Preview of Grade 7

Multi-Step Equations and Inequalities

CONCEPT CONNECTION

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Chapter Project Online
KEYWORD: MS8CA Ch11

The best length for a surfboard depends on a surfer's body type and skill level. An algebraic inequality can be used to represent possible surfboard lengths.

Surfer
California coast

ARE YOU READY?

✓ Vocabulary

Choose the best term from the list to complete each sentence.

1. __?__ are mathematical operations that undo each other.

2. To solve an equation you need to __?__.

3. A(n) __?__ is a mathematical statement that two expressions are equivalent.

4. A(n)__?__ is a mathematical statement that two ratios are equivalent.

isolate the variable

equation

proportion

inverse operations

expression

Complete these exercises to review skills you will need for this chapter.

✓ Add Whole Numbers, Decimals, Fractions, and Integers

Add.

5. $24 + 16$
6. $-34 + (-47)$
7. $35 + (-61)$
8. $-12 + (-29) + 53$

9. $2.7 + 3.5$
10. $\frac{2}{3} + \frac{1}{2}$
11. $5.87 + 10.6$
12. $\frac{8}{9} + \frac{9}{11}$

✓ Evaluate Expressions

Evaluate each expression for $a = 7$ and $b = -2$.

13. $a - b$
14. $b - a$
15. $\frac{b}{a}$
16. $2a + 3b$

17. $\frac{-4a}{b}$
18. $3a - \frac{8}{b}$
19. $1.2a + 2.3b$
20. $-5a - (-6b)$

✓ Solve Equations by Dividing

Solve.

21. $8x = -72$
22. $-12a = -60$
23. $\frac{2}{3}y = 16$
24. $-12b = 9$

25. $12 = -4x$
26. $13 = \frac{1}{2}c$
27. $-2.4 = -0.8p$
28. $\frac{3}{4} = 6x$

✓ Solve Proportions

Solve.

29. $\frac{3}{4} = \frac{x}{24}$
30. $\frac{8}{9} = \frac{4}{a}$
31. $\frac{12}{5} = \frac{15}{c}$
32. $\frac{y}{50} = \frac{35}{20}$

33. $\frac{2}{3} = \frac{18}{w}$
34. $\frac{35}{21} = \frac{d}{3}$
35. $\frac{7}{13} = \frac{h}{195}$
36. $\frac{9}{15} = \frac{27}{p}$

Unpacking the Standards

The information below "unpacks" the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

California Standard	Academic Vocabulary	Chapter Concept
Preview of Grade 7 AF1.3 **Simplify** numerical expressions by **applying** properties of rational numbers (e.g., identity, inverse, **distributive, associative, and commutative) and **justify** the process used.** (Lessons 11-2, 11-3)	**simplify** rewrite in a simpler form **applying** using **justify** give a reason for	You use mathematical properties to write expressions in a simpler form. You give reasons for each step when you simplify expressions.
Preview of Grade 7 AF4.0 Students **solve** simple linear equations and **inequalities over** the rational numbers. (Lessons 11-6, 11-7)	**solve** find the value or values of a variable that make an equation or inequality true **over** for the set of	You find the set of values that make an inequality true. ***Example:*** $x + 4 < 10$ $x + 4 - 4 < 10 - 4$ $x < 6$ Any number less than 6 is a solution of the inequality.
Preview of Grade 7 AF4.1 Solve **two-step** linear equations and inequalities in one variable **over the rational numbers,** interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results. (Lessons 11-1, 11-8) (Lab 11-1)	**two-step** refers to an equation or inequality that can be solved by using two operations	You solve two-step linear equations and inequalities. ***Example:*** $2x + 5 = 13$ $2x + 5 - 5 = 13 - 5$ $2x = 8$ $\frac{2x}{2} = \frac{8}{2}$ $x = 4$
Preview of Algebra I 5.0 Students solve **multistep** problems, including word problems, **involving** linear equations and linear inequalities **in one variable** and provide justification for each step. (Lessons 11-3, 11-4)	**multistep** needing more than one step **involving** needing the use of	You solve equations when finding the solution involves two or more steps.

Standards 6AF1.2, 6AF3.1, 6AF3.2, and 7AF1.1 are also covered in this chapter. To see standards 6AF1.2, 6AF3.1, and 6AF3.2 unpacked, go to Chapter 1, p. 4 (6AF1.2), and Chapter 10, p. 484 (6AF3.1 and 6AF3.2).

Reading and Writing Math

Study Strategy: Prepare for Your Final Exam

Math is a cumulative subject, so your exam will cover all of the material you have learned from the beginning of the course. Being prepared is the key for you to be successful on your exam.

2 weeks before the final exam

- Review lesson notes and vocabulary.
- Look at previous exams and homework. Rework problems that I answered incorrectly or that I did not complete.
- Make a list of all formulas, rules, and important steps.
- Create a practice exam using problems from the book that are similar to problems from the previous tests.

1 week before the final exam

- Take the practice exam and check it. For each problem I miss, find two or three similar problems and work those.
- Look over each chapter's Study Guide: Review.
- Quiz a friend or myself on the formulas and major points from my list.
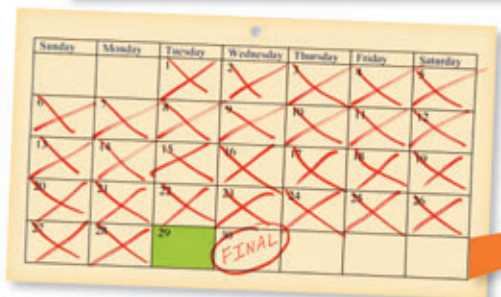
1 day before the final exam

- Make sure I have sharpened pencils with erasers and any other math tools I may need.
- Review any problem areas one last time.

FINAL

Try This

1. Create a timeline that you will use to study for your final exam.

Model Two-Step Equations

KEY

| + | = positive variable |
| − | = negative variable |

+ = 1   − = −1

REMEMBER
- + + − = 0
- + + − = 0
- In an equation, the expressions on both sides of the equal sign are equivalent.

In Lab 2-5, you learned how to solve one-step equations using algebra tiles. You can also use algebra tiles to solve two-step equations. When solving a two-step equation, it is easiest to perform addition and subtraction before multiplication and division.

Activity

1 Use algebra tiles to model and solve $2p + 2 = 10$.

$2p + 2 = 10$
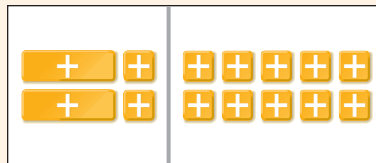
Model the equation.

$$2p + 2 = 10$$
$$\underline{-2 \quad -2}$$
$$2p \quad = \quad 8$$
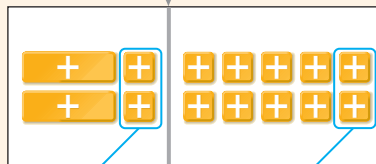
Remove 2 yellow tiles from each side of the mat.

$$\frac{2p}{2} = \frac{8}{2}$$
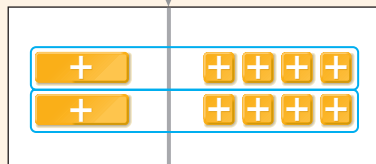
Divide each side into 2 equal groups.

$$p = 4$$
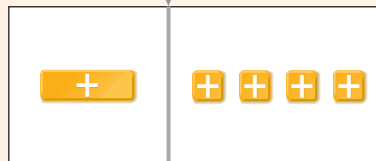
The solution is p = 4.

2 Use algebra tiles to model and solve $3n + 6 = -15$.

$3n + 6 = -15$

Model the equation.

$3n + 6 + (-6) = -15 + (-6)$

It is not possible to remove 6 yellow tiles from each side of the mat. Add 6 red tiles to each side.

$3n = -21$

Remove zero pairs from the left side of the mat.

$\dfrac{3n}{3} = \dfrac{-21}{3}$

Divide each side into 3 equal groups.

$n = -7$

The solution is $n = -7$.

Think and Discuss

1. When you add a value to one side of an equation, why do you also have to add the same value to the other side?

2. When you solved $3n + 6 = -15$ in the activity, why were you able to remove six yellow unit tiles and six red unit tiles from the left side of the equation?

3. Model and solve $3x - 5 = 10$. Explain each step.

4. How would you check the solution to $3n + 6 = -15$ using algebra tiles?

Try This

Use algebra tiles to model and solve each equation.

1. $4 + 2x = 20$

2. $3r + 7 = -8$

3. $-4m + 3 = -25$

4. $-2n - 5 = 17$

5. $10 = 2j - 4$

6. $5 + r = 7$

7. $4h + 2h + 3 = 15$

8. $-3g = 9$

9. $5k + (-7) = 13$

11-1 Solving Two-Step Equations

Why learn this? You can solve a two-step equation to determine the monthly cost of a tennis-club membership. (See Example 3.)

When you solve equations that have one operation, you use an inverse operation to isolate the variable.

$$\begin{aligned} n + 7 &= 15 \\ -7 & \quad -7 \\ \hline n &= 8 \end{aligned}$$

You can also use inverse operations to solve equations that have more than one operation.

$$\begin{aligned} 2x + 3 &= 23 \\ -3 & \quad -3 \\ \hline 2x &= 20 \end{aligned}$$

Use the inverse of multiplying by 2 to isolate x.

$$\frac{2x}{2} = \frac{20}{2}$$
$$x = 10$$

E X A M P L E 1 **Solving Two-Step Equations Containing Multiplication**

Solve.

Reasoning

Helpful Hint

Reverse the order of operations when solving equations that have more than one operation.

A $2n + 5 = 13$

$$\begin{aligned} 2n + 5 &= 13 \\ -5 & \quad -5 \\ \hline 2n &= 8 \end{aligned}$$
Subtract 5 from both sides.

$$\frac{2n}{2} = \frac{8}{2}$$
Divide both sides by 2.
$$n = 4$$

B $19 = -3p - 8$

$$\begin{aligned} 19 &= -3p - 8 \\ +8 & \quad\quad +8 \\ \hline 27 &= -3p \end{aligned}$$
Add 8 to both sides.
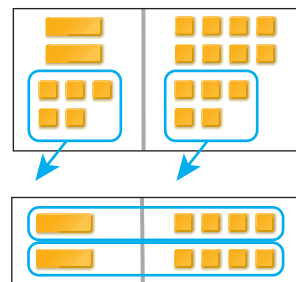
$$\frac{27}{-3} = \frac{-3p}{-3}$$
Divide both sides by −3.
$$-9 = p$$

Check

$$\begin{aligned} 19 &= -3p - 8 \\ 19 &\overset{?}{=} -3(-9) - 8 \quad \text{\textit{Substitute −9 for p.}} \\ 19 &\overset{?}{=} 27 - 8 \\ 19 &\overset{?}{=} 19 \checkmark \quad\quad\quad \text{\textit{−9 is a solution.}} \end{aligned}$$

EXAMPLE 2 **Solving Two-Step Equations Containing Division**

Solve.

A $8 + \dfrac{m}{4} = 17$

$$8 + \dfrac{m}{4} = 17$$
$$\underline{-8 \qquad\quad -8} \qquad\qquad \textit{Subtract 8 from both sides.}$$
$$\dfrac{m}{4} = 9$$
$$(4)\dfrac{m}{4} = (4)9 \qquad\qquad \textit{Multiply both sides by 4.}$$
$$m = 36$$

B $3 = \dfrac{u}{6} - 12$

$$3 = \dfrac{u}{6} - 12$$
$$\underline{+12 \qquad\quad +12} \qquad\qquad \textit{Add 12 to both sides.}$$
$$15 = \dfrac{u}{6}$$
$$(6)15 = (6)\dfrac{u}{6} \qquad\qquad \textit{Multiply both sides by 6.}$$
$$90 = u$$

EXAMPLE 3 *Fitness Application*

A new one-year membership at Vista Tennis Center costs $160. A registration fee of $28 is paid up front, and the rest is paid monthly. How much do new members pay each month?

| registration fee | plus | monthly cost | is | $160 |

Let *m* represent the monthly cost.

| $28 | + | 12m | = | $160 |

$$28 + 12m = 160$$
$$\underline{-28 \qquad\qquad -28} \qquad\qquad \textit{Subtract 28 from both sides.}$$
$$12m = 132$$
$$\dfrac{12m}{12} = \dfrac{132}{12} \qquad\qquad \textit{Divide both sides by 12.}$$
$$m = 11$$

New members pay $11 per month for a one-year membership.

Think and Discuss

1. Explain how you decide which inverse operation to use first when solving a two-step equation.

2. Tell the steps you would follow to solve $-1 + 2x = 7$.

11-1 **Exercises**

California Standards Practice
Preview of Grade 7 ➤ **AF4.1**

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Homework Help Online
KEYWORD: MS8CA 11-1
Parent Resources Online
KEYWORD: MS8CA Parent

GUIDED PRACTICE

See Example ① Solve.

1. $3n + 8 = 29$ **2.** $-4m - 7 = 17$ **3.** $2 = -6x + 4$

See Example ② Solve.

4. $12 + \frac{b}{6} = 16$ **5.** $\frac{y}{8} - 15 = 2$ **6.** $10 = -8 + \frac{n}{4}$

See Example ③ **7.** A coffee shop sells a ceramic refill mug for $8.95. Each refill costs $1.50. Last month Rose spent $26.95 on a mug and refills. How many refills did she buy?

INDEPENDENT PRACTICE

See Example ① Solve. Check each answer.

8. $5x + 6 = 41$ **9.** $-9p - 15 = 93$ **10.** $-2m + 14 = 10$

11. $-7 = 7d - 8$ **12.** $-7 = -3c + 14$ **13.** $12y - 11 = 49$

See Example ② Solve.

14. $24 + \frac{h}{4} = 10$ **15.** $\frac{k}{5} - 13 = 4$ **16.** $-17 + \frac{q}{8} = 13$

17. $24 = \frac{m}{10} + 32$ **18.** $-9 = 15 + \frac{v}{3}$ **19.** $\frac{m}{-7} - 14 = 2$

See Example ③ **20.** Each Saturday, a gym holds a 45-minute yoga class. The weekday yoga classes last 30 minutes. The number of weekday classes varies. Last week, the yoga classes totaled 165 minutes. How many weekday yoga classes were held?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP22.

Translate each equation into words, and then solve the equation.

21. $6 + \frac{m}{3} = 18$ **22.** $3x + 15 = 27$ **23.** $2 = \frac{n}{5} - 4$

Solve.

24. $18 + \frac{y}{4} = 12$ **25.** $5x + 30 = 40$ **26.** $\frac{s}{12} - 7 = 8$

27. $-10 + 6g = 110$ **28.** $-8 = \frac{z}{7} + 2$ **29.** $46 = -6w - 8$

30. $15 = -7 + \frac{r}{3}$ **31.** $-20 = -4p - 12$ **32.** $7 + 2r = 5$

33. Consumer Math A long-distance phone company charges $1.01 for the first 25 minutes of a call, and then $0.09 for each additional minute. A call cost $9.56. How long did it last?

34. The school purchased baseball equipment and uniforms for a total cost of $1,836. The equipment cost $612, and the uniforms were $25.50 each. How many uniforms did the school purchase?

35. If you double the number of calories per day that the U.S. Department of Agriculture recommends for children who are 1 to 3 years old and then subtract 100, you get the number of calories per day recommended for teenage boys. Given that 2,500 calories are recommended for teenage boys, how many calories per day are recommended for children?

36. According to the U.S. Department of Agriculture, children who are 4 to 6 years old need about 1,800 calories per day. This is 700 calories more than half the recommended calories for teenage girls. How many calories per day does a teenage girl need?

37. Hector consumed 2,130 calories from food in one day. Of these, he consumed 350 calories at breakfast and 400 calories having a snack. He also ate 2 portions of one of the items shown in the table for lunch and the same for dinner. What did Hector eat for lunch and dinner?

38. ⭐ **Challenge** There are 30 mg of cholesterol in a box of macaroni and cheese. This is 77 mg minus $\frac{1}{10}$ the number of milligrams of sodium it contains. How many milligrams of sodium are in a box of macaroni and cheese?

YOUR TOTAL COMES TO $76.41 AND 17,843 CALORIES.

As a service to health-conscious customers, many grocery stores have installed scanners that calculate the total number of calories purchased.

Calorie Counter		
Food	**Portion**	**Calories**
Stir-fry	1 cup	250
Enchilada	1 whole	310
Chicken sandwich	1 whole	345
Tomato soup	1 cup	160

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Web Extra!
KEYWORD: MS8CA Health

SPIRAL STANDARDS REVIEW   MG1.3, ⬅ MG2.2

39. **Multiple Choice** For which equation is $x = -2$ a solution?

   Ⓐ $2x + 5 = 9$ Ⓑ $8 = 10 - x$ Ⓒ $\frac{x}{2} + 3 = 2$ Ⓓ $-16 = -4x - 8$

40. **Short Response** A taxi cab costs $1.25 for the first mile and $0.25 for each additional mile. Write an equation for the total cost of a taxi ride, where x is the number of miles. How many miles can be traveled in the taxi for $8.00?

Find the measure of the third angle in each triangle, given two angle measures.
(Lesson 9-7)

41. 49°, 74°   42. 115°, 37°   43. 17°, 21°

44. A triangular prism has a base with an area of 18 in^2 and a height of 9 in. What is the volume of the prism? (Lesson 10-8)

11-2 Simplifying Algebraic Expressions

Why learn this? You can use an expression with more than one term to describe the length of a talent show.

Individual skits at the talent show can last up to x minutes each, and group skits can last up to y minutes each. The expression $7x + 9y$ represents the maximum length of the talent show if 7 individuals and 9 groups perform.

Vocabulary
term
coefficient

In the expression $7x + 9y$, $7x$ and $9y$ are *terms*. A **term** can be a number, a variable, or a product of numbers and variables. Terms in an expression are separated by plus or minus signs.

Caution!

A variable by itself, such as y, has a coefficient of 1. So $y = 1y$.

In the term $7x$, 7 is called the *coefficient*. A **coefficient** is a number that is multiplied by a variable in an algebraic expression.

Coefficient — 7x — **Variable**

Like terms are terms with the same variable raised to the same power. The coefficients do not have to be the same. Constants, like 5, $\frac{1}{2}$, and 3.2, are also like terms.

Like Terms	$3x$ and $2x$	w and $\frac{w}{7}$	5 and 1.8
Unlike Terms	$5x^2$ and $2x$ *The exponents are different.*	$6a$ and $6b$ *The variables are different.*	3.2 and n *Only one term contains a variable.*

EXAMPLE 1 Identifying Like Terms

Identify like terms in the list.

$$5a \quad \frac{t}{2} \quad 3y^2 \quad 7t \quad x^2 \quad 4z \quad k \quad 4.5y^2 \quad 2t \quad \frac{2}{3}a$$

Look for like variables with like powers.

$$\boxed{5a} \quad \boxed{\frac{t}{2}} \quad \boxed{3y^2} \quad \boxed{7t} \quad x^2 \quad 4z \quad k \quad \boxed{4.5y^2} \quad \boxed{2t} \quad \boxed{\frac{2}{3}a}$$

Helpful Hint

Use different shapes or colors to indicate sets of like terms.

Like terms: $\quad 5a$ and $\frac{2}{3}a \qquad \frac{t}{2}$, $7t$, and $2t \qquad 3y^2$ and $4.5y^2$

To simplify an algebraic expression that contains like terms, combine the terms. Combining like terms is like grouping similar objects.
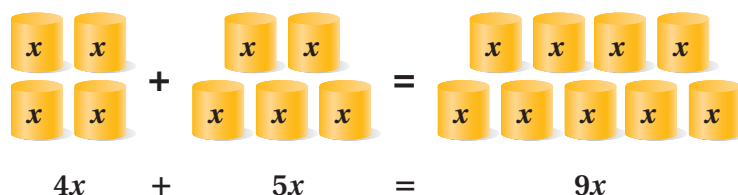
$$4x \quad + \quad 5x \quad = \quad 9x$$

To combine like terms that have variables, use the Distributive Property.

$$4x + 5x = (4 + 5)x = 9x$$

EXAMPLE 2 Simplifying Algebraic Expressions

Reasoning

Simplify. Justify your steps using the Commutative, Associative, and Distributive Properties when necessary.

A $7x + 2x$

$7x + 2x$ *7x and 2x are like terms.*

$(7 + 2)x = 9x$ *Distributive Property*

B $5x^3 + 3y + 7x^3 - 2y - 4x^2$

$5x^3 + 3y + 7x^3 - 2y - 4x^2$ *Identify like terms.*

$5x^3 + 7x^3 + 3y - 2y - 4x^2$ *Commutative Property*

$(5 + 7)x^3 + (3 - 2)y - 4x^2$ *Distributive Property*

$12x^3 + y - 4x^2$

C $2(a + 2a^2) + 2b$

$2(a + 2a^2) + 2b$

$2a + 4a^2 + 2b$ *Distributive Property*

There are no like terms to combine.

EXAMPLE 3 *Geometry Application*

Remember!

To find the perimeter of a figure, add the lengths of the sides.

See Lesson 10-1, p. 486.

Write an expression for the perimeter of the quadrilateral. Then simplify the expression.
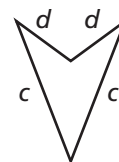
$c + d + d + c$ *Write an expression using the side lengths.*

$c + c + d + d$ *Identify and group like terms.*

$(1 + 1)c + (1 + 1)d$ *Distributive Property*

$2c + 2d$

Think and Discuss

1. Explain whether $5x$, $5x^2$, and $5x^3$ are like terms.

2. Explain how you know when an expression cannot be simplified.

GUIDED PRACTICE

See Example **1** Identify like terms in each list.

1. $6b$ $5x^2$ $4x^3$ $\frac{b}{2}$ x^2 $2e$

2. $12a^2$ $4x^3$ b $4a^2$ $3.5x^3$ $\frac{5}{6}b$

See Example **2** Simplify. Justify your steps using the Commutative, Associative, and Distributive Properties when necessary.

3. $5x + 3x$

4. $6a^2 - a^2 + 16$

5. $4a^2 + 5a + 14b$

See Example **3** **6.** **Geometry** Write an expression for the perimeter of the rectangle. Then simplify the expression.

$5n$ $6b$ $6b$ $5n$
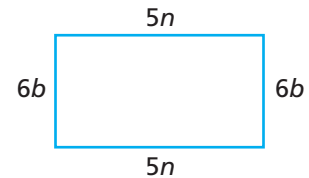
INDEPENDENT PRACTICE

See Example **1** Identify like terms in each list.

7. $2b$ b^6 b x^4 $3b^6$ $2x^2$

8. 6 $2n$ $3n^2$ $6m^2$ $\frac{n}{4}$ 7

9. $10k^2$ m 3^3 $\frac{p}{6}$ $2m$ 2

10. 6^3 y^3 $3y^2$ 6^2 y $5y^3$

See Example **2** Simplify. Justify your steps using the Commutative, Associative, and Distributive Properties when necessary.

11. $3a + 2b + 5a$

12. $5b + 7b + 10$

13. $a + 2b + 2a + b + 2c$

14. $y + 4 + 2x + 3y$

15. $q^2 + 2q + 2q^2$

16. $18 + 2d^3 + d + 3d$

See Example **3** **17.** **Geometry** Write an expression for the perimeter of the given figure. Then simplify the expression.

$3a$ $3a$ $2n$ $2n$ $5a$
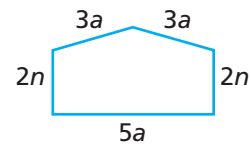
PRACTICE AND PROBLEM SOLVING

Simplify each expression.

18. $4x + 5x$

19. $32y - 5y$

20. $4c^2 + 5c + 2c$

21. $5d^2 - 3d^2 + d$

22. $5f^2 + 2f + f^2$

23. $7x + 8x^2 - 3y$

24. $p + 9q + 9 + 14p$

25. $6b + 6b^2 + 4b^3$

26. $a^2 + 2b + 2a^2 + b + 2c$

27. **Geometry** Write an expression for the perimeter of the given triangle. Then evaluate the perimeter when n is 1, 2, 3, 4, and 5.

n		1	2	3	4	5
Perimeter						
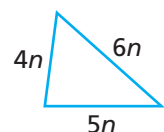
$4n$ $6n$ $5n$

28. Critical Thinking Determine whether the expression $9m^2 + k$ is equal to $7m^2 + 2(2k - m^2) + 5k$. Use properties to justify your answer.

29. Multi-Step Brad makes d dollars per hour as a cook at a deli. The table shows the number of hours he worked each week in June.

Hours Brad Worked	
Week	Hours
1	21.5
2	23
3	15.5
4	19

   a. Write and simplify an expression for the amount of money Brad earned in June.

   b. Evaluate your expression from part **a** for $d = \$9.50$.

   c. What does your answer to part **b** represent?

30. Business Ashley earns $8 per hour working at a grocery store. Last week she worked h hours bagging groceries and twice as many hours stocking shelves. Write and simplify an expression for the amount Ashley earned.

31. Reasoning The terms $3x$, $23x^2$, $6y^2$, $2x$, y^2 and one other term can be written in an expression which, when simplified, equals $5x + 7y^2$. Identify the term missing from the list and write the expression.

32. What's the Question? At one store, a pair of jeans costs $29 and a shirt costs $25. At another store, the same kind of jeans costs $26 and the same kind of shirt costs $20. The answer is $29j - 26j + 25s - 20s = 3j + 5s$. What is the question?

**33. Write About It** Describe the steps for simplifying the expression $2x + 3 + 5x - 15$.

34. Challenge A rectangle has a width of $x + 2$ and a length of $3x + 1$. Write and simplify an expression for the perimeter of the rectangle.

SPIRAL STANDARDS REVIEW AF1.2, AF2.1, AF3.1, AF3.2

35. Multiple Choice Translate "six times the sum of x and y" and "five less than y." Which algebraic expression represents the sum of these two verbal expressions?

   Ⓐ $6x + 5$ Ⓑ $6x + 2y - 5$ Ⓒ $6x + 5y + 5$ Ⓓ $6x + 7y - 5$

36. Multiple Choice The side length of a square is $2x + 3$. Which expression represents the perimeter of the square?

   Ⓐ $2x + 12$ Ⓑ $4x + 6$ Ⓒ $6x + 7$ Ⓓ $8x + 12$

Evaluate the expression $9y - 3$ for each given value of the variable. (Lesson 1-5)

37. $y = 2$ **38.** $y = 6$ **39.** $y = 10$ **40.** $y = 18$

Convert each measure. (Lesson 4-9)

41. 23 cm to millimeters   **42.** 600 m to kilometers   **43.** 0.5 g to milligrams

11-3 Solving Multi-Step Equations

Why learn this? You can solve a multi-step equation to determine the number of comic books in a collection. (See Example 3.)

Some types of equations require more than two steps to solve. These equations are known as multi-step equations. The first step in solving some multi-step equations is to combine like terms.

E X A M P L E   1   **Combining Like Terms to Solve Equations**

Solve $7n - 1 - 2n = 14$.

$$7n - 1 - 2n = 14$$
$$5n - 1 = 14 \qquad \text{Combine like terms.}$$
$$\underline{+1 \qquad +1} \qquad \text{Add 1 to both sides.}$$
$$5n = 15$$
$$\frac{5n}{5} = \frac{15}{5} \qquad \text{Divide both sides by 5.}$$
$$n = 3$$

You may need to use the Distributive Property to solve an equation that has parentheses. Multiply each term inside the parentheses by the factor that is outside the parentheses. Then combine like terms.

E X A M P L E   2   **Using the Distributive Property to Solve Equations**

Remember!

The Distributive Property states that $a(b + c) = ab + ac$. For instance, $2(3 + 5) = 2(3) + 2(5)$.

See Lesson 1-4, p. 21.

Solve $3(z - 1) + 8 = 14$.

$$3(z - 1) + 8 = 14$$
$$3(z) - 3(1) + 8 = 14 \qquad \text{Distribute 3 on the left side.}$$
$$3z - 3 + 8 = 14 \qquad \text{Simplify.}$$
$$3z + 5 = 14 \qquad \text{Combine like terms.}$$
$$\underline{-5 \qquad -5} \qquad \text{Subtract 5 from both sides.}$$
$$3z = 9$$
$$\frac{3z}{3} = \frac{9}{3} \qquad \text{Divide both sides by 3.}$$
$$z = 3$$

EXAMPLE **3** **PROBLEM SOLVING APPLICATION**

PROBLEM SOLVING

Reasoning

Jamal owns twice as many comic books as Levi owns. Brooke owns 6 more than the number of comic books Jamal and Levi own combined. Brooke owns 30 comic books. How many does Levi own?

1 Understand the Problem

Rewrite the question as a statement.

• Find the number of comic books that Levi owns.

List the **important information:**

• Jamal owns 2 times as many comic books as Levi owns.
• The number of comic books Jamal owns plus the number Levi owns added to 6 equals the number Brooke owns.
• Brooke owns 30 comic books.

2 Make a Plan

Let c represent the number of comic books Levi owns. Then $2c$ represents the number Jamal owns.

Jamal's books	+	Levi's books	+	6	=	Brooke's books
$2c$	+	c	+	6	=	30

Solve the equation $2c + c + 6 = 30$ for c.

3 Solve

$$2c + c + 6 = 30$$

$$3c + 6 = 30 \quad \text{\textit{Combine like terms.}}$$

$$\underline{-6 \quad -6} \quad \text{\textit{Subtract 6 from both sides.}}$$

$$3c = 24$$

$$\frac{3c}{3} = \frac{24}{3} \quad \text{\textit{Divide both sides by 3.}}$$

$$c = 8$$

Levi owns 8 comic books.

4 Look Back

Make sure that your answer makes sense in the original problem. Levi has 8 comic books. Jamal has $2(8) = 16$. Brooke has $8 + 16 + 6 = 30$.

Think and Discuss

1. List the steps required to solve $-n + 5n + 3 = 27$.

2. Describe how to solve the equations $4x + 2 = 10$ and $4(x + 2) = 10$. Are the solutions the same or different? Explain.

11-3 **Exercises**

California Standards Practice
Preview of Grade 7 ➟ **AF1.3** and Algebra I ➟ **5.0**

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GUIDED PRACTICE

See Example **1** Solve.

1. $14n + 2 - 7n = 37$ **2.** $10x - 11 - 4x = 43$ **3.** $1 = -3 + 4p - 2p$

See Example **2**   **4.** $12 - (x + 3) = 10$ **5.** $15 = 2(q + 4) + 3$ **6.** $5(m - 2) + 36 = -4$

See Example **3**   **7.** Keisha read twice as many books this year as Ben read. Subtracting 5 from the number of books Keisha and Ben read combined gives the number of books Sheldon read. Sheldon read 10 books. How many books did Ben read?

INDEPENDENT PRACTICE

See Example **1** Solve.

8. $b + 18 + 3b = 74$ **9.** $10x - 3 - 2x = 4$

10. $18w - 10 - 6w = 50$ **11.** $19 = 5n + 7 - 3n$

12. $-27 = -3p + 15 - 3p$ **13.** $-x - 8 + 14x = -34$

See Example **2**   **14.** $2(x + 4) + 6 = 22$ **15.** $1 - 3(n + 5) = -8$

16. $43 - 14(p + 7) = -97$ **17.** $18 + 60n - 32 = 76$

18. $0 = 9\left(k - \frac{2}{3}\right) + 33$ **19.** $6(t - 2) - 76 = -142$

See Example **3**   **20.** Abby ran 3 times as many laps as Karen. Adding 4 to the number of laps Abby and Karen ran combined gives the number of laps Jill ran. Jill ran 8 laps. How many laps did Karen run?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP22.

Solve.

21. $0.5x + 7 + 1.5x = 5$ **22.** $4(t - 8) + 20 = 5$ **23.** $63 = 8w + 26 - 27$

24. $17 = -5(3 + w) + 7$ **25.** $\frac{1}{4}a - 12 + \frac{7}{4}a = 4$ **26.** $9 = -(r - 5) + 11$

27. $2(b - 3) = -30$ **28.** $8.44 = 0.4h + 6.92 + 0.4h$ **29.** $57 = -25x + 180 - 16x$

30. **Consumer Math** Three friends ate dinner at a restaurant. The friends decided to add a 15% tip and then split the bill evenly. Each friend paid $10.35. What was the total bill for dinner before tip?

31. Ann earns 1.5 times her normal hourly pay for each hour that she works over 40 hours in a week. Last week she worked 51 hours and earned $378.55. What is her normal hourly pay?

32. **Geometry** The base angles of an isosceles triangle are congruent. The measure of each of the base angles is twice the measure of the third angle. Find the measures of all three angles.

33. Consumer Math Patrice used a $15 gift certificate when she purchased a pair of sandals. After 8% sales tax was applied to the price of the sandals, the $15 was deducted. Patrice had to pay a total of $12 for the sandals. How much did the sandals cost before tax?

34. Science To convert temperatures between degrees Celsius and degrees Fahrenheit, you can use the formula $F = \frac{9}{5}C + 32$. The table shows the melting points of various elements.

  a. What is the melting point in degrees Celsius of gold?

  b. What is the melting point in degrees Celsius of hydrogen?

35. On his first two social studies tests, Billy made an 86 and a 93. What grade must Billy make on the third test to have an average of 90 for all three tests?

Melting Points of Elements

Gold	Hydrogen	Lead	Silver
1,946°F	−432.2°F	621°F	1,762°F

Temperature (°F): 2,100 / 1,800 / 1,500 / 1,200 / 900 / 600 / 300 / 0 / −300 / −600
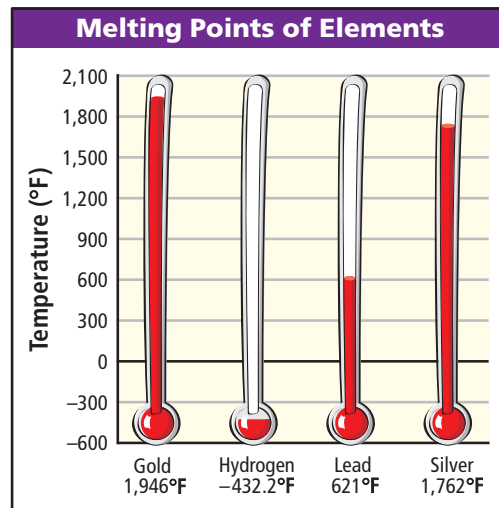
36. What's the Question? Three friends shared a taxi ride from the airport to their hotel. After adding a $7.00 tip, the friends divided the cost of the ride evenly. If solving the equation $\frac{1}{3}(c + \$7.00) = \$11.25$ gives the answer, what is the question?

37. Write About It Describe two different ways you could solve the equation $3(x + 4) = 15$.

38. Challenge Are the solutions to the following equations the same? Explain.
$$\frac{3y}{4} + 2 = 4 \text{ and } 3y + 8 = 16$$

39. Multiple Choice Solve $\frac{1}{4}(2x - 2) = 7$.

  Ⓐ $x = 15$ Ⓑ $x = 18$ Ⓒ $x = 20$ Ⓓ $x = 21$

40. Multiple Choice For which equation(s) is $x = 3$ a solution?

  I $2x - 5 + 3x = 10$ **II** $\frac{1}{2}(-x + 7) = 2$ **III** $\frac{-4x}{6} = 2$ **IV** $6.3x - 2.4 = 16.5$

  Ⓐ I only   Ⓑ I and II   Ⓒ I, II, and III   Ⓓ I, II, and IV

Solve each equation. (Lesson 2-5)

41. $x + 4 = -2$ **42.** $-7 = y - 4$ **43.** $\frac{z}{2} = -9$ **44.** $-6d = 54$

45. A bag contains 20 slips of paper numbered 1 to 20. Abby draws a slip of paper from the bag at random, replaces it, and then selects a second slip. What is the probability that she will select even-numbered slips on both draws? (Lesson 8-6)

11-4 Solving Equations with Variables on Both Sides

Why learn this? You can determine how many weeks you would have to rent a video game console to make buying the console worth the price. (See Example 3.)

Some problems require you to solve equations that have the same variable on both sides of the equal sign. To solve this kind of problem, you need to get the terms with variables on one side of the equal sign.

EXAMPLE 1 **Using Inverse Operations to Group Terms with Variables**

Group the terms with variables on one side of the equal sign, and simplify.

A $6m = 4m + 12$

$$6m = 4m + 12$$
$$\underline{-\,4m -\,4m }$$
$$2m = 12$$

Subtract 4m from both sides.
Simplify.

B $-7x - 198 = 5x$

$$-7x - 198 = 5x$$
$$\underline{+\,7x +\,7x}$$
$$-198 = 12x$$

Add 7x to both sides.
Simplify.

EXAMPLE 2 **Solving Equations with Variables on Both Sides**

🔵 **Reasoning**

Solve.

A $5n = 3n + 26$

$$5n = 3n + 26$$
$$\underline{-\,3n -\,3n }$$
$$2n = 26$$

$$\frac{2n}{2} = \frac{26}{2}$$
$$n = 13$$

Subtract 3n from both sides.
Simplify.

Divide both sides by 2.

Check $5n = 3n + 26$

$$5(13) \stackrel{?}{=} 3(13) + 26$$
$$65 = 65 \checkmark$$

Substitute 13 for n.

65 is a solution.

Solve.

B $19 + 7n = -2n + 37$

$$19 + 7n = -2n + 37$$

$$\underline{+2n \quad +2n}$$ *Add 2n to both sides.*

$$19 + 9n = 37$$ *Simplify.*

$$\underline{-19 -19}$$ *Subtract 19 from both sides.*

$$9n = 18$$ *Simplify.*

$$\frac{9n}{9} = \frac{18}{9}$$ *Divide both sides by 9.*

$$n = 2$$

C $\frac{5}{9}x = \frac{4}{9}x + 9$

$$\frac{5}{9}x = \frac{4}{9}x + 9$$

$$\underline{-\frac{4}{9}x \quad -\frac{4}{9}x}$$ *Subtract $\frac{4}{9}x$ from both sides.*

$$\frac{1}{9}x = \phantom{\frac{4}{9}x + }9$$ *Simplify.*

$$(9)\frac{1}{9}x = (9)9$$ *Multiply both sides by 9.*

$$x = 81$$

E X A M P L E 3 *Consumer Math Application*

Mari can buy a video game console for \$72.45 and rent a game for \$7.95 per week, or she can rent a console and the same game for a total of \$22.44 per week. How many weeks would Mari need to rent both the video game and the console to pay as much as she would if she had bought the console and rented the game instead?

Let w represent the number of weeks.

$$22.44w = 72.45 + 7.95w$$

$$\underline{-7.95w -7.95w}$$ *Subtract 7.95w from both sides.*

$$14.49w = 72.45$$ *Simplify.*

$$\frac{14.49w}{14.49} = \frac{72.45}{14.49}$$ *Divide both sides by 14.49.*

$$w = 5$$

Mari would need to rent the video game and the console for 5 weeks to pay as much as she would have if she had bought the console.

Think and Discuss

1. Explain how you would solve $\frac{1}{2}x + 7 = \frac{2}{3}x - 2$.

2. Describe how you would decide which variable term to add or subtract on both sides of the equation $-3x + 7 = 4x - 9$.

11-4 Exercises

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Preview of Algebra I ⟶ 5.0

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GUIDED PRACTICE

See Example 1 **Group the terms with variables on one side of the equal sign, and simplify.**

1. $5n = 4n + 32$ **2.** $-6x - 28 = 4x$ **3.** $8w = 32 - 4w$

See Example 2 **Solve.**

4. $4y = 2y + 40$ **5.** $8 + 6a = -2a + 24$ **6.** $\frac{3}{4}d + 4 = \frac{1}{4}d + 18$

See Example 3 **7. Consumer Math** Members at the Star Theater pay $30.00 per month plus $1.95 for each movie. Nonmembers pay the regular $7.95 admission fee. How many movies would both a member and a nonmember have to see in a month to pay the same amount?

INDEPENDENT PRACTICE

See Example 1 **Group the terms with variables on one side of the equal sign, and simplify.**

8. $12h = 9h + 84$ **9.** $-10p - 8 = 2p$ **10.** $6q = 18 - 2q$

11. $-4c - 6 = -2c$ **12.** $-7s + 12 = -9s$ **13.** $6 + \frac{4}{5}a = \frac{9}{10}a$

See Example 2 **Solve.**

14. $9t = 4t + 120$ **15.** $42 + 3b = -4b - 14$ **16.** $\frac{6}{11}x + 4 = \frac{2}{11}x + 16$

17. $1.5a + 6 = 9a + 12$ **18.** $32 - \frac{3}{8}y = \frac{3}{4}y + 5$ **19.** $-6 - 8c = 3c + 16$

See Example 3 **20. Consumer Math** Members at a swim club pay $5 per lesson plus a one-time fee of $60. Nonmembers pay $11 per lesson. How many lessons would both a member and a nonmember have to take to pay the same amount?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP22.

Solve. Check each answer.

21. $3y + 7 = -6y - 56$ **22.** $\frac{7}{8}x + 6 = \frac{3}{8}x + 14$

23. $5r + 6 - 2r = 7r - 10$ **24.** $-10p + 8 = 7p + 12$

25. $9 + 5r = -17 - 8r$ **26.** $8k + 70 = -7k + 10$

27. Reasoning A choir is singing at a festival. On the first night, 12 choir members were absent, so the choir stood in 5 equal rows. On the second night, only 1 member was absent, so the choir stood in 6 equal rows. The same number of people stood in each row each night. How many members are in the choir?
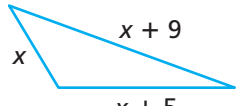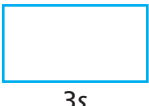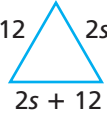
28. Consumer Math Jaline can purchase tile at a store for $0.99 per tile and rent a tile saw for $24. At another store, she can borrow the tile saw for free if she buys tile there for $1.49 per tile. How many tiles must she buy for the cost to be the same at both stores?

The figures in each pair have the same perimeter. Find the value of each variable.

29.

30.

31. Recreation A rock-climbing gym charges nonmembers $18 per day to use the wall plus $7 per day for equipment rental. Members pay an annual fee of $400 plus $5 per day for equipment rental. How many days must both a member and a nonmember use the wall in one year so that both pay the same amount?

32. Multi-Step Two families drove from Denver to Cincinnati. After driving 582 miles the first day, the Smiths spread the rest of the trip equally over the next 3 days. The Chows spread their trip equally over 6 days. The distance the Chows drove each day was equal to the distance the Smiths drove each of the three days.

  a. How many miles did the Chows drive each day?

  b. How far is it from Denver to Cincinnati?

33. What's the Error? To combine terms in the equation $-8a - 4 = 2a + 34$, a student wrote $-6a = 38$. What is the error?

34. Write About It If the same variable is on both sides of an equation, must it have the same value on each side? Explain your answer.

35. Challenge Combine terms before solving the equation $12x - 4 - 12 = 4x + 8 + 8x - 24$. Do you think there is just one solution to the equation? Why or why not?

SPIRAL STANDARDS REVIEW   MG1.1, MG1.2, MG2.2

36. Multiple Choice For which equation is $x = 0$ NOT a solution?

  Ⓐ $3x + 2 = 2 - x$ Ⓑ $2.5x + 3 = x$ Ⓒ $-x + 4 = 3x + 4$ Ⓓ $6x + 2 = x + 2$

37. Extended Response One calling plan offers long-distance calls for $0.03 per minute. Another plan costs $2.00 per month but offers long-distance service for $0.01 per minute. Write and solve an equation to find the number of long-distance minutes for which the two plans would cost the same. Write your answer in a complete sentence.

Find the measure of the angle that is complementary to each given angle. (Lesson 9-4)

38. 32°   **39.** 89°   **40.** 13°   **41.** 45°

Given the radius or diameter, find the area of each circle. Use $\frac{22}{7}$ as an estimate for π. (Lesson 10-5)

42. $r = 35$ in.   **43.** $d = 14$ m   **44.** $r = 28$ cm   **45.** $d = 21$ yd

READY TO GO ON?

Quiz for Lessons 11-1 Through 11-4

11-1 Solving Two-Step Equations

Solve.

1. $-4x + 6 = 54$

2. $15 + \frac{y}{3} = 6$

3. $\frac{z}{8} - 5 = -3$

4. $-33 = -7a - 5$

5. $-27 = \frac{r}{12} - 19$

6. $-13 = 11 - 2n$

7. A taxi service charges an initial fee of $1.50 plus $1.50 for every mile traveled. A taxi ride costs $21.00. How many miles did the taxi travel?
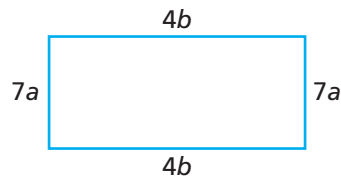
11-2 Simplifying Algebraic Expressions

Simplify each expression.

8. $2y - 2y^2 + 5y^2$

9. $x - 4 + 7x - 9$

10. $10 + b - 6a - 9b$

11. Write an expression for the perimeter of the given figure. Then simplify the expression.

4b

7a ⬚ 7a

4b

11-3 Solving Multi-Step Equations

Solve.

12. $3(3b + 2) = -30$

13. $13 = 2c + 3 + 5c$

14. $\frac{1}{2}(8w - 6) = 17$

15. $6(x - 4) + 10 = 28$

16. $6m - 4m + 8 = 34$

17. $15 = 5 + 9p - 4p$

18. Peter used a $5.00 gift certificate to help pay for his lunch. After adding a 15% tip to the cost of his meal, Peter still had to pay $2.36 in cash. How much did Peter's meal cost?

11-4 Solving Equations with Variables on Both Sides

Solve.

19. $12m = 3m + 108$

20. $\frac{7}{8}n - 3 = \frac{5}{8}n + 12$

21. $1.2x + 3.7 = 2.2x - 4.5$

22. $-7 - 7p = 3p + 23$

23. $2.3q - 16 = 5q + 38$

24. $\frac{3}{5}k + \frac{7}{10} = \frac{11}{15}k - \frac{2}{5}$

25. One shuttle service charges $10 for pickup and $0.10 per mile. Another shuttle service has no pickup fee but charges $0.35 per mile. Find the number of miles for which the cost of the two shuttle services is the same.

Focus on Problem Solving

Solve

• **Write an equation**

When you are asked to solve a problem, be sure to read the entire problem before you begin solving it. Sometimes you will need to perform several steps to solve the problem, and you will need to know all of the information in the problem before you decide which steps to take.

Read each problem and determine what steps are needed to solve it. Then write an equation that can be used to solve the problem.

1 Martin can buy a pair of inline skates and safety equipment for $49.50. At a roller rink, Martin can rent a pair of inline skates for $2.50 per day, but he still needs to buy safety equipment for $19.50. How many days would Martin have to skate in order to pay as much to rent skates and buy safety equipment as he would have to pay to buy both?

2 Christopher draws caricatures at the local mall. He charges $5 for a simple sketch and $15 for a larger drawing. In one day, Christopher earned $175. He drew 20 simple sketches that day. How many larger drawings did he make?

3 Book-club members are required to buy a minimum number of books each year. Leslee bought 3 times the minimum. Denise bought 7 more than the minimum. Together, they bought 23 books. What is the minimum number of books?

4 Coach Willis has won 150 games during his career. This is 10 more than $\frac{1}{2}$ as many games as Coach Gentry has won. How many games has Coach Gentry won?

5 The perimeter of an isosceles triangle is 4 times the length of the shortest side. The longer sides are 4.5 ft longer than the shortest side. What is the length of each side of the triangle?

6 Miss Rankin's class has raised $100.00 for a class trip. The class needs to collect a total of $225.00. How many $0.50 carnations must the class sell to reach its goal?

11-5 Introduction to Inequalities

Vocabulary
inequality
algebraic inequality
solution set
compound inequality

Why learn this? You can use an inequality to describe the number of students in an auditorium. (See Example 1.)

An **inequality** is a statement that compares two expressions by using one of the following symbols: $<$, $>$, $\leq$, $\geq$, or $\neq$.

Symbol	Meaning	Word Phrases
$<$	Is less than	Fewer than, below
$>$	Is greater than	More than, above
$\leq$	Is less than or equal to	At most, no more than
$\geq$	Is greater than or equal to	At least, no less than

EXAMPLE 1 Writing Inequalities

Write an inequality for each situation.

A **There are at least 25 students in the auditorium.**

number of students ≥ 25 *"At least" means greater than or equal to.*

B **No more than 150 people can occupy the room.**

room capacity ≤ 150 *"No more than" means less than or equal to.*

An inequality that contains a variable is an **algebraic inequality**. A value of the variable that makes the inequality true is a solution of the inequality. An inequality may have more than one solution. Together, all of the solutions are called the **solution set**.

You can graph the solutions of an inequality on a number line. If the variable is "greater than" or "less than" a number, then that number is indicated with an open circle.

This open circle shows that 5 is not a solution.

$a > 5$
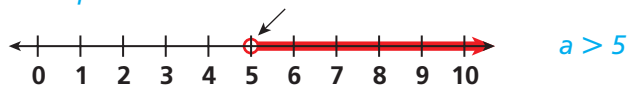
If the variable is "greater than or equal to" or "less than or equal to" a number, that number is indicated with a closed circle.

This closed circle shows that 3 is a solution.

$b \leq 3$

EXAMPLE **2** **Graphing Simple Inequalities**

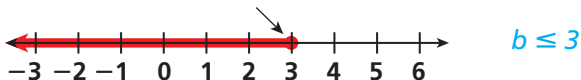Graph each inequality.

A $x > -2$

Draw an open circle at −2. The solutions are values of x greater than −2, so shade to the right of −2.

B $-1 \geq y$
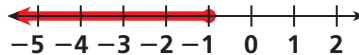
Draw a closed circle at −1. The solutions are −1 and values of y less than −1, so shade to the left of −1.

Writing Math

The compound inequality $-2 < y$ and $y < 4$ can be written as $-2 < y < 4$.

A **compound inequality** is the result of combining two inequalities. The words *and* and *or* are used to describe how the two parts are related.

$x > 3$ or $x < -1$

x is either greater than 3 or less than −1.

$-2 < y$ and $y < 4$

y is both greater than −2 and less than 4. y is between −2 and 4.

EXAMPLE **3** **Graphing Compound Inequalities**

Graph each compound inequality.

A $s \geq 0$ or $s < -3$
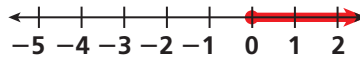
Graph s ≥ 0.
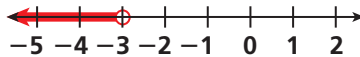
Graph s < −3.
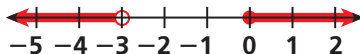
Include the solutions shown by either graph.

Reading Math

$1 < p$ is the same as $p > 1$.

B $1 < p \leq 5$

Graph 1 < p.

Graph p ≤ 5.
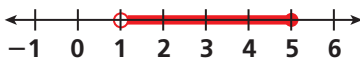
Include the solutions that the graphs have in common.

Think and Discuss

1. Compare the graphs of the inequalities $y > 2$ and $y \geq 2$.

2. Explain how to graph each type of compound inequality.

11-5 **Exercises**

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GUIDED PRACTICE

See Example 1 **Write an inequality for each situation.**

1. No more than 18 people are allowed in the gallery at one time.

2. There are fewer than 8 fish in the aquarium.

3. The water level is above 45 inches.

See Example 2 **Graph each inequality.**

4. $x < 3$ 5. $\frac{1}{2} \geq r$ 6. $2.8 < w$ 7. $y \geq -4$

See Example 3 **Graph each compound inequality.**

8. $a > 2$ or $a \leq -1$ 9. $-4 < p \leq 6$ 10. $-2 \leq n < 0$

INDEPENDENT PRACTICE

See Example 1 **Write an inequality for each situation.**

11. The temperature is below 40°F.

12. There are at least 24 pictures on the roll of film.

13. No more than 35 tables are in the cafeteria.

14. Fewer than 250 people attended the rally.

See Example 2 **Graph each inequality.**

15. $s \geq -1$ 16. $y < 0$ 17. $n \leq -3$

18. $2 < x$ 19. $-6 \leq b$ 20. $m < -4$

See Example 3 **Graph each compound inequality.**

21. $p > 3$ or $p < 0$ 22. $1 \leq x \leq 4$ 23. $-3 < y < -1$

24. $k > 0$ or $k \leq -2$ 25. $n \geq 1$ or $n \leq -1$ 26. $-2 < w \leq 2$

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP23.

Graph each inequality or compound inequality.

27. $z \leq -5$ 28. $3 > f$ 29. $m \geq -2$

30. $3 > y$ or $y \geq 6$ 31. $-9 < p \leq -3$ 32. $q > 2$ or $-1 > q$

Write each statement using inequality symbols.

33. The number c is between -2 and 3.   34. The number y is greater than -10.

Write an inequality shown by each graph.

35.
```
 ←─┼──○──┼──┼──┼──○──┼──┼──┼→
  −4 −3 −2 −1  0  1  2  3  4
```

36.
```
 ←──┼──┼──┼──┼──○──┼──┼──┼──┼→
   −5 −4 −3 −2 −1  0  1  2  3
```
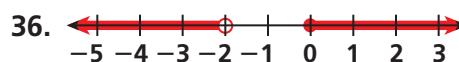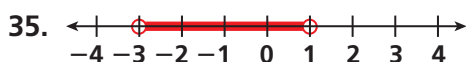
The portion of the earth's surface that lies beneath the ocean and consists of continental crust is the continental margin. The continental margin is divided into the continental shelf, the continental slope, and the continental rise.

Continental shelf
Continental slope
Continental rise

Abyssal plain
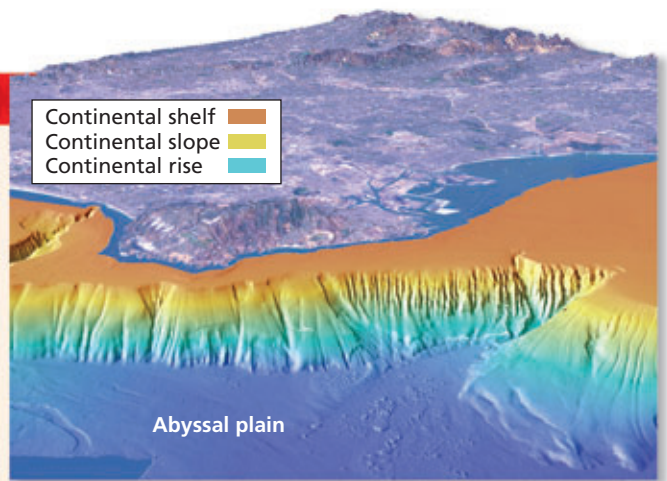
37. The continental shelf begins at the shoreline and slopes toward the open ocean. The depth of the continental shelf can reach 200 meters. Write a compound inequality for the depth of the continental shelf.

38. The continental slope begins at the edge of the continental shelf and continues down to the flattest part of the ocean floor. The depth of the continental slope ranges from about 200 meters to about 4,000 meters. Write a compound inequality for the depth of the continental slope.

39. The bar graph shows the depth of the ocean in various locations as measured by different research vessels. For each vessel, write a compound inequality that shows the ranges of depth that it measured.

40. ⭐ **Challenge** Water freezes at 32°F and boils at 212°F. Write three inequalities to show the ranges of temperatures for which water is a solid, a liquid, and a gas.

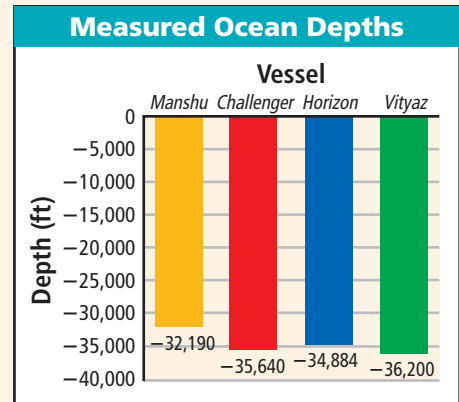*Deep Flight* is designed to explore the ocean in underwater flights.

Measured Ocean Depths

Vessel

| | Manshu | Challenger | Horizon | Vityaz |

Depth (ft)

−32,190 −35,640 −34,884 −36,200

41. Multiple Choice Which inequality represents *a number that is greater than −4 and less than 3*?

Ⓐ $-4 \geq n \geq 3$ Ⓑ $-4 < n < 3$ Ⓒ $-4 > n > 3$ Ⓓ $-4 \leq n \leq 3$

42. Multiple Choice Which inequality is shown by the graph?

−5 −4 −3 −2 −1 0 1 2 3 4 5
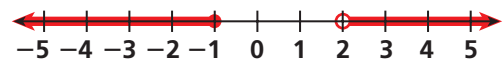
Ⓐ $x < -1$ or $x \leq 2$ Ⓑ $x < -1$ or $x \geq 2$ Ⓒ $x \leq -1$ or $x < 2$ Ⓓ $x \leq -1$ or $x > 2$

43. Mateo drove 472 miles in 8 hours. What was his average rate of speed? (Lesson 5-2)

Use cross products to solve each proportion. (Lesson 5-4)

44. $\dfrac{x}{5} = \dfrac{9}{30}$ **45.** $\dfrac{12}{14} = \dfrac{36}{n}$ **46.** $\dfrac{16}{y} = \dfrac{12}{75}$ **47.** $\dfrac{w}{24} = \dfrac{34}{60}$

11-6 Solving Inequalities by Adding or Subtracting

Why learn this? You can solve an inequality to determine the high temperature on a day when the temperature is different than the day before. (See Example 3.)

In Joshua Tree National Park, a day of snow can be followed by a day of sun.

When you add or subtract the same number on both sides of an inequality, the resulting statement will still be true.

$$-2 < 5$$
$$\underline{+7 \quad +7}$$
$$5 < 12$$

You can find solution sets of inequalities the same way you find solutions of equations, by isolating the variable.

EXAMPLE 1 Solving Inequalities by Adding

Solve. Then graph each solution set on a number line.

A $x - 12 > 32$

$$x - 12 > 32$$
$$\underline{+12 \quad +12} \qquad \text{Add 12 to both sides.}$$
$$x \qquad > \quad 44$$

Draw an open circle at 44. Solutions are values of x greater than 44, so shade to the right of 44.

Check According to the graph, 55 should be a solution.

$$x - 12 > 32$$
$$55 - 12 \overset{?}{>} 32 \qquad \text{Substitute 55 for } x.$$
$$43 > 32 \ \checkmark \qquad \text{55 is a solution.}$$

B $-14 \geq y - 8$

$$-14 \geq y - 8$$
$$\underline{+8 \qquad +8} \qquad \text{Add 8 to both sides.}$$
$$-6 \geq y$$
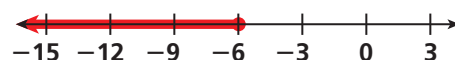
Draw a closed circle at -6. Solutions are -6 and values of y less than -6, so shade to the left of -6.

As shown in Example 1A, you can check the solution to an inequality by choosing any number in the solution set and substituting it into the original inequality.

EXAMPLE 2 **Solving Inequalities by Subtracting**

Solve. Check each answer.

A $c + 9 < 20$

$$
\begin{aligned}
c + 9 &< 20 \\
-9 \quad &-9 \qquad \text{\textit{Subtract 9 from both sides.}}\\
c \quad &< 11
\end{aligned}
$$

Check

$$
\begin{aligned}
c + 9 &< 20 \\
0 + 9 &\overset{?}{<} 20 \qquad \text{\textit{0 is less than 11. Substitute 0 for c.}}\\
9 &\overset{?}{<} 20 \checkmark
\end{aligned}
$$

B $-2 < x + 16$

$$
\begin{aligned}
-2 &< x + 16 \\
-16 \quad &-16 \qquad \text{\textit{Subtract 16 from both sides.}}\\
-18 &< x
\end{aligned}
$$

Check

$$
\begin{aligned}
-2 &< x + 16 \\
-2 &\overset{?}{<} 0 + 16 \qquad \text{\textit{0 is greater than −18. Substitute 0 for x.}}\\
-2 &\overset{?}{<} 16 \checkmark
\end{aligned}
$$

EXAMPLE 3 **_Weather Application_**

Sunday's high temperature of 52°F was at least 20°F higher than Monday's high temperature. What was Monday's high temperature?

Sunday's high	was at least	20°F higher than	Monday's high.
52	≥	20 +	t

$$
\begin{aligned}
52 &\geq 20 + t \\
-20 \quad &-20 \qquad \text{\textit{Subtract 20 from both sides.}}\\
32 &\geq t \qquad \quad \text{\textit{Rewrite the inequality.}}\\
t &\leq 32
\end{aligned}
$$

Monday's high temperature was at most 32˚F.

Think and Discuss

1. **Compare** solving addition and subtraction equations with solving addition and subtraction inequalities.

2. **Describe** how to check whether −36 is a solution of $s − 5 > 1$.

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GUIDED PRACTICE

See Example 1 **Solve. Then graph each solution set on a number line.**

1. $x - 9 < 18$ **2.** $y - 11 \geq -7$ **3.** $4 \geq p - 3$

See Example 2 **Solve. Check each answer.**

4. $n + 5 > 26$ **5.** $b + 21 \leq -3$ **6.** $9 \leq 12 + k$

See Example 3 **7. Weather** Yesterday's high temperature was 30°F. Tomorrow's weather forecast includes a high temperature that is no more than 12°F warmer than yesterday's. What high temperatures are forecast for tomorrow?

INDEPENDENT PRACTICE

See Example 1 **Solve. Then graph each solution set on a number line.**

8. $s - 2 > 14$ **9.** $m - 14 < -3$ **10.** $b - 25 > -30$

11. $c - 17 \leq -6$ **12.** $-25 > y - 53$ **13.** $71 \leq x - 9$

See Example 2 **Solve. Check each answer.**

14. $w + 16 < 4$ **15.** $z + 9 > -3$ **16.** $p + 21 \leq -4$

17. $26 < f + 32$ **18.** $65 > k + 54$ **19.** $n + 29 \geq 25$

See Example 3 **20.** Clark scored at least 12 points more than Josh scored. Josh scored 15 points. How many points did Clark score?

21. Science Adriana is helping track bird populations. She counted 8 fewer birds on Tuesday than on Thursday. She counted at most 32 birds on Thursday. How many birds did Adriana count on Tuesday?

PRACTICE AND PROBLEM SOLVING

Solve.

22. $k + 3.2 \geq 8$ **23.** $a - 13 > -10$ **24.** $c - 1\frac{1}{4} < 6\frac{1}{2}$

25. $-20 \geq 18 + m$ **26.** $7.02 < x + 4$ **27.** $g + 3\frac{2}{3} < 10$

28. $-109 > r - 58$ **29.** $5.9 + w \leq 21.6$ **30.** $n - 21.6 > 26$

31. $-150 \leq t + 92$ **32.** $y + 1\frac{1}{8} \geq 4\frac{3}{4}$ **33.** $v + 0.9 \leq 1.5$

34. Consumer Math To get a group discount for baseball tickets, Marco's group must have at least 20 people. The group needs at least 7 more people to sign up. How many have signed up so far?

35. Recreation Mila is 62 inches tall. She needs a surfboard of length ℓ that is at least 12 inches longer than her height. Write and solve an inequality to find the values of ℓ that are suitable lengths for Mila's surfboard.
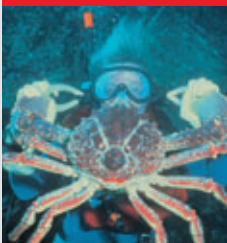
36. **Transportation** The *shinkansen*, or bullet train, of Japan travels at an average speed of 162.3 miles per hour. It has a top speed of 186 miles per hour. At most, how many more miles per hour can the train travel beyond its average speed before it reaches its maximum speed?

37. **Science** The giant spider crab, the world's largest crab, lives off the southeastern coast of Japan. Giant spider crabs can grow as much as 3.6 meters across. A scientist finds one that could still grow another 0.5 m across. How wide is the giant spider crab that he found?

38. The line graph shows the number of miles Amelia rode her bike in each of the last four months. She wants to ride at least 5 miles more in May than she did in April. At least how many miles does Amelia want to ride in May?

Distance Biked
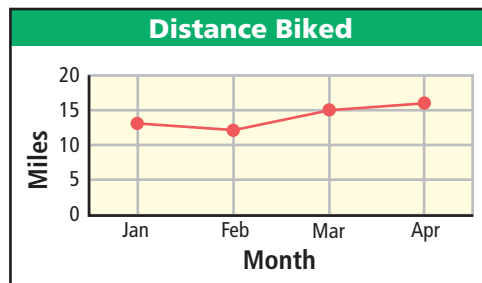
39. **Science** The average human ear can detect sounds that have frequencies between 20 hertz and 20,000 hertz. The average dog ear can detect sounds with frequencies of up to 30,000 hertz greater than those a human ear can detect. Up to how many hertz can a dog hear?

40. **Choose a Strategy** If five days ago was the day after Saturday, what was the day before yesterday?

41. **Write About It** Explain how to solve and check the inequality $n - 9 < -15$.

42. **Challenge** Solve the inequality $x + (4^2 - 2^3)^2 > -1$.

SPIRAL STANDARDS REVIEW   SDAP1.1, SDAP3.4

43. **Multiple Choice** Which inequality has the following graphed solution?

$$-5 \ -4 \ -3 \ -2 \ -1 \ \ 0 \ \ 1 \ \ 2 \ \ 3 \ \ 4 \ \ 5$$
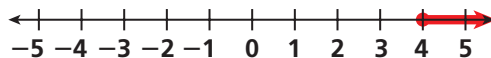
Ⓐ $x - 2 \geq -2$ Ⓑ $x + 3 \geq 7$ Ⓒ $x - 3 \leq 1$ Ⓓ $x + 5 < 9$

44. **Short Response** The highest-paid employee at the movie theater is the manager, who earns $10.25 per hour. The lowest-paid employees earn $3.90 less per hour than the manager. Write and graph a compound inequality to show all the other hourly wages earned at the movie theater.

Find the mean, median, mode, and range of each data set. (Lesson 7-1)

45. 20, 18, 16, 20, 15, 13, 17   46. 0.4, 0.9, 1.2, 0.6, 0.3, 1.4

47. Find the probability of rolling an even number on a number cube 2 times in a row. (Lesson 8-6)

11-7 Solving Inequalities by Multiplying or Dividing

Who uses this? Melon farmers can solve inequalities to determine how many watermelons they must sell to make a profit. (See Example 3.)

When you multiply or divide both sides of an inequality by the same positive number, the statement will still be true. However, when you multiply or divide both sides by the same *negative* number, you need to reverse the direction of the inequality symbol for the statement to be true.

MELONS $5.00

A number line marked at −4, −2, 0, 2, 4.
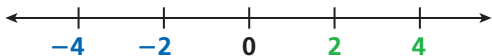
$2 < 4$	*2 is less than 4.*
$(-1)2 \;\square\; (-1)4$	*Multiply both sides by −1.*
$-2 \;\square\; -4$	*Use the number line to determine the correct inequality symbol.*
$-2 > -4$	*Reverse the inequality symbol because −2 is greater than −4.*

EXAMPLE 1 Solving Inequalities by Multiplying

Solve.

A $\dfrac{x}{11} < 3$

$$\frac{x}{11} < 3$$

$$(11)\frac{x}{11} < (11)3 \qquad \textit{Multiply both sides by 11.}$$

$$x < 33$$

B $4.8 \leq \dfrac{r}{-6}$

$$4.8 \leq \frac{r}{-6}$$

$$(-6)4.8 \geq (-6)\frac{r}{-6} \qquad \textit{Multiply both sides by −6, and reverse the inequality symbol.}$$

$$-28.8 \geq r$$

Check $\quad 4.8 \leq \dfrac{r}{-6}$

$$4.8 \overset{?}{\leq} \frac{-30}{-6} \qquad \textit{−30 is less than −28.8. Substitute −30 for r.}$$

$$4.8 \leq 5 \;\checkmark$$

EXAMPLE 2 **Solving Inequalities by Dividing**

Solve. Check each answer.

A $4x > 9$

$4x > 9$

$\dfrac{4x}{4} > \dfrac{9}{4}$ *Divide both sides by 4.*

$x > \dfrac{9}{4}$, or $2\dfrac{1}{4}$

Check

$4x > 9$

$4(3) \overset{?}{>} 9$ *3 is greater than $2\frac{1}{4}$. Substitute 3 for x.*

$12 \overset{?}{>} 9$ ✔

B $-60 \geq -12y$

$-60 \geq -12y$

$\dfrac{-60}{-12} \leq \dfrac{-12y}{-12}$ *Divide both sides by -12, and reverse the inequality symbol.*

$5 \leq y$

Check

$-60 \geq -12y$

$-60 \overset{?}{\geq} -12(10)$ *10 is greater than 5. Substitute 10 for y.*

$-60 \overset{?}{\geq} -120$ ✔

EXAMPLE 3 *Agriculture Application*

Reasoning

It cost the Schmidts $517 to raise watermelons. How many watermelons must they sell at $5 apiece to make a profit?

To make a profit, the Schmidts need to earn more than $517. Let w represent the number of watermelons they must sell.

$5w > 517$ *Write an inequality.*

$\dfrac{5w}{5} > \dfrac{517}{5}$ *Divide both sides by 5.*

$w > 103.4$

The Schmidts cannot sell 0.4 watermelon, so they need to sell at least 104 watermelons to earn a profit.

Think and Discuss

1. Compare solving multiplication and division equations with solving multiplication and division inequalities.

2. Explain how you would solve the inequality $0.5y > 4.5$.

11-7 **Exercises**

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GUIDED PRACTICE

See Example **1** Solve.

1. $\frac{w}{8} < -4$ **2.** $\frac{z}{-6} \geq 7$ **3.** $-4 < \frac{p}{-12}$

See Example **2** Solve. Check each answer.

4. $3m > -15$ **5.** $16 > -8y$ **6.** $25c \leq 200$

See Example **3** **7.** It cost Deirdre $212 to make candles. How many candles must she sell at $8 apiece to make a profit?

INDEPENDENT PRACTICE

See Example **1** Solve.

8. $\frac{s}{5} > 1.4$ **9.** $\frac{m}{-4} < -13$ **10.** $\frac{b}{6} > -30$

11. $\frac{c}{-10} \leq 12$ **12.** $\frac{y}{9} < 2.5$ **13.** $\frac{x}{1.1} \geq -1$

See Example **2** Solve. Check each answer.

14. $6w < 4$ **15.** $-5z > -3$ **16.** $15p \leq -45$

17. $-9f > 27$ **18.** $20k < 30$ **19.** $-18n \geq 180$

See Example **3** **20.** Attendance at a museum more than tripled from Monday to Saturday. On Monday, 186 people went to the museum. How many people went to the museum on Saturday?

21. It cost George $678 to make wreaths. How many wreaths must he sell at $15 apiece to make a profit?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP23.

Solve.

22. $\frac{a}{65} \leq -10$ **23.** $0.4p > 1.6$ **24.** $-\frac{m}{5} < -20$

25. $\frac{2}{3}y \geq 12$ **26.** $\frac{x}{-9} \leq 3$ **27.** $\frac{g}{2.1} > 0.3$

28. $\frac{r}{6} \geq \frac{2}{3}$ **29.** $4w \leq 1\frac{1}{2}$ **30.** $-10n < 10^2$

31. $-t > -4$ **32.** $-\frac{y}{12} < 4$ **33.** $5.6v \geq -14$

34. A community theater group produced 8 plays over the last two years. The group's goal for the next two years is to produce at least $1\frac{1}{2}$ times as many plays as they did in the two previous years. How many plays does the group want to produce in the next two years?

35. Tammy is going to a family reunion 350 miles away. She plans to travel no faster than 70 miles per hour. What is the least amount of time it will take her to get there?

36. Social Studies Of the total U.S. population, about 874,000 people are Pacific Islanders. The graph shows where most of these Americans live.

a. According to the graph, less than 10% of Pacific Islanders live in the Midwest. How many Pacific Islanders live in the Midwest?

b. According to the graph, between 10% and 20% of Pacific Islanders live in the South. How many Pacific Islanders live in the South?

Where in the U.S. Do Pacific Islanders Live?

Midwest
Northwest
South
West
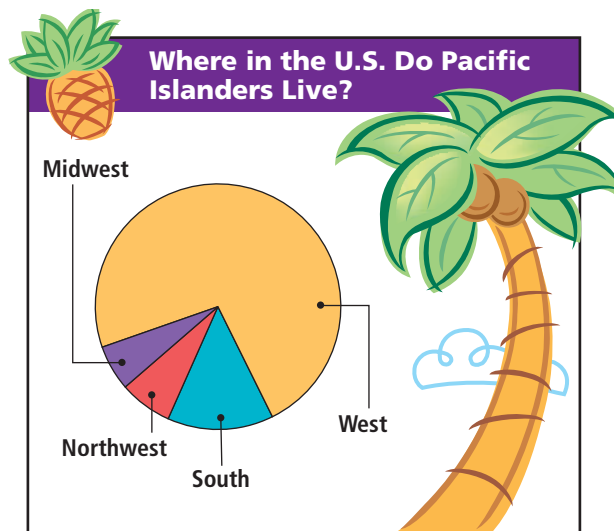
Source: USA Today

37. Seventh-graders at Mountain Middle School have sold 360 subscriptions to magazines. This is $\frac{3}{4}$ of the number of subscriptions that they need to sell to reach their goal. How many total subscriptions must they sell to beat their goal?

38. Recreation Malcolm has saved $362 to spend on his vacation. He wants to have at least $35 a day available to spend. How many days of vacation does Malcolm have enough money for?

39. Write a Problem Write a word problem that can be solved using the inequality $\frac{x}{2} \geq 7$. Solve the inequality.

40. Write About It Explain how to solve the inequality $\frac{n}{-8} < -40$.

41. Challenge Use what you have learned about solving multi-step equations to solve the inequality $4x - 5 \leq 7x + 4$.

SPIRAL STANDARDS REVIEW　　　　　　　NS1.4, SDAP3.2,　　SDAP3.3

42. Multiple Choice Solve $\frac{x}{4} > -2$.

Ⓐ $x > -8$ 　　　　Ⓑ $x < -8$ 　　　　Ⓒ $x < 8$ 　　　　Ⓓ $x > 8$

43. Gridded Response It cost John and Jamie $150 to grow tomatoes. They sell each tomato for $0.50. How many tomatoes must they sell to make a profit?

Find the percent of each number. (Lesson 6-4)

44. 45% of 200 　　**45.** 112% of 50 　　**46.** 14% of 150 　　**47.** 1% of 350

48. In 16 tries, Sondra made 9 baskets. What is the experimental probability that Sondra will make a basket the next time she tries? (Lesson 8-2)
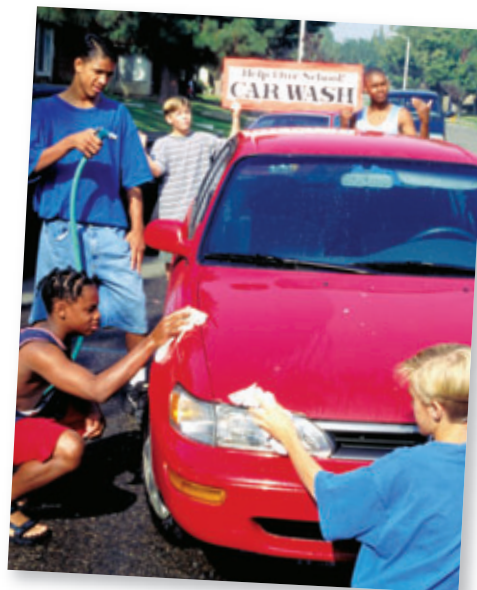
11-8 Solving Two-Step Inequalities

Why learn this? You can solve a two-step inequality to determine how much money band members need to raise at a car wash. (See Example 2.)

When you solve two-step equations, you can use the order of operations in reverse to isolate the variable. You can use the same process when solving two-step inequalities.

EXAMPLE 1 Solving Two-Step Inequalities

Solve. Then graph each solution set on a number line.

Remember!

Draw a closed circle when the inequality includes the point and an open circle when it does not include the point.

See Lesson 12-5, p. 570.

A $\dfrac{x}{5} - 15 < 10$

$$\dfrac{x}{5} - 15 < 10$$
$$\underline{+15 +15}\qquad \textit{Add 15 to both sides.}$$
$$\dfrac{x}{5} < 25$$
$$(5)\dfrac{x}{5} < (5)25 \qquad \textit{Multiply both sides by 5.}$$
$$x < 125$$

Check $\quad \dfrac{x}{5} - 15 < 10$

$$\dfrac{0}{5} - 15 \overset{?}{<} 10 \qquad \textit{0 is less than 125. Substitute 0 for x.}$$
$$-15 < 10 \text{ ✔}$$

B $42 \le \dfrac{y}{-9} + 30$

$$42 \le \dfrac{y}{-9} + 30$$
$$\underline{-30 -30}\qquad \textit{Subtract 30 from both sides.}$$
$$12 \le \dfrac{y}{-9}$$
$$-9(12) \ge (-9)\dfrac{y}{-9} \qquad \textit{Multiply both sides by −9, and reverse the inequality symbol.}$$
$$-108 \ge y$$

Solve. Then graph each solution set on a number line.

C $3x - 12 \geq 9$

$$3x - 12 \geq \quad 9$$
$$\underline{\quad + 12 \quad + 12} \qquad \textit{Add 12 to both sides.}$$
$$3x \qquad \geq \quad 21$$

$$\frac{3x}{3} \geq \frac{21}{3} \qquad \textit{Divide both sides by 3.}$$
$$x \geq 7$$

D $10 > -4y + 6$

$$10 > -4y + 6$$
$$\underline{- 6 \qquad \quad - 6} \qquad \textit{Subtract 6 from both sides.}$$
$$4 > -4y$$

$$\frac{4}{-4} < \frac{-4y}{-4} \qquad \textit{Divide both sides by } -4, \textit{ and reverse the inequality symbol.}$$
$$-1 < y$$

EXAMPLE 2 *School Application*

The 83 members of the Newman Middle School Band are trying to raise at least $5,000 to buy new percussion instruments. They have already raised $850. How much should each student still raise, on average, to meet the goal?

Let d represent the average amount each student should still raise.

$$83d + 850 \geq 5{,}000 \qquad \textit{Write an inequality.}$$
$$\underline{\quad - 850 \quad - 850} \qquad \textit{Subtract 850 from both sides.}$$
$$83d \qquad \geq 4{,}150$$

$$\frac{83d}{83} \geq \frac{4{,}150}{83} \qquad \textit{Divide both sides by 83.}$$
$$d \geq 50$$

On average, each band member should raise at least $50.

Think and Discuss

1. Tell how you would solve the inequality $8x + 5 < 20$.

2. Explain why the *greater than or equal to* symbol was used in the inequality in Example 2.

11-8 **Exercises**

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GUIDED PRACTICE

See Example **1** **Solve. Then graph each solution set on a number line.**

1. $5x + 3 < 18$

2. $-19 \geq \frac{z}{7} + 23$

3. $3y - 4 \geq 14$

4. $\frac{m}{4} - 2 > -3$

5. $42 \leq -11p - 13$

6. $\frac{n}{-3} - 4 > 4$

See Example **2** **7.** Three students collected more than \$93 washing cars. They used \$15 to reimburse their parents for cleaning supplies. Then they divided the remaining money equally. How much did each student earn?

INDEPENDENT PRACTICE

See Example **1** **Solve. Then graph each solution set on a number line.**

8. $5s - 7 > -42$

9. $\frac{b}{2} + 3 < 9$

10. $19 \leq -2q + 5$

11. $-8c - 11 \leq 13$

12. $\frac{y}{-4} + 6 > 10$

13. $\frac{x}{9} - 5 \leq -8$

14. $\frac{r}{-2} - 9 > -14$

15. $44 \geq 13j + 18$

16. $\frac{d}{13} - 12 > 27$

See Example **2** **17.** Rico has \$5.00. Bagels cost \$0.65 each, and a small container of cream cheese costs \$1.00. What is the greatest number of bagels Rico can buy if he also buys one small container of cream cheese?

18. The 35 members of a drill team are trying to raise at least \$1,200 to cover travel costs to a training camp. They have already raised \$500. How much should each member still raise, on average, to meet the goal?

PRACTICE AND PROBLEM SOLVING

Extra Practice
See page EP23.

Solve.

19. $32 \geq -4x + 8$

20. $0.5 + \frac{n}{5} > -0.5$

21. $1.4 + \frac{c}{3} < 2$

22. $1 < \frac{3}{4}b - 2$

23. $12 + 2w - 8 \leq 20$

24. $5k + 6 - k \geq -14$

25. $\frac{s}{2} + 9 > 12 - 15$

26. $4t - 3 - 10t < 15$

27. $\frac{d}{2} + 1 + \frac{d}{2} \leq 5$

28. Mr. Monroe keeps a bag of small prizes to distribute to his students. He likes to keep at least twice as many prizes in the bag as he has students. The bag currently has 79 prizes in it. Mr. Monroe has 117 students. How many more prizes does he need to buy?

29. Manny needs to buy 5 work shirts that are each the same price. After he uses a \$20 gift certificate, he can spend no more than \$50. What is the maximum amount that each shirt can cost?

30. **Business** Darcy earns a salary of \$1,400 per month, plus a commission of 4% of her sales. She wants to earn a total of at least \$1,600 this month. What is the least amount of sales she needs?

31. Multi-Step The bar graph shows how many students from Warren Middle School participated in a reading challenge each of the past four years. This year, the goal is for at least 10 more students to participate than the average number of participants from the past four years. What is the goal for this year?

Reading Challenge Participants
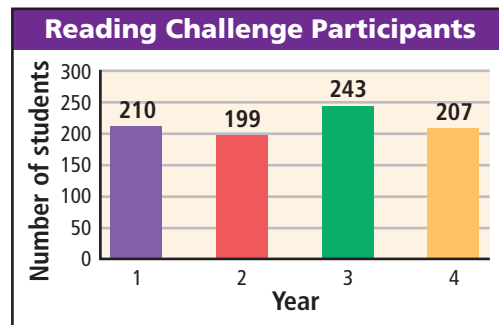
32. Consumer Math Michael wants to buy a belt that costs $18. He also wants to buy some shirts that are on sale for $14 each. He has $70. At most, how many shirts can Michael buy together with the belt?

33. Earth Science A granite rock contains the minerals feldspar, quartz, and biotite mica. The rock has $\frac{1}{3}$ as much biotite mica as quartz. The rock is at least 30% quartz. What percent of the rock is feldspar?

Feldspar

Quartz

Biotite mica

Granite

34. What's the Error? A student's solution to the inequality $\frac{x}{-9} - 5 > 2$ was $x > 63$. What error did the student make in the solution?

35. Write About It Explain how to solve the inequality $4y + 6 < -2$.

36. Challenge A student scored 92, 87, and 85 on three tests. She wants her average score for five tests to be at least 90. What is the lowest score the student can get, on average, on her fourth and fifth tests?

SPIRAL STANDARDS REVIEW   NS2.1, NS2.2, ⚷ SDAP3.1

37. Multiple Choice Which inequality has the following graphed solution?

Ⓐ $2x - 5 > 1$ Ⓑ $-x + 3 < 6$ Ⓒ $3x - 12 < -3$ Ⓓ $-5x - 2 > -13$

38. Gridded Response Gretta earns $450 per week plus a 10% commission on book sales. How many dollars of books must she sell to earn at least $650 per week?

Multiply or divide. Write each answer in simplest form. (Lessons 4-4 and 4-5)

39. $\frac{3}{5} \cdot \frac{10}{21}$ **40.** $1\frac{1}{6} \cdot 5\frac{1}{4}$ **41.** $\frac{7}{8} \div \frac{7}{12}$ **42.** $5\frac{7}{10} \div 3$

43. Jamie flips a coin and rolls a number cube. Use a tree diagram to find all of the possible outcomes. What is the probability of getting heads on the coin and 3 on the number cube? (Lesson 8-4)

READY TO GO ON?

Quiz for Lessons 11-5 Through 11-8

✔️ **11-5** Introduction to Inequalities

Write an inequality for each situation.

1. Gary has at least 25 blue T-shirts.

2. The room can hold no more than 50 people.

Graph each inequality.

3. $b > -1$ **4.** $5 \leq t$ **5.** $-3 \geq x$

Graph each compound inequality.

6. $5 \geq p$ and $p > -1$ **7.** $-8 > g$ or $g \geq -1$ **8.** $-4 \leq x < 0$

✔️ **11-6** Solving Inequalities by Adding or Subtracting

Solve. Then graph each solution set on a number line.

9. $28 > m - 4$ **10.** $8 + c \geq -13$ **11.** $-1 + v < 1$

12. $5 \leq p - 3$ **13.** $-8 > f + 1$ **14.** $-7 - w < 10$

15. A group of climbers are at an altitude of at most 17,500 feet. They are on their way to the top of Mount Everest, which is at an altitude of 29,035 feet. How many more feet do they have left to climb?

✔️ **11-7** Solving Inequalities by Multiplying or Dividing

Solve. Check each answer.

16. $-8s > 16$ **17.** $\frac{x}{-2} \leq 9$ **18.** $-7 \leq \frac{b}{3}$

19. $\frac{c}{-3} \geq -4$ **20.** $28 > 7h$ **21.** $6y < -2$

✔️ **11-8** Solving Two-Step Inequalities

Solve. Then graph each solution set on a number line.

22. $2x - 3 > 5$ **23.** $3 \geq -2d + 4$ **24.** $3g - 2 - 10g > 5$

25. $14 < -4a + 6$ **26.** $3.6 + 7.2k < 25.2$ **27.** $3z - 2 \leq 13$

28. A concert is being held in a gymnasium that can hold no more than 450 people. The bleachers seat 60 people. There will also be 26 rows of chairs set up. At most, how many people can sit in each row?

29. The 23 members of the Westview Journalism Club are trying to raise at least $2,100 to buy new publishing design software. The members have already raised $1,180. How much should each student still raise, on average, to meet the goal?

CONCEPT CONNECTION

Classy Music Ricky is learning to play the guitar and is thinking about taking classes at one of the schools listed in the table.

1. Last year, Ricky spent $590 on guitar lessons. If he takes classes at Main Street Music and spends the same amount this year, how many classes can he take?

2. How many classes would Ricky have to take in order for the cost at Main Street Music to be the same as the cost at SoundWorks?

School	Cost of Lessons
Main Street Music	Annual registration fee: $50 $12 per class
SoundWorks	Annual registration fee: $14 $16.50 per class
Town Hall	No annual registration fee $18 per class

3. Ricky plans to buy a new guitar this year. He expects to pay $139 for it. His total budget for the guitar and the classes is $600. Write and solve an inequality to find the maximum number of classes Ricky will be able to take if he goes to Main Street Music and stays within his budget.

4. Write and solve inequalities to find the maximum number of classes Ricky will be able to take with a $600 budget if he goes to the other schools. Assuming the three schools are equal in other respects, which of the schools should he choose? Why?

Game Time

Flapjacks

Five pancakes of different sizes are stacked in a random order. How can you get the pancakes in order from largest to smallest by flipping portions of the stack?

To find the answer, stack five disks of different sizes in no particular order. Arrange the disks from largest to smallest in the fewest number of moves possible. Move disks by choosing a disk and flipping over the whole stack from that disk up.
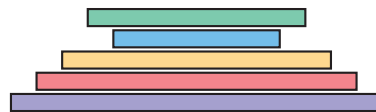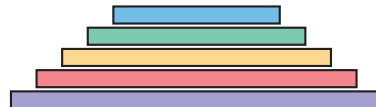
Start with a stack of five.
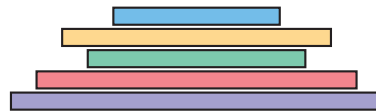
Flip the stack from the second disk up.

Now flip the stack from the third disk up.

Finally, flip the stack from the second disk up.

At most, it should take $3n - 2$ turns, where n is the number of disks, to arrange the disks from largest to smallest. The five disks above were arranged in three turns, which is less than $3(5) - 2 = 13$. Try it on your own.

Leaping Counters

Remove all but one of the counters from the board by jumping over each counter with another and removing the jumped counter. The game is over when you can no longer jump a counter. A perfect game would result in one counter being left in the center of the board.

A complete copy of the rules and a game board are available online.

go.hrw.com
**Game Time Extra**
KEYWORD: MS8CA Games

Materials
- **8 tongue depressors**
- **2 pieces of electrical wire**
- **scissors**
- **markers**

It's in the Bag!

PROJECT ▸ **Wired for Multi-Step Equations**

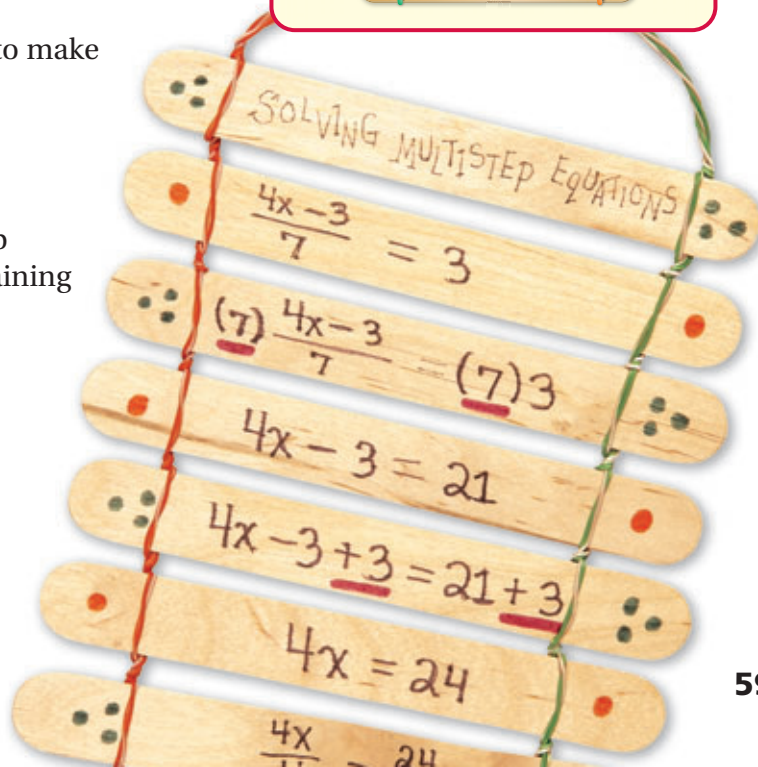These "study sticks" will help you sort out the steps in solving equations.

Directions

1 Twist a piece of electrical wire around each end of a tongue depressor. Twist the wire tightly so that it holds the tongue depressor securely. **Figure A**

2 Slide another tongue depressor between the ends of the wires. Slide it down as far as possible and then twist the wires together to hold this tongue depressor securely. **Figure B**

3 Continue in the same way with the remaining tongue depressors.

4 Twist the wires together at the top to make a handle. Trim the wires as needed.
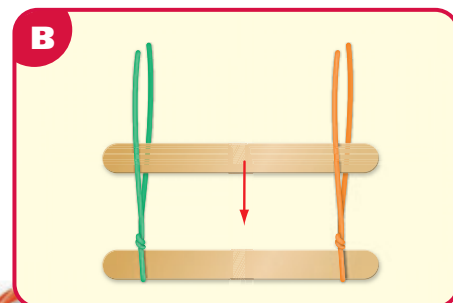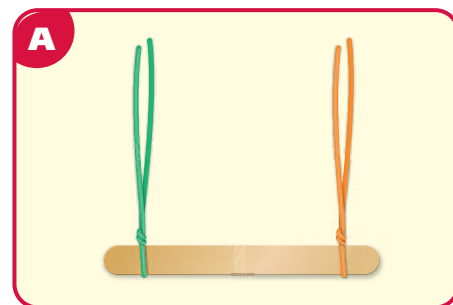
Taking Note of the Math

Write the title of the chapter on the top tongue depressor. On each of the remaining tongue depressors, write the steps for solving a sample multi-step equation.

SOLVING MULTISTEP EQUATIONS

$$\frac{4x-3}{7} = 3$$

$$(7)\frac{4x-3}{7} = (7)3$$

$$4x - 3 = 21$$

$$4x - 3 + 3 = 21 + 3$$

$$4x = 24$$

$$\frac{4x}{} $$

Vocabulary

algebraic inequality 572

coefficient 558

compound inequality 573

inequality 572

solution set 572

term 558

Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ states that two quantities either are not equal or may not be equal.

2. A(n) ___?___ is a combination of more than one inequality.

3. Together, the solutions of an inequality are called the ___?___.

11-1 Solving Two-Step Equations (pp. 554–557)

Preview of 7AF4.1

EXAMPLE

- Solve $6a - 3 = 15$.

  $6a - 3 = 15$

  $6a - 3 + 3 = 15 + 3$ *Add 3 to both sides.*

  $6a = 18$

  $\dfrac{6a}{6} = \dfrac{18}{6}$ *Divide both sides by 6.*

  $a = 3$

EXERCISES

Solve.

4. $-5y + 6 = -34$

5. $9 + \dfrac{z}{6} = 14$

6. $-8 = \dfrac{w}{-7} + 13$

7. $4x + 5 = 21$

8. $7b - 4 = 10$

9. $\dfrac{k}{9} + 10 = 12$

10. $\dfrac{a}{5} - 7 = -4$

11. $-5p + 6 = 26$

11-2 Simplifying Algebraic Expressions (pp. 558–561)

Preview of 7AF1.3; 6AF1.2, 6AF3.1, 6AF3.2

EXAMPLES

Simplify each expression.

- $6a + 4a$

  $(6 + 4)a$

  $10a$

- $4x^3 + 5y + 8x^3 - 4y - 5x^2$

  $4x^3 + 5y + 8x^3 - 4y - 5x^2$

  $12x^3 + y - 5x^2$

EXERCISES

Simplify each expression.

12. $7b^2 + 8 + 3b^2$

13. $12a^2 + 4 + 3a^2 - 2$

14. $x^2 + x^3 + x^4 + 5x^2$

15. $x^2 + 7x - 9 + 5x^2$

16. $10p + 4p - 7p + 5$

11-3 Solving Multi-Step Equations (pp. 562–565)

EXAMPLE

■ Solve $\frac{1}{7}(4x - 3) = 3$.

$$\frac{1}{7}(4x - 3) = 3$$

$(7)\frac{1}{7}(4x - 3) = (7)3$ *Multiply.*

$$4x - 3 = 21$$

$4x - 3 + 3 = 21 + 3$ *Add 3 to both sides.*

$$4x = 24$$

$\frac{4x}{4} = \frac{24}{4}$ *Divide both sides by 4.*

$$x = 6$$

EXERCISES

Solve.

17. $7a + 4 - a = 6$ **18.** $9 = \frac{1}{4}(6j - 18)$

19. $\frac{1}{3}(8b - 5) = 9$ **20.** $52 = -9 + 16y - 19$

21. $4(x + 1) - 3 = 29$ **22.** $9k + k - 4 = 6$

23. Noelle biked twice as many miles as Leila. Adding 2 to the number of miles Noelle and Leila biked combined gives the number of miles Dani biked. Dani biked 17 miles. How many miles did Leila bike?

11-4 Solving Equations with Variables on Both Sides (pp. 566–569)

EXAMPLE

■ Solve $8a = 3a + 25$.

$$8a = 3a + 25$$

$8a - 3a = 3a - 3a + 25$ *Subtract.*

$$5a = 25$$

$\frac{5a}{5} = \frac{25}{5}$ *Divide.*

$$a = 5$$

EXERCISES

Solve.

24. $-6b + 9 = 12b$

25. $5 - 7c = -3c - 19$

26. $18m - 14 = 12m + 2$

27. $20 - 2x = x - 40$

28. $10x + 6 = 54 + 9x$

29. $18 - \frac{1}{2}y = 3 + \frac{1}{3}y$

11-5 Introduction to Inequalities (pp. 572–575)

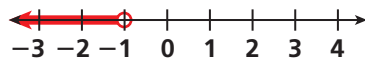### EXAMPLES

Write an inequality for each situation.

■ You have to be at least 17 years old to drive a car in New Jersey.
age of driver ≥ 17

■ There can be at most 60 people on the bus.
number of people ≤ 60

■ Graph $x < -1$.

EXERCISES

Write an inequality for each situation.

30. A bridge's load limit is at most 9 tons.

31. The large tree in the park is more than 200 years old.

32. The moving truck holds at least 2 tons of furniture.

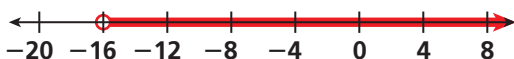**33.** The hole is not yet 49 feet deep.

Graph each inequality.

34. $y \geq 3$

35. $-2 \leq k < -1$

11-6 Solving Inequalities by Adding or Subtracting (pp. 576–579)

Preview of 7AF4.0

EXAMPLE

Solve. Graph the solution set.

- $b + 6 > -10$

$$b + 6 > -10$$
$$b + 6 - 6 > -10 - 6$$
$$b > -16$$

EXERCISES

Solve. Graph each solution set.

36. $r - 16 > 9$ **37.** $-14 \geq 12 + x$

38 $\frac{3}{4} + g < 8\frac{3}{4}$ **39.** $\frac{5}{6} > \frac{2}{3} + t$

40. $7.46 > r - 1.54$ **41.** $u - 58 \geq -124$

42. The Wildcats scored at least 13 more points than the Stingrays scored. The Stingrays scored 25 points. How many points did the Wildcats score?

11-7 Solving Inequalities by Multiplying or Dividing (pp. 580–583)

Preview of 7AF4.0

EXAMPLE

Solve.

- $\frac{m}{-4} \geq 3.8$

$$\frac{m}{-4} \geq 3.8$$
$$(-4)\frac{m}{-4} \leq (-4)3.8 \quad \textit{Multiply and reverse the inequality symbol.}$$
$$m \leq -15.2$$

EXERCISES

Solve.

43. $\frac{n}{-8} > 6.9$ **44.** $-18 \leq -3p$

45. $\frac{k}{13} < -10$ **46.** $-5p > -25$

47. $2.3 \leq \frac{v}{1.2}$ **48.** $\frac{c}{-11} < -3$

49. $6g < -42$ **50.** $12x \geq 6$

51. It cost Carlita $204 to make beaded purses. How many purses must Carlita sell at $13 apiece to make a profit?

11-8 Solving Two-Step Inequalities (pp. 584–587)

Preview of 7AF4.1

EXAMPLE

Solve. Graph the solution set.

- $\frac{k}{3} - 18 > 24$

$$\frac{k}{3} - 18 > 24$$
$$\frac{k}{3} - 18 + 18 > 24 + 18$$
$$\frac{k}{3} > 42$$
$$(3)\frac{k}{3} > (3)42$$
$$k > 126$$

EXERCISES

Solve. Graph each solution set.

52. $-7b - 16 > -2$ **53.** $38 + \frac{d}{6} < 37$

54. $15 - 4n + 9 \leq 40$ **55.** $\frac{y}{-3} + 18 \geq 12$

56. $\frac{c}{3} + 7 > -11$ **57.** $32 \geq 4x - 8$

58. $18 + \frac{h}{6} \geq -8$ **59.** $14 > -2t - 6$

60. $-3 < \frac{w}{-4} + 10$ **61.** $\frac{y}{7} + 3.9 \leq 8.9$

62. Luis has $53.55. T-shirts cost $8.95 each, and a belt costs $16.75. How many T-shirts can Luis buy if he also buys a new belt?

Simplify. Justify your steps using the Commutative, Associative, and Distributive Properties when necessary.

1. $12x + 5x - x + 7$

2. $6 + 3n + 8 - 2n$

3. $x^3 + 2x^3 - x + 12x$

Solve.

4. $3y - 8 = 16$

5. $\frac{x}{3} + 12 = -4$

6. $\frac{a}{6} - 7 = -4$

7. $-7b + 5 = -51$

8. $\frac{1}{3}(5y - 4) = 7$

9. $8r + 7 - 13 = 58$

10. $6 = \frac{1}{5}(12s - 6)$

11. $3(4t - 10) = -54$

12. $-14q = 4q - 126$

13. $5p + 24 = p - 96$

14. $9 - 6k = 3k - 54$

15. $-36d = -70d + 340$

16. The bill for the repair of a computer was $179. The cost of the parts was $44, and the labor charge was $45 per hour. How many hours did it take to repair the computer?

Write an inequality for each situation.

17. You must be more than 4 ft tall to go on the ride.

18. You cannot go more than 65 miles per hour on Route 18.

Graph each inequality.

19. $a < -2$

20. $-5 < d$ and $d \le 2$

21. $c > -1$ or $c < -5$

22. $b \ge 3$

Solve. Then graph each solution set on a number line.

23. $n + 8 < -9$

24. $n - 124 > -59$

25. $-40 > \frac{x}{32}$

26. $-3y \le -48$

27. Rosa wants to save at least $125 to buy a new skateboard. She has already saved $46. How much more does Rosa need to save?

28. Gasoline costs $2.75 a gallon. At most, how many gallons can be bought for $22.00?

Solve. Then graph each solution set on a number line.

29. $m - 7.8 \le 23.7$

30. $18z > -8$

31. $\frac{w}{-4.9} \le 3.4$

32. $-15 < 4a + 9$

33. $2.8 - \frac{c}{4} \ge 7.4$

34. $\frac{d}{5} - 8 > -4$

35. The seventh-grade students at Fulmore Middle School are trying to raise at least $7,500 for the local public library. So far, each of the 198 students has raised an average of $20. How much more money must each seventh-grader collect, on average, to reach the goal?

STRATEGIES FOR SUCCESS

Any Question Type: Use a Diagram

Diagrams are a helpful tool. If a diagram is included in a test item, study it closely as it may contain useful information. Sometimes it is helpful to draw your own diagram.

EXAMPLE 1

Multiple Choice A small circle is inside a large circle. The diameter of the small circle is 10 feet. If the circumference of the large circle is 4 times the circumference of the small circle, what is the radius of the large circle?

    Ⓐ 20 ft   Ⓑ 30 ft   Ⓒ 40 ft   Ⓓ 120 ft

Draw a diagram to help you visualize the problem.
Draw two circles and label them with all the information given in the problem.

Diameter = 10 ft

The circumference of the large circle is 4 times the circumference of the small circle, so the diameter of the large circle is 4 times the diameter of the small circle. The diameter of the large circle is 4 · 10 = 40 feet, so its radius is 20 feet.

Choice A is correct.
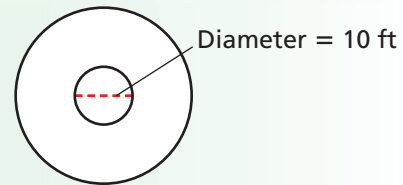
EXAMPLE 2

Short Response $\triangle ABC$ is similar to $\triangle FDE$. Find the missing length.

A
    15 in.   E
9 in.   ? 8 in
        52°
B   12 in.   C F 6 in. D
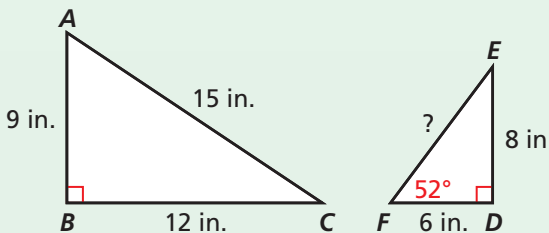
These triangles do not look similar and are not drawn to scale, but the information in the problem says that they are.

Set up a proportion to find the missing length and solve for x. $\frac{x}{6} = \frac{15}{9}$

The unknown side length is 10 in.

HOT TIP! If you have trouble understanding what a test item is asking, draw a diagram to help you visualize the question.

Read each test item and answer the questions that follow.

Item A
Multiple Choice The temperature at the ski lodge was 21°F at 9:00 P.M. At sunrise, the temperature was 34°F. How many degrees did the temperature rise overnight?

- Ⓐ 54°F
- Ⓒ 13°F
- Ⓑ 25°F
- Ⓓ 4°F

1. What information will help you solve the problem?

2. Sketch a diagram to help you solve this problem. Be sure to label the diagram with all of the information you know.

Item B
Short Answer Prove that the two rectangles below are similar. Explain your reasoning.

M ____6 cm____ N

A ___7.5 cm___ B

3 cm

15 cm

D _____ C P _____ O
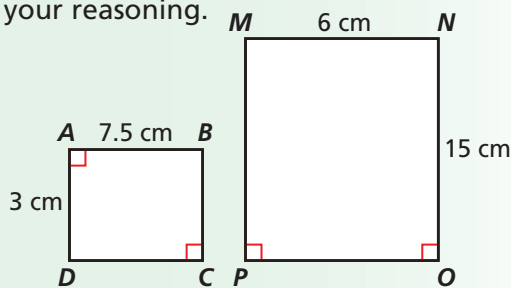
3. What information can you get from the diagram to help you prove that the figures are similar?

4. Do you think the drawings accurately illustrate the given information? If not, why?

5. What is the length of $\overline{DC}$?

Item C
**Gridded Response** The longest side of a triangle is 14.4 centimeters. Its shortest side is 5.9 centimeters shorter than the longest side. If the perimeter of the triangle is 35.2 centimeters, what is the length of the third side?

6. How do you determine the perimeter of a triangle?

7. Sketch a diagram of the triangle. Explain how sketching the diagram can help you answer the problem.

8. Tell how you would fill in your response to this test item on a grid.

Item D
Multiple Choice Which angle pairs are vertical angles?
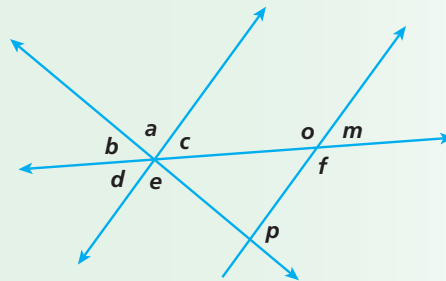
- Ⓐ ∠a and ∠f
- Ⓒ ∠b and ∠e
- Ⓑ ∠b and ∠c
- Ⓓ ∠c and ∠d

9. Which answer choice can you immediately eliminate? Why?

10. How can you use the diagram to help you eliminate the other choices?

11. Explain which answer choice is correct.

MASTERING THE STANDARDS

Cumulative Assessment, Chapters 1–11

Multiple Choice

1. Nolan has 7 red socks, 3 black socks, 10 white socks, and 5 blue socks in a drawer. If Nolan chooses one sock, replaces it, and then chooses a second sock, what is the probability that he will choose 2 white socks?

  (A) $\frac{3}{20}$ (C) $\frac{2}{5}$

  (B) $\frac{4}{25}$ (D) $\frac{19}{25}$

2. Of the 10,500 books in the school library, $\frac{2}{5}$ of the books are fiction. Given that 30% of the remaining books are biographies, how many books are biographies?

  (A) 4,200   (C) 1,260

  (B) 2,940   (D) 1,890

3. There are 126 girls and 104 boys attending a luncheon. Each person at the luncheon writes his or her name on a piece of paper and puts the paper in a barrel. One name is randomly selected from the barrel to win a new MP3 player. What is the probability the person selected is male?

  (A) 45.2%   (C) 82.5%

  (B) 54.8%   (D) Not here

4. A trapezoid has two bases, b_1 and b_2, and height h. For which values of b_1, b_2, and h is the area of the trapezoid equal to 16 in^2?

  (A) $b_1 = 8$ in., $b_2 = 4$ in., $h = 2$ in.

  (B) $b_1 = 5$ in., $b_2 = 3$ in., $h = 4$ in.

  (C) $b_1 = 2$ in., $b_2 = 8$ in., $h = 6$ in.

  (D) $b_1 = 2$ in., $b_2 = 4$ in., $h = 4$ in.

5. There were 18 teachers and 45 students registered to participate in a 5K walk-a-thon. Which ratio accurately compares the number of students to the number of teachers?

  (A) 1:5   (C) 3:15

 (B) 5:2   (D) 18:45

6. What is $7 + (-6)$?

  (A) -13 (C) 1

  (B) -1 (D) 13

7. Martha buys a surfboard that costs $405 for 40% off. How much money does she save?

  (A) $243   (C) $24

  (B) $162   (D) $17

8. The total number of students in seventh grade at Madison Middle School is expected to increase by 15% from year three to year four. What will enrollment be in year four?

Seventh-Grade Enrollment

  (A) 42   (C) 345

  (B) 295   (D) 238

9. The two figures are similar. Which proportion can be used to find the missing side length?

4 mm, 4 mm, 6 mm, 6 mm, x

18 mm, 18 mm, 27 mm, 27 mm, 9 mm
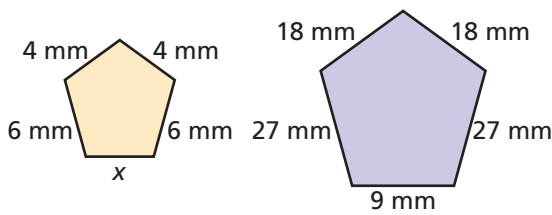
Ⓐ $\frac{4}{x} = \frac{18}{27}$

Ⓒ $\frac{6}{x} = \frac{9}{18}$

Ⓑ $\frac{6}{x} = \frac{27}{4}$

Ⓓ $\frac{4}{x} = \frac{18}{9}$

HOT TIP! Create and use a number line to help you order rational numbers quickly.

10. Which rational number is greater than $-3\frac{1}{3}$ but less than $-\frac{4}{5}$?

Ⓐ -0.4

Ⓒ -0.19

Ⓑ $-\frac{22}{5}$

Ⓓ $-\frac{9}{7}$

11. Which equation could be used to find the circumference in inches of a circle with a radius of 5 inches?

Ⓐ $C = 5 \cdot \pi$

Ⓒ $C = 5^2 \cdot \pi$

Ⓑ $C = 10 \cdot \pi$

Ⓓ $C = 10^2 \cdot \pi$

Gridded Response

12. What is the least common multiple of 6, 8, and 10?

13. Freddy counted the number of bats he saw each night for one week. What is the median of the data set?

Number of Bats Spotted
42, 21, 36, 28, 40, 21, 31

14. What is the probability of flipping a coin and getting tails and then rolling a number greater than or equal to 4 on a 6-sided number cube? Write your answer as a decimal.

Short Response

15. Nancy is stenciling 5-inch-wide stars, end-to-end, around her rectangular bedroom. Her bedroom is $12\frac{3}{4}$ feet wide and $15\frac{1}{4}$ feet long. How many whole stars will Nancy stencil?

16. A restaurant offers a choice of 3 main dishes (roast beef, chicken, or fish), 3 vegetables (broccoli, carrots, and corn), and 2 sides (soup or salad). Use a tree diagram to find all the possible outcomes when choosing one main dish, one vegetable, and one side.

17. Hallie is baking 5 batches of bran muffins for the bake sale. Each batch requires $1\frac{2}{3}$ cups of flour. Hallie has $8\frac{1}{4}$ cups of flour. Does she have enough flour to make five batches? Explain your answer.

Extended Response

18. There are 5 blue tiles, 7 red tiles, and 8 yellow tiles in a jar.

a. If you pick a tile without looking, what is the probability of picking a blue tile? Express this probability as a percent, a fraction, and a decimal.

b. If you pick a tile without looking, what is the probability of NOT picking a yellow tile? Write your answer in simplest form.

c. You conduct an experiment in which you pick a tile out of the jar 50 different times. Each time you record the color of tile and then replace the tile before you pick another. How many times would you expect to pick a blue tile? Explain.