending my life studying. This past summer, I was given an opportunity to do math research. The group I worked with looked into flows on simplicial complexes. Due to a communication error between the students in the group and the advisor, I was able to learn a very important lesson in research: read, read, read. While I may not have found any useful results mathematically, I did find a useful result personally: I thoroughly enjoyed participating in the math research, much more than I did the physics research the previous two summers.

Given that I am majoring in physics in addition to math, I have not been able to take as many math electives as the majority of math majors. For this reason, I do not yet have well defined research interests. Thus, I am looking to attend a graduate school with a fairly large math department with faculty doing research in a variety of areas, something that The Ohio State University will be able to offer me. Because it was an

algebra course that piqued my interest, I do have some interest in furthering my knowledge in that area. Because the research I did this past summer in topology was enjoyable, I would also like to learn more about topology, too. The combinatorics class that I am currently enrolled in has been designed to show that combinatorial methods are applicable to many different fields of mathematics, thus, combinatorics would also be a desirable field of study due to its ability to allow me to continue to do work in many different fields. Simply put, I have a diverse set of mathematical interests and, thus, I am seeking a large, diverse program in which to work.

My educational goal is to complete a PhD. It is not, however, the degree that drives me but the math. I want to learn as much as I can about mathematics and to do my personal best to make a contribution to mathematics so that others can build on this work to further advance the study of math. I realize the best way to do this is by furthering my studies in a good graduate program. My professional goal is to make a contribution to math and to help spread my love of math to others. Thus, I would love to become a research mathematician and professor. Within the past year, I have had the opportunity to experience both teaching (as a teaching assistant) and participating in math research. I can see myself doing both for the rest of my life.

I believe I am prepared to pursue a PhD for one simple reason: I have always wanted to pursue a PhD. At first, the reason was quite juvenile. I had a competitive spirit and simply wanted to be the best at everything I did. I wanted to be in the best classes, I wanted to get the best grades, and I wanted to get the highest possible degree in my field. However, as time has passed, my reason has evolved. In middle school and high school, because I enjoyed helping my peers with their math homework, my desired career became an educator. Since I knew I could not put up with teaching anyone younger than college aged, I wanted to become a professor. Hence, getting a PhD became a means to an end. Finally, I got to college, and my reason for wanting to pursue a PhD evolved even more. While I have always enjoyed math, my love for it has grown even more since entering college. Before college, math classes were sequential. Every course built off of the previous one. In college, I have been able to see the various branches of math and the connections that exist between them that make math complex and interesting. Thus, I now want to pursue a PhD out of my love and curiosity for math. The other reasons are still present. I still want to be the best in everything I do, and I still want to be a professor, but my main reason for wanting to pursue a PhD is to learn as much as I can about the subject I love so much.

# Personal Statement



## Learning Experiences

I am a senior majoring in mathematics at Zhejiang University(ZJU). In 2009, I was admitted as a freshman to the Chu Kochen Honors College of ZJU, a place for elites designed to educate the top 5% of the students. Students in the CKC College faced intensive coursework and a very competitive environment since membership in the college is performance dependent. Nevertheless, I did very well in all my classes, ranking the 3rd among 269 in my first two years, and winning First Class Scholarship for two consecutive years. After careful consideration, I decided to follow my interest and chose mathematics as my major at the end of my first year.

### Student Seminars in ZJU: Leading Me on the Right Path

During the summer of my freshman year, I signed up for two student seminars, Point Set Topology and Algebra organized by Prof. S.C. Wong of Zhejiang University. This was a turning point in my mathematical education, as I was gradually lead to the world of modern mathematics by them. In a typical seminar, regular lectures were given by students themselves instead of teachers. I signed for more than 5 talks per seminar, and my presentation skills gradually perfected. Moreover, from learnings in the seminars, I realized the importance of working out examples and problems. Not only do they solidify one's current learnings, good examples often motivate further studies.

For instance, in a seminar using M.Artin's *Algebra* as textbook, I came upon an example showing that the Riemann Surface for the algebraic equation  $y^2=x(x-1)(x-2)$  was a torus by explicitly computing its locus in  $P^2$ . However, when I tried to do the same for  $y^2=x(x-1)(x-2)(x-3)$ , I found the locus becomes a torus with two points identified, and the original method fails. Naturally, I was very curious on knowing what the surface was for the latter equation. This motivated me in my later study of complex curves, where I found out that the correct answer is a torus using Riemann-Hurwitz formula. In fact, I recently learned a remedy for Artin's method: By applying the "blowing up" surgery to the singular points of the locus(twice), we separates the ramified points and obtain the torus without singularity. The seminars continued throughout my second year, covering a wide range of topics listed in my CV. Together with the progression of problem solving and asking new questions, they have lead me on the right path of learning mathematics.

### Experience in Hong Kong: Efforts in Geometry and Topology

During my third year from 2011.9 to 2012.6, I went to the University of Hong Kong as an exchange student with tuition paid by the Li&Fung Scholarship. This year brought up my interest

and efforts in various areas of geometry, where I took advanced courses and studied more topics on my own.

First of all, motivated by learnings from the previous year, I attended a course in my first semester about Riemann Surfaces taught by Prof. Naming Mok of HKU. The lecture was for first-year graduate students, and I was the only third-year undergrad in class. During the semester, we went through the Riemann-Roch theorem and the Mittag-Leffler problem in the language of sheaf cohomology. The course gave me a good understanding on basic tools like divisors and sheafs, which provides me a solid foundation for further study on complex algebraic geometry.

Another topic I studied intensively was algebraic topology . Throughout the year, I gradually built my foundations by working on the first three chapters of Hatcher's *Algebraic Topology*. I also enrolled in two advanced undergrad courses in HKU, *Manifold Theory* by Prof. Si Ye Wu, and *Geometric Topology* by Prof P.P.Wong. Among other things, the courses each introduced its own homology theory, de Rahm and singular, completed the picture of homology theory from a topological viewpoint. I then intended to pursue a more rigorous study of homology from an algebraic viewpoint, so I also enrolled in a class on Homological Algebra. It turns out that the algebraic machinery can provide new perspectives to old problems. For example, after studying derived functors, I learned a sheaf theoretic proof of de Rahm's theorem by constructing acyclic resolutions. This approach is conceptually cleaner, and can be copied to give new results such as the Dobeault Theorem.

I also learned Riemannian Geometry on my own in Hong Kong. Other than the basics, one of the most important techniques I learned was the use of moving frames. This method used by Chern is a powerful tool to analyzes the local data of a manifold, as I have seen from reading some research-level articles. For example, in an ongoing class taught by Prof. Hong Wei Xu , I studied papers by Yau on *pinching and classification of embeddings with constant mean curvature*, which used moving frames to obtain powerful results. I am certain that this will be of great assistance to my future learning.

My hard put efforts in geometry paid off. In the summer of 2012, I was selected as one of the 15 finalists to take the oral exam for the S.T.Yau's College Math competition on the Geometry and Topology section. This further strengthened my confidence to take a further step in studying geometry.

## **Future Planning**

I have since long made up my mind to go further on the training of becoming a mathematician. I have prepared myself well for my future graduate studies. Not only did I do well in my undergraduate courses, I also have plenty of undergraduate research experience listed in detail in my CV. I am clear on my interests and certain about my choice to continue my learnings.

My interests lie in many possible directions. Currently, differential geometry is my primary area of interest. I am especially interested in topics where geometry works together with other

branches of mathematics. For one instance, I enjoyed my current learnings on Mean Curvature Flows, where techniques from PDE and geometry combine together to classify singularities and asymptotic behavior of flows. For another, I'm also keen on the topic of pinching problems and rigidity problems of submanifolds, which I'm currently taking a course on. Besides differential geometry, I also have a good interest in complex geometry. My prior readings on Forster, as well as the first two chapters of Griffiths and Harris's book will give me a quicker grasp on the subject.

Ohio State University has one of the finest research groups in geometry in the world. It has many first-class mathematicians working in differential geometry and complex geometry like prof Fangyang Zhen. I hope that I would have the opportunity to study and make progress in Ohio State University.

It is essential to me to academically pursue an area in which I am passionate about. I enjoy mathematics because of the creativity and determination that is required to be successful in the field, and I like working on a problem for an extended amount of time to finally have the satisfaction in finding a solution. Because I am constantly fascinated by the elegance of mathematics and by the connections between and within its different subfields, I desire to continue exploring the discipline and conduct my own mathematical research. Ultimately, my goal is to become a research mathematician at a university and specialize in either topology or algebra.

I was first introduced to topology while participating in the REU program at Indiana University this past summer. The project I was assigned involved gluing  $n$  cubes together and representing the gluing as a triple of permutations in the symmetric group of  $n$  letters. Given a triple of permutations, I worked on finding a method for determining when the corresponding gluing was a 3-manifold. Initially, I approached the problem using only algebraic and combinatorial techniques. My mentor, Professor Christopher Judge, suggested using the fundamental group instead, and I found the problem could also be solved using this topological concept. I think that these links between topology and other fields of mathematics are attractive because combining approaches to a problem can often lead to a simpler solution. In my graduate topology course I was fascinated when I saw the fundamental theorem of algebra proved as an easy corollary of the computation of the fundamental group of the circle. I am excited to continue studying more examples of how topology is used to find elegant proofs of theorems that at first appear difficult to prove.

I am also interested in abstract algebra, so much that I wrote a research paper on the history of abstract algebra for a history of mathematics course. I investigated the origins of algebraic structures because I wished to know what factors and problems motivated the abstraction of these objects. While writing this paper, I also became inspired by the prominent female mathematician Emmy Noether and her influence in the field. Despite being a Jewish female in an era and place where women and Jews struggled to assert their roles in mathematics and the sciences, she revolutionized abstract algebra, especially in ring, group, and field theory. I enjoy learning about the complex relationships between the objects, which I have encountered this fall through Galois theory. It is interesting to see theorems involving automorphisms of groups used to describe properties of field extensions. Because of the intricate connections between the different algebraic structures, I want to study and carry out research in algebra in graduate school.

For the past two fall semesters, I have participated in a class designed to prepare students for the Putnam competition, and I received a score of 10 on the Putnam exam in December 2011. The Putnam problems engage my attention because they require a great amount of originality and persistence to be solved. One Putnam problem I find particularly intriguing involves inventing a finite game such that the probability of winning is an irrational number. It uses the binary representation of the given irrational decimal in order to define the finite game. This problem stayed on my mind for a couple days after I saw it because of the neat trick used in the solution. Working on Putnam problems has taught me to approach problems from different perspectives and think more innovatively.

Participating in the Putnam competition has allowed me to hone my problem solving abilities, which I also improved during the REU program this past summer. I benefited from focusing on one problem for eight weeks because it tested my patience and required that I employ my creative thinking skills. If I had had more time to work on the problem, I would have analyzed the following question: Does the limit of the ratio of the number of gluings of  $n$  cubes that are 3-manifolds to the number of gluings of  $n$  cubes as  $n$  approaches infinity exist? This question is important because if this limit exists, we would know the approximate probability that a random gluing of  $n$  cubes is a 3-manifold for large  $n$ . As part of the REU program, I also had the opportunity to attend weekly seminars on topics in various fields of mathematics, which made me excited to discover new areas that I have yet to study.

Attending the Ohio State University would allow me to continue pursuing my goal of becoming a mathematician at a research university. The department's strengths in both algebra and topology would help me choose the field in which I desire to specialize. Professor Michael Davis and Professor Ronald Solomon are examples of faculty members with whom I would like to work. Professor Davis' work combining geometry and topology and Professor Solomon's research in the theory of finite groups are areas in which I see myself conducting my own research. I know I am ready and prepared to continue my studies in graduate school and into mathematical research. The rigorous, diverse menu of math classes that I have pursued as an undergraduate have established a solid mathematical foundation that I have tested, built on and expanded through participation in competitions and research.



## LETTER OF PURPOSE

---

PhD Applicant

I aspire to earn a PhD in mathematics and to conduct societally significant research as a professor at a college or university. This path weaves together three strong interests of mine that have been developing steadily for years: mathematics, the environment, and education. The Ohio State University PhD program provides excellent opportunities to develop further in each area.

Though it has taken me some time to recognize it, I have long been a mathematician at heart. Although I majored in biology at Carleton, I took math courses wherever they fit into my schedule. I delighted in discovering interdisciplinary applications and pursued an independent study of probability models of DNA sequence evolution with math professor Bob Dobrow. In my post-college years, my interest in math continued to crop up persistently. Whether designing experiments in the genetics lab at Harvard Medical School or balancing forces in a rustic rigging system with the Student Conservation Association, I gravitated to quantitative analysis. In late 2011, Donella Meadows' book *Thinking in Systems*<sup>1</sup> piqued my interest in math as a lens for understanding and protecting the natural world. Mindful of my life-long inclination towards math and eager to explore the intersection between math and sustainability, I embarked on the Smith post-bac program.

My time at Smith has deepened my love for math and my desire to pursue a mathematical career. Immediately upon my arrival, I learned of an exciting opportunity in environmental modeling with theoretical hydrologist Andrew Guswa. I joined his research team this past summer and helped develop tools that support the valuation of ecosystem services such as fresh water supply and flood mitigation. Data scarcity poses a considerable barrier to the quantification of ecosystem services worldwide, so we devised a stochastic model that downscales temporally coarse rainfall data to improve predictions of the rainfall-runoff water balance. This project, which was completed in consultation to the Natural Capital Project, gave me valuable practice in applying math to real-world problems and collaborating with partners in and outside of academia. I provide further details on this and two other formative Smith research experiences in a supplemental document.

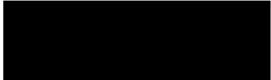
---

<sup>1</sup> Meadows, Donella. *Thinking in Systems: A Primer*. Ed. Diana Wright. White River Junction, Vermont: Chelsea Green Publishing, 2008.

The post-bac program has offered experience not only in applied mathematics but also in the foundational areas of algebra and analysis. I am drawn to the abstraction and rigor that connects concepts throughout mathematics, and my appreciation for the structure of the field as a whole has matured. For example, our recent studies in real analysis have stimulated my interest in measure theory as the basis of probability, and I plan to study it independently in the coming months. I look forward to deepening my grounding in mathematics through graduate coursework, learning not only tools for application but also the theory that makes them work.

In the long term, I aspire to conduct research and to teach as a professor at a college or university. I hope to create innovative mathematical tools that support the sustainability of our society – for example, by improving climate forecasts, natural resource stewardship, or human infrastructure efficiency. The teaching component of graduate work and professorship also appeals to me. I have consistently enjoyed working as a TA, tutor, and substitute teacher. At Carleton College, my interests and promise in teaching were recognized with the Jefferson Natural Sciences Teaching Award. Whether holding a TA session, studying with classmates, or meeting with my research group, I have a tendency to jump up to the chalkboard and build a common understanding by visually representing our ideas. This inclination towards communication will help me both as an educator and as a research collaborator.

The Ohio State University PhD program is an excellent fit for me in several ways. The strength of the faculty in many fields including applied mathematics, mathematical biology, and probability would provide great opportunities to explore these interests. The integrated mathematics department makes it possible to explore multiple areas before settling on a thesis topic. Furthermore, working with students as a teaching associate would help me to grow as an educator. For all these reasons, I would be delighted to pursue my graduate education at Ohio State University.



## Statement of Objectives

Since entering college as a mathematics major, I've suspected that I would pursue a PhD in mathematics. Over the past four years at Oberlin College, my experiences have helped solidify these plans. I've sought ways to extend my education outside of Oberlin's normal course offerings, through independent study and the Budapest Semesters in Mathematics program. I've also had two summer opportunities to prepare myself for graduate school, at the Cornell University Summer Math Institute and the Williams College SMALL Research Experience for Undergraduates. These experiences have challenged and excited me, and prepared me to pursue a future in mathematics.

My first exposure to academia came the summer after my sophomore year, at the Cornell Summer Math Institute. Before this point, I knew that I enjoyed mathematics, but I had very little exposure to what might lie in my future. Within two days of arriving at Cornell, all of my previous notions of graduate school and academia were broken down, and I was left questioning myself and what I wanted. It took the next eight weeks to rebuild my self confidence. We spent each day attending an algebra course and working on research projects in functional analysis. The schedule was demanding, but not impossible; it was intended as a rough estimate of the life of a graduate student. As a student at a small liberal arts college, this experience was invaluable. At Cornell, I was able to talk with current graduate students and hear what they had to say about graduate school, both good and bad. Their advice, warnings, and anecdotes reshaped my understanding of what graduate study entails. As I developed a stronger idea of the demands of graduate school, I reevaluated myself and my goals. By the end of the summer, graduate study was no longer an uncertain possibility for my future, but a concrete plan.

A few short weeks after leaving Cornell I arrived in Budapest. Following my summer in Ithaca, I was eager to take a class load that wouldn't be possible at Oberlin, both in number and rigor of courses. Many of the courses were difficult, yet engaging, and presented mathematics from a new perspective. It was wonderful to escape the boundaries of Oberlin's small department and take classes from different professors. The semester challenged me, academically and personally, but I finished it confident in my abilities. My experience at Cornell showed me what graduate study demands; that fall taught me I am capable of it.

My first real research experience came this summer in the Williams College SMALL REU. My group's project was on Algebraic and Geometric Combinatorics, advised by Elizabeth Beazley, a topic much closer in line with my mathematical interests than functional analysis. Our goal was to generalize a projection on the affine Grassmannian due to Berg, Jones, and Vazirani to one on parabolic quotients of affine Weyl groups in other classical Lie types. The original projection is described as a bijection in several different models for realizing the elements of the affine Grassmannian, both combinatorial and geometric, which have analogues in other the other types. We were able to successfully generalize the projection to type  $C$ , interpreting the map in each of the models. Many of the results of our research are available in the paper we wrote, *Bijjective Projections on Affine Weyl Groups*, available on arXiv at <http://arxiv.org/abs/1212.0771>; section 6 in particular highlights my contributions. Throughout the summer I focused my work on the geometric model with great success. We were able to geometrically

realize the bijection as an explicit projection between alcoves, which allowed us to prove deeper properties of the projection beyond those evident from the combinatorics alone. A number of our results for type  $C$  extend beyond the scope of the paper we were hoping to generalize.

Besides writing a paper, our group has focused on presenting our research at conferences. In late July, we gave a talk at the Young Mathematicians Conference at The Ohio State University and earned second prize for the quality and content of our presentation. At YMC, I was able to meet mathematics students from around the country, as well as get a feel for OSU's math program. Besides attending at YMC, we are presenting again in January at the Joint Mathematics Meetings, and we've also applied to the Formal Power Series and Algebraic Combinatorics conference next June in Paris, France.

My summer at SMALL was more than just nine weeks of research. It was also nine weeks which I spent surrounded by top mathematics students from around the country. I was awed by their caliber, but also by my ability to hold my own working with them. While I had not necessarily taken the same level of coursework as some of my peers, what I knew, I knew well, and I was able to quickly learn anything I was missing. As a researcher, I was among equals. This reassured me that despite coming from a small college with no graduate program, I am well prepared to succeed in graduate school among students who benefited from opportunities which I lacked, such as the chance to take graduate courses. This experience also gave me the chance to engage in real, meaningful research. Unlike the previous summer, I felt there were useful applications to our work and that it was not far off from research I might do in graduate school. The most important lessons I learned this summer were not specific to the problem we were working on, but rather pertinent to mathematical research in any area. This summer taught me how to better effectively engage with a problem and make progress toward a solution. Most importantly, I learned that mathematical research is something I enjoy and want to keep doing.

I have also sought opportunities to extend my Oberlin education past the course offerings, which I've exhausted. I've engaged in independent study in three different contexts in the past year, studying combinatorics during Oberlin's Winter Term and representation theory of the symmetric group as a private reading last spring, and this fall I was invited to do an honors project under the supervision of Susan Jane Colley. For my project, I am studying commutative algebra at the graduate level. I've found these opportunities for independent study beneficial in a number of ways: I've learned interesting and useful material, gained the ability to learn independently, and practiced communicating mathematics through presentations of what I've learned to my advising professor.

I am drawn to OSU not only because of the strength and size of the program as a whole, but also because of its strength in my field of choice, algebraic geometry, and related fields. I'm drawn to algebraic geometry due to the deep relationship between algebra and geometry, which compares with the interplay between combinatorics, algebra, and geometry in my summer research, and I'm eager to engage in research on my own. I think OSU would be an excellent place for me to do this, especially coming from a small liberal arts college; the size and breadth of the department would expose me to fields I didn't encounter as an undergraduate. I've worked hard to prepare myself for graduate study and I'm excited to begin this next step.

**Name:** [REDACTED]

***Personal Statement – PhD in Mathematics, Ohio State University, Fall 2013 Admission***

I grew up to love Mathematics ever since I was in junior high school in a small town called Sidoarjo, probably famous for its Sidoarjo mud-flow disaster. Although my school in Indonesia only had basic amenities, I was able to learn calculus when I was 13, thanks to my dedicated teacher, Ms. Revi, who privately taught and gave me a book to read. It was my first encounter with higher level Mathematics, which I thoroughly enjoy every day. I took part in numerous Mathematics competitions under her encouragement, eventually attaining a gold medal in the national round of Indonesian Mathematics Olympiad, the first from my town. It was also thanks to her support that I was awarded a full academic scholarship to study in Singapore, and here I am studying pure Mathematics at National Univ. Singapore. There was a time when studying Mathematics at advanced level in a reputable institution in the US was just a far-fetched dream for me, but now I feel ready to take one step further to fulfil my dream as a mathematician by applying to the PhD program at Ohio State University (OSU).

My main area of mathematical interest is in analysis, more specifically in harmonic and functional analysis and their interactions. My first acquaintance with this field was when I had a research experience in my university on Fourier analysis at the beginning of my third year. Under the guidance of my mentor, I studied Fourier transforms in  $L^p$  and Schwartz space rigorously. I got a chance to study important objects in harmonic analysis, from maximal functions to covering lemmas, Riesz-Thorin interpolation to convolution operator, and many others. I was particularly amazed by the power of such interpolation theorems to interpolate boundedness of operator  $T: L^p \rightarrow L^q$ , giving interesting results such as Hausdorff-Young and convolution inequalities. I was also fascinated by how complex-analytic technique (Phragmen-Lindelof principle) is used to prove it. Throughout, my mentor introduced me to various classical books such as 'Introduction to Harmonic Analysis' [Katznelson], and 'Introduction to Fourier Analysis on Euclidean Spaces' [Stein]. They were beautiful. Finally, I got a chance to study some old papers on  $p$ -multiplier, where inclusion relations between various function spaces, and boundedness of relevant operators are studied. It then culminated in a talk that I gave to some honours students. It really was an enjoyable experience, and I begin to seriously consider this as my field of interest.

Subsequently, I had privilege to do summer research abroad. I was awarded a competitive international scholarship by Math. Sci. Inst., Australian National Univ. (ANU) to do a summer research, where I studied Sobolev spaces and its applications to second-order elliptic PDE, under the guidance of Prof. John Urbas. It was illuminating for me to see PDE problem cast as an operator acting on suitable function spaces, thereby allowing functional-analytic methods to work. I explored various properties of Sobolev spaces, applied that to study the questions of existence and regularity of solutions. I also had a glimpse at aspects of ~~operator theory~~ ~~linear~~ ~~PDE~~ and Holder-type estimates on nonlinear equations. In 2012, I was awarded a joint international scholarship by Imperial College London (ICL) and NUS to pursue research in areas of advanced Mathematics not available in my university. There are only 2 recipients of this award in the whole university. My project was on spectral theory of Schrodinger operator, in which I learnt about properties of its spectra, and explored various conditions for essential self-adjointness of the operator, discreteness of the spectra, among others. Throughout I got a taste of various important techniques in operator theory and functional analysis. It was interesting to see how problems motivated from Physics are solved using techniques from functional analysis and PDE theory. Overall, these programmes allow me to see interconnections between various branches of analysis, and I become even more intrigued to pursue further research in this area.

I am currently doing another summer research in ANU with a full scholarship under mentorship of Dr. Pierre Portal. At the moment I am reviewing some background materials on harmonic and functional analysis, such as BMO, Hardy spaces and singular integral theory, and some results on holomorphic functional calculus of differential operator in  $L^p$ . It is hoped that soon we could do a small project and survey papers of more recent interests on Hardy and BMO spaces associated with operators [J. Amer. Math. Soc., Vol. 18, No. 4 (Oct 2005), p943-973]. With the help of my mentor, I also enrol in an online workshop on 'Operator Semigroup and Dispersive Equations' (organized by KIT), where I learn functional-analytic approach to dispersive evolution equation, such as Schrodinger and semilinear wave equations. It is an interactive experience, where participants learn from a series of lectures and take turn to submit solutions. It also allows me to see how semigroup theory and functional analysis interact to provide a natural framework to study evolution equations.

Meanwhile, I continue to pursue my passion for research by writing an honours thesis on Riemann-Henstock approach to Feynman path integral under mentorship of Prof. Tuan Seng Chew, where delicate gauges and estimates are used to make rigorous a type of functional integral under the framework of Henstock integral. In my free time, I like to read papers which are accessible at my level of understanding from local journals and try to improve on it. Some of my small results have appeared in the regional journals (one already appeared, another one accepted). On a few occasions, I give talks in honours student seminar on various topics in analysis I found interesting. This is something that I enjoy doing, and I hope of doing it more professionally at a higher level as a mathematician.

I also continue to read advanced textbooks either independently or under directions. In particular, I should mention that I have done almost every question from advanced books like 'Measure and Integral' [Wheeden-Zygmund], 'Real Analysis' [Royden], 'Real and Complex Analysis' [Rudin], 'Partial Differential Equations' [Evans], 'Functional Analysis' [Rudin], and 'Analysis on Manifolds' [Munkres]. I am currently in the process of finishing 'Functions of One Complex Variable I' [Conway] as part of my honours coursework. For the past semester, I did a directed reading focusing on the seminal paper ' $H^p$  Spaces of Several Variables' [Fefferman, Stein], using the book 'Real Harmonic Analysis' [Auscher] as references. They both widen my horizons to powerful and beautiful results in harmonic analysis.

I really feel that doing research in mathematics is my cup of tea. In particular, OSU is a perfect fit since there are a few experts working in the area of my interests. I am very much interested to work with Prof. J. McNeal, whose research area is in complex analysis. He has produced many outstanding papers on the study of Bergman kernel,  $\bar{\partial}$ -operator, and other related areas, all of which involve careful estimates and original ideas in several complex variables as well as other areas such as harmonic analysis. As I look at the type of work he and his PhD students did for research recently (related to Bergman kernel), I feel that it is of the type of Maths that I would like to pursue further. The work of other professor, such as that of Prof. O. Costin on Borel summability and Schrodinger operator is also one that is interesting for me to study further.

It is my sincere hope that I could be admitted to do mathematics research in a stimulating, yet challenging environment such as OSU. Upon obtaining a PhD, I hope of getting a post-doctoral job in a university, where I could fulfil my passion to do research and teach. It is my dream that someday after becoming a successful mathematician, I could contribute back to my country, Indonesia, to improve its research and education in Mathematics. I hope of being able to build a vibrant community of mathematicians from Indonesia in a distant future, which is virtually non-existent at the moment. And as a first step, I certainly hope to be among a handful of Indonesians who could complete their PhD in this university.