Preparing for Mathematics Olympiad

All views expressed here are my personal & are from my 3 year experience on search of good study material for young mathematicians. I have also given a rough grouping of some useful books.

A. **Why to Prepare**: To pursue Mathematics as a field of research later in life, starting with Undergraduate Courses at Chennai Mathematical Institute, Indian Statistical Institute or any university abroad.

B. **Traits Needed**: Love & Devotion for beauty of mathematics

C. **When to start:** One can appear for Regional Math Olympiad (RMO) from class VIII to Class XI *(not for class XII from year 2014 onwards)*. I can't comment when one should start as every child is special & different.

NOTE REGARDING ALL BOOKS: *I have read many, but not all of these books [as few are still in my "To Read" list] In case you can't find any of these books in market (out of stock or its very costly) simply e-mail me at gaurishkorpal01@gmail.com and I will help you to get one or can suggest an equivalent book.*

D. **School Mathematics v/s Olympiad Mathematics:** **"**School mathematics" (or "IIT Mathematics" for class XI & XII students) or "Olympiad Mathematics" seem to be of different nature but at core, they are actually same thing i.e. "Mathematics". Only our prospective is different in both cases.
Here I can point out some similarities in both to convince you:

1. *NCERT Mathematics Textbook for Class IX [NCF – 2005]* is fantastic book to start for Olympiad mathematics as it touches nearly all topics (like *geometry*, *polynomials, Number Theory (rational - irrational numbers)*, *Introduction to mathematical modelling*) which we study at advanced level for Olympiads.
   FOR EXAMPLE: I was spell bound by chapter - "*Introduction to Euclid's Axioms*" and I ended up reading *"Euclid's Window by Leonard Mlodinow "* & *"Fun & Fundamentals of Mathematics by Narlikar"*
2. *NCERT Mathematics Textbook for Class X [NCF – 2005]* also consists of basics of "Number Theory" topics like "*Euclid's division algorithm*". Moreover the appendices on "*Proofs in Mathematics*" & "*Mathematical Modelling*" are worth reading even at later stages.
   FOR EXAMPLE: The discussion on 'Proof by Contradiction' is awesome.
3. *NCERT Mathematics Textbook for Class XI[NCF – 2005]* includes some of most fundamental & important topics of Olympiad mathematics like "*Set Theory*", "*Principle of Mathematical Induction*", "*Summation of Series*", "*Binomial Theorem*" & "*Permutation & Combinations*". Also appendices on "*Infinite Series*" & "*Mathematical Modelling"* are worth reading.
4. *NCERT Mathematics Textbook for Class XII [NCF – 2005]* major focus in on Calculus, but still its appendices on "*Proofs in Mathematics*" & "*Mathematical Modelling*" are worth reading.
   FOR EXAMPLE: In Appendix – 1 (Proofs in Mathematics), there are proofs for two beautiful theorems of Number theory (a) Prime Numbers are infinite & (b) $2^{2^n} + 1$ is not prime.

E. **Suggested Readings to get an insight of "Beauty of mathematics"** :

- *History of Development of Mathematics* :

1. Men of Mathematics (Volume 1 & 2) *by E. T. Bell*
2. The Mathematical Experience *by P. J. Davis & R. Hersh*
3. Euclid's Window : The story of geometry from parallel lines to hyper space *by Leonard Mlodinow*
4. Fermat's Last Theorem *by Simon Singh*
5. Journey Through Genius *by William Dunham*
6. Notebooks of Srinivasa Ramanujan @ http://www.imsc.res.in/~rao/ramanujan/NotebookFirst.htm
   Must peep into the notebooks of Genius, many parts of which are still topic of research

- *Biographies or Autobiographies of some Great Mathematicians*:

1. The Man who knew infinity *by Robert Kanigel* [biography of Srinivasa Ramanujan Iyenger]
2. The Man who loved only numbers *by Paul Hoffmann* [biography of Paul Erdὸs]
3. A Beautiful Mind *by Sylvia Nasar* [biography of John Nash]
4. Adventures of a Mathematician *by Stanislaw M Ulam* [autobiography]
5. A Mathematician's Apology *by GH Hardy* [autobiography]

F. Step by Step Learning:

Here "Levels" have been marked by me as per maturity level of mind needed in my opinion to understand what's written, in these books. Also order of books in each level again specifies increasing order of difficulty of book in that level. You can keep on switching between various levels and books as per your comfort level as every child is different.

Level – 1 [a] (Expanding Horizons)

Book	Remarks
Mathematical Circles (Russian Experience) *by Fomin, Genkin, Itenberg*	Thought provoking [for VIII & IX class]
A Mathematical Mosaic: Patterns & Problem-Solving *by Ravi Vakil*	My favourite!
Arithmetic and Algebra: Numbers and the beginnings of Algebra *by Shirali*	Ideal for beginners.
First Steps in Number Theory: A Primer on Divisibility *by S.A. Shirali*	Discusses congruences in good detail
Hands-on Geometry *by Christopher M. Freeman*	Step-by-step guide to learn construction
Non-Routine Problems in Mathematics *by AMTI (Editor: V. K. Krishnan)*	Lovely book but has few wrong solutions
The Cartoon Guide to Calculus *by Larry Gonick*	An illustrative guide to elementary calculus
What is Mathematics ? *by Richard Courant and Herbert Robbins*	This book will be your friend for 4 years

Level – 1 [b] (Introduction to Higher mathematics with "Little Mathematics Library")

These books include short introductory material (without much detail) on various topics which can help student to get an idea of different fields of mathematics, written especially for high school students preparing for Olympiad.
All these can be downloaded legally from http://mirtitles.org/2012/09/06/little-mathematics-library-taking-stock/

Topic			Book
Geometry	2 D Geometry	General	Lobachevskian Geometry *by Smogorzhevsky*
			Remarkable Curves *by Markushevich*
		Constructions	Dividing Line Segment in Given Ratio *by Beskin*
			Geometrical Constructions using Compasses Only *by Kostovskii*
			The Ruler in Geometrical Constructions *by Smogorzhevsky*
		Proofs	Proof in Geometry *by Fetisov*
			Induction in Geometry *by Yaglom & Golovina*
		Analytical Geometry	Method of Coordinates *by Smogorzhevsky*
	3 D Geometry		Stereographic Projection *by Rosenfeld & Sergeeva*
			Images Of Geometric Solids *by Beskin*
Algebra	General		Fascinating Fractions *by Beskin*
			The Method of Mathematical Induction *by Sominskii*
			Algebraic Equations of Arbitrary Degrees *by Kurosh*
	Complex Numbers		Complex Numbers and Conformal Mappings *by Markushevich*
	Number Theory		Fundamental Theorem of Arithmetic *by Kaluzhnin*
			Solving Equations In Integers *by Gelfond*
	Inequalities		Systems of Linear Inequalities *by Solodovnikov*
			Inequalities *by Korovkin*
	Combinatorics		Pascal's Triangle *by Uspenskii*
			The Mote Carlo Method *by Sobol*
Calculus	General		Calculus of Rational Functions *by Shilov*
			Plotting Graphs *by Shilov*
	Differential Calculus		Differentiation Explained *by Boltyansky*
			Method Of Successive Approximations *by Vilenkin*
	Integral Calculus		Areas and Logarithms *by Markushevich*
Miscellaneous	Economics		Elements Of Game Theory *by Venttsel*
	Programming		Posts Machine *by Upensky*
	Research		Godel's Incompleteness Theorem *by Uspensky*

Level – 2 [a] (Building basics: Learning Theory)

CAUTION : Don't stick to one topic & keep switching (as per your wisdom). Learning should be natural & effortless as most of textbooks are for undergraduate levels, so doesn't worry if you don't understand something in one go.

Topic			Text Book
Geometry			1.The Foundations of Geometry *by David Hilbert* 2. Geometry Revisited *by H. S. M. Coxeter and S. L.Greitzer* 3. Triangles: Constructions & Inequalities *by Subramanian & Murlidharan [AMTI]* 4. The Elements of Coordinate Geometry *by S. L. Loney* 5. Projective Geometry *by H. S. M. Coxeter* 6. Geometric Transformations (Part – I,II,III) *by I. M. Yaglom*
Trigonometry			Plane Trigonometry (Part-1 & 2) *by S. L. Loney*
Algebra	General		1. Higher Algebra *by Hall & Knight* 2. Discrete Mathematics with Graph Theory *by Goodaire & Parmenter* 3. generatingfunctionology *by H. S. Wilf*
	Sequence & Series		A Primer on Number Sequences *by S.A. Shirali*
	Combinatorics		1. An Introduction to Combinatorics *by Alan Slomson* 2. Combinatorics : Theory & Applications *by V. Krishnamurthy*
	Linear Algebra		Introduction to Linear Algebra *by Gilbert Strang* OR An Introduction to Linear Algebra *by V. Krishnamurthy, V. P. Mainra & J. L. Arora*
	Inequalities		Inequalities – An Approach Through Problems *by B. J. Venkatachala* OR Inequalities- A Mathematical Olympiad Approach *by Manfrino, Ortega, Delgado*
	Number Theory	General	Elementary Number Theory *by David Burton*
		Diophantine Equations	An introduction to Diophantine Equations – A Problem Based Approach *by Titu Andreescu, Ion Cucurezeanu & Dorin Andrica*
	Probability		Probability Theory (First Steps) *by E. S. Wentzel*
Analysis			Calculus (Vol. 1) *by Tom M. Apostol* OR Introduction to Calculus & Analysis (Vol. 1) *by Richard Courant & Fritz John*
Functional Equations			Functional Equations and How to Solve Them *by Christopher G Small*
Iterations			Adventures in Iterations (Vol. 1) *by S. A. Shirali*
Chaos			1. Chaos *by James Gleick* 2. Videos on Chaos (9 Chapters) at http://www.chaos-math.org/en

Level – 2 [b] (Optional Further Investigations - for specific topic lovers only)

Topic		Text Book
Geometry		1. Geometry *by Pogorelov* 2. Introduction to Geometry *by H. S. M. Coxeter* 3. Non-Euclidean Geometry *by H. S. M. Coseter*
Algebra	General	1. Algebra *by Michael Artin* 2. Higher Algebra *by A. Kurosh*
	Inequalities	Inequalities *by G. H. Hardy, J. E. Littlewood & G. Polya*
	Combinatorics	A course in Combinatorics *by J. H. van Lint & R. M. Wilson*
	Number Theory	An Introduction to the Theory of Numbers *by G.H. Hardy*
	Linear Algebra	Linear Algebra *by Hoffman Kenneth , Ray Kunze*
	Probability	1. The Theory of Probability *by B. V. Gnedenko* 2. An Introduction to Probability Theory & its Applications *by William Feller*
Analysis		Calculus (Vol. 2) *by Tom M. Apostol* OR Introduction to Calculus & Analysis (Vol. 2) *by Richard Courant & Fritz John*
Functional Equations		Lectures on Functional Equations & their applications *by J. Aczel*
Iterations		Adventures in Iterations (Vol. 2) *by S. A. Shirali*
Chaos		Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise *by M. Schroeder*

Level – 3 (Learning General Approach for Problem Solving)

Book	Remarks
How to Solve it *by G. Polya*	Classics on "Learning Problem Solving" Also solve "The Stanford Mathematics Problem Book" to practise the concepts taught in these books.
Mathematics and Plausible Reasoning (Vol 1 & 2) *by G. Polya*	
Mathematical Discovery *by G. Polya*	
Techniques of problem solving *by S. G. Krantz*	Covers a wide range of topics.
Solving Mathematical Problems *by Terence Tao*	Short & beautifully written book
Problem Solving Through Recreational Mathematics *by Averbach & Chein*	Innovative way of learning!
Problem Solving Strategies *by Arthur Engel*	Must read its first three chapters.
The Art and Craft of Problem Solving *by Paul Zeitz* (along with Student's Manual & Instructor's Manual)	First four chapters are worth reading. Many classical problems are there.
How to Solve it - Modern Heuristics *by Michalewicz & Fogel*	----

Level – 4 (Problem Solving Ideas for specific Topics)

All these books have a basic theme of "classification" of problems according to "useful ideas"

Book	Remarks
Polynomials *by E. J. Barbeau*	Good collection of Challenging problems
Pell's Equations *by E. J. Barbeau*	
Introduction to Functional Equations Theory & Problem Solving Strategies for Mathematical Competitions & beyond *by Costas Efthimiou*	Also includes introduction to "Iteration" & "Chaos"
Functional Equations – A Problem Solving Approach *by B. J. Venkatachala*	Unique type of classification of problems
Trigonometric Functions : Problem Solving Approach *by Panchishkin & Shavgulidze*	Awesome
Number Theory Structures, Examples, and Problems *by Titu Andreescu and Dorin Andrica*	Good books for brushing up of basics
A Path to Combinatorics for Undergraduates: Counting Strategies *by Andreesu & Feng*	
The Cauchy–Schwarz Master Class: An Introduction to art of Mathematical Inequalities *by J. M. Steele*	Grand Finale of Inequalities
Aspects of Combinatorics: A wide Ranging Introduction *by Victor Bryant*	Assorted ideas...
Straight Lines & Curves *by Vasilyev & Gutenmacher*	Awesome
Problems & Solutions in Euclidian Geometry *by M. N. Aref & William Wernick*	-----

Level – 5 (Refining Problem Solving Art by Learning Tricks)

Book	Remarks
Adventures in Problem Solving *by S.A. Shirali* *The book Challenge & Thrill of pre College Mathematics by C. R. Pranesachar, B. J. Venkatachala, K. N. Ranganathan & V. Krishnamurthy supplements this book as it consists some interesting topics in Geometry (like Erdὸs-Mordell Theorem) & Combinatorics (like Ramsey's Theorem) which have been posed as exercise in this book.	Problems based upon interesting concepts. Also teaches how to get aid from computer. * The supplementary book suggested here has many typos & some weird topics like Synthetic Division (pg 504), linear equations in 4 variables (pg 438) which can be found in classic texts like "Higher Algebra by Hall & Knight".
Mathematical Olympiad Challenges *by T. Andreescu & R. Gelca*	These focus on developing tricks into methods and methods into mastery. Each section begins with a theme of problem solving with one or two examples that are easy if we apply the theme, and then gives a whole bunch of problems that need to be solved by variants of the basic theme.
Mathematical Olympiad Treasures *by T. Andreescu & B. Enescu*	
Problems from the book *by Titu Andreescu & Gabriel Dospinescu*	
Mathematical Miniatures *by Titu Andreescu & Svetoslav Savchev*	Contains really challenging problems divided into 50 ideas. Also a beautifully organised book!
Problem - Solving Through Problems *by L. C. Larson*	Don't have solutions for all Problems proposed
Winning Solution *by Edward Lozansky & Cecil Rousseau*	It consists of many difficult solved (& unsolved) problems along with many new theorems.

Level - 6 (Practising for Confidence)

Book	Remark
Gems – Junior *by V. Seshan [AMTI]*	331 RMO Level Problems
Gems – Inter *by S. R. Santhanam [AMTI]*	148 INMO Level Problems
Problem Primer for Olympiads *by Pranesachar, Venkatachala & Yogananda*	110 IMO Level Problems
Test of Mathematics at 10+2 Level *by Indian Statistical institute*	Good Subjective Questions [without solutions]
Problems in Plane Geometry *by I.F. Sharygin*	A good collection
101 Problems in Algebra *by Titu Andreescu and Zuming Feng*	Problems from the training sessions of USA IMO Team
102 Combinatorial Problems *by Titu Andreescu and Zuming Feng*	
103 Trigonometry Problems *by Titu Andreescu and Zuming Feng*	
104 Number Theory Problems *by Titu Andreescu, Zuming Feng and Dorin Andrica*	
Selected Problems & Theorems in Elementary Mathematics *by Shklyarsky, Yaglom*	Arithmetic & Algebra Problems
Fifty Challenging problems in Probability *by F. Mosteller*	Awesome

Level – 7 (Practising for Perfection)

Book	Remark
The Wohascum County Problem Book *by G. T. Gilbert, M. I. Krusemeyer, and L. C. Larson*	Carefully selected 130 problems.
The Math Problems Notebook *by Valentin Boju and Louis Funar*	Ultimate Practise.
360 Problems from Mathematical Contests *by Titu Andreescu and Dorin Andrica*	A decent source of practise
IMO Compendium by Du˘*san Djuki´c, Vladimir Jankovi´c, Ivan Mati´c & Nikola Petrovi´*	Past IMO till 2004
Putnam & Beyond *by by Titu Andreescu and Razvan Gelca*	Problems from various nations & IMO
The USSR Olympiad Problem Book *by D.O. Shklarsky. N.N.Chentzov and I.M.Yaslom*	Mother of all Problem books.

G. Strengthening of Brain by Recreational Mathematics:

Book	Remark
Creativity of Ramanujan [For Primary & Middle School] *by P. K. Srinivasan [Association of Mathematics Teachers of India (AMTI)]*	Fascinating insights of Ramanujan's Notebooks (mainly "Magic Squares")
The Wonder World of Kaprekar Numbers *by AMTI (Editor – R. Athmaraman)*	Generating Special number patterns
Fun and Fundamentals of Mathematics *by J. V. Narlikar and M. Narlikar*	Indianised versions of all classical puzzles from around the world. I loved reading this!!
Mathematical Recreations & Essays *by W. W. R. Ball & H. S. M. Coxeter*	Similar to the book by *Narlikar*
The Moscow Puzzles: 359 Mathematical Recreations *by Boris Kordemsky*	A USSR classic!
Algebra Can be Fun *by Ya. I. Perelman*	Another USSR Classic
Puzzles to Puzzle You *by Shakuntala Devi*	One of many awesome books by "Human Computer"
The Unexpected Hanging & other mathematical Diversions *by M. Gardner*	Collection of Puzzles from "Scientific American" journal
Wheels, Life & Other Mathematical Amusements *by M. Gardener*	These add new spirit to puzzle solving
Entertaining Mathematical Puzzles *by M. Gardener*	
Challenging Mathematical Teasers *by J. A. H. Hunter*	Really challenging problems with elementary solutions
Good Old-Fashioned Challenging Puzzles *by H. E. Dudeney*	Good classifications
What is the name of this book? *by R. M. Smullyan*	Awesome puzzles
Mathematical Puzzles of Sam Loyd *by Sam Loyd (edited by M. Gardener)*	Classic puzzles from puzzle master
Dimensions *(Videos made for inducing understanding of fourth dimension)*	Visit : www.dimensions-math.org

H. Suggested readings for those who still want more Serious Mathematics (for fun!):

Book	Remark
Geometry and the Imagination *by D. Hilbert*	A classic !
A treatise on Problems of Maxima and Minima solved by Algebra *by Ramchundra*	Mind blowing book! *(AMTI)*
The Penguin Dictionary of Curious & Interesting Numbers *by David Wells*	A concise collection of various interesting terms generally not found in textbooks
The Penguin Dictionary of Curious & Interesting Geometry *by David Wells*	
Proofs Without Words (Vol. I & II) *by Roger B. Nelsen*	Just "see" simple proofs
Mathematical Gems (Vol. I,II,III) *by Ross Honsberger*	Higher level study of various topics of mathematics
Mathematical Diamonds *by Ross Honsberger*	
Ingenuity in mathematics *by Ross Honsberger*	
100 Great Problems of Elementary mathematics *by Heinrich Dorrie*	Classical Problems
The Book of Numbers *by J. H. Conway and R. K. Guy*	Fabulous book
The Art of Counting *by Paul Erdὸs*	A collection of work of Paul Erdὸs
Proofs from the book *by M. Aigner and G. M. Ziegler*	Dedicated to Paul Erdὸs
Echoes from Resonance - Number Theory *by S. A. Shirali & C. S. Yogananda*	Articles from Resonance Journal
Excursions in Calculus: An Interplay of the continuous and the discrete *by Young*	-----
Geometric Etudes in Combinatorial Mathematics *by Alexander Soifer*	-----
Art gallery Theorems & Algorithms *by Rourke*	Research work
The Drunkard's Walk: How Randomness Rules our Lives *by Leonard Mlodinow*	Enjoyable reading.
Symmetry – A Journey into the Patterns of Nature *by Marcus du Sautoy*	Beautifully written.
The Code Book: How to make it, break it, hack it, crack it *by Simon Singh*	Enjoyable reading.

I. Web Resources:

1. Cut The Knot (free maths resources): http://www.cut-the-knot.org
2. Questions from Past Mathematical Competitions can be downloaded from: http://artofproblemsolving.com
3. A collection of 4,100 Olympiad problems and about 1,700 other problems : http://mks.mff.cuni.cz/kalva/
4. Shyam Sunder Gupta's Recreational Mathematics resources : http://shyamsundergupta.com
5. The Prime Puzzles & Problems Connection by Carlos Rivera: http://www.primepuzzles.net/
6. Math Forum Library: http://mathforum.org/library
7. Visual Calculus Portal: http://archives.math.utk.edu/visual.calculus/
8. Wolfram Mathworld [a good source of reference] : http://mathworld.wolfram.com
9. Terence Tao's Blog: http://terrytao.wordpress.com
10. Shailesh Shirali's Blog: http://Joyofmathshirali.blogspot.in
11. Vipul Naik's Website: http://www.vipulnaik.com

J. Useful Mathematics Periodicals:

Mathematics periodicals play a vital role, as they keep you updated and a chance to go through different prospective of good writers (e.g. I learnt "Principle of Inclusion & Exclusion" through an article written by B. Sury in "At Right Angles" Magazine)

1. At Right Angles : www.teachersofindia.org/en/periodicals/at-right-angles *[A goldmine for high school students]*
2. Bona Mathematica *by Bhaskaracharya Pratishthana, 56/14, Vishnupant Damle Path, Erandavana, Pune*
3. Mathematical Reflections : http://www.awesomemath.org/
4. The Mathematics Student *by Indian Mathematical Society, Department of Mathematics, University of Pune*
5. The Mathematics Teacher *by The Association of Mathematics Teachers of India, Chennai , India*
6. Crux Mathematicorum : http://cms.math.ca/crux/ *[older issues are available online]*

K. Useful Softwares:

1. *Computer Algebra System*:
   - i. Mathematica (paid)
   - ii. Sage (free)
2. *Interactive Geometry Software*:
   - i. Geometer's Sketchpad (paid)
   - ii. GeoGebra (free)
3. *Spread Sheet:*
   - i. Microsoft Office Excel (paid)
   - ii. Libre Office Calc (free)

– Gaurish Korpal
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