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Poroelastodynamics: Linear Models, Analytical Solutions, and Numerical Methods

Martin Schanz
Institute of Applied Mechanics, Graz University of Technology

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Abstract

This article presents an overview on poroelastodynamic models and some analytical solutions. A brief summary of Biot's theory and on other poroelastic dynamic governing equations is given. There is a focus on dynamic formulations and the quasi-static case is not considered at all. Some analytical solutions for special problems, fundamental solutions, and Green's functions are discussed. The numerical realization with two different methodologies, namely the Finite Element Method and the Boundary Element Method, is reviewed.

1 Introduction

In many branches of engineering porous materials play an important role, in e.g., the petroleum industry, chemical engineering, soil mechanics, and in recent years also in bio-mechanics. The attempt to capture the physical behavior of a two- or more-phase material is quite old. First ideas can be even traced back to Leonhard Euler in the 18th century [61]. The theory of continuum mechanics for a one phase material is well established (see, e.g., Truesdell and Toupin [207]). The extension of continuum mechanics to more than one phase results in mixture theories (see, e.g., [103, 104, 12]). A historical treatment can be found in the review article by de Boer [62] or in the monograph [61] of the same author including the scientific (and indeed dramatic) dispute of von Terzaghi and Fillunger on the uplift and capillary problems. Despite this dispute, both von Terzaghi and Fillunger may be seen as the pioneers of two branches in today's established porous theories.

Based on the work of von Terzaghi, a theoretical description of porous materials saturated by a viscous fluid was presented by Biot [22] in 1941, whereas going back to Fillunger's work the mixture theory based theories have been developed, especially the Theory of Porous Media. This theory is based on the axioms of continuum theories of mixtures [207, 37] extended by the concept of volume fractions by Bowen [34, 35] and others [88, 89].

The above cited theories are first established as quasi-static theories, i.e., no inertia effects have been taken into account. The dynamic version of a two-phase theory, which is the focus of this article, will be called in the following poroelastodynamics. Mostly, the starting point of poroelastodynamics is attributed to the two papers by Biot [24, 25] in 1956. However, first works on poroelastodynamics are those of Frenkel [93] in 1944 from which a reprint [92] exists. Further work in the Russian scientific community based on this pioneering work is reviewed by Nikolaevskiy [158]. The connection of Frenkel's work to Biot's theory is presented by Pride and Garambois [173], where it is shown that both researchers have developed the same theory. Biot [24] has even cited Frenkel's original work. Nowadays most publications refer to Biot's papers. The uniqueness of Biot's theory is discussed by Deresiewicz and Skalak [79]. The same author published a series of papers on boundary or interface conditions related to a porous body [70, 71, 78, 72, 74, 75, 73, 80, 76, 77].

The extension to a nearly saturated poroelastic material has been presented by Aifantis [2] and Wilson and Aifantis [226] for the quasi-static case. There are also several other theories for partially saturated media or media with more phases, e.g., [157, 213, 186]. The dynamic extension of Biot's theory to three phases has been published by Vardoulakis and Beskos [208]. Further development of this model to describe the dynamic behavior of rocks inclusively wave propagation phenomena has been provided by Beskos [18], Beskos et al. [21, 20], and Vgenopoulou et al. [212]. In these publications, the third phase handles the fissures in the rocks which are separately treated from the normal pores. Local heterogeneities in porous materials have been treated by Wei and Muraleetharan [214, 215] in the framework of a two-phase mixture theory.

A state of the art overview on the theory of poroelasticity, its numerical approximation, and applications may be found in the proceedings of the Biot conferences. Up to now, this event took place three times, first in Louvain-la-Neuve [205], second in Grenoble [9], and third in Norman [1]. There are also two special journal issues: One in the International Journal of Solids and Structures (**35** (34-35), 1998) edited by A.H.-D. Cheng et al. and the other in Soil

Dynamics and Earthquake Engineering (**26** (6-7), 2006) edited by M.D. Trifunac. Here, only the poroelastodynamic work is reviewed not considering the much more extensive literature on consolidation.

First, the different versions of dynamic theories are recalled and the basic equations are given. These theories are compared with the help of a one-dimensional wave propagation example. Then analytical solutions are presented with focus on fundamental solutions and Green's functions. Last, two numerical procedures – the Finite and Boundary Element Method – are reviewed. Certainly, it is not possible to make a complete review of the available literature and on all techniques.

Throughout this paper, the summation convention is applied over repeated indices and Latin indices receive the values 1,2, and 1,2,3 in two-dimensions (2-d) and three-dimensions (3-d), respectively. Commas $()_{,i}$ denote spatial derivatives and dots $\dot{()}$ denote the time derivative. The Kronecker delta is denoted by δ_{ij}. Further, any index (sub- or superscript) f refers to the interstitial fluid and s to the skeleton.

2 Biot's theory

Based on the work of von Terzaghi, a theory of porous materials containing a viscous fluid was presented by Biot [22]. In the following years, Biot extended his theory to anisotropic cases [23] and also to poroviscoelasticity [26]. The dynamic extension was published in two papers, one for the low frequency range [24] and the other for the high frequency range [25]. A collection of Biot's papers on porous materials has been published by Tolstoy [206]. Among the significant findings in the theory for poroelastodynamics was the identification of three waves for a 3-d continuum, two compressional waves and one shear wave. This extra compressional wave, known as the slow longitudinal wave, has been experimentally confirmed by Plona [170].

In Biot's theory a fully saturated material is considered, i.e., an elastic skeleton with a statistical distribution of interconnected pores is modeled. This porosity is denoted by

$$\phi = \frac{V^f}{V} \ , \tag{1}$$

where V^f is the volume of the interconnected pores contained in a sample of bulk volume V. The sealed pores are considered as part of the solid. Full saturation is assumed leading to $V = V^f + V^s$ with V^s the volume of the solid.

As the aim of the paper is to model wave propagation phenomena, it is sufficient to formulate a linear kinematic equation. To do so, first, the solid displacements u_i^s and the fluid displacements u_i^f are introduced. Alternatively to the fluid displacements, a relative displacement $w_i = \phi \left(u_i^f - u_i^s \right)$, the so-called seepage displacement, may be used. The relation of the solid/fluid strain to the solid/fluid displacement is chosen linear, respectively

$$\varepsilon_{ij}^s = \frac{1}{2} \left(u_{i,j}^s + u_{j,i}^s \right) \qquad \varepsilon_{kk}^f = u_{k,k}^f \tag{2}$$

assuming small deformation gradients. In (2), the components of the solid and fluid strain tensors are denoted by ε_{ij}^s and ε_{kk}^f, respectively.

In a two-phase material not only each constituent, the solid and the fluid, may be compressible on a microscopic level but also the skeleton itself possesses a structural compressibility. If the compression modulus of one constituent is much larger than the compression modulus of the bulk material this constituent is assumed to be materially incompressible. A common example for a materially incompressible solid constituent is soil. In this case, the individual grains are much stiffer than the skeleton itself. Hence, beside the general model assuming all constituents as compressible also idealised versions modeling some constituent as incompressible exist. In the following, a brief summary of the basic equations will be given. The compressible model, some incompressible models, and some simplified models will be presented.

2.1 Compressible model

If the constitutive equations are formulated for the elastic solid and the interstitial fluid, a partial stress formulation is obtained [23]

$$\sigma^s_{ij} = 2G\varepsilon^s_{ij} + \left(K - \frac{2}{3}G + \frac{Q^2}{R}\right)\varepsilon^s_{kk}\delta_{ij} + Q\varepsilon^f_{kk}\delta_{ij} \tag{3a}$$

$$\sigma^f = -\phi p = Q\varepsilon^s_{kk} + R\varepsilon^f_{kk}\,. \tag{3b}$$

The respective stress tensors are denoted by σ^s_{ij} and $\sigma^f\delta_{ij}$. The elastic skeleton is assumed to be isotropic and homogeneous with the two elastic material constants bulk modulus K and shear modulus G. In the above, the sign conventions for stress and strain follow that of elasticity, namely, tensile stress and strain is denoted positive. Therefore, in equation (3b) the pore pressure p is the negative hydrostatic stress in the fluid σ^f. The coupling between the solid and the fluid is characterized by the two parameters Q and R. Considerations of constitutive relations at micro mechanical level as given by Detournay and Cheng [82] shows that these parameters can be calculated from the bulk moduli of the constituents by

$$R = \frac{\phi^2 K^f K^{s2}}{K^f\left(K^s - K\right) + \phi K^s\left(K^s - K^f\right)} \tag{4}$$

$$Q = \frac{\phi\left(\left(1-\phi\right)K^s - K\right)K^f K^s}{K^f\left(K^s - K\right) + \phi K^s\left(K^s - K^f\right)}\,, \tag{5}$$

where K^s denotes the bulk modulus of the solid grains and K^f the bulk modulus of the fluid.

An alternative representation of the constitutive equation (3) is used in Biot's earlier work [22]. There, the total stress $\sigma_{ij} = \sigma^s_{ij} + \sigma^f\delta_{ij}$ is introduced and with Biot's effective stress coefficient

$$\alpha = 1 - \frac{K}{K^s} = \phi\left(1 + \frac{Q}{R}\right) \tag{6}$$

the constitutive equation with the solid displacement u^s_i and the pore pressure p

$$\sigma_{ij} = 2G\varepsilon^s_{ij} + \left(K - \frac{2}{3}G\right)\varepsilon^s_{kk}\delta_{ij} - \alpha\delta_{ij}p = \sigma^{effective}_{ij} - \alpha\delta_{ij}p \tag{7a}$$

is obtained. The effective stress $\sigma_{ij}^{effective}$ is identified in (7a) in its generalized form. Additional to the total stress σ_{ij}, the variation of fluid volume per unit reference volume ζ is introduced

$$\zeta = \alpha \varepsilon_{kk}^{s} + \frac{\phi^2}{R} p \, . \tag{7b}$$

This variation of fluid ζ is defined by the mass balance over a reference volume, i.e., by the continuity equation

$$\dot{\zeta} + q_{i,i} = 0 \tag{8}$$

with the specific flux $q_i = \phi \left(\dot{u}_i^f - \dot{u}_i^s \right) = \dot{w}_i$. Equation (8) shows that ζ describes the motion of the fluid relative to the solid.

The next step is to state the balances of momentum. For a two-phase material the balance of momentum can be formulated for the mixture or by separating the solid skeleton from the fluid for each constituent.

The partial balances are used by Biot [24] resulting in a formulation with the unknowns solid displacement u_i^s and fluid displacement u_i^f

$$\sigma_{ij,j}^{s} + (1-\phi) f_i^s = (1-\phi)\rho_s \ddot{u}_i^s - \rho_a \left(\ddot{u}_i^f - \ddot{u}_i^s \right) - \frac{\phi^2}{\kappa} \left(\dot{u}_i^f - \dot{u}_i^s \right) \tag{9a}$$

$$\sigma_{,i}^{f} + \phi f_i^f = \phi \rho_f \ddot{u}_i^f + \rho_a \left(\ddot{u}_i^f - \ddot{u}_i^s \right) + \frac{\phi^2}{\kappa} \left(\dot{u}_i^f - \dot{u}_i^s \right) \, . \tag{9b}$$

The first balance equation (9a) is that for the solid grains and the second (9b) is that for the interstitial fluid. In equation (9), the body forces in the solid skeleton f_i^s and in the fluid f_i^f are introduced. Further, the respective densities are denoted by ρ_s and ρ_f. To describe the dynamic interaction between the fluid and the skeleton an additional density the apparent mass density ρ_a has been introduced by Biot [24]. It can be written as $\rho_a = C\phi\rho_f$ where C is a factor depending on the geometry of the pores and the frequency of excitation. At low frequency, Bonnet and Auriault [30] measured $C = 0.66$ for a assembly of glass spheres. In higher frequency ranges, a functional dependence (a combination of Kelvin functions) of C on frequency has been proposed based on conceptual porosity structures, e.g., in [25] and [30]. Another relation to determine this apparent mass density has been suggested by Berryman [15]

$$\rho_a = \phi \rho_f (1-a) \qquad a = 1 - r \left(1 - \frac{1}{\phi} \right) , \tag{10}$$

which he found in good agreement with experiments. In (10), the parameter r is determined by the form of the particles and vary in the range $0 \le r \le 1$.

The factor ϕ^2/κ in front of the damping term in (9) is usually denoted by b. Here, the simplification of a frequency independent, respectively time independent, value is taken which is only valid in low frequency range. Further, the factor ϕ^2/κ is valid only in case of circular pores when κ denotes the permeability. More general models with a frequency dependence of κ may be found, e.g., in the works of Biot [25], Johnson et al. [123], or Auriault et al. [8]. For a correct formulation of (9) the frequency dependence of this factor has to be transformed to time domain and a convolution in time between the respective time-dependent factor and the damping term has to be performed.

The third above mentioned balance of momentum for the mixture is formulated in Biot's earlier work [22] for quasi-statics and in [24] for dynamics. This dynamic equilibrium is given by

$$\sigma_{ij,j} + F_i = \rho_s (1-\phi)\, \ddot{u}_i^s + \phi\rho_f \ddot{u}_i^f \;, \tag{11}$$

with the bulk body force per unit volume $F_i = (1-\phi) f_i^s + \phi f_i^f$. It is obvious that adding the two partial balances (9a) and (9b) results in the balance of the mixture (11).

In most papers using the total stress formulation, the constitutive assumption for the fluid transport in the interstitial space is given by Darcy's law. Here, it is also used, however, with the balance of momentum in the fluid (9b) Darcy's law is already given. Rearranging (9b) and taking the definition of the flux $q_i = \phi\left(\dot{u}_i^f - \dot{u}_i^s\right)$ as well as $\sigma^f = -\phi p$ into account the dynamic version of Darcy's law

$$q_i = -\kappa\left(p_{,i} + \frac{\rho_a}{\phi}\left(\ddot{u}_i^f - \ddot{u}_i^s\right) + \rho_f \ddot{u}_i^f - f_i^f\right) \tag{12}$$

is achieved.

Aiming at the equation of motion, the constitutive equations have to be combined with the corresponding balances of momentum. The kinematic conditions (2) are first inserted in the constitutive equations. Next, the degrees of freedom must be determined. There are several possibilities:

i) To use the solid displacement u_i^s and the fluid displacement u_i^f with six (four) unknowns in 3-d (2-d) [24].

ii) Alternatively the solid displacement u_i^s and the seepage displacement w_i with also six (four) unknowns in 3-d (2-d) can be used [231]. Beside the seepage displacement also sometimes the seepage velocity, i.e., the time derivative of w_i is applied.

iii) A combination of the pore pressure p and the solid displacement u_i^s with four (three) unknowns in 3-d (2-d) can be established. As shown by Bonnet [29], this choice is sufficient.

In the following, the first choice will be denoted by u_i^s-u_i^f-formulation, the second by u_i^s-w_i-formulation, and the third by u_i^s-p-formulation.

u_i^s-u_i^f-formulation First, the equations of motion for a poroelastic body are presented for the unknowns solid displacement u_i^s and fluid displacement u_i^f. Inserting in (9) the constitutive equations (3) written for the partial stress tensors yields a set of equations of motion in time domain

$$\begin{aligned} G u_{i,jj}^s + \left(K + \frac{1}{3}G\right) u_{j,ij}^s + Q\left(\frac{Q}{R} u_{j,ji}^s + u_{j,ji}^f\right) + (1-\phi) f_i^s \\ = (1-\phi)\rho_s \ddot{u}_i^s - \rho_a\left(\ddot{u}_i^f - \ddot{u}_i^s\right) - \frac{\phi^2}{\kappa}\left(\dot{u}_i^f - \dot{u}_i^s\right) \end{aligned} \tag{13a}$$

$$R\left(\frac{Q}{R} u_{j,ji}^s + u_{j,ji}^f\right) + \phi f_i^f = \phi\rho_f \ddot{u}_i^f + \rho_a\left(\ddot{u}_i^f - \ddot{u}_i^s\right) + \frac{\phi^2}{\kappa}\left(\dot{u}_i^f - \dot{u}_i^s\right) \;. \tag{13b}$$

u_i^s-w_i-formulation Second, for this representation in all of the above equations u_i^f is changed to w_i. With this unknown, the dynamic equilibrium for the mixture (12) reads

$$\sigma_{ij,j} + F_i = \rho \ddot{u}_i^s + \rho_f \ddot{w}_i \, , \tag{14}$$

using the bulk density $\rho = \rho_s (1 - \phi) + \phi \rho_f$. The dynamic version of Darcy's law (12) is then

$$\dot{w}_i = q_i = -\kappa \left(p_{,i} + \rho_f \ddot{u}_i^s + \frac{1}{\phi} \left(\frac{\rho_a}{\phi} + \rho_f \right) \ddot{w}_i - f_i^f \right) \, . \tag{15}$$

Inserting in the partial balances of momentum (9) the constitutive equations (7) reformulated with w_i and using the time integrated form of the continuity equation (8), i.e., $\zeta = -w_{i,i}$, the system of governing equations

$$G u_{i,jj}^s + \left(K + \frac{1}{3} G + \alpha^2 \frac{R}{\phi^2} \right) u_{j,ij}^s + \alpha \frac{R}{\phi^2} w_{j,ji} + F_i = \rho \ddot{u}_i^s + \rho_f \ddot{w}_i \tag{16a}$$

$$\alpha \frac{R}{\phi} u_{j,ji}^s + \frac{R}{\phi} w_{j,ji} + \phi f_i^f = \phi \rho_f \ddot{u}_i^s + \left(\frac{\rho_a}{\phi} + \rho_f \right) \ddot{w}_i + \frac{\phi}{\kappa} \dot{w}_i \tag{16b}$$

is obtained.

u_i^s-p-formulation Third, the respective equations of motion are presented for the pore pressure p and the solid displacement u_i^s as unknowns. To achieve this formulation the fluid displacement u_i^f or the relative fluid/solid displacement w_i has to be eliminated. In order to do this, Darcy's law (15) has to be rearranged to obtain w_i. Since the seepage displacement is given as second order time derivative in (15) and the flux is related to its first order time derivative this rearrangement is performed in Laplace domain. A transformation to Laplace domain results in

$$\hat{w}_i = -\underbrace{\frac{\kappa \rho_f \phi^2 s^2}{\phi^2 s + s^2 \kappa (\rho_a + \phi \rho_f)}}_{\beta} \frac{1}{s^2 \rho_f} \left(\hat{p}_{,i} + s^2 \rho_f \hat{u}_i^s - \hat{f}_i^f \right) \, . \tag{17}$$

In equation (17), the abbreviation β is defined for further usage and $\mathscr{L}\{f(t)\} = \hat{f}(s)$ denotes the Laplace transform, with the complex variable s. Moreover, vanishing initial conditions for u_i^s and u_i^f are assumed here and in the following. Now, the final set of differential equations for the displacement $\hat{u}_i^s$ and the pore pressure $\hat{p}$ is obtained by inserting the constitutive equations (7) into the Laplace transformed dynamic equilibrium (14) and the continuity equation (8) with $\hat{w}_i$ from equation (17). This leads to the final set of differential equations for the displacement $\hat{u}_i^s$ and the pore pressure $\hat{p}$

$$G \hat{u}_{i,jj}^s + \left(K + \frac{1}{3} G \right) \hat{u}_{j,ij}^s - (\alpha - \beta) \hat{p}_{,i} - s^2 (\rho - \beta \rho_f) \hat{u}_i^s = \beta \hat{f}_i^f - \hat{F}_i \tag{18a}$$

$$\frac{\beta}{s \rho_f} \hat{p}_{,ii} - \frac{\phi^2 s}{R} \hat{p} - (\alpha - \beta) s \hat{u}_{i,i}^s = \frac{\beta}{s \rho_f} \hat{f}_{i,i}^f \, . \tag{18b}$$

Certainly, the same elimination process of w_i can be performed in Fourier domain resulting in a similar set of equations.

This set of equations describes the behavior of a poroelastic continuum completely as well as the u_i^s-u_i^f-formulation (13) and the u_i^s-w_i-formulation (16). Contrary to both latter formulations an analytical representation of (18) in time domain is only possible for $\kappa \to \infty$. This represent a negligible friction between solid and interstitial fluid.

2.2 Incompressible models

As mentioned in the beginning, several incompressible versions of Biot's theory can be formulated depending on which constituent is modelled incompressible. In total three cases exist:

i) only the solid grains are incompressible,

ii) only the fluid is incompressible, or

iii) the combination of both.

The respective conditions for such incompressibilities are [82]

$$\frac{K}{K^s} \ll 1 \quad \text{incompressible solid,} \qquad \frac{K}{K^f} \ll 1 \quad \text{incompressible fluid .} \tag{19}$$

Inserting these conditions of incompressibility (19) in the material parameter R, Q, and α from (4), (5), and (6), respectively, it becomes obvious that the first two incompressible models can be handled as the compressible model with different material data. These material data are given in the following:

- Incompressible solid grains $K/K^s \ll 1$

$$\alpha \approx 1 \qquad R \approx K^f \phi \qquad Q \approx K^f (1-\phi) \tag{20}$$

- Incompressible fluid $K/K^f \ll 1$

$$\alpha \text{ unchanged} \qquad R \approx \frac{\phi^2 K^s}{1-\phi-\dfrac{K}{K^s}} \qquad Q \approx \frac{\phi(\alpha-\phi) K^s}{1-\phi-\dfrac{K}{K^s}} \tag{21}$$

These limiting values of the material data can be inserted in the constitutive equations (3) or (7) and, hence, no special treatment is required. These two models are sometimes referred to as hybrid models [63] and need no further consideration.

The third case has to be treated separately:

- Both constituents are assumed to be incompressible $K/K^s \ll 1$ and $K/K^f \ll 1$

$$\alpha \approx 1 \qquad R \to \infty \qquad Q \to \infty \qquad \text{but} \qquad \frac{Q}{R} = \frac{1-\phi}{\phi} \tag{22}$$

The relation $R, Q \to \infty$ expresses that the values of R and Q become large, however, due to physical reasons they are in any case limited. But, the condition that R becomes large is used to neglect in (7b) the influence of the pore pressure. This condition and $\alpha = 1$ results in the constitutive equations

$$\sigma_{ij} = 2G\varepsilon^s_{ij} + \left(K - \frac{2}{3}G\right)\varepsilon^s_{kk}\delta_{ij} - \delta_{ij}p \tag{23a}$$

$$\zeta = \varepsilon^s_{kk} \tag{23b}$$

for incompressible constituents using the total stress formulation. In (23a), the effective stress principle as formulated by von Terzaghi is recovered. The second equation (23b) shows that this special modeling of a porous continuum relates the variation of fluid volume directly to the volumetric solid strain. Further, there is no longer any constitutive assumption for the pore pressure p.

For the partial stress formulation (3), a different point of view must be considered because inserting the infinite values of Q and R in the constitutive law (3) results in an infinite stress. A possible formulation for incompressible constituents has been published by Biot [23]

$$(1-\phi)\,\varepsilon^s_{kk} + \phi\varepsilon^f_{kk} = 0\,, \tag{24}$$

i.e., it is assumed that the dilatation of the bulk material vanishes. Realizing the relation

$$\frac{Q}{R} = \frac{1-\phi}{\phi} \qquad \Rightarrow \qquad \frac{Q}{R}\varepsilon^s_{kk} + \varepsilon^f_{kk} = 0 \tag{25}$$

also in the partial stress formulation incompressible constituents can be included resulting in the constitutive assumptions

$$\sigma^s_{ij} = 2G\varepsilon^s_{ij} + \left(K - \frac{2}{3}G\right)\varepsilon^s_{kk}\delta_{ij} \tag{26a}$$

$$\sigma^f = -\phi p = R\left(\frac{Q}{R}\varepsilon^s_{kk} + \varepsilon^f_{kk}\right) \stackrel{!}{=} 0\,. \tag{26b}$$

To achieve the zero value in equation (26b), the condition that the value R becomes large but is limited must be used. Equation (26b) makes no longer any sense and shows again that for both constituents modeled incompressible the pore pressure p is not governed by a constitutive law.

Comparing the incompressible constitutive equations (23) and (26), it becomes obvious that they do not coincide whereas the underlying compressible models are equal. Especially, in (26) an uncoupling of the solid and the fluid in the constitutive assumption occurs in contrast to (23). Keeping in mind that the starting point of both models is different, either it is the incompressibility condition (19) or the assumption of vanishing bulk dilatation (24), the different results are not really a contradiction.

It must be remarked that with an assumption of incompressibility different wave propagation phenomena may be lost. Especially in the case with both constituents modeled incompressible the fast compressional wave has an infinite velocity which is unphysical [184]. In the following, the expression 'incompressible' will denote the case when both constituents are modeled incompressible.

u_i^s-u_i^f-incompressible formulation Inserting in (13) the incompressibility condition (24) yields the governing equations

$$Gu^s_{i,jj} + \left(K + \frac{1}{3}G\right) u^s_{j,ij} + (1-\phi) f_i^s = (1-\phi)\rho_s \ddot{u}_i^s - \rho_a \left(\ddot{u}_i^f - \ddot{u}_i^s\right) - \frac{\phi^2}{\kappa}\left(\dot{u}_i^f - \dot{u}_i^s\right) \quad (27a)$$

$$\phi f_i^f = \qquad \phi\rho_f \ddot{u}_i^f + \rho_a \left(\ddot{u}_i^f - \ddot{u}_i^s\right) + \frac{\phi^2}{\kappa}\left(\dot{u}_i^f - \dot{u}_i^s\right) \quad (27b)$$

using the solid displacement u_i^s and fluid displacement u_i^f as unknowns. Due to the uncoupling of the fluid and solid in the constitutive assumptions (26), in equations (27) only the coupling by the acceleration and damping terms remain. Further, the second equation (27b) is no longer an independent equation which is related to the fact that the pore pressure has no constitutive law. As an additional equation the incompressibility condition (24) has to be used.

u_i^s-w_i-incompressible formulation In the derivation of the compressible u_i^s-w_i-formulation the total stress is used. The incompressible form of it (23a) has still the pore pressure as variable. However, for incompressible constituents the relation between the solid strain and the pore pressure is missing. Hence, an incompressible u_i^s-w_i-formulation can not be established. This is also true if the second incompressible model used for the u_i^s-u_i^f-formulation is applied. There, the solid displacement is related to the fluid displacement by a constant value (25) such that a relative displacement can no longer be a variable.

u_i^s-p-incompressible formulation If the solid displacement and the pore pressure are used as unknowns a sufficient set of differential equations is obtained. Inserting in (18) simply the conditions (22), i.e., setting $\alpha = 1$ and taking the limit $R \to \infty$, the equations of motion under the assumption of incompressible constituents are achieved resulting in

$$G\hat{u}^s_{i,jj} + \left(K + \frac{1}{3}G\right)\hat{u}^s_{j,ij} - (1-\beta)\hat{p}_{,i} - s^2(\rho - \beta\rho_f)\hat{u}_i^s = \beta\hat{f}_i^f - \hat{F}_i \quad (28a)$$

$$\frac{\beta}{s\rho_f}\hat{p}_{,ii} - (1-\beta)s\hat{u}^s_{i,i} = \frac{\beta}{s\rho_f}\hat{f}^f_{i,i}\,. \quad (28b)$$

The equation for the pore pressure (28b) shows that this variable is no longer a degree of freedom. Integrating of (28b) yields the gradient of the pore pressure which can then be eliminated in (28a). The pore pressure is in this case only determined by the deformation of the solid skeleton and no longer by any deformation of the fluid.

2.3 Simplified models

The u_i^s-u_i^f-formulation (13) or the u_i^s-w_i-formulation (16) leads with a usual time discretisation, e.g., Newmark, straight forward to a FEM based time stepping procedure. Unfortunately, these formulations have more unknowns per node than the u_i^s-p-formulation (18) which exists only in Laplace or Fourier domain. Hence, under the view point of computer storage a u_i^s-p-formulation in time domain is desirable.

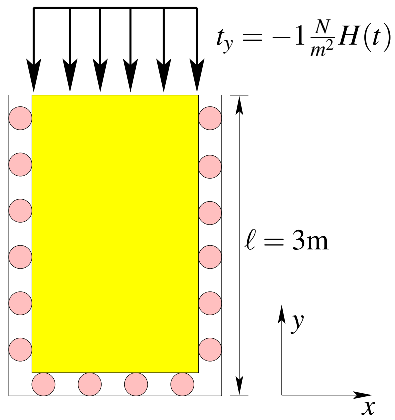


Figure 1: Geometry and boundary conditions of the poroelastic column

The reason why a u_i^s-p–formulation is not possible in time domain is found in the generalized Darcy's law (15) where an elimination of the fluid displacement u_i^f or of the relative displacement w_i is not possible. An elimination is possible if the second time derivative of the seepage displacement w_i is neglected in the equilibrium (14) and in the dynamic version of Darcy's law (15). This results in the simplified dynamic equilibrium

$$\sigma_{ij,j} + F_i = \rho \ddot{u}_i^s , \tag{29}$$

and the simplified dynamic version of Darcy's law

$$\dot{w}_i = q_i = -\kappa \left(p_{,i} + \rho_f \ddot{u}_i^s - f_i^f \right) . \tag{30}$$

Now, the simplified balance of momentum for the fluid (30) can be used to replace w_i in the equations (7) and (8). Rearranging them yields the governing set of differential equations for the unknowns solid displacement u_i^s and pore pressure p in the time domain

$$Gu^s_{i,jj} + \left(K + \frac{1}{3}G \right) u^s_{j,ij} - \alpha p_{,i} - \rho \ddot{u}_i^s = -F_i \tag{31a}$$

$$\kappa p_{,ii} - \frac{\phi^2}{R}\dot{p} - \alpha \dot{u}^s_{i,i} + \kappa \rho_f \ddot{u}^s_{i,i} = \kappa f^f_{i,i} . \tag{31b}$$

This simplification has been published by Zienkiewicz et al. [233]. From the same group another even more simplified version has been published [231] where a static version of Darcy's law is used. There also the inertia of the solid displacement in Darcy's law (30) is neglected.

In [233], the limitations of the different simplifications are discussed with the help of an analytical 1-d example. The conclusion in this publication is that in soil mechanics or geomechanical applications with mostly low frequency acceleration the complete Biot theory does not significantly differ from the simplified form. To demonstrate the effect of the simplifications on wave propagation phenomena the semi-analytical solution of a poroelastic column as depicted in Figure 1 is calculated. Due to the boundary conditions a one-dimensional problem is given which is solved in Laplace domain using standard techniques for ordinary differential equations

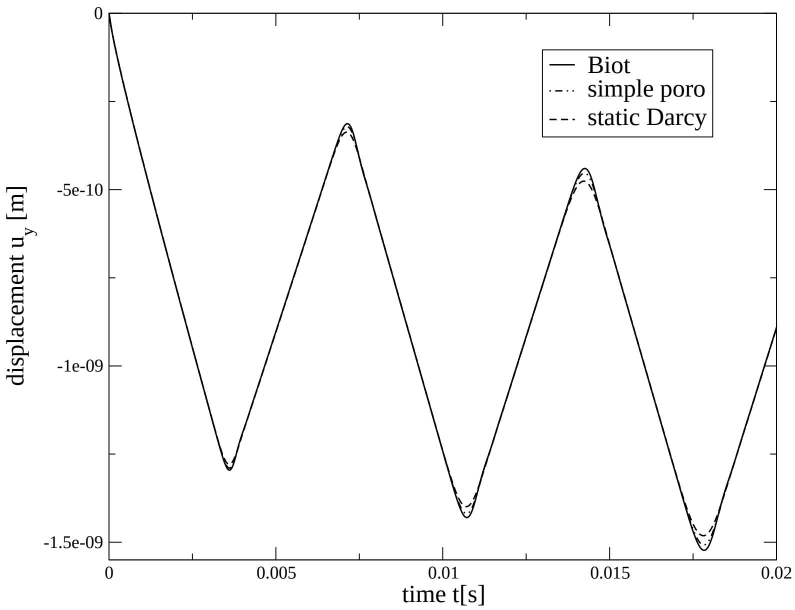


Figure 2: Comparison of Biot's theory (denoted 'Biot'), the simplified theory (denoted 'simple poro'), and the theory using a static Darcy's law (denoted 'static Darcy'): Displacement response of a poroelastic column due to a Heaviside stress load

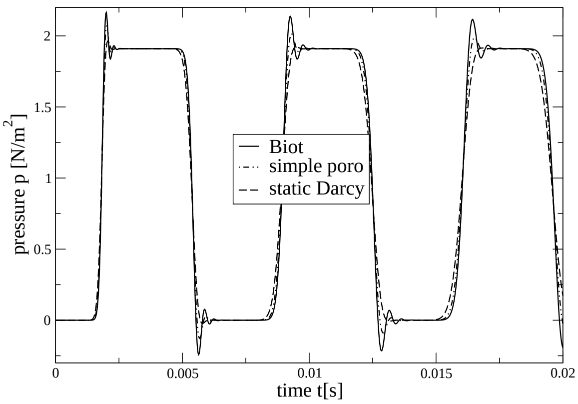
(for details see [181] in case of the complete Biot model and [185] for the simplified model). The inverse transformation is performed with the convolution quadrature method [145, 146]. The used material data are those of a soil [131] which are taken from literature and are given in Table 1. The displacement of the midpoint at the top of the column is plotted versus time in

Table 1: Material data of a water saturated soil (taken from [131])

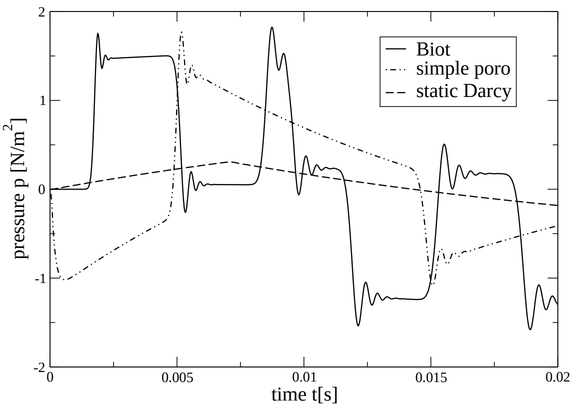
	$K[\frac{N}{m^2}]$	$G[\frac{N}{m^2}]$	$\rho\ [\frac{kg}{m^3}]$	$\rho_f\ [\frac{kg}{m^3}]$	ϕ	$R\ [\frac{N}{m^2}]$	α	$\kappa\ [\frac{m^4}{Ns}]$
soil	$2.1 \cdot 10^8$	$9.8 \cdot 10^7$	1884	1000	0.48	$1.2 \cdot 10^9$	0.98	$3.55 \cdot 10^{-9}$

Figure 2 for Biot's theory, for the simplified theory (31), and for those equations with a static Darcy law. The differences are visible but they are small. Especially, the simplified model (31) does not deviate too much from the full solution.

The main differences in these theories concern the slow compressional wave. This highly dispersive wave can be made visible if the permeability is increased, i.e., the viscosity of the fluid is reduced. The pressure calculated at the bottom of the column is plotted versus time in Figure 3. The response corresponding to the realistic permeability is shown in picture 3(a) and for the increased permeability in picture 3(b). In both cases, the results of the three models differ. In the realistic case only the pressure jumps at the wave arrival are not so sharp for the simplified model compared to the full model. But, for the reduced permeability only the full



(a) $\kappa = 3.55 \cdot 10^{-9} \frac{m^4}{Ns}$



(b) $\kappa = 1 \cdot 10^{-5} \frac{m^4}{Ns}$

Figure 3: Comparison of Biot's theory (denoted 'Biot'), the simplified theory (denoted 'simple poro'), and the theory using a static Darcy's law (denoted 'static Darcy'): Pressure response of a poroelastic column at the bottom for different permeabilities κ

Biot model shows a correct behavior with visible wave fronts. Both other simplified theories result in a completely unphysical behavior, i.e., they model the slow compressional wave wrong.

For completeness it should be remarked that Bowen and Lockett [36] have studied under what conditions a consolidation theory, i.e., all inertia terms are neglected, can be used. It has been found that even in the long time behavior for, e.g., harmonic loadings the inertia can not be neglected. For a poroelastic slab under impulse loading a similar study has been presented by Yeh and Tsay [228]. In this parametric study the FEM is used and the boundary conditions and length to thickness ratio of the slab are varied.

3 Mixture theory and micromechanical based approaches

Biot's theory has been developed mainly in an intuitive way. Beside this approach it is quite natural to use the basic principles of continuum mechanics and apply them to a mixture, hence to use the axioms of continuum theories of mixtures [207, 37]. This approach goes back to Fillunger [91] even when this pioneering work has been written before the above mentioned principles has been established. Based on the work of Fillunger, a quite remarkable contribution in this development has been published by Heinrich and Desoyer [112, 113]. The mixture theory approach has to be extended by the concept of volume fractions as suggested by Bowen [34, 35]. Thus this approach, known as the Theory of Porous Media (TPM), proceeds from the assumption of immiscible and superimposed continua with internal interactions. A lot of contributions to the development of this theory is found, e.g., by de Boer and Ehlers [64], de Boer [61], Ehlers [88, 89]. However, most of these publications deal only with the quasi-static case and a dynamic formulation is only published for incompressible constituents by Diebels and Ehlers [83], by de Boer et al. [65], or by Liu et al. [139].

Remarks on the equivalence of both theories, Biot's and TPM, are found in the work of Bowen [35]. In this paper, he showed that the Biot theory is a special case of a linearized theory of mixtures with constant volume fractions. Bowen called this the case of 'frozen volume fraction'. To achieve equivalence between both approaches the parameter Q introduced by Biot (see (3)) has to be zero, which means that the interaction between both constituents in the constitutive equations is neglected. Further, this comparison has been made only for the quasi-static formulation. For the dynamic equations in the u_i^s-u_i^f-formulation Ehlers and Kubik [90] compared and discussed the linear versions of both theories claiming that they are equivalent if Biot's apparent mass density is assumed to be zero. In [183] again a comparison of both theories but now for the u_i^s-p-formulation has been presented where the main findings confirm the other papers. Both approaches coincide in case of incompressible constituents and if the apparent mass density is neglected. In this case the different material data can be matched. In case of compressible constituents the final equation of the u_i^s-p-formulation in TPM is [183]

$$G\hat{u}^s_{i,jj} + \left(K + \frac{1}{3}G\right)\hat{u}^s_{j,ji} - \left(n_0^F + z^S n_0^S - \beta^T\right)\hat{p}_{,i} - s^2\left(\rho - \beta^T\rho_f\right)\hat{u}_i^s = \beta^T\hat{f}_i^f - \hat{F}_i\,, \tag{32a}$$

$$\frac{\beta^T}{s\rho_f}\hat{p}_{,ii} - \frac{n_0^F s}{K_f}\hat{p} - \left(n_0^F + z^S n_0^S - \beta^T\right)s\hat{u}^s_{i,i} = \frac{\beta^T}{s\rho_f}\hat{f}^f_{i,i}\,. \tag{32b}$$

In (32), all clearly identifiable parameters are denoted as in section 2. The abbreviation β^T is

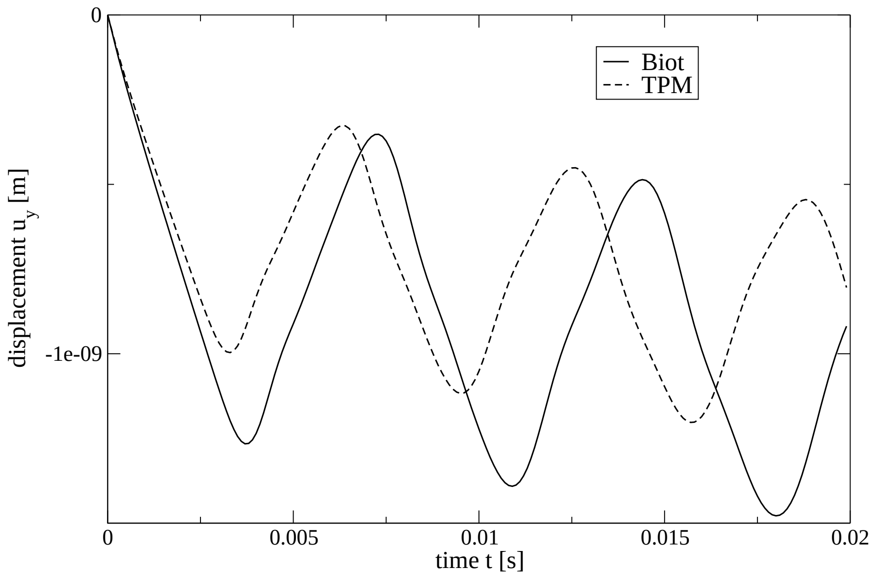


Figure 4: Comparison of TPM and Biot: Response of a poroelastic column due to a Heaviside stress load

the same as in (17) if $\rho_a = 0$ is set. Further, the porosity is denoted by $n_0^F = \phi$ and $n_0^S = 1 - \phi$ where the subscript $()_0$ indicates that the porosity has to be constant. The governing differential operator in Biot's theory (18) and TPM (32) is the same but there is still an open question how to relate the two material data, R and α of Biot's theory, to some data of the TPM, especially how to determine z^S.

The bulk modulus of the fluid is introduced in (32b) instead of the universal gas constant and the absolute temperature compared to the original work [183]. This follows a suggestion by Gurevich [105] and is in accordance with the model given by Coussy [56]. The displacement response of the same poroelastic column as in the previous section (see Figure 1) has been computed. It is plotted in Figure 4 versus time. The results of both theories show the same behavior but have different amplitudes. Not only from Figure 4, it can be concluded that the question remains how the compressible TPM can be formulated such that it coincides with Biot's theory, i.e., how α and R can be related to any parameter (or combination of parameters) in TPM. Further, Gurevich [105] has shown that the low frequency limit of the TPM does not result in a Gassmann type wave velocity. As a consequence of these works on the equivalence of both theories, it may be stated that even though both approaches are similar the theories differ strongly how they model the solid-fluid interaction.

Wilmanski [224] has proposed a Simple Mixture Model which is a linearized form of a poroelastodynamic theory based on the mixture theory. Different to the TPM this theory takes all dynamic effects, except the apparent mass density, into account [225]. The linearized version can be formulated with the porosity n as

$$\begin{aligned}
&\frac{\partial \rho^f}{\partial t} + \rho_0^f \dot{u}_{i,i}^f = 0 \qquad \left| \frac{\rho^f - \rho_0^f}{\rho_0^f} \right| \ll 1 \\
&\rho_0^f \ddot{u}_i^f + \kappa^K \operatorname{grad} \rho^f + \beta^K \left(n - n_E\right)_{,i} + \pi \left(\dot{u}_i^f - \dot{u}_i^s \right) = 0 \\
&\rho_0^s \ddot{u}_i^s - \lambda^s u_{j,ij} + \mu^s \left(u_{i,jj}^s + u_{j,ij}^s \right) + \beta^K \left(n - n_E\right)_{,i} - \pi \left(\dot{u}_i^f - \dot{u}_i^s \right) = 0 \\
&\frac{\partial \left(n - n_E\right)}{\partial t} + \Phi \left(\dot{u}_{i,i}^f - \dot{u}_{i,i}^s \right) e + \frac{n - n_E}{\tau_n} = 0 \, , \quad n_E = n_0 \left(1 + \delta \, u_{i,i}^s\right) , \quad \left| \frac{n - n_0}{n_0} \right| \ll 1 \, .
\end{aligned} \tag{33}$$

In (33), the subscript $()_0$ denotes the constant reference values of the respective data and $\kappa^K, \lambda^s, \mu^s, \beta^K, \pi, \tau_n, \delta, \Phi$ are constant material parameters. These equations coincide with Biot's if $Q = 0$ and $\rho_a = 0$ is assumed in Biot's theory and if the parameter β^K is set to zero in (33) which governs the change of porosity. The latter can safely be done in a linear formulation [218]. As seen above, the same restrictions have been found by Bowen [35] in his comparison with Biot's theory. Hence, (33) can be interpreted as an extension of Bowen [35]. Wilmanski [223] demonstrated that the different values of Q, i.e., the different description of fluid-structure coupling in the models, have a remarkable effect on the attenuation of the compressional waves.

Also by Wilmanski [221] it has been shown that in Biot's theory the relative acceleration terms connected with the apparent mass density violate the principle of material objectivity [217] and the coupling in the constitutive assumptions by the parameter Q violates the second law of thermodynamics [219]. However, it is possible to construct a non-linear model [220, 222] which satisfies these two principles of continuum mechanics and whose linearization leads to Biot's model. Hence, the Biot theory can be considered as a thermodynamically sound way of a linear description of the dynamics of saturated porous media [223]. Nevertheless, Wilmanski uses the Simple Mixture Model to study wave properties in porous media because he stated that the Simple Mixture Model incorporates all essential features from more complex theories. The differences are, as stated above, only the amount of attenuation.

In (33), the change of porosity n is modeled as given in the last equation of (33). This additional and expectable degree of freedom is not visible in Biot's model. Further, in the TPM due to the linearization only a constant porosity is found. But, Wilmanski [223] state that this change of porosity is inherently in Biot's theory (see also [3]). In a different approach based on a variational principle Lopatnikov and Cheng [140] show that this change of porosity, which is also modeled in the non-linear TPM, results in a semi-linear set of governing equations. Semi-linear expresses that the equations are achieved by a linearization, however, the non-linearity concerning the porosity has been kept. In a complete linear theory this effect can not explicitly be modeled. The dynamic extension of this poroelastic model is found in [141]. According to the authors, a complete linearization of their model results in Biot's theory. An advantage of this model is the clear relation of the macroscopic material data to micromechanical data. This relation is presented also with experimental data by Cheng and Abousleiman [51].

The above listed theories are in principle macroscopic theories even if in some of them a microscopic motivation is used to model some effects. There exist also micro-mechanical approaches where homogenization techniques are used to find the macroscopic governing equations. Burridge and Keller [38] derived governing equations for porous continua based on a two scale homogenization. These equations coincide with Biot's equations if the viscosity of the interstitial fluid is assumed to be of small order. If the dimensionless viscosity is of order one a viscoelastic model is achieved. Later Auriault [7] has also presented such a two scale homogenization for a porous body. This results in governing equations of Biot type [8].

Based on the mixture theory a boundary layer theory has been proposed by Mei and Foda [154]. For sufficient high frequencies which are pertinent to ocean waves or seismic waves, a boundary layer of Stoke's type has been shown to exist near the surface of the solid. Outside this layer, the fluid and the solid skeleton move together according to the laws of a single phase elastic medium with changed material data. Hence, this approximate theory simplifies any calculation to an elastodynamic problem and a diffusion problem for the pore fluid pressure in the boundary layer.

4 Analytical solutions

In the following, analytical or semi-analytical solutions of the above mentioned poroelastodynamic theories are listed. It is remarked that the paper only presents dynamic poroelasticity and does not include the widespreaded quasi-static consolidation theory. The interested reader may consider, e.g., for quasi-static fundamental solutions the paper by Cheng and Detournay [53] or for a literature review on poroelastic Green's functions the introduction in [162, 161]. Further, much more analytical solutions for porous media, especially in the viewpoint of geomechanics may be found in the review article by Selvadurai [187].

The following nomenclature is used: A *fundamental solution* is the solution due to an impulse in an unbounded full space and a *Green's function* takes additionally some boundary conditions into account, e.g., the solution of a stress free half space.

4.1 Fundamental solutions

A mandatory requirement for every Boundary Element (BE) Formulation is the knowledge of fundamental solutions. These solutions solve the underlying differential equation with the inhomogeneity of a Dirac distribution. Physically spoken, the response of a system due to a unit impulse is wanted. These solutions exist for a lot of linear problems.

A first approach to develop fundamental solutions for poroelastodynamics has been published by Burridge and Vargas [39] for the u_i^s-w_i-formulation. As inhomogeneity they chose only one point force in the solid which is not sufficient for the usage as fundamental solution in a BE formulation. Further, the asymptotic solution for large times has been presented. Later, Norris [159] derived time harmonic fundamental solutions for the same formulation using a point force in the solid as well as a point force in the fluid. He also obtained explicit asymptotic approximations for the far-field displacements, as well as those for low and high frequency responses. Philippacopoulos [169] published also fundamental solutions for the u_i^s-w_i-formulation in frequency domain in a cylindrical coordinate system but using as Burridge and Vargas only a point force in the solid skeleton. Also for the u_i^s-w_i-formulation Manolis and Beskos [150] published fundamental solutions in Laplace domain. Note, there is a correction of this paper published in [151]. There, the authors essentially changed the unknown relative fluid to solid displacement to the absolute fluid displacement, i.e., fundamental solutions for the u_i^s-u_i^f-formulation are provided. Additionally to the derivation of these solutions the analogy between poroelasticity and thermoelasticity is highlighted. As shown there, this analogy is only possible for the u_i^s-p-formulation. This was confirmed by Bonnet [29] when he presented the fundamental solution for the u_i^s-p-formulation in frequency domain. Bonnet converted the three-dimensional thermoelastic solutions of Kupradze [132] to poroelasticity and, additionally, he provided the two-dimensional solutions. In the u_i^s-p-formulation the inhomogeneity is the force F_i and a source term. This source term must be added to the continuity equation (8) and appears, finally, in the pressure equation (18b) as right hand side. Further, Bonnet concluded that the u_i^s-p-formulation is sufficient and the u_i^s-u_i^f-formulation is overdetermined. It should be mentioned, however, that in Bonnet's paper [29] there is some inconsistency regarding the sign of the time variation assumed for the harmonic variables which in the poroelastic equations is different to that of the thermoelastic ones. This is corrected by Domínguez [84, 85].

Kaynia and Banerjee [128] published also fundamental solutions for the u_i^s-u_i^f-formulation in Laplace domain which are derived in a different way compared to Norris [159] and Manolis and Beskos [150]. There the limits of a high- and low-permeable media are presented even in time domain using an analytical inverse transformation. The technique of Norris [159] to find fundamental solutions is used by Zimmerman and Stern [238] for a potential formulation corresponding to the u_i^s-p-formulation. Recently, a so-called 2.5-d fundamental solution for the u_i^s-p-formulation has been presented by Lu et al. [143]. Such a 2.5-d solution can be used if a 3-d load is applied to an infinitely long structure with a uniform cross-section, i.e., the structure is essentially 2-d. For this technique the reader may consider, e.g., [147, 230].

Boutin et al. [33] published fundamental solutions for the micromechanical based theory of Auriault et al. [8] which is equal to Biot's. The motivation for this remake of the former work [29] was to have a clearer insight in the necessary loads for deriving fundamental solutions. For this purpose, the governing equations have been rewritten to get a symmetric operator.

In all of the above cited papers fundamental solutions are given in transformed domains. Contrary, a time domain fundamental solution was presented by Wiebe and Antes [216] for the u_i^s-u_i^f-formulation. However, in these solutions the viscous coupling of the solid and fluid is neglected. Without this restriction Chen presented in two papers fundamental solutions for the u_i^s-p-formulation, i.e., for a 2-d continuum in [46] and a for 3-d continuum in [45]. These solutions are achieved from the corrected Laplace domain solutions of Bonnet [29] by an analytical inverse transformation resulting partly in an integral which must be solved numerically.

The above discussed fundamental solutions have been developed for the general case of the complete Biot theory. Fundamental solutions have also been derived for either the simplified theory (31) and for the incompressible models (27a) or (28a). In the Laplace domain for the simplified model (31) fundamental solutions are presented by Schanz and Struckmeier [185] and for the incompressible case for both representations (27) and (28) by Schanz and Pryl [184]. Solutions for hybrid models, i.e., only one of both constituent is incompressible, can be achieved by inserting the respective parameters in the compressible fundamental solutions. The respective time dependent fundamental solutions for the simplified version of Biot's theory using only the static version of Darcy's law has been derived by Gatmiri and Kamalian [95]. An even more simpler approach where additionally to the static Darcy's law also incompressible constituents are modeled is presented in 2-d by Gatmiri and Nguyen [96] and in 3-d by Kamalian et al. [124]. All these time domain solutions are found by an analytical inverse Laplace transform of the Laplace domain solutions.

In the above, fundamental solutions are discussed for the case of an isotropic poroelastic material. Clearly, also anisotropy can be taken into account for the elastic skeleton as well as for the pores, i.e., the permeability may be a tensor function (see, e.g., [23, 27, 50]). A fundamental solution in the dynamic case for a transversely isotropic poroelastic material is published by Kazi-Aoual et al. [129]. The final solution can be given in form of a semi-infinite integral over a highly complicated function. Norris [160] published a very abstract derivation of fundamental solutions for general anisotropic poroelasticity (as well for piezoelectric and thermoelastic solids). However, no explicit final formulas are given. Based on this work, Hanyga [108] developed asymptotic time-domain fundamental solutions in a neighborhood of the wave fronts.

4.2 Half-space solutions

Paul [164, 165] may be the first who investigated a two-dimensional problem involving the disturbance of a poroelastic half-space loaded by an impulsive line load [164] or a circular uniform surface load [165]. However, surface tractions and displacement response were considered only for the solid constituent. The solutions have been achieved by Laplace- and/or Fourier-transform where the inverse transformation is performed with the technique of Cagniard. The results consist of highly complicated integrals even for the long time limit. Gazetas and Petrakis [97] computed the compliance of a poroelastic half-space for rocking motions of an infinitely long rigid strip that permitted complete drainage at the contact surface. The solution has been found in the frequency domain. In the extension of these works, Halpern and Christiano [106] published the steady state half-space solution which considers both solid and fluid tractions to be applied on the surface. The required inverse transformation was performed numerically contrary to Paul [164]. In a companion paper, Halpern and Christiano [107] gave the half-space solution with a rigid plate resting on the surface. A series of papers for different loadings of the same problem of a rigid plate on the half-space were published by Jin and Liu [121, 118, 119]. Zeng and Rajapakse [229] developed a solution for a buried rigid disk. However, the final solution consists of several integral equations which are solved numerically. The surface displacements and stresses of a half-space loaded by an incident compression or shear wave has been studied in [138] under the simplifying assumption of an inviscid fluid, i.e., the waves are no longer dispersive in this study.

Lamb's problem for poroelastic materials has been solved by Philippacopoulos [166] in the frequency-wavenumber domain. In a comment to this solution, Sharma [192] mentioned that the boundary conditions have to be chosen different, i.e., not only the total stress vector has to be prescribed but also the pore pressure. Clearly, with these different loading conditions different results are achieved. Philippacopoulos [166] got displacements smaller than the dry (elastic) solution whereas Sharma [192] got for some dissipation values larger amplitudes. The solution for a layered half-space with a loading applied on a rigid disk is given in frequency domain in cylindrical coordinates by Philippacopoulos [167].

Contrary, to the above listed solutions where the load has been applied on the surface, Senjuntichai and Rajapakse [189] published the solution for a 2-d poroelastic half-space with a buried load, i.e., a Green's function is given which is in principle useable in BE formulations. This solution is found by Fourier transform with respect to the horizontal coordinate and time. The vertical coordinate is not transformed. The inverse transformation is performed numerically. With the same technique the solution for a plane multi-layered poroelastic medium loaded by a buried time-harmonic load or fluid source has been found by Rajapakse and Senjuntichai [175]. A discussion on the solution technique used in this paper has been published by Kausel [127]. Bougacha et al. [32] have been presented a similar but only semi-analytical solution of the same problem with applications given in [31]. Contrary to the solution of Rajapakse and Senjuntichai [175], in the deduction of this solution a Finite Element approach is used for the approximation in the horizontal coordinate. Hence, only the solutions of Senjuntichai and Rajapakse [189] and Rajapakse and Senjuntichai [175] can be used as a Green's function modeling the semi-infinite extension of a half-space exactly. A solution in 3-d for a half-space loaded by a buried load is given in cylindrical coordinates for the u_i^s-p-formulation by Jin and Liu [120] and for the u_i^s-w_i-formulation by Philippacopoulos [168]. Both solutions are found in frequency domain by superposition with the full-space fundamental solution. The wave propagation in a layered half-space is treated by an exact stiffness formulation in the wavenumber-frequency domain by Degrande et al. [69]. This technique is in the same publication also applied to unsaturated soils.

4.3 Solutions for special problems

Some analytical solutions for dynamic one-dimensional problems have been found. For example, Grag et al. [102] examined the response of an infinitely long fluid saturated soil column subjected to a Heaviside step function velocity boundary condition at one end. There, a closed form solution was obtained for the limiting cases of zero and infinite fluid drag. Grag's solution in Laplace domain with a subsequent numerical inversion led to a solution of Biot's equations [115] and was compared to a 1-d Finite Element solution in [114]. A solution in frequency domain of a finite 1-d column loaded at the top by total stress and pore pressure was presented by Cheng et al. [54] for comparison with a Boundary Element solution. Based on the Theory of Porous Media an analytical 1-d solution for an infinitely long column was deduced for incompressible constituents [65]. Finally, for a more general material case of a partially saturated dual-porosity medium, a 1-d solution in Laplace domain is available from Vgenopoulou and Beskos [209].

It is of interest to observe that none of the above cited theoretical one-dimensional wave propagation investigations seems to have examined the problem of impact or step loading and produced the dynamic wave behavior. For a step loading such a solution has been published by Schanz and Cheng [181]. This solution is deduced in Laplace domain and the time response is achieved by applying the Convolution Quadrature Method as proposed by Lubich [145, 146]. Due to the usage of this technique every time history for the load can be considered. In a strong sense this solution should be called semi-analytical. The extension of this semi-analytical solution to a poroviscoelastic material based on Biot's theory can be found in [182].

With this 1-d solution the passage of both compressional waves can be observed. Of the other impact or step loading solutions reported in the literature from [209, 65], only monotonic decay was observed. This could be caused by the use of a extremely dissipative material or, as in case of the solution from de Boer et al. [65], because an incompressible model is used.

A common analytical solution for the validation of numerical procedures is the quasi-static behavior of a borehole. This solution is found, e.g., in [81, 82]. The dynamic version is unfortunately much more complicated. Under plain strain conditions a solution in Laplace domain with a numerical inverse transformation is given by Senjuntichai and Rajapakse [188]. An extension to a viscoelastic solid skeleton and with partial sealed boundary conditions may be found in [227]. A solution in 3-d in cylindrical coordinates is given in the frequency-wavenumber domain by Lu and Jeng [142]. The authors perform also a numerical inverse transformation, hence this and the before mentioned solutions may be denoted semi-analytical. The semi-analytical borehole solution of fissured rocks is given in Laplace domain with a subsequent inverse transformation by Vgenopoulou and Beskos [210, 211]. The microseismicity caused by the injection of the borehole fluids into the surrounding porous reservoir is studied analytically in [87].

The first work on scattering in a porous media has been presented by Berryman [16]. The same problem is treated by Mei et al. [155] but using the simplified poroelastic model of the first author. The scattering within a borehole, is treated by Hasheminejad and Hosseini [110]. The first author has presented several papers on nearly analytical solutions for scattering where either a porous scatterer or cylinders are involved (see, e.g., [111, 109]). The scattering and the diffraction of plane shear waves by a shallow circular-arc canyon has been solved by Liang et al. [136]. The scattering of waves by a circular crack in a porous media is studied by Galvin and Gurevich [94]. The dynamic stress intensity factor for a mode I loading of a penny-shaped crack in an infinite poroelastic solid is calculated in [122].

Another application is the steady state response of a half-space due to a moving load. Theodorakopoulos [197] presented an analytical solution in Fourier domain for plain strain conditions and an incompressible solid constituent. For the same problem but using the more simpler poroelastic model of Mei and Foda [154] a solution has been published by Theodorakopoulos et al. [202] and [156]. In 3-d and for a rectangular moving load a solution is given in Fourier domain with a numerical inverse transformation by Cai et al. [41]. A flexible plate on a saturated soil layer is treated in cylindrical coordinates by Chen et al. [48]. A similar problem, the vertical vibration of an elastic strip resting on a half-space with variable distance to a bedrock, is calculated with an infinite series solution by Cai et al. [42].

With similar techniques as the aforementioned solutions the pressure on a rigid cantilever wall connected to a poroelastic soil is calculated by Theodorakopoulos et al. [200] for Biot's theory. The same problem modeled with the simplified model of Mei and Foda [154] is as well treated by Theodorakopoulos et al. [201]. Both solutions are determined in Fourier domain assuming plain strain conditions, a spatial constant acceleration as excitation, and an incompressible solid. Two extensions of this problem have been published. The same problem as sketched above but with a flexible wall is treated by Lanzoni et al. [133]. The extension to a pair of rigid walls has been published in [198] and with an additional rotational degree of freedom at the footing of the wall in [199].

In structural engineering special solutions for structural elements are often used as there are beams or plates. Such structural elements based on a poroelastic constitutive equation have been developed on the basis of Biot's theory. In the monograph by Cederbaum et al. [43] the vibrations, large deflections, and the stability of poroelastic beams are discussed. A plate theory based on the Kirchhoff assumptions and using Biot's theory has been published by Theodorakopoulos and Beskos [203, 204]. In the book by Cederbaum et al. [43] as well a plate theory is given based on Kirchhoff's hypotheses but the diffusion in the plate is restricted to the in-plane direction. Based on the Mindlin hypotheses a poroelastic plate theory has been developed in [40].

5 Numerical methods

Beside analytical solutions for poroelastodynamic problems there are numerically solutions available. Those methodologies are based on several different numerical techniques which are also used for other classes of problems. Hence, the most popular method in solid mechanics the Finite Element Method (FEM) has also been extended to poroelastodynamics. Further, the Boundary Element Method (BEM) has also been applied due to its ability to treat semi-infinite domains correctly. Clearly, also Finite Differences or Finite Volume Methods exist for poroelastodynamics. Here, the review is restricted to the first two mentioned methodologies.

5.1 Finite Element Method

The standard book on FE formulations for poroelasticity has been presented by Lewis and Schrefler [134] but it is mainly restricted to consolidation. The most complete presentation for poroelastodynamics but with a strong focus on geomechanics has been published by Zienkiewicz et al. [237]. In principle, this monograph summarizes all the effort this group has been published on this subject. In the following, it will become obvious that there is a second large field of application beside geomechanics – sound absorbing materials, i.e., the acoustical behavior of porous materials. For this field of application there exists only a monograph on the basic modeling approaches [4] but no monograph on the numerical development.

5.1.1 Formulations based on Biot's theory

Ghaboussi and Wilson [100] seem to be the first who have formulated a FEM for dynamical processes in poroelastic media. They used the u_i^s-w_i-formulation and showed as well the corresponding variational formulation. There is even a small example for a 1-d model of a poroelastic half-space. Twelve years later Zienkiewicz and Shiomi [231] published FE formulations for the u_i^s-u_i^f-formulation as well as for the cheaper u_i^s-p-formulation. Cheap means in this context less storage due to less degrees of freedom per node. As discussed earlier in this paper, the u_i^s-p-formulation in time domain can only be obtained if some terms corresponding to the fluid acceleration can be neglected. Zienkiewicz and Shiomi [231] used the simplified model (31). As mentioned before, this approach is a good approximation, e.g., for applications in earthquake engineering [233]. Further, Zienkiewicz et al. [236] give an overview of the different formulations with their respective simplifications and present the accompanying FE formulations.

The applicability of the different FE formulations based on different sets of unknowns is also studied by Simon et al. [194]. The same group discussed the usage of different polynomial shape functions in the formulations in [193]. Both studies were performed with a comparison on the basis of a one-dimensional model.

The above cited formulations always solve the complete system of coupled equations in one step. There is also an approach for a staggered solution presented in [234]. Some discussions on the stability of this numerical scheme may be found in [235] and the application of a time step control based on an error estimator for dynamic problems is presented in [232].

5.1.2 Formulations based on the mixture theory

The above cited FE formulations are based on Biot's theory. Certainly, as the governing differential operator is the same also for the mixture theory, the above methodologies can be transformed to these representations of a poroelastodynamic model. The mixture theory for a porous material as proposed by Bowen [35] is used in a FE formulation by Prevost [171, 172]. A hybrid model with an incompressible solid constituent is applied as well as a formulation with both constituents modeled incompressible is presented. Both formulations are based on the velocities of the solid and fluid as unknowns. Contrary to this approach Diebels and Ehlers [83] presented a formulation based on the Theory of Porous Media, using the three fields of unknowns solid displacements, fluid displacements, and pore pressure. In this formulation incompressible constituents are assumed. A totally different approach in the time discretisation is made by Chen et al. [49] where a time discontinuous Galerkin procedure is proposed instead of a Newmark scheme. The presented formulation is based on the Theory of Porous Media and an incompressible solid skeleton is assumed, i.e., a hybrid model is chosen.

There are not much approaches available to take large deformations into account in a dynamic poroelastic formulation. The first one seems to be presented by Manzari [153] based on an extension of the simplified Biot theory (31). In this formulation an incompressible solid skeleton is assumed and only the principle of virtual work is given. A FE formulation is presented by Li et al. [135] where the elastic skeleton is treated by a Neo-Hooke material. They use a solid velocity - pore pressure formulation based on the simplified Biot theory (31).

5.1.3 Formulations for acoustics

To model the acoustical behavior of porous structures, e.g., absorbers, often the so-called equivalent fluid approach is used [4]. In this approach, the shear wave is neglected as well as the slow compressional wave. Hence, a Helmholtz equation has to be solved where the influence of the porous material is modeled as a complex valued compression modulus and a complex valued density (see, e.g., [44]).

A porous formulation has been published for a rigid skeleton by Craggs [58]. Later, more detailed poroelastic approaches based on the complete Biot theory have been presented by Kang and Bolton [125], Coyette and Wynendaele [57], and Panneton and Atalla [163]. In these publications the degrees of freedom are the displacements of the solid skeleton and the displacements of the fluid, i.e., a u_i^s-u_i^f-formulation is used. A u_i^s-p-formulation in time domain for acoustics has been presented by Göransson [101] where a symmetric FE formulation has been established by using additionally to the pore pressure a fluid potential [101]. To formulate this approach in time domain the fluid acceleration has been neglected, i.e., the simplified model (31) has been used.

However, this is not necessary because in acoustics usually not the time response but the frequency response is required. Hence, a formulation in frequency domain is necessary where also the fluid inertia can easily be taken into account. Such a formulation has been published by Atalla et al. [5]. In this publication, also the coupling of the poroelastic FEM with a FEM for the surrounding air modelled by the Helmholtz equation is presented. The field of application is the determination of the acoustic behavior of layered porous materials as sound absorbers (see also, e.g., [28]). In [5], the formulation is based on the partial stresses in the weak form of the governing equations. The problem with formulating the boundary conditions in such a formulation is discussed in [66]. The same formulation but based on the total stress is presented in [6]. There, the boundary conditions appear naturally in a physical form. However, the coupling conditions to the surrounding air become more complicated. A comparison of these different approaches to model the sound radiation of porous materials in a cavity is found in [11].

To improve the numerical behavior of these FE formulations, especially to model thin structures as plate like absorbers, Hörlin et al. [117] use hierarchical finite elements. Later, this has been extended to a *hp*-version in [116]. These formulations work in the frequency domain. Beside these two publications another paper on the same topic has been published by Rigobert et al. [177].

Contrary to all above mentioned FE approaches Bermúdez et al. [13, 14] presented a formulation based on Raviart-Thomas-elements. In [13] a porous media with a rigid skeleton is treated, whereas in [14] the skeleton can be elastic but the used model is non-dissipative. The latter model is derived by a homogenization approach.

5.1.4 Semi-infinite domains

In the above, it can be seen that the poroelastic model is often used in semi-infinite domains where one essential physical feature is the Sommerfeld radiation condition. Hence, the numerical methods should be able to model the artificial boundary of the mesh such that no wave reflection occurs as well that the amplitudes of the waves decrease with traveling distance. Degrande and De Roeck [67] have published absorbing boundary condition to model parts of this physical condition. The corresponding FE formulation is given in [68]. Khalili et al. [130] presented a coupling of the frequency domain FE formulation (u_i^s-u_i^f-formulation) with a 1-d analytical solution to model the infinite space. The frequency responses shown confirm that simply by setting either a zero displacement boundary condition or a free boundary deteriorate the solution if not a very large mesh (discretised domain) is used. However, the presented infinite element works only in one direction and is, therefore, only a good model for wave guides, e.g., a half space on a bedrock. A more sophisticated version of infinite elements has been presented by Shao and Lan [191]. There, a truncated series of waves is used where the corresponding wave velocity is found by minimizing the reflection coefficient. Another approach is the so-called double asymptotic approximation which has been extended from elastodynamics [98, 99] to poroelastodynamics by Qi and Geers [174]. This formulation may be regarded as a kind of Boundary Element approach.

5.2 Boundary Element Method

A more natural treatment of semi-infinite domains is the Boundary Element Method. The efficiency of BEM in dealing with such domains has long been recognized by researchers and engineers. There is a large number of applications (see, e.g., Beskos [17, 19]). A survey on the history of BEM itself is given by Cheng and Cheng [52]. The modeling of poroelastic domains with BEM is reviewed by Chopra [55].

For modeling transient behavior, two approaches exist in pure boundary based formulations: the solution in the time domain by time-stepping [152] and the solution in the integral transform domain via Laplace or Fourier transform with subsequent inverse transformation [59]. The time stepping approach is sometimes impossible due to the lack of a time dependent fundamental solution which is often only available in the transformed domain. Therefore, first poroelastodynamic BE formulations based on Biot's theory have been published in Laplace domain by Manolis and Beskos [150] for the u_i^s-u_i^f-formulation (see the correction of this paper in [151]). For of the u_i^s-p-formulation BE formulations in frequency domain have been published by Cheng et al. [54] and Domínguez [85]. A non-singular formulation in frequency domain has been reported by Zimmerman and Stern [238] for modeling wave scattering. The non-singular integral equation is achieved by subtracting the elastostatic integral equation of the interior of the scatterer from the poroelastic one. Based on the same regularization technique a so-called 2.5-d BE formulation has recently been published by Lu et al. [144].

However, it is more natural to work in the time domain and observe the phenomenon as it evolves. Such a time domain formulation was developed by Wiebe and Antes [216] with the restriction of vanishing damping between the solid skeleton and the fluid. Another time dependent formulation was proposed by Chen and Dargush [47] based on the analytical inverse transformation of the Laplace domain fundamental solutions. But, as the authors admit, this formulation is highly CPU-time demanding. Alternatively, Schanz [178, 179, 180] utilizes the Convolution Quadrature Method proposed by Lubich [145, 146] to establish a time stepping BE formulation for poroelastodynamics based on the Laplace transformed fundamental solutions.

Besides the above cited BE formulations where fundamental solutions are used which fulfill the governing differential equation other approaches sometimes called Domain Boundary Element Method can be found. There, mostly the fundamental solutions of elastostatics (Kelvin solution) are used and that part of the differential operator which is not satisfied is tackled as right hand side. This results in a domain integral (see, e.g., [196]). For poroelastodynamics taking even plasticity effects into account such a formulation has been published by Soares et al. [195]. However, only the simplified model sketched in section 2.3 using a static Darcy's law is used.

Applications of poroelastic BE formulation in the frequency domain can be found, among others, in the analysis of dam-reservoir systems. First two-dimensional calculations are presented by Maeso and Domínguez [148] and Domínguez and Maeso [86]. Recently, the same approach is used in 3-d calculations by Aznárez et al. [10]. The response of axisymmetric foundations in poroelastic media in frequency domain by BEM has been presented by Dargush and Chopra [60]. Another application with a poroelastodynamic BE formulation is given by Maeso et al. [149] for a pile group in soil. In tunneling, there is a publication by Kattis et al. [126] using instead of Biot's theory the simplification of Mei and Foda [154].

All the above mentioned BE formulations are so-called direct methods. An indirect method is also possible in poroelastodynamics and has been developed by Rajapakse and Senjuntichai [176]. An application to an embedded rigid foundation in a poroelastic half-space is presented in [190]. The scattering of shear waves by a canyon is treated in [137] also by an indirect poroelastic BE formulation.

References

[1] Y. N. Abousleiman, A. H.-D. Cheng, and F.-J. Ulm, editors. *Poromechanics III, Biot Centennial (1905–2005) Proceedings of the Third Biot Conference on Poromechanics*, Leiden, London, New York, Philadelphia, Singapore, 2005. A.A. Balkema Publishers.

[2] E. C. Aifantis. On the problem of diffusion in solids. *Acta Mech.*, 37(3–4):265–296, 1980.

[3] B. Albers and K. Wilmanski. Modeling acoustic waves in saturated poroelastic media. *J. Engrg. Mech., ASCE*, 131(9):974–985, 2005. doi: 10.1061/(ASCE)0733-9399(2005)131:9(974). URL `http://link.aip.org/link/?QEM/131/974/1`.

[4] J.F. Allard. *Propagation of Sound in Porous Media*. Elsevier Applied Science, London, New York, 1993.

[5] N. Atalla, R. Panneton, and P. Debergue. A mixed displacement-pressure formulation for poroelastic materials. *J. Acoust. Soc. Am.*, 104(3):1444–1452, 1998.

[6] N. Atalla, M. A. Hamdi, and R. Panneton. Enhanced weak integral formulation for the mixed (u,p) poroelastic equations. *J. Acoust. Soc. Am.*, 109(6):3065–3068, 2001.

[7] J.-L. Auriault. Dynamic behaviour of a porous medium saturated by a Newtonian fluid. *Int. J. Engng. Sci.*, 18(6):775–785, 1980.

[8] J.-L. Auriault, L. Borne, and R. Chambon. Dynamics of porous saturated media, checking of the generalized law of Darcy. *J. Acoust. Soc. Am.*, 77(5):1641–1650, 1985.

[9] J.-L. Auriault, C. Geindreau, P. Royer, J. F. Bloch, C. Boutin, and J. Lewandowska, editors. *Poromechanics II, Proceedings of the Second Biot Conference on Poromechanics*, Lisse (Niederlande), 2002. Balkema at Swets & Zeitlinger.

[10] J. J. Aznárez, O. Maeso, and J. Domínguez. BE analysis of bottom sediments in dynamic fluid-structure interaction problems. *Eng. Anal. Bound. Elem.*, 30(2):124–136, 2006.

[11] F.-X. Bécot and F. Sgard. On the use of poroelastic materials for the control of the sound radiated by a cavity backed plate. *J. Acoust. Soc. Am.*, 120(4):2055–2066, 2006.

[12] A. Bedford and D. S. Drumheller. Theories of immiscible and structured mixtures. *Int. J. Engng. Sci.*, 21(8):863–960, 1983.

[13] A. Bermúdez, J. L. Ferrín, and A. Prieto. A finite element solution of acoustic propagation in rigid porous media. *Int. J. Numer. Methods. Engrg.*, 62(10):1295–1314, 2005.

[14] A. Bermúdez, J. L. Ferrín, and A. Prieto. Finite element solution of new displacement/pressure poroelastic models in acoustics. *Comput. Methods Appl. Mech. Engrg.*, 195(17-18):1914–1932, 2006.

[15] J. G. Berryman. Confirmation of Biot's theory. *Appl. Phy. Lett.*, 37(4):382–384, 1980.

[16] J. G. Berryman. Scattering by a spherical inhomogeneity in a fluid-saturated porous medium. *J. Math. Phys.*, 26(6):1408–1419, 1985.

[17] D. E. Beskos. Boundary element methods in dynamic analysis. *AMR*, 40(1):1–23, 1987.

[18] D. E. Beskos. Dynamics of saturated rocks. I: Equations of motion. *J. Engrg. Mech., ASCE*, 115(5):982–995, 1989.

[19] D. E. Beskos. Boundary element methods in dynamic analysis: Part II (1986-1996). *AMR*, 50(3):149–197, 1997.

[20] D. E. Beskos, C. P. Providakis, and H. S. Woo. Dynamics of saturated rocks. III: Rayleigh waves. *J. Engrg. Mech., ASCE*, 115(5):1017–1034, 1989.

[21] D. E. Beskos, I. Vgenopoulou, and C. P. Providakis. Dynamics of saturated rocks. II: Body waves. *J. Engrg. Mech., ASCE*, 115(5):996–1016, 1989.

[22] M. A. Biot. General theory of three-dimensional consolidation. *J. Appl. Phys.*, 12(2): 155–164, 1941.

[23] M. A. Biot. Theory of elasticity and consolidation for a porous anisotropic solid. *J. Appl. Phys.*, 26(2):182–185, 1955.

[24] M. A. Biot. Theory of propagation of elastic waves in a fluid-saturated porous solid.I. Low-frequency range. *J. Acoust. Soc. Am.*, 28(2):168–178, 1956.

[25] M. A. Biot. Theory of propagation of elastic waves in a fluid-saturated porous solid.II. Higher frequency range. *J. Acoust. Soc. Am.*, 28(2):179–191, 1956.

[26] M. A. Biot. Theory of deformation of a porous viscoelastic anisotropic solid. *J. Appl. Phys.*, 27(5):459–467, 1956.

[27] M. A. Biot. Mechanics of deformation and acoustic propagation in porous media. *J. Appl. Phys.*, 33(4):1482–1498, 1962.

[28] J.S. Bolton, N.-M. Shiau, and Y.J. Kang. Sound transmission through multi-panel structures lined with elastic porous materials. *J. Sound Vibration*, 191(3):317–347, 1996.

[29] G. Bonnet. Basic singular solutions for a poroelastic medium in the dynamic range. *J. Acoust. Soc. Am.*, 82(5):1758–1762, 1987.

[30] G. Bonnet and J.-L. Auriault. Dynamics of saturated and deformable porous media: Homogenization theory and determination of the solid-liquid coupling coefficients. In N. Boccara and M. Daoud, editors, *Physics of Finely Divided Matter*, pages 306–316. Springer Verlag, Berlin, 1985.

[31] S. Bougacha, J. M. Roësset, and J. T. Tassoulas. Dynamic stiffness of foundations on fluid-filled poroelastic stratum. *J. Engrg. Mech., ASCE*, 119(8):1649–1662, 1993.

[32] S. Bougacha, J. T. Tassoulas, and J. M. Roësset. Analysis of foundations on fluid-filled poroelastic stratum. *J. Engrg. Mech., ASCE*, 119(8):1632–1648, 1993.

[33] C. Boutin, G. Bonnet, and P. Y. Bard. Green functions and associated sources in infinite and stratified poroelastic media. *Geophys. J. Roy. Astron. Soc.*, 90:521–550, 1987.

[34] R. M. Bowen. Incompressible porous media models by use of the theory of mixtures. *Int. J. Engng. Sci.*, 18(9):1129–1148, 1980.

[35] R. M. Bowen. Compressible porous media models by use of the theory of mixtures. *Int. J. Engng. Sci.*, 20(6):697–735, 1982.

[36] R. M. Bowen and R. R. Lockett. Inertial effects in poroelasticity. *J. of Appl. Mech.*, 50: 334–342, 1983.

[37] R.M. Bowen. Theory of mixtures. In A.C. Eringen, editor, *Continuum Physics*, volume III, pages 1–127. Acadenic Press, New York, 1976.

[38] R. Burridge and J. B. Keller. Poroelasticity equations derived from microstructure. *J. Acoust. Soc. Am.*, 70(4):1140–1146, 1981.

[39] R. Burridge and C. A. Vargas. The fundamental solution in dynamic poroelasticity. *Geophys. J. Roy. Astron. Soc.*, 58:61–90, 1979.

[40] A. Busse, M. Schanz, and H. Antes. A poroelastic Mindlin plate. *Proc. Appl. Math. Mech.*, 3(1):260–261, 2003.

[41] Y. Cai, H. Sun, and C. Xu. Steady state response of poroelastic half-space soil medium to a moving rectangular load. *Internat. J. Solids Structures*, 44(22–23):7183–7196, 2007.

[42] Y. Q. Cai, Y. M. Cheng, S. K. A. Au, C. J. Xu, and X. H. Ma. Vertical vibration of an elastic strip footing on saturated soil. *Int. J. Numer. Anal. Methods Geomech.*, 32(5): 493–508, 2008.

[43] G. Cederbaum, L. Li, and K. Schulgasser. *Poroelastic Structures*. Elsevier, Amsterdam, 2000.

[44] Y. Champoux and J.F. Allard. Dynamic tortuosity and bulk modulus in air-saturated porous media. *J. Appl. Phys.*, 70(4):1975–1979, 1991.

[45] J. Chen. Time domain fundamental solution to Biot's complete equations of dynamic poroelasticity. Part II: Three-dimensional solution. *Internat. J. Solids Structures*, 31(2): 169–202, 1994.

[46] J. Chen. Time domain fundamental solution to Biot's complete equations of dynamic poroelasticity. Part I: Two-dimensional solution. *Internat. J. Solids Structures*, 31(10): 1447–1490, 1994.

[47] J. Chen and G. F. Dargush. Boundary element method for dynamic poroelastic and thermoelastic analysis. *Internat. J. Solids Structures*, 32(15):2257–2278, 1995.

[48] S. L. Chen, L. Z. Chen, and J. M. Zhang. Dynamic response of a flexible plate on saturated soil layer. *Soil Dyn. Earthquake Eng. J.*, 26(6–7):637–647, 2006.

[49] Z. Chen, H. Steeb, and S. Diebels. A time-discontinuous Galerkin method for the dynamical analysis of porous media. *Int. J. Numer. Anal. Methods Geomech.*, 30(11):1113–1134, 2006.

[50] A. H.-D. Cheng. Material coefficients of anisotropic poroelasticity. *Int. J. Rock Mech. Min. Sci.*, 34(2):199–205, 1997.

[51] A. H.-D. Cheng and Y. Abousleiman. Intrinsic poroelasticity constants and a semilinear model. *Int. J. Numer. Anal. Methods Geomech.*, 32(7):803–831, 2008.

[52] A. H.-D. Cheng and D.T. Cheng. Heritage and early history of the boundary element method. *Eng. Anal. Bound. Elem.*, 29(3):268–302, 2005.

[53] A. H.-D. Cheng and E. Detournay. On singular integral equations and fundamental solutions of poroelasticity. *Internat. J. Solids Structures*, 35(34-35):4521–4555, 1998.

[54] A. H.-D. Cheng, T. Badmus, and D.E. Beskos. Integral equations for dynamic poroelasticity in frequency domain with BEM solution. *J. Engrg. Mech., ASCE*, 117(5):1136–1157, 1991.

[55] M. Chopra. Poro-elasticity using BEM. In A. J. Kassab and M. H. Aliabadi, editors, *Coupled Field Problems*, volume 11 of *Advances in Boundary Elements*, pages 211–240. WIT-Press, 2001.

[56] O. Coussy. *Poromechanics*. John Wiley & Sons, Chichester, 2004.

[57] J.P. Coyette and H. Wynendaele. A finite element model for predicting the acoustic transmission characteristics of layered structures. In R.J. Bernhard and J.S. Bolton, editors, *Proc. of Inter-Noise 95*, pages 1279–1282, New York, 1995. Noise Control Foundation.

[58] A. Craggs. A finite element for rigid porous absorbing materials. *J. Sound Vibration*, 61 (1):101–111, 1978.

[59] T. A. Cruse and F. J. Rizzo. A direct formulation and numerical solution of the general transient elastodynamic problem, I. *Aust. J. Math. Anal. Appl.*, 22(1):244–259, 1968.

[60] G. F. Dargush and M. B. Chopra. Dynamic analysis of axisymmetric foundations on poroelastic media. *J. Engrg. Mech., ASCE*, 122(7):623–632, 1996.

[61] R. de Boer. *Theory of Porous Media*. Springer-Verlag, Berlin, 2000.

[62] R. de Boer. Highlights in the historical development of the porous media theory: Toward a consistent macroscopic theory. *AMR*, 49(4):201–262, 1996.

[63] R. de Boer. The thermodynamic structure and constitutive equations for fluid-saturated compressible and incompressible elastic porous solids. *Internat. J. Solids Structures*, 35 (34-35):4557–4573, 1998.

[64] R. de Boer and W. Ehlers. The development of the concept of effective stresses. *Acta Mech.*, 83(1-2):77–92, 1990.

[65] R. de Boer, W. Ehlers, and Z. Liu. One-dimensional transient wave propagation in fluid-saturated incompressible porous media. *Arch. Appl. Mech.*, 63(1):59–72, 1993.

[66] P. Debergue, R. Panneton, and N. Atalla. Boundary conditions for the weak formulation of the mixed (u, p) poroelasticity problem. *J. Acoust. Soc. Am.*, 106(5):2383–2390, 1999.

[67] G. Degrande and G. De Roeck. An absorbing boundary condition for wave propagation in saturated poroelastic media – Part I: Formulation and efficiency evaluation. *Soil Dyn. Earthquake Eng. J.*, 12(7):411–421, 1993.

[68] G. Degrande and G. De Roeck. An absorbing boundary condition for wave propagation in saturated poroelastic media – Part II: Finite element formulation. *Soil Dyn. Earthquake Eng. J.*, 12(7):423–432, 1993.

[69] G. Degrande, G. De Roeck, and P. Van Den Broeck. Wave propagation in layered dry, saturated and unsaturated poroelastic media. *Internat. J. Solids Structures*, 35(34–35): 4753–4778, 1998.

[70] H. Deresiewicz. The effect of boundaries on wave propagation in a liquid filled porous solid: I. Reflection of plane waves at a free plane boundary (non-dissipative case). *Bull. Seismol. Soc. Am.*, 50(4):599–607, 1960.

[71] H. Deresiewicz. The effect of boundaries on wave propagation in a liquid filled porous solid: II. Love waves in a porous layer. *Bull. Seismol. Soc. Am.*, 51(1):51–59, 1961.

[72] H. Deresiewicz. The effect of boundaries on wave propagation in a liquid filled porous solid. IV. Surface waves in a half space. *Bull. Seismol. Soc. Am.*, 52(3):627–638, 1962.

[73] H. Deresiewicz. The effect of boundaries on wave propagation in a liquid filled porous solid. VII. Surface waves in a half space in the presence of a liquid layer. *Bull. Seismol. Soc. Am.*, 54(1):425–430, 1964.

[74] H. Deresiewicz. The effect of boundaries on wave propagation in a liquid filled porous solid. V. Transmission across a plane interface. *Bull. Seismol. Soc. Am.*, 54(1):409–416, 1964.

[75] H. Deresiewicz. The effect of boundaries on wave propagation in a liquid filled porous solid. VI. Love waves in a double surface layer. *Bull. Seismol. Soc. Am.*, 54(1):417–423, 1964.

[76] H. Deresiewicz. The effect of boundaries on wave propagation in a liquid filled porous solid. IX. Love waves in a porous internal stratum. *Bull. Seismol. Soc. Am.*, 55(5):919–923, 1965.

[77] H. Deresiewicz and A. Levy. The effect of boundaries on wave propagation in a liquid filled porous solid. X. Transmission through a stratified medium. *Bull. Seismol. Soc. Am.*, 57(3):381–391, 1967.

[78] H. Deresiewicz and J. T. Rice. The effect of boundaries on wave propagation in a liquid filled porous solid. III. Reflection of plane waves at a free plane boundary (general case). *Bull. Seismol. Soc. Am.*, 52(3):595–625, 1962.

[79] H. Deresiewicz and R. Skalak. On uniqueness in dynamic poroelasticity. *Bull. Seismol. Soc. Am.*, 53(4):783–788, 1963.

[80] H. Deresiewicz and B. Wolf. The effect of boundaries on wave propagation in a liquid filled porous solid. VIII. Reflection of plane waves at an irregular boundary. *Bull. Seismol. Soc. Am.*, 54(5):1537–1561, 1964.

[81] E. Detournay and A. H.-D. Cheng. Poroelastic response of a borehole in a non-hydrostatic stress field. *Int. J. Rock Mech. Min. Sci.*, 25(3):178–182, 1988.

[82] E. Detournay and A. H.-D. Cheng. *Fundamentals of Poroelasticity*, volume II of *Comprehensive Rock Engineering: Principles, Practice & Projects*, chapter 5, pages 113–171. Pergamon Press, 1993.

[83] S. Diebels and W. Ehlers. Dynamic analysis of a fully saturated porous medium accounting for geometrical and material non-linearities. *Int. J. Numer. Methods. Engrg.*, 39(1): 81–97, 1996.

[84] J. Domínguez. An integral formulation for dynamic poroelasticity. *J. of Appl. Mech.*, 58: 588–591, 1991.

[85] J. Domínguez. Boundary element approach for dynamic poroelastic problems. *Int. J. Numer. Methods. Engrg.*, 35(2):307–324, 1992.

[86] J. Domínguez and O. Maeso. Earthquake analysis of arch dams. II: Dam-water-foundation interaction. *J. Engrg. Mech., ASCE*, 119(3):513–530, 1993.

[87] I. Edelmann. An analytical interpretation of liquid injection induced microseismicity in porous reservoirs. *Soil Dyn. Earthquake Eng. J.*, 26(6–7):566–573, 2006.

[88] W. Ehlers. Constitutive equations for granular materials in geomechanical context. In K. Hutter, editor, *Continuum Mechanics in Environmental Sciences and Geophysics*, CISM Courses and Lecture Notes, No. 337, pages 313–402. Springer-Verlag, Wien, 1993.

[89] W. Ehlers. Compressible, incompressible and hybrid two-phase models in porous media theories. *ASME: AMD-Vol.*, 158:25–38, 1993.

[90] W. Ehlers and J. Kubik. On finite dynamic equations for fluid-saturated porous media. *Acta Mech.*, 105(1-4):101–117, 1994.

[91] P. Fillunger. Der Auftrieb von Talsperren, Teil I-III. *Österr. Wochenschrift öffentl. Baudienst*, pages 532–570, 1913.

[92] J. Frenkel. On the theory of seismic and seismoelectric phenomena in a moist soil. *J. Engrg. Mech., ASCE*, 131(9):879–887, 2005. doi: 10.1061/(ASCE)0733-9399(2005)131:9(879). URL `http://link.aip.org/link/?QEM/131/879/1`.

[93] J. Frenkel. On the theory of seismic and seismoelectric phenomena in a moist soil. *J. Phys. (USSR)*, III(5):230–241, 1944.

[94] R.J. Galvin and B. Gurevich. Scattering of a longitudinal wave by a circular crack in a fluid-saturated porous media. *Internat. J. Solids Structures*, 44(22–23):7389–7398, 2007.

[95] B. Gatmiri and M. Kamalian. On the fundamental solution of dynamic poroelastic boundary integral equations in the time domain. *Int. J. Geomech.*, 2(4):381–398, 2002.

[96] B. Gatmiri and K.-V. Nguyen. Time 2D fundamental solution for saturated porous media with incompressible fluid. *Commun. Numer. Meth. Engng.*, 21(3):119–132, 2005.

[97] G. Gazetas and E. Petrakis. Offshore caissons on porous saturated soil. In S. Parkash, editor, *Proc. of Int. Conf. on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics*, pages 381–386, University of Missouri-Rolla, Rolla, 1981.

[98] T. L. Geers. Residual potential and approximate methods for three-dimensional fluid-structure interaction problems. *J. Acoust. Soc. Am.*, 49(5B):1505–1510, 1971.

[99] T. L. Geers. Doubly asymptotic approximations for transient motions of submerged structures. *J. Acoust. Soc. Am.*, 64(5):1500–1508, 1978. doi: 10.1121/1.382093. URL `http://link.aip.org/link/?JAS/64/1500/1`.

[100] J. Ghaboussi and E. L. Wilson. Variational formulation of dynamics of fluid-saturated porous elastic solids. *J. Engrg. Mech., ASCE*, 98(EM4):947–963, 1972.

[101] P. Göransson. A 3-D, symmetric, finite element formulation of the Biot equations with application to acoustic wave propagation through an elastic porous medium. *Int. J. Numer. Methods. Engrg.*, 41(1):167–192, 1998.

[102] S. K. Grag, A. H. Nafeh, and A. J. Good. Compressional waves in fluid-saturated elastic porous media. *J. Appl. Phys.*, 45(5):1968–1974, 1974.

[103] A. E. Green and P. M. Naghdi. A theory of mixtures. *Arch. Rational Mech. Anal.*, 24(4): 243–263, 1967.

[104] A. E. Green and P. M. Naghdi. A note on mixtures. *Int. J. Engng. Sci.*, 6(11):631–635, 1968.

[105] B. Gurevich. Comparison of the low-frequency predicitons of Biot's and de Boer's poroelasticity theories with Gassmann's equation. *Appl. Phy. Lett.*, 91(9):091919, 2007.

[106] M. R. Halpern and P. Christiano. Response of poroelastic halfspace to steady-state harmonic surface tractions. *Int. J. Numer. Anal. Methods Geomech.*, 10:609–632, 1986.

[107] M. R. Halpern and P. Christiano. Steady-state harmonic response of a ridgid plate bearing on a liquid-saturated poroelastic halfspace. *Earthquake Eng. Struct. Dyn.*, 14:439–454, 1986.

[108] A. Hanyga. Time-domain poroelastic green's functions. *J. Comp. Acoustics*, 11(3):491–501, 2003.

[109] S.M. Hasheminejad and R. Avazmohammadi. Acoustic diffraction by a pair of poroelastic cylinders. *Z. Angew. Math. Mech.*, 86(8):589–605, 2006.

[110] S.M. Hasheminejad and H. Hosseini. Nonaxisymmetric interaction of a spherical radiator in a fluid-filled permeable borehole. *Internat. J. Solids Structures*, 45(1):24–47, 2008.

[111] S.M. Hasheminejad and S. Mehdizadeh. Acoustic radiation from a finite spherical source placed in fluid near a poroelastic sphere. *Arch. Appl. Mech.*, 74(1-2):59–74, 2004.

[112] G. Heinrich and K. Desoyer. Hydromechanische Grundlagen für die Behandlung von stationären und instationären Grundwasserströmungen. *Arch. Appl. Mech.*, 23(2):73–84, 1955.

[113] G. Heinrich and K. Desoyer. Hydromechanische Grundlagen für die Behandlung von stationären und instationären Grundwasserströmungen. *Arch. Appl. Mech.*, 24(2):81–84, 1956.

[114] M. S. Hiremath, R. S. Sandhu, L. W. Morland, and W. E. Wolfe. Analysis of one-dimensional wave propagation in a fluid-saturated finite soil column. *Int. J. Numer. Anal. Methods Geomech.*, 12:121–139, 1988.

[115] S. J. Hong, R. S. Sandhu, and W. E Wolfe. On Grag's solution of Biot's equations for wave propagation in a one-dimensional fluid-saturated elastic porous solid. *Int. J. Numer. Anal. Methods Geomech.*, 12:627–637, 1988.

[116] N.-E. Hörlin. 3D hierarchical *hp*-FEM applied to elasto-acoustic modelling of layered porous media. *J. Acoust. Soc. Am.*, 285(1–2):341–363, 2005.

[117] N.-E. Hörlin, M. Nordström, and P. Göransson. A 3-D hierarchical FE formulation of Biot's equations for elasto-acoustic modelling of porous media. *J. Acoust. Soc. Am.*, 245 (4):633–652, 2001.

[118] B. Jin and H. Liu. Horizontal vibrations of a disk on a poroelastic half-space. *Soil Dyn. Earthquake Eng. J.*, 19(4):269–275, 2000.

[119] B. Jin and H. Liu. Rocking vibrations of rigid disk on saturated poroelastic medium. *Soil Dyn. Earthquake Eng. J.*, 19(7):469–472, 2000.

[120] B. Jin and H. Liu. Dynamic response of a poroelastic half space to horizontal buried loading. *Internat. J. Solids Structures*, 38(44-45):8053–8064, 2001.

[121] B. Jin and H. Liu. Vertical dynamic response of a disk on a saturated poroelastic half-space. *Soil Dyn. Earthquake Eng. J.*, 18(6):437–443, 1999.

[122] B. Jin and Z. Zhong. Dynamic stress intensity factor (Mode I) of a penny-shaped crack in infinite poroelastic solid. *Int. J. Engng. Sci.*, 40(6):637–646, 2002.

[123] D. L. Johnson, J. Koplik, and R. Dashen. Theory of dynamic permeability and tortuosity in fluid-saturated porous media. *Journal of Fluid Mechanics*, 176:379–402, 1987.

[124] M. Kamalian, B. Gatmiri, and M. J. Sharahi. Time domain 3D fundamental solutions for saturated poroelastic media with incompressible constituents. *Commun. Numer. Meth. Engng.*, 24(9):749–759, 2008.

[125] Y.J. Kang and J.S. Bolton. Finite element modeling of isotropic elastic porous materials coupled with acoustical finite elements. *J. Acoust. Soc. Am.*, 98(1):635–643, 1995.

[126] S. E. Kattis, D. E. Beskos, and A. H.-D. Cheng. 2D dynamic response of unlined and lined tunnels in poroelastic soil to harmonic body waves. *Earthquake Eng. Struct. Dyn.*, 32(1):97–110, 2003.

[127] E. Kausel. Discussion on 'dynamic response of a multi-layered poroelastic medium'. *Earthquake Eng. Struct. Dyn.*, 25(10):1165–1167, 1996.

[128] A. M. Kaynia and P. K. Banerjee. Fundamental solutions of Biot's equations of dynamic poroelasticity. *Int. J. Engng. Sci.*, 31(5):817–830, 1993.

[129] M. N. Kazi-Aoual, G. Bonnet, and P. Jouanna. Green's functions in an infinite transversely isotropic saturated poroelastic medium. *J. Acoust. Soc. Am.*, 84(5):1883–1889, 1988.

[130] N. Khalili, M. Yazdchi, and S. Valliappan. Wave propagation analysis of two-phase saturated porous media using coupled finite-infinite element method. *Soil Dyn. Earthquake Eng. J.*, 18(8):533–553, 1999.

[131] Y. K. Kim and H. B. Kingsbury. Dynamic characterization of poroelastic materials. *Exp. Mech.*, 19(7):252–258, 1979.

[132] V. D. Kupradze. *Potential Methods in the Theory of Elasticity*. Israel Program for Scientific Translations, Jerusalem, 1965.

[133] L. Lanzoni, E. Radi, and A. Tralli. On the seismic response of a flexible wall retaining a viscous poroelastic soil. *Soil Dyn. Earthquake Eng. J.*, 27(9):818–842, 2007.

[134] R. W. Lewis and B. A. Schrefler. *The Finite Element Method in the Static and Dynamic Deformation and Consolidation of Porous Media*. John Wiley and Sons, Chichester, 1998.

[135] C. Li, R. I. Borja, and R. A. Regueiro. Dynamics of porous media at finite strain. *Comput. Methods Appl. Mech. Engrg.*, 193(36–38):3837–3870, 2004.

[136] J. Liang, Z. Ba, and V. W. Lee. Diffraction of plane SV waves by a shallow cicular-arc canyon in a saturated poroelastic half-space. *Soil Dyn. Earthquake Eng. J.*, 26(6–7): 582–610, 2006.

[137] J. Liang, H. You, and V. W. Lee. Scattering of SV waves by a canyon in a fluid-saturated, poroelastic layered half-space, modeled using the indirect boundary element method. *Soil Dyn. Earthquake Eng. J.*, 26(6–7):611–625, 2006.

[138] C.-H. Lin, V. W. Lee, and M. D. Trifunac. The reflection of plane waves in a poroelastic half-space saturated with inviscid fluid. *Soil Dyn. Earthquake Eng. J.*, 25(3):205–223, 2005.

[139] Z. Liu, J. Bluhm, and R. de Boer. Inhomogeneous plane waves, mechanical energy flux, and energy dissipation in a two-phase porous medium. *Z. Angew. Math. Mech.*, 78(9): 617–625, 1998.

[140] S. L. Lopatnikov and A. H.-D. Cheng. Variational formulation of fluid infiltrated porous material in thermal and mechanical equilibrium. *Mech. Matls.*, 34(11):685–704, 2002.

[141] S. L. Lopatnikov and A. H.-D. Cheng. Macroscopic Lagrangian formulation of poroelasticity with porosity dynamics. *J. Mech. Phys. Solids*, 52(12):2801–2839, 2004.

[142] J.-F. Lu and D.-S. Jeng. Dynamic analysis of an infinite cylindrical hole in a saturated poroelastic medium. *Arch. Appl. Mech.*, 76(5–6):263–276, 2006.

[143] J.-F. Lu, D.-S. Jeng, and S. Williams. A 2.5-D dynamic model for a saturated porous medium: Part I. Green's function. *Internat. J. Solids Structures*, 45(2):378–391, 2008.

[144] J.-F. Lu, D.-S. Jeng, and S. Williams. A 2.5-D dynamic model for a saturated porous medium: Part II. Boundary element method. *Internat. J. Solids Structures*, 45(2):359–377, 2008.

[145] C. Lubich. Convolution quadrature and discretized operational calculus. I. *Numer. Math.*, 52(2):129–145, 1988.

[146] C. Lubich. Convolution quadrature and discretized operational calculus. II. *Numer. Math.*, 52(4):413–425, 1988.

[147] J. E. Luco, H. L. Wong, and F. C. P. De Barros. Three-dimensional response of a cylindrical canyon in a layered half-space. *Earthquake Eng. Struct. Dyn.*, 19(6):799–817, 1990.

[148] O. Maeso and J. Domínguez. Earthquake analysis of arch dams. I: Dam-foundation interaction. *J. Engrg. Mech., ASCE*, 119(3):496–512, 1993.

[149] O. Maeso, J. J. Aznárez, and F. García. Dynamic impedances of piles and group of piles in saturated soils. *Comput. & Structures*, 83(10–11):769–782, 2005.

[150] G. D. Manolis and D. E. Beskos. Integral formulation and fundamental solutions of dynamic poroelasticity and thermoelasticity. *Acta Mech.*, 76(1-2):89–104, 1989. Errata [151].

[151] G. D. Manolis and D. E. Beskos. Corrections and additions to the paper "Integral formulation and fundamental solutions of dynamic poroelasticity and thermoelasticity". *Acta Mech.*, 83(3-4):223–226, 1990.

[152] W. J. Mansur. *A Time-Stepping Technique to Solve Wave Propagation Problems Using the Boundary Element Method*. Phd thesis, University of Southampton, 1983.

[153] M. T. Manzari. On finite deformation dynamic analysis of saturated soils. *Arch. Mech.*, 48(2):281–310, 1996.

[154] C. C. Mei and M. A. Foda. Wind-induced response in a fluid-filled poroelastic solid with a free surface – a boundary layer theory. *Geophys. J. Roy. Astron. Soc.*, 66(3):597–631, 1981.

[155] C. C. Mei, B. I. Si, and D. Cai. Scattering of simple harmonic waves by a circular cavity in a fluid-infiltrated poroelastic medium. *Wave Motion*, 6(3):265–278, 1984.

[156] C. C. Mei, B. I. Si, and Y.-S. Chen. Dynamic response in a poroealstic ground induced by a moving air pressure. *Wave Motion*, 7(2):129–141, 1985.

[157] K. K. Muraleetharan and C. Wei. Dynamic behaviour of unsaturated porous media: Governing equations using the theory of mixtures with interfaces (TMI). *Int. J. Numer. Anal. Methods Geomech.*, 23(13):1579–1608, 1999.

[158] V. N. Nikolaevskiy. Biot–frenkel poromechanics in russia (review). *J. Engrg. Mech., ASCE*, 131(9):888–897, 2005. doi: 10.1061/(ASCE)0733-9399(2005)131:9(888). URL `http://link.aip.org/link/?QEM/131/888/1`.

[159] A. N. Norris. Radiation from a point source and scattering theory in a fluid-saturated porous solid. *J. Acoust. Soc. Am.*, 77(6):2012–2023, 1985.

[160] A. N. Norris. Dynamic Green's functions in anisotropic piezoelectric, thermoelastic and poroelastic solids. *Proc. R. Soc. Lond. A*, 447:175–188, 1994.

[161] E. Pan. Static Green's functions in multilayered half spaces. *Appl. Math. Modelling*, 21 (8):509–521, 1997.

[162] E. Pan. Green's function in layered poroelastic half-spaces. *Int. J. Numer. Anal. Methods Geomech.*, 23(13):1631–1653, 1999.

[163] R. Panneton and N. Atalla. An efficient finite element scheme for solving the three-dimensional poroelasticity problem in acoustics. *J. Acoust. Soc. Am.*, 101(6):3287–3298, 1997.

[164] S. Paul. On the displacements produced in a porous elastic half-space by an impulsive line load (non-dissipative case). *Pure and Appl. Geophysics*, 114(4):605–614, 1976.

[165] S. Paul. On the disturbance produced in a semi-infinite poroelastic medium by a surface load. *Pure and Appl. Geophysics*, 114(4):615–627, 1976.

[166] A. J. Philippacopoulos. Lamb's problem for fluid-saturated, porous media. *Bull. Seismol. Soc. Am.*, 78(2):908–923, 1988.

[167] A. J. Philippacopoulos. Axisymmetric vibrations of disk resting on saturated layered half-space. *J. Engrg. Mech., ASCE*, 115(10):2301–2322, 1989.

[168] A. J. Philippacopoulos. Buried point source in a poroelastic half-space. *J. Engrg. Mech., ASCE*, 123(8):860–869, 1997.

[169] A. J. Philippacopoulos. Spectral Green's dyadic for point sources in poroelastic media. *J. Engrg. Mech., ASCE*, 124(1):24–31, 1998.

[170] T. J. Plona. Observation of a second bulk compressional wave in porous medium at ultrasonic frequencies. *Appl. Phy. Lett.*, 36(4):259–261, 1980.

[171] J. H. Prevost. Nonlinear transient phenomena in saturated porous media. *Comput. Methods Appl. Mech. Engrg.*, 30(1):3–18, 1982.

[172] J. H. Prevost. Wave propagation in fluid-saturated porous media: An efficient finite element procedure. *Soil Dyn. Earthquake Eng. J.*, 4(4):183–202, 1985.

[173] S. R. Pride and S. Garambois. Electroseismic wave theory of Frenkel and more recent developments. *J. Engrg. Mech., ASCE*, 131(9):898–907, 2005. doi: 10.1061/(ASCE)0733-9399(2005)131:9(898). URL `http://link.aip.org/link/?QEM/131/898/1`.

[174] Q. Qi and T. L. Geers. Double asymptotic approximations for transient poroelastodynamics. *J. Acoust. Soc. Am.*, 102(3):1361–1371, 1997.

[175] R. K. N. D. Rajapakse and T. Senjuntichai. Dynamic response of a multi-layered poroelastic medium. *Earthquake Eng. Struct. Dyn.*, 24:703–722, 1995.

[176] R. K. N. D. Rajapakse and T. Senjuntichai. An indirect boundary integral equation method for poroelasticity. *Int. J. Numer. Anal. Methods Geomech.*, 19(9):587–614, 1995.

[177] S. Rigobert, N. Atalla, and F.C. Sgard. Investigation of the convergence of the mixed displacement-pressure formulation for three-dimensional poroelastic materials using hierarchical elements. *J. Acoust. Soc. Am.*, 114(5):2607–2617, 2003.

[178] M. Schanz. Dynamic poroelasticity treated by a time domain boundary element method. In T. Burczynski, editor, *IUTAM/IACM/IABEM Symposium on Advanced Mathematical and Computational Mechanic Aspects of the Boundary Element Method*, pages 303–314. Kluwer Academic Publishers, Dordrecht, Boston, London, 2001.

[179] M. Schanz. Application of 3-d Boundary Element formulation to wave propagation in poroelastic solids. *Eng. Anal. Bound. Elem.*, 25(4-5):363–376, 2001.

[180] M. Schanz. *Wave Propagation in Viscoelastic and Poroelastic Continua: A Boundary Element Approach*, volume 2 of *Lecture Notes in Applied Mechanics*. Springer-Verlag, Berlin, Heidelberg, New York, 2001.

[181] M. Schanz and A. H.-D. Cheng. Transient wave propagation in a one-dimensional poroelastic column. *Acta Mech.*, 145(1-4):1–18, 2000.

[182] M. Schanz and A. H.-D. Cheng. Dynamic analysis of a one-dimensional poroviscoelastic column. *J. of Appl. Mech.*, 68(2):192–198, 2001.

[183] M. Schanz and S. Diebels. A comparative study of Biot's theory and the linear Theory of Porous Media for wave propagation problems. *Acta Mech.*, 161(3-4):213–235, 2003.

[184] M. Schanz and D. Pryl. Dynamic fundamental solutions for compressible and incompressible modeled poroelastic continua. *Internat. J. Solids Structures*, 41(15):4047–4073, 2004.

[185] M. Schanz and V. Struckmeier. Wave propagation in a simplified modeled poroelastic continuum: Fundamental solutions and a time domain boundary element formulation. *Int. J. Numer. Methods. Engrg.*, 64(13):1816–1839, 2005.

[186] B. A. Schrefler, L. Simoni, L. Xikui, and O. C. Zienkiewicz. Mechanics of partially saturated porous media. In C.S. Desai and G. Gioda, editors, *Numerical Methods and Constitutive Modelling in Geomechanics*, CISM courses, pages 169–209. Springer-Verlag, Wien, 1990.

[187] A.P.S. Selvadurai. The analytical method in geomechanics. *AMR*, 60(3):87–106, 2007.

[188] T. Senjuntichai and R. K. N. D. Rajapakse. Transient response of a circular cavity in a poroelastic medium. *Int. J. Numer. Anal. Methods Geomech.*, 17(6):357–383, 1993.

[189] T. Senjuntichai and R. K. N. D. Rajapakse. Dynamic Green's functions of homogeneous poroelastic half-plane. *J. Engrg. Mech., ASCE*, 120(11):2381–2404, 1994.

[190] T. Senjuntichai, S. Mani, and R. K. N. D. Rajapakse. Vertical vibration of an embedded rigid foundation in a poroelastic soil. *Soil Dyn. Earthquake Eng. J.*, 26(6–7):626–636, 2006.

[191] X.-M. Shao and Z.-L. Lan. Finite element methods for the equations of waves in fluid-saturated porous media. *Chin. J. Geophys.*, 43(2):291–306, 2000.

[192] M. D. Sharma. Comments on “Lamb’s problem for fluid-saturated porous media”. *Bull. Seismol. Soc. Am.*, 82(5):2263–2273, 1992.

[193] B. R. Simon, J. S. S. Wu, and O. C. Zienkiewicz. Evaluation of higher order, mixed and Hermitean finite element procedures for dynamic analysis of saturated porous media using one-dimensional models. *Int. J. Numer. Anal. Methods Geomech.*, 10(5):483–499, 1986.

[194] B. R. Simon, J. S. S. Wu, O. C. Zienkiewicz, and D. K. Paul. Evaluation of u-w and u-p FEM for the response of saturated porous media using one-dimensional models. *Int. J. Numer. Anal. Methods Geomech.*, 10(5):461–482, 1986.

[195] D. Soares, J. C. F. Telles, and W. J. Mansur. A time-domain boundary element formulation for the dynamic analysis of non-linear porous media. *Eng. Anal. Bound. Elem.*, 30(5): 363–370, 2006.

[196] J. C. F. Telles. *The Boundary Element Method Applied to Inelastic Problems*. Springer-Verlag, Berlin, 1983.

[197] D. D. Theodorakopoulos. Dynamic analysis of a poroelastic half-plane soil medium under moving loads. *Soil Dyn. Earthquake Eng. J.*, 23(7):521–533, 2003.

[198] D. D. Theodorakopoulos. Dynamic pressures on a pair of rigid walls retaining poroelastic soil. *Soil Dyn. Earthquake Eng. J.*, 23(1):41–51, 2003.

[199] D. D. Theodorakopoulos and D. E. Beskos. Dynamic pressures on a pair of rigid walls experiencing base rotation and retaining poroelastic soil. *Engineering Structures*, 25(3): 359–370, 2003.

[200] D. D. Theodorakopoulos, A. P. Chassiakos, and D. E. Beskos. Dynamic pressures on rigid cantilever walls retaining poroelastic soil media. Part I: First method of solution. *Soil Dyn. Earthquake Eng. J.*, 21(4):315–338, 2001.

[201] D. D. Theodorakopoulos, A. P. Chassiakos, and D. E. Beskos. Dynamic pressures on rigid cantilever walls retaining poroelastic soil media. Part II: Second method of solution. *Soil Dyn. Earthquake Eng. J.*, 21(4):339–364, 2001.

[202] D. D. Theodorakopoulos, A. P. Chassiakos, and D. E. Beskos. Dynamic effects of moving load on a poroelastic soil medium by an approximate method. *Internat. J. Solids Structures*, 41(7):1801–1822, 2004.

[203] D.D. Theodorakopoulos and D.E. Beskos. Flexural vibrations of fissured poroelastic plates. *Arch. Appl. Mech.*, 63(6):413–423, 1993.

[204] D.D. Theodorakopoulos and D.E. Beskos. Flexural vibrations of poroelastic plates. *Acta Mech.*, 103:191–203, 1994.

[205] J.-F. Thimus, A. H.-D. Cheng, O. Coussy, and E. Detournay, editors. *Poromechanics – A Tribute to Maurice A. Biot*, Rotterdam, 1998. A.A. Balkema.

[206] I. Tolstoy, editor. *Acoustics, Elasticity and Thermodynamics of Porous Media: Twenty-One Papers by M. A. Biot*. Acoustical Society of America, 1991.

[207] C. Truesdell and R.A. Toupin. The classical field theories. In S. Flügge, editor, *Handbuch der Physik*, volume III/1, pages 226–793. Springer-Verlag, Berlin, 1960.

[208] I. Vardoulakis and D. E. Beskos. Dynamic behavior of nearly saturated porous media. *Mech. Matls.*, 5(1):87–108, 1986.

[209] I. Vgenopoulou and D. E. Beskos. Dynamic behavior of saturated poroviscoelastic media. *Acta Mech.*, 95(1-4):185–195, 1992.

[210] I. Vgenopoulou and D.E. Beskos. Dynamics of saturated rocks. IV: Column and borehole problems. *J. Engrg. Mech., ASCE*, 118(9):1795–1813, 1992.

[211] I. Vgenopoulou and D.E. Beskos. Dynamic poroelastic soil column and borehole problem analysis. *Soil Dyn. Earthquake Eng. J.*, 11(6):335–345, 1992.

[212] I. Vgenopoulou, D. E. Beskos, and I. Vardoulakis. High frequency wave propagation in nearly saturated porous media. *Acta Mech.*, 85(1–2):115–123, 1990.

[213] C. Wei and K. K. Muraleetharan. A continuum theory of porous media saturated by multiple immiscible fluids: I. Linear poroelasticity. *Int. J. Engng. Sci.*, 40(16):1807–1833, 2002.

[214] C. Wei and K. K. Muraleetharan. Acoustical characterization of fluid-saturated porous media with local heterogenities: Theory and application. *Internat. J. Solids Structures*, 43(5):982–1008, 2006.

[215] C. Wei and K. K. Muraleetharan. Linear viscoelastic behavior of porous media with non-uniform saturation. *Int. J. Engng. Sci.*, 45(2–8):698–715, 2007.

[216] Th. Wiebe and H. Antes. A time domain integral formulation of dynamic poroelasticity. *Acta Mech.*, 90(1-4):125–137, 1991.

[217] K. Wilmanski. Some questions on material objectivity arising in models of porous materials. In P. Podio-Guidugli and M. Brocato, editors, *Rational Continua, Classical and New*, pages 149–161. Springer-Verlag, Milano Berlin Heidelberg, 2001.

[218] K. Wilmanski. Thermodynamics of multicomponent continua. In J. Majewski and R. Teisseyre, editors, *Earthquake Thermodynamics and Phase Transformations in the Earth's Interior*, pages 567–655. Academic press, San Diego, 2001.

[219] K. Wilmanski. Thermodynamical admissibility of Biot's model of poroelastic saturated materials. *Arch. Mech.*, 54(5–6):709–736, 2002.

[220] K. Wilmanski. On microstructural tests for poroelastic materials and corresponding Gassmann-type relations. *Geotechnique*, 54(9):593–603, 2004.

[221] K. Wilmanski. Elastic modelling of surface waves in single and multicomponent systems. In C. G. Lai and K. Wilmanski, editors, *Surface Waves in Geomechanics: Direct and Inverse Modelling for Soils and Rocks*, volume 481 of *CISM*, chapter 5, pages 203–276. Springer-Verlag, Wien, 2005.

[222] K. Wilmanski. Tortuosity and objective relative accelerations in the theory of porous media. *Proc. R. Soc. Lond. A*, 461(2057):1533–1561, 2005.

[223] K. Wilmanski. A few remarks on Biot's model and linear acoustics of poroelastic saturated materials. *Soil Dyn. Earthquake Eng. J.*, 26(6-7):509–536, 2006.

[224] K. Wilmanski. *Thermodynamics of Continua*. Springer-Verlag, Berlin Heidelberg New York, 1998.

[225] K. Wilmanski. A thermodynamic model of compressible porous materials with the balance equation of porosity. *Transp. Porous Media*, 32(1):21–47, 1998.

[226] R. K. Wilson and E. C. Aifantis. On the theory of consolidation with double porosity. *Int. J. Engng. Sci.*, 20:1009–1035, 1982.

[227] K. H. Xie, G.-B. Liu, and Z.-Y. Shi. Dynamic response of partially sealed circular tunnel in viscoelastic saturated soil. *Soil Dyn. Earthquake Eng. J.*, 24(12):1003–1011, 2004.

[228] F.-H. Yeh and H.-S. Tsay. Dynamic behavior of a poroelastic slab subjected to uniformly distributed impulsive loading. *Comput. & Structures*, 67(4):267–277, 1998.

[229] X. Zeng and R. K. N. D. Rajapakse. Vertical vibrations of a rigid disk embedded in a poroelastic medium. *Int. J. Numer. Anal. Methods Geomech.*, 23(15):2075–2095, 1999.

[230] L. P. Zhang and A. K. Chopra. 3-dimensional analysis of spatially varying ground motions around a uniform canyon in a homogeneous half-space. *Earthquake Eng. Struct. Dyn.*, 20 (10):911–926, 1991.

[231] O. C. Zienkiewicz and T. Shiomi. Dynamic behaviour of saturated porous media; the generalized Biot formulation and its numerical solution. *Int. J. Numer. Anal. Methods Geomech.*, 8:71–96, 1984.

[232] O. C. Zienkiewicz and Y. M. Xie. A simple error estimator and adaptive time stepping procedure for dynamic analysis. *Earthquake Eng. Struct. Dyn.*, 20:871–887, 1991.

[233] O. C. Zienkiewicz, C. T. Chang, and P. Bettess. Drained, undrained, consolidating and dynamic behaviour assumptions in soils. *Geotechnique*, 30(4):385–395, 1980.

[234] O. C. Zienkiewicz, E. Hinton, K. H. Leung, and R. L. Taylor. Staggered time marching schemes in dynamic soil analysis and a selective explicit extrapolation. In *Proc. of the Second Symposium on Innovative Numerical Analysis for the Engineering Sciences*. University of Virginia Press, 1980.

[235] O. C. Zienkiewicz, D. K. Paul, and A. H. C. Chan. Unconditionally stable staggered solution procedure for soil-pore fluid interaction problems. *Int. J. Numer. Methods. Engrg.*, 26:1039–1055, 1988.

[236] O. C. Zienkiewicz, A. H. C. Chan, M.Pastor, D. K.Paul, and T. Shiomi. Static and dynamic behaviour of soils: A rational approach to quantitative solutions. I. fully saturated problems. *Proc. R. Soc. Lond. A*, 429:285–309, 1990.

[237] O. C. Zienkiewicz, A. H. C. Chan, M. Pastor, B. A. Schrefler, and T. Shiomi. *Computational Geomechanics with Special Reference to Earthquake Engineering*. John Wiley & Sons, Chichester, New York, Weinheim, Brisbane, Singapore, Toronto, 1999.

[238] D. Zimmerman and M. Stern. Boundary element solution of 3-D wave scatter problems in a poroelastic medium. *Eng. Anal. Bound. Elem.*, 12(4):223–240, 1993.