Preparing High School Geometry Teachers to Teach the Common Core

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Overview

In today's talk we will address:
1. Changes in Curriculum due to Common Core State Standards
2. State of Teacher Preparedness
3. Examples of New Approaches based on the Connecticut Core Geometry Curriculum
4. Our experiences working with in-service and pre-service teachers

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What is new (for most teachers) in the Common Core?

- Renewed emphasis on reasoning and proof
- Transformations as the foundation for congruence and similarity
- Formal Geometric Constructions
- Locus approach to Conic Sections

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Challenges for Pre-service and In-service Teachers

- Many attended secondary school after the mid-80's (when proof began to be deemphasized)
- Many career changers never had any geometry courses
- Proof and construction were topics often either absent or minimized in their geometry courses.
- Even for those who did have proofs in their geometry courses, a transformational approach is likely to be unfamiliar.

What are some of the roots of these challenges?

NCTM Curriculum and Evaluation Standards for School Mathematics (March 1989)

Topics to Receive Increased Attention:	Topics to Receive Decreased Attention
Integration across topics at all grade levels	Euclidean geometry as a complete axiomatic system
Coordinate and transformational approaches	Proofs of incidence and betweenness theorems
The development of short sequences of theorems	Geometry from a synthetic viewpoint
Deductive arguments expressed orally and in sentence or paragraph form	Two-column proofs
Computer-based explorations of 2-D and 3-D figures	Inscribed and circumscribed polygons
Three-dimensional geometry	Theorems for circles involving segment ratios
Real-world applications and modeling	Analytic geometry as a separate course

Van Hiele: Levels of Geometric Thinking and Phases of Instruction (1959,1984,1986)

- Level 1: *Visual* Identify shapes according to their appearance.
- Level 2: *Descriptive/Analytic* recognize shapes by their properties
- Level 3: *Abstract/Relational* can form abstract definitions, distinguish between necessary and sufficient sets of conditions, and sometimes provide logical arguments
- Level 4: *Formal Deduction* establish theorems within an axiomatic system
- Level 5: *Rigor/Metamathematical* reason about mathematical systems

Readiness for high school geometry

70% of students begin high school geometry at Level 0 or 1 only those who enter at level 2 or higher have a good chance of becoming competent with proof by the end of the year.

What do today's Geometry teachers need to know and be able to do? (Herbst and Kosko, 2014)

- Design a problem or task to pose to students
- Evaluate a students' constructed responses, particularly student-created definitions, explanations, arguments, and solutions to problems
- Create an answer key or rubric for a test
- Translate students' mathematical statements into conventional vocabulary

What did we notice with our teachers?

- It was not clear that they understood an axiomatic system
- They had little experience with proof
- They had little experience with constructions
- They did not see how they might apply previously used strategies in a new proof
- Our goal for HS geometry students should be VH level 4, to attain that goal our teachers need to be at VH level 5.

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Connecticut Core High School Mathematics curriculum

- Origins: Secondary School Reform Act of 2010
- Aligned with Common Core
- Algebra 1 written in 2009; pilot tested 2010-2013
- Geometry and Algebra 2 written 2015
- Writers from State University and Community College System with help from high school teachers

Connecticut Core Geometry: Key Features

- Follows structure of Algebra 1: Unit/Investigation/Activity
- Transformational approach as specified in Common Core
- Use of a variety of tools: compass/staightedge, coordinates, software (including Geogebra)
- In general, students will first discover properties using drawings, manipulatives, and/or software before writing a formal proof.
- Variable scaffolding on proofs to meet needs of diverse students.

Transformations and the Common Core State Standards (CCSSM)

- Major shift in the axiomatic foundations for the study of plane geometry at the high school level
- Congruence and similarity defined in terms of transformations
- Assumption that students have had rich experiences with transformations in Grade 8.

Euclid (ca. 300 BCE)

- For over 2000 years Euclid's *Elements* was considered the most authoritative treatment of geometry.
- In Book I, Proposition 4 asserts the SAS criterion for congruent triangles.
- SSS is proved in Proposition 8 and ASA in Proposition 25.
- However, the proofs of Propositions 4 and 8 both employed the controversial technique of superposition.

David Hilbert ca. 1900 CE

- Reformulated Euclidean geometry by filling in gaps to make the system more rigorous.
- Made SAS a postulate, from which he was able to prove the other congruence theorems including SSS and ASA.
- Hilbert's approach with minor variations developed by G. D. Birkhoff formed the basis of the postulates used by the School Mathematics Study Group (SMSG) and other texts from the new math era of the 1950's and 1960's.
- To make the material more accessible to students ASA and SSS have usually been postulated along with SAS.

What CCSS Says about Transformations

- The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions (*isometries*): translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally).
- In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other.
- This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles.

Connecticut Core Geometry Approach to Transformations

- In Unit 1 students discover the properties of isometries. These properties become postulates.
- In Unit 2 these postulates are used to prove the SAS, ASA, and SSS congruence theorems.
- In Unit 4 properties of dilations are discovered and postulated.
- Then these postulates are used to prove the SAS, ASA, and SSS similarity theorems.

Transformational Postulates

Isometry Postulate (applies to Translations, Rotations, Reflections and Glide Reflections): All isometries preserve distance and angle measure. Pairs of parallel lines are mapped onto parallel lines. Midpoints of segments are mapped onto midpoints.

Translation Postulate: Under a translation by the vector from X to Y, a line parallel to $\overleftrightarrow{XY}$ is mapped onto itself. A line that is not parallel to $\overleftrightarrow{XY}$ is mapped onto another line that is parallel to itself.

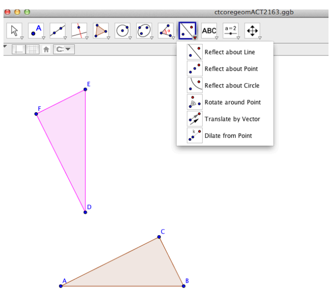
Rotation Postulate: Under a rotation about a point P, the point P is mapped onto itself. A line through point P is mapped onto another line through P. If the angle of rotation is 180° a line through P is mapped onto itself.

Reflection Postulate: Under a reflection about line l every point on l is mapped onto itself. A line that is parallel to l is mapped onto another line that is parallel to l.

Dilation Postulate: Dilations preserve angle measure and betweenness. They map parallel lines onto parallel lines and midpoints onto midpoints. The length of the image of a segment is the length of the segment times the scale factor. If a line passes through the center of dilation it is mapped onto itself. If a line l does not pass through the center of dilation it is mapped onto a line l' parallel to l.

Experiment with Transforming Congruent Figures

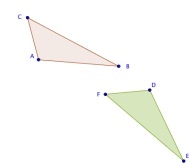
Task: Map ΔABC onto ΔDEF



SAS Congruence Theorem

Given ΔABC and ΔDEF with $AB = DE$, $AC = DF$, and $m\angle BAC = m\angle EDF$

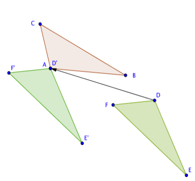
Prove $\Delta ABC \cong \Delta DEF$



SAS Congruence Theorem

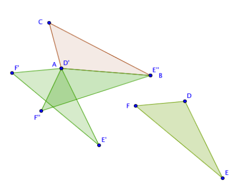
Step 1: If A and D do not coincide then translate ΔDEF by the vector from D to A.

Now D' is the same point as A.



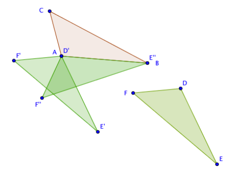
SAS Congruence Theorem

Step 2: If $\overline{AB}$ and $\overline{D'E'}$ do not coincide then rotate $\Delta D'E'F'$ about point A through $\angle E'AB$.



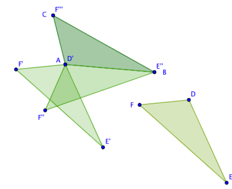
SAS Congruence Theorem

Step 3: $\overline{D''E''}$ will now coincide with $\overline{AB}$ since we were given that $AB = DE$.



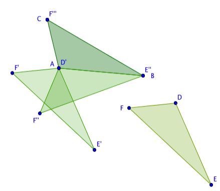
SAS Congruence Theorem

Step 4: If F'' is on the opposite side of $\overleftrightarrow{AB}$ from C, reflect $\Delta D''E''F''$ over $\overleftrightarrow{AB}$.



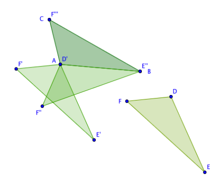
SAS Congruence Theorem

Step 5: $\angle E'''D'''F'''$ will now coincide with $\angle BAC$ since we were given $m\angle BAC = m\angle EDF$, and $\overline{D'''F'''}$ will coincide with $\overline{AC}$ since we were given $AC = DF$.



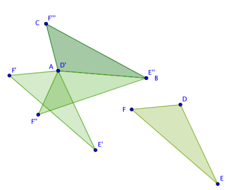
SAS Congruence Theorem

Step 6: Because F''' coincides with C and E''' coincides with B. $\overline{E'''F'''}$ must coincide with $\overline{BE}$. Otherwise we would have two line segments $\overline{E'''F'''}$ and $\overline{BE}$ passing through the same two points, which is impossible.



SAS Congruence Theorem

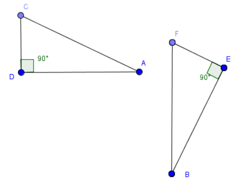
Step 7: Since ΔABC is the image of ΔDEF under an isometry, $\Delta ABC \cong \Delta DEF$.



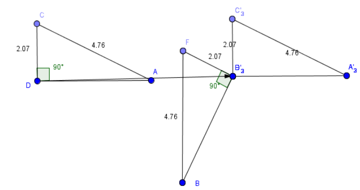
Another Example

- We used transformations to prove the HL Congruence Theorem: If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.
- Our strategy is to use transformations to create an isosceles triangle with the congruent legs as an altitude.

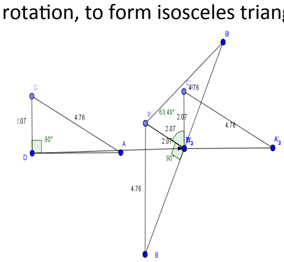
In one case Δ*ADC* and Δ*BEF* have opposite orientations.



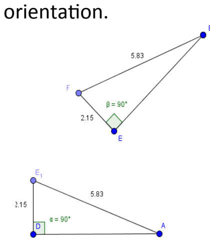
Start with a translation



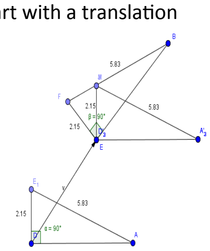
Then a rotation, to form isosceles triangle *BFA''*



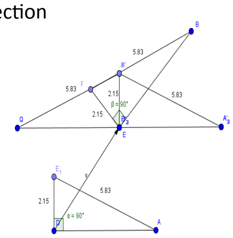
In this case the triangles have the same orientation.



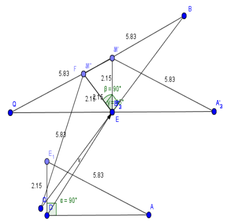
Start with a translation



Then a reflection



And finally a rotation.



CT Core Geometry Approach

- Use hands-on techniques and dynamic geometry software to develop conjectures.
- Prove the theorem or develop a model of the general case.
- Apply the theorem or general model to solve other problems.

Developing a Conjecture

Use a ruler and protractor to make measurements on the figure below.

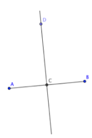
1. Measure the distances CA and CB. What do you notice?
2. C is the __________ of $\overline{AB}$.
3. Measure $\angle DCB$. What do you notice?
4. $\overleftrightarrow{DC}$ is ______________ to $\overline{AB}$.
5. $\overleftrightarrow{DC}$ is the ____________ ____________ of $\overline{AB}$.
6. Measure the distances DA and DB. What do you notice?
7. Now place more points E, F, and G on $\overleftrightarrow{DC}$.
8. Measure these distances

   EA = __________ EB = __________

   FA = __________ FB = __________

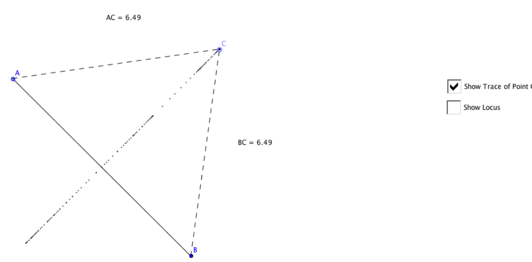
   GA = __________ GB = __________

   What do you notice?
9. Make a conjecture about all points that lie on $\overleftrightarrow{DC}$



10. Now place a point H in the plane that is not on $\overleftrightarrow{DC}$. Measure HA and HB. What do you notice?
11. Try to find a point J in the plane that is not on $\overleftrightarrow{DC}$ so that $JA = JB$. What do you notice?
12. Make a conjecture about all points that are equidistant from points A and B.

Exploring Locus Through an Applet



CCSS.Math.Content.HSG.CO.C.9 : Prove theorems about lines and angles. *Points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* *

Proof of the Perpendicular Bisector Theorem

The Perpendicular Bisector Theorems says that the locus of points that are equidistant from the endpoints of a segment is the perpendicular bisector of the segment.

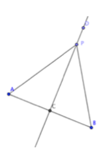
To prove this theorem we need to prove two things:

(1) If a point lies on the perpendicular bisector of a line segment, then it is equidistant from the endpoints of the segment, and
(2) If a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of the segment.

Prove part (1):

Given: $\overleftrightarrow{DC}$ is the perpendicular bisector of $\overline{AB}$
P lies on $\overleftrightarrow{DC}$

Prove: $PA = PB$



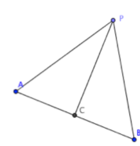
*http://www.corestandards.org/Math/Content/HSG/CO/

Proof of the Perpendicular Bisector Theorem

Prove part (2)

Given: $PA = PB$
C is the midpoint of $\overline{AB}$

Prove: $\overline{PC} \perp \overline{AB}$



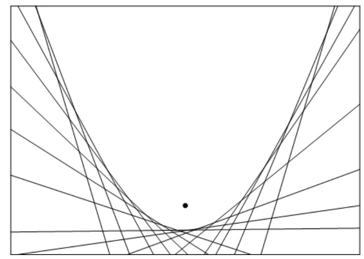
Translate between the geometric description and the equation for a conic section

CCSS.Math.Content.HSG.GPE.A.1
Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

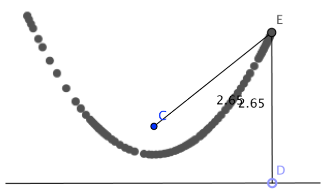
CCSS.Math.Content.HSG.GPE.A.2
Derive the equation of a parabola given a focus and directrix.

CCSS.Math.Content.HSG.GPE.A.3
(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Hands-On Approach



Exploration Using GeoGebra



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CT Core Geometry Professional Development

August 2015: Four day sessions at two locations for a total of 24 hours

* Conducted by authors and experienced high school teachers
* Served approximately 80 teachers
* Three hours on each of the 8 units in the course

November and December 2015: 3 hour Saturday morning "users conferences"

MATH 328 Curriculum and Technology in Secondary School Mathematics II

* required of undergraduate mathematics education majors
* companion to MATH 327 which focuses on algebra
* Van Hiele Level 4 course
* prerequisite for MATH 383 College Geometry, a Van Hiele Level 5 course

Preservice Teacher Feedback

Engaging

Performance tasks make great real-world connections

Having both "hands-on" and "technology" investigations has developed a deeper understanding of "appropriate use of tools".

Plenty of proofs but supported with models of how to differentiate.

Students have advanced on the Van Hiele levels.

"I feel like I am now prepared to teach Geometry"

Web Sites

Our course materials may be found at www.ctcorestandards.org.
Click on Materials for Teachers, then Mathematics, then Geometry.

Euclid's Elements may be read online at aleph0.clarku.edu/~djoyce/java/elements/elements.html