this length, plus the thickness of the base plate, allows elongation so that the plate can be securely fastened. The sleeve will also allow the smaller-diameter anchor bolts to be bent to fit the predrilled holes in the base plate if there is slight misalignment.

If a sleeve is used, it may or may not be filled with grout after the base plate is attached and the anchor nut tightened. There are major differences of opinion on this:

1. Some think the sleeve should not be grouted so that stress reversals will produce strain changes over a length of bolt rather than locally.
2. Some think that after the bolt is tightened to a proof load (about 70 percent of yield) no strains of any magnitude are developed unless the moment is large enough to separate the base plate from the grout bed.

In any case, if the sleeve is grouted, the distance to develop subsequent strains is limited to roughly the thickness of the base plate. The question is of little importance where no stress reversals occur because the sleeve is used only for alignment in this case and the nut is usually made only snug-tight (about one-fourth turn from tight).

Anchor studs are available that are screwed into expanding sleeves that have been placed in predrilled holes in the footing to a depth of 75 to 300 mm . The studs may expand the sleeve against the concrete, or the sleeve may be driven down over a steel wedge to produce expansion, after which the anchor is screwed in place. Anchor studs can only be tightened a limited amount since the elongation distance is the base plate thickness. They are primarily used for anchoring equipment into permanent position.

Base plate anchor bolts are designed for any tension and/or shear forces that develop when overturning moments are present. Both bolt diameter and depth of embedment require analysis, although the latter is not specifically indicated in most (including ACI) building codes. Where a column has no moment a pair of anchor bolts is used, with the size being somewhat arbitrarily selected by the designer. Some additional information on anchor bolts may be found in Ueda et al. (1991, with references).

## 8-7 PEDESTALS

A pedestal is used to carry the loads from metal columns through the floor and soil to the footing when the footing is at some depth in the ground. The purpose is to avoid possible corrosion of the metal from the soil. Careful backfill over the footing and around the pedestal will be necessary to avoid subsidence and floor cracks. If the pedestal is very long, a carefully compacted backfill will provide sufficient lateral support to control buckling. The ACI (Art. 7.3 and 318.1) limits the ratio of unsupported length $L_{u}$ to least lateral dimension $h$ as

$$
\frac{L_{u}}{h} \leq 3
$$

for pedestals. The problem is to identify the unsupported length $L_{u}$ correctly when the member is embedded in the soil.

The code allows both reinforced and unreinforced pedestals. Generally the minimum percentage of steel for columns of $0.01 A_{\text {col }}$ of Art. $10.9^{8}$ should be used even when the pedestal

[^0]

Figure 8-9 Pedestal details (approximate). Note that vertical steel should always be designed to carry any tension stresses from moment or uplift
is not designed as a reinforced column-type element. Rather, when the pedestal is designed as an unreinforced member, the minimum column percent steel ( 4 to 8 bars) is arbitrarily added. When steel base plates are used, this reinforcement should terminate about 70 to 90 mm from the pedestal top in order to minimize point loading on the base plate.

Steel should be liberally added at the top, as in Fig. 8-9, to avoid spalls and to keep the edges from cracking. Room must be left, however, to place the anchor bolts necessary to hold the bearing plate and column in correct position. The anchor bolts should be inside the spiral or tie reinforcement to increase the pullout resistance.

Pedestals are usually considerably overdesigned, since the increase in materials is more than offset by reduced design time and the benefit of the accrued safety factor.

Pedestals can usually be designed as short columns because of the lateral support of the surrounding soil. They may be designed for both axial load and moment, but this feature is beyond the scope of this text. For the rather common condition of the pedestal being designed as a simply supported column element interfacing the superstructure to the footing, the following formula may be used:

$$
\begin{equation*}
P_{u}=\phi\left(0.85 f_{c}^{\prime} A_{c}+A_{s} f_{y}\right) \tag{8-13}
\end{equation*}
$$

where $\quad P_{u}=$ factored ultimate column design load, kN or kips
$A_{c}=$ net area of concrete in pedestal $\left(A_{g}-A_{s}\right)$ for unreinforced pedestals $A_{s}=0.0$ and $A_{c}=$ total concrete area
$A_{s}=$ area of reinforcing steel if designed as a reinforced column
$f_{y}=$ yield strength of rebar steel
$\phi=0.70$ for tied and 0.75 for spiral reinforcement; 0.65 for nonreinforced pedestals


Figure E8-5a, b
Example 8-5. Design a pedestal and bearing plate for the following conditions:

$$
\begin{gathered}
D=800 \mathrm{kN} \quad L=625 \mathrm{kN} \quad P=1425 \mathrm{kN} \\
\mathbf{W} 310 \times 107 \mathrm{column} \quad d=311 \mathrm{~mm} \quad b_{f}=306 \mathrm{~mm} \\
F_{y}=250 \mathrm{MPa}(A 36 \mathrm{steel}) \text { for both column and bearing plate } \\
\text { Concrete: } f_{c}^{\prime}=24 \mathrm{MPa} ; \quad f_{y}=400 \mathrm{MPa}(\text { Grade }=400) \\
\text { Soil: } q_{a}=200 \mathrm{kPa}
\end{gathered}
$$

## Solution.

Step 1. We will set dimensions of the pedestal for the base plate but increase (shoulder it out) 50 mm to allow bearing for the floor slab as illustrated in Fig. E8-5a. First, find areas $A_{1}$ and $A_{2}$ :

$$
A_{2}=\frac{P}{0.175 f_{c}^{\prime}}=\frac{1425}{0.175 \times 24 \times 1000}=0.3393 \mathrm{~m}^{2}\left(1000 \text { converts } f_{c}^{\prime} \text { to } \mathrm{kPa}\right)
$$

Next,

$$
A_{1}=\frac{1}{A^{2}}\left(\frac{P}{0.35 f_{c}^{\prime}}\right)^{2}=\frac{1}{0.3393}\left(\frac{1425}{8400}\right)^{2}=0.0848 \mathrm{~m}^{2}
$$

or

$$
A_{1}=\frac{P}{0.7 f_{c}^{\prime}}=\frac{1425}{16800}=0.0848 \mathrm{~m}^{2}
$$

Use a plate area $A_{1} \geq 0.0848 \mathrm{~m}^{2}$.
Use a pedestal with $A_{2} \geq 0.3393 \mathrm{~m}^{2}$. For the pedestal try

$$
B^{2}=0.3393 \rightarrow B=\sqrt{0.3393}=0.582 \mathrm{~m}
$$

Let us use $\boldsymbol{B}=\mathbf{0 . 6 0 0} \mathrm{m}$;

$$
A_{2}=0.60 \times 0.60=0.36 \mathrm{~m}^{2}>0.3393 \mathrm{~m}^{2} \quad \text { O.K. }
$$

Looking at the column dimensions, let us try a plate of

$$
\begin{aligned}
& C=d+25=311+25=336 \rightarrow \mathbf{3 3 5} \mathrm{~mm} \\
& B=b_{f}+25=306+25=331 \rightarrow \mathbf{3 3 0} \mathrm{~mm}
\end{aligned}
$$

Check the furnished area, that is,

$$
A_{1}=0.335 \times 0.330=0.1106 \mathrm{~m}^{2}>0.0848 \mathrm{~m}^{2} \quad O . K .
$$

The allowable concrete bearing stress (base plate area $\mathrm{A}_{1}<\mathrm{A}_{2}$ ) is

$$
F_{p}=0.35 f_{c}^{\prime} \sqrt{\frac{A_{2}}{A_{1}}}=0.35(24) \sqrt{\frac{0.360}{0.111}}=15.13 \mathrm{MPa}<0.7 f_{c}^{\prime}
$$

Let us check:

$$
A_{1} F_{p}=0.111(15.13)(1,000)=1679>1425 \mathrm{kN} \quad \text { O.K. }
$$

Step 2. Find the plate thickness $t_{p}$ :

$$
\begin{aligned}
& m=\frac{335-0.95 d}{2}=\frac{335-0.95(311)}{2}=19.8 \mathrm{~mm} \\
& n=\frac{330-0.80 b_{f}}{2}=\frac{330-0.80(306)}{2}=42.6 \mathrm{~mm} \\
& L=d+b_{f}=311+306=617 \mathrm{~mm}=0.617 \mathrm{~m} \\
& X=\frac{4 P}{L^{2} F_{p}}=\frac{4(1425)}{0.617^{2}(15.13 \times 1,000)}=0.9896 \rightarrow 0.99 \\
& \lambda=\min \left(1.0, \frac{2 \sqrt{X}}{1+\sqrt{1-X}}\right)=\min \left(1.0, \frac{2 \sqrt{0.99}}{1+\sqrt{1-0.99}}\right)=\min (1.0,1.81) \\
& \lambda=1.0
\end{aligned}
$$

$$
\lambda n^{\prime}=1.0(0.25) \sqrt{d_{b} b_{f}}=1.0(0.25) \sqrt{311 \times 306}=77.1 \mathrm{~mm}
$$

$$
v=\max \left(m, n, \lambda n^{\prime}\right)=\max (19.8,42.6,77.1)=77.1 \mathrm{~mm}
$$

$$
f_{p}=1425 / A_{1}=1425 / 0.1106=12884 \mathrm{kPa}=12.88 \mathrm{MPa}
$$

The plate thickness is

$$
t_{p}=2 v \sqrt{\frac{f_{p}}{F_{y}}}=2(77.1) \sqrt{\frac{12.88}{250}}=35.0 \mathrm{~mm}
$$

Use a base plate of $335 \times 330 \times 35 \mathrm{~mm}$.
Step 3. Design pedestal reinforcement.
Top area $=600 \times 600 \mathrm{~mm}=360000 \mathrm{~mm}^{2}$
Use minimum of $0.01 A_{\text {col }} \rightarrow A_{s}=0.01(360000)=3600 \mathrm{~mm}^{2}$


Anchor bolt pattern

Figure E8-5c

Choose eight No. 25 bars, providing $8(500)=4000 \mathrm{~mm}^{2}$, which is greater than $3600 \mathrm{~m}^{2}$ and therefore is acceptable. Place bars in pattern shown in Fig. E8-5b.

Step 4. Design anchor rods/bolts. Theoretically no anchorage is required, however, we will arbitrarily use enough to carry $0.1 \times P$ in shear:

$$
P_{v}=0.1(1425)=142.5 \mathrm{kN}
$$

Use standard size bolt holes, and from Table 1D of AISC (1989), obtain $F_{v}=70 \mathrm{MPa}$ (10 ksi) for A307 grade steel.

Using $25-\mathrm{mm}$ diameter bolts, we have.

$$
P_{\text {bolt }}=0.7854\left(0.025^{2}\right)(70)(1000)=34 \mathrm{kN} / \text { bolt }
$$

No. of bolts required $=142.5 / 34=4.15$ bolts $\rightarrow$ use 4 bolts
Place anchor bolts in the pattern shown on Fig. E8-5c. Use anchor bolt steel of A-307 grade (or better).

## 8-8 BASE PLATE DESIGN WITH OVERTURNING MOMENTS

It is sometimes necessary to design a base plate for a column carrying moment as well as axial force. The AISC and other sources are of little guidance for this type of design. Only a few pre-1970s steel design textbooks addressed the problem. The Gaylord and Gaylord (1972) textbook provided a design alternative using a rectangular pressure distribution as used in this section. Most designs were of the ( $\mathrm{P} / \mathrm{A}) \pm(\mathrm{Mc} / \mathrm{I})$ type but were generally left to the judgment of the structural engineer. The author will present two methods for guidance.

METHOD 1. For small eccentricity where eccentricity $e_{x}=M / P$, with small $e_{x}$ arbitrarily defined as less than $C / 2$ and $C$ is the base plate length (dimensions $B, C$ ) as shown in Fig. 8-10. In this case we make the following definitions:

$$
\begin{aligned}
C^{\prime} & =C-2 e_{x} \\
A_{p} & =\text { effective plate area }=B \times C^{\prime} \\
A_{\mathrm{ftg}} & =\text { area of supporting member (footing or pedestal) } \\
P_{\mathrm{col}} & =\text { column axial load } \\
M & =\text { column moment } \\
e_{x} & =M / P_{\mathrm{col}}=\text { eccentricity }
\end{aligned}
$$

From these we may compute the following:

$$
\begin{aligned}
\mathrm{FAC} & =0.35 \sqrt{\frac{A_{\mathrm{ftg}}}{A_{p}}} \leq 0.7 \\
F_{p} & =\frac{(0.35+\mathrm{FAC})}{2} \cdot f_{c}^{\prime}
\end{aligned}
$$

This $F_{p}$ is the average allowable bearing pressure to be used for design purposes.
By trial, find the footing (or pedestal) dimensions to obtain the footing area $A_{\text {ftg }}$.
By trial find the base plate dimensions so that the effective plate area

$$
B \times C^{\prime} \times F_{p} \geq P_{\mathrm{col}}
$$

Use minimum of 2
bolts or anchor rods

(a) Assumptions for column base plate with small eccentricity. $F_{p}$ depends on area of supporting member (pedestal or footing).

## Usual tension

bolt location

(b) Assumptions for large eccentricity.

Figure 8-10 Base plates with eccentricity due to column moment. Bolt pattern is usually symmetrical about column center line when base plate moments are from wind or earthquake.

The assumption of a constant $F_{p}$ across the effective plate area is made. If you obtain a set of dimensions of $B \times C^{\prime}$, you can compute the actual contact stress $F_{\mathrm{pa}} \leq F_{p}$. Usually one makes this calculation since a trial case of $B \times C^{\prime} \times F_{p}=P_{\text {col }}$ is next to impossible. When you have a case of $F_{\mathrm{pa}}=P_{\mathrm{col}} /\left(B \times C^{\prime}\right) \leq F_{p}$, you have a solution. You may not have the "best" solution, and you might try several other combinations to obtain the minimum plate mass. Clearly a limitation is that the plate must be larger than the column footprint by about 25 mm ( 1 inch) in both dimensions to allow room for fillet-welding of the column to the base plate in the fabricating shop.

When you have found a $B \times C \times F_{\mathrm{pa}}$ combination that works, the $\sum M=0$ condition for statics is automatically satisfied. The center of the distance $C^{\prime}$ is always $e_{x}$ from the column center (or $x$ axis).

Computation of base plate thickness requires using the distances $m$ and $n$ as in Example 8-5 except that $L$ is not computed because it has no significance here.

A computer program (the author uses STDBASPL-one suggested on your diskette) is most useful, as it finds several combinations of $B \times C$ that work, computes the resulting mass, and outputs sufficient data for performing any necessary statics checks. The program outputs the largest thickness computed. This distance is not checked against available rolled plate thicknesses since the steel fabricator may have thicker plate stock on hand and it may be more economical to substitute than to order a small quantity for the specific project.

## Example 8-6.

Given. A W $360 \times 162(\mathbf{W ~} 14 \times 109)$ column carries a 500 kN axial load and has a moment of $100 \mathrm{kN} \cdot \mathrm{m}$. The footing dimensions are $1.5 \times 2 \mathrm{~m}$. Refer to Fig. E8-6a.

$$
\begin{array}{lr}
\text { Column dimensions (from AISC, 1992): } & \begin{aligned}
& d=364 \mathrm{~mm} \text { (rounded to } 365 \mathrm{~mm} \text { ) } \\
& \text { Concrete } f_{c}^{\prime}=24 \mathrm{MPa} b_{f}=371 \mathrm{~mm} \text { (rounded to } 375 \mathrm{~mm} \text { ) } \\
& F_{y}=250 \mathrm{MPa} \text { (base plate) }
\end{aligned} \text { (b) }
\end{array}
$$

Required. Find a minimum weight (or mass) base plate.
I used computer output (but you can make several trials and get the same results) to find

$$
e_{x}=\frac{M}{P_{\mathrm{col}}}=\frac{100}{500}=0.200 \mathrm{~m}
$$

Try a plate $B=450 \mathrm{~mm} \times \mathrm{C}=500 \mathrm{~mm}$.

$$
\begin{aligned}
C^{\prime} & =500 / 1000-2(0.2)=0.100 \mathrm{~m} \\
B \times C^{\prime} & =0.450 \times 0.10=0.045 \mathrm{~m}^{2} \\
\mathrm{FAC} & =0.35 \sqrt{\frac{1.5 \times 2}{0.045}}=2.86>0.7 \mathrm{use} 0.7 \\
F_{p} & =\frac{(0.35+0.7)}{2} \cdot 24=12.6 \mathrm{MPa} \\
\text { Actual } F \mathrm{pa} & =\frac{P_{\mathrm{col}}}{A_{p}}=\frac{500}{(1000 \times 0.045)}=11.111 \mathrm{MPa}
\end{aligned}
$$

The factor 1000 converted 500 kN to 0.50 MN .
The "actual" column dimensions are used to compute plate thickness based on $m$ and $n$. Thus,

$$
\begin{aligned}
m & =(500-0.95 \times 364) / 2=77.1 \mathrm{~mm} \\
n & =(450-0.80 \times 371) / 2=76.6 \mathrm{~mm}
\end{aligned}
$$



Figure E8-6a


Figure E8-6b

Using the larger value, we obtain the plate thickness as

$$
t_{p}=2 m \sqrt{\frac{F_{\mathrm{pa}}}{F_{y}}}=2(77.1) \sqrt{\frac{11.11}{250}}=32.51 \mathrm{~mm}
$$

The plate weight $/$ mass (steel mass $=7850 \mathrm{~kg} / \mathrm{m}^{3}$ or $490 \mathrm{lb} / \mathrm{ft}^{3}$ ) is $\mathrm{W}=0.500 \times 0.450 \times 0.0325 \times$ $7850=57.42 \mathrm{~kg}$

Summarizing, we have
Base plate: $\mathbf{5 0 0} \times \mathbf{4 5 0} \times \mathbf{3 2 . 5} \mathrm{mm}$
and $W=57.42 \mathrm{~kg}$
The reader should verify that $\sum F_{v}=0$ and that about the column centerline $\sum M_{\mathrm{cl}}=0$.
Theoretically no base plate anchorage bolts are required, however, at least two $20-\mathrm{mm}$ diameter A307 grade bolts would be used in the web zone of the plate as shown in Fig. E8-6b.

METHOD 2. Base plate design for large eccentricity. When the eccentricity is such that it falls outside one-half the depth of the column, it is necessary to resort to a different type of solution, one that uses heel bolts in tension. The heel bolts may fall behind the heel flange or be centered on both sides of it. The bolt pattern depends on the bolt force required for equilibrium and the designer's prerogative. By making the bolt option part of a computer program interactive either case can be analyzed. The general procedure is as follows (refer to Fig. 8-11):

$$
\begin{align*}
\sum F_{v} & =T+P-R=0 \quad T=R-P  \tag{a}\\
\sum M_{\mathrm{cl}} & =0=T \times X_{T}+R \times X_{R}-M \tag{b}
\end{align*}
$$

Noting that $R=B\left(F_{p}\right) k d$ and substituting Eq. (a) for $T$, we obtain a quadratic equation of the form $(a) k d^{2}+(b) k d+c=0$ from which we can solve for the depth of the rectangular stress block $k d$. This approach is illustrated by the next example.

Figure 8-11 Base plate with a large overturning moment and small axial load $P$. The resulting $e_{x}=M / P$ is such that $2 e_{x}>d / 2$.


There are two additional considerations for large eccentricity. First, the allowable bearing stress $F_{p}=0.35 f_{c}^{\prime}$. This result must occur as part of the solution of the quadratic equation. Second, it is always necessary to check to see whether the bolt tension force $T$ in these equations controls the design of the base plate thickness.

Example 8-7.

## Given.

Column load $P=90 \mathrm{kN} \quad M=175 \mathrm{kN} \cdot \mathrm{m}$
Column: W $360 \times 134 \quad d=356 \mathrm{~mm} \quad b_{f}=369 \mathrm{~mm}$
$f_{c}^{\prime}=21 \mathrm{MPa} \quad F_{y}=250 \mathrm{MPa}$ (refer to Fig. E8-7a)
Pedestal dimensions: $L=700 \mathrm{~mm} \quad B=610 \mathrm{~mm}$
Initial trial base plate: $B=610 \mathrm{~mm} \quad C=700 \mathrm{~mm}$
Computed eccentricity $e_{x}=M / P=175 / 90=1.944 \mathrm{~m} \gg 0.356 / 2$
The computer program (and hand calculations as well) requires that you "guess" at a set of initial dimensions. If the resulting computed $k d$ is outside the compression flange, that solution is not a

good one, so the trial base plate is reduced and a new trial is initiated. This process continues until a solution is obtained where the $k d$ zone is at least partly under the compression flange. The minimum width is, of course, at least the width of the column +25 mm ( 1 in .) rounded to an even multiple of 5 mm (or inches in integers).

Required. Design a column base plate for the given conditions. Refer to Fig. E8-7a.
Solution. Computer program STDBASPL was again used. A solution can be obtained using

$$
C=550 \mathrm{~mm} \quad \text { and } \quad B=450 \mathrm{~mm}
$$

Take $F_{p}=0.35 f_{c}^{\prime}=0.35(21)=7.35 \mathrm{MPa}$
Convert $B, C$ to meters $\rightarrow C=0.550 \mathrm{~m} \quad B=0.450 \mathrm{~m}$
Define $A 1=T-\operatorname{arm}=C / 2-0.030=0.550 / 2-0.030=0.245 \mathrm{~m}$
The 0.030 provides adequate edge clearance for bolts up to 25 mm in diameter (AISC, 1989) to resist the computed $T$ force. In this example as many as four bolts can be put into the $0.450-\mathrm{m}$ width depending on bolt diameter. Alternatively, of course, we can redefine $A 1=d / 2$ and use four bolts (two on each side of the heel flange), producing the following:

$$
\begin{aligned}
& R=B\left(F_{p}\right) k d=0.45(7.35) k d=3.3075 k d \\
& \text { Arm } A 2=C / 2-k d / 2=0.55 / 2-k d / 2=0.275-k d / 2 \\
& \begin{array}{c}
\sum F_{v}=0 \quad \text { gives } \quad T=R-90 / 1000=0 \\
\\
\quad T=3.3075 k d-0.09(\mathrm{MN})
\end{array} \\
& \sum M_{\mathrm{cl}}=0=T \times A 1+R \times A 2-M=0(\text { units of } \mathrm{MN} \cdot \mathrm{~m})
\end{aligned}
$$

Substituting,

$$
(3.3075 k d-0.09)(0.245)+(3.3075 k d)(0.275-k d / 2)-0.175=0
$$

Collecting terms, we obtain

$$
-1.654 k d^{2}+1.720 k d-0.197=0
$$

Solving for $k d$, we find

$$
k d=0.131 \mathrm{~m}
$$

Checking, we see that

$$
\begin{gathered}
R=3.3075(0.131) \times 1000=433.3 \mathrm{kN} \\
\sum F_{v}=0: P_{\mathrm{col}}+T-R=0 \\
T=433.3-90=343.3 \mathrm{kN} \\
\sum M_{c 1}=0: T \times A 1+R \times A 2-175=? \\
343.3(0.245)+433.3[0.275-.1311 / 2]-175=? \\
84.1+90.8-175=174.9-175 \approx 0 \quad \text { O.K. }
\end{gathered}
$$

Find the required number of bolts and plate thickness for bending produced by bolt tension. In the AISC (1989) text, Table J3.5 indicates we can use bolts of either 22- or $25-\mathrm{mm}$ diameter. We would simply increase the plate length if larger bolts are needed. From Table 8-5 we see that a 20P2.5 Grade B bolt can carry 169 kN for a total of $2 \times 169=338<343.3 \mathrm{kN}$ (but O.K.). If Grade B bolts are not available, use alternatives of either A572 or A588 bolt material.

Figure E8-7b


No. of bolts required: $T / T_{b}=343.3 / 290=1.2 \rightarrow$ use 2 bolts Maximum bolt spacing of $12 d=12 \times 25=300 \mathrm{~mm}$ does not control Next find the base plate thickness.

$$
\begin{aligned}
m & =[550-0.95(356)] / 2=105.9 \mathrm{~mm} \leftarrow \text { controls } \\
n & =[450-0.80(369)] / 2=77.4 \mathrm{~mm}<105.9 \\
t_{p} & =2 m \sqrt{\frac{F_{p}}{F_{y}}}=2(105.9) \sqrt{\frac{7.35}{250}}=36.3 \mathrm{~mm}
\end{aligned}
$$

For bending (point $A$ of Fig. E8-7b) caused by bolt force, we calculate

$$
\begin{aligned}
T & =343.3 \mathrm{kN} \\
\text { Arm } & =97-30=67 \mathrm{~mm}=0.067 \mathrm{~m} \\
M & =0.067(343.3) / 1000=0.023 \mathrm{MN} \cdot \mathrm{~m}\left(\text { since } F_{b} \text { is in } \mathrm{MPa}\right) \\
F_{b} \mathrm{~S} & =M \rightarrow F_{b}=0.75 F_{y}=0.75(250)=187.5 \mathrm{MPa} \\
\mathrm{~S} & =B t_{p}^{2} / 6=0.45 t_{p}^{2} / 6 \\
t_{p}^{2} & =\frac{6 M}{0.45 \times 187.5} \\
t_{p} & \left.=\sqrt{\frac{6 \times 0.023}{0.45 \times 187.5}}=0.0404 \mathrm{~m} \rightarrow 40.4 \mathrm{~mm} \rightarrow \text { controls (greater than } 36.3\right)
\end{aligned}
$$

Provide the following base plate:

$$
B=\mathbf{4 5 0} \mathrm{mm} \quad C=550 \mathrm{~mm} \quad t_{p} \geq \mathbf{4 0 . 4} \mathrm{mm}
$$

Use two 20P2.5 A-307 Grade B bolts (see Table 8-4) if available
Plate mass $\approx(0.45)(0.55)(0.0404)(7850)=78.5 \mathrm{~kg}$

## Comments.

a. The plate mass is calculated for purposes of comparison since the thickness $t_{p}$ must be a value produced by the steel mills (will probably be 45 mm ).
$b$. This solution is adequate if bolts of required length (or end anchorage) can be obtained.
c. The $30-\mathrm{mm}$ edge distance is the minimum required depending on how the plate is cut.
$d$. It may be possible to reduce the pedestal area beneath the base plate to the plate dimensions unless other factors govern.

It should be evident that two design firms can come up with different size base plates (unless they are both using the same computer program) that would be considered acceptable. It should also be evident that these designs are a mixture of "ultimate" and "working stress" designs. It is still a common practice to use $P / A \pm M c / I$, giving a triangular pressure diagram for this design. One should be aware, however, that the plate toe will always bend and redistribute the compression stresses so that the rectangular compressive pressure block is more realistic.

In most cases the overturning moment is attributable to wind, so that even though the preceding base plate designs considered a moment clockwise about the axis of rotation, the base plate will be symmetrically attached. That is, the same number of heel bolts are placed in the toe region. Also, be aware that with moment the bolts must either have locking washers or be tightened to sufficient tension not to work loose during wind (and stress) reversals. The bolt tension produces additional compression stress in the base plate, so that the sum of the stress from overturning and from bolt tightening may be a rather high value on the order of 0.7 to $0.8 f_{c}^{\prime}$. In this case the user should check stresses. It may be necessary to redesign the base plate using $0.3 f_{c}^{\prime}$ instead of $0.35 f_{c}^{\prime}$. With a computer program it is only necessary to edit one line to change from 0.35 to $0.30 f_{c}^{\prime}$. Alternatively, one can simply increase the plate dimensions-say, 30 mm for the toe, 30 mm for the heel, and 50 mm for the sides ( 25 mm on each side).

Finally, note that the AISC design manual does not consider prying action for base plates, however, it does for tee hangers and connections. You probably should consider prying action as well for the base plate with large moment. The equation for this purpose is

$$
t_{\mathrm{bp}} \geq \sqrt{\frac{8 T_{b} b^{\prime}}{p F_{y}}}
$$

where $\quad T_{b}=$ bolt force, kN (or MN) or kips
$b^{\prime}=\frac{1}{2}$ the distance from column flange to bolt line, $m$ or in.
$p=$ tributary width of base plate per bolt, m or in.
$F_{y}=$ yield strength of base plate, $\mathrm{kPa}\left(\mathrm{MPa}\right.$ if $T_{b}$ in MN$)$ or ksi

If the base plate thickness $t$ from previous computations is less than the $t_{\mathrm{bp}}$ just computed, the thickness probably should be increased to $t_{\mathrm{bp}}$.

## 8-9 RECTANGULAR FOOTINGS

Rectangular footings are necessary where square footings cannot be used because of space limitations. They may be used where an overturning moment is present to produce a more


Figure 8-12 Placement of steel in short direction of a rectangular footing based on ACI Code Art. 15.4.4.
economical footing. The design is quite similar to that for a square footing. The depth will be controlled by shear, except that wide-beam action will probably control if the $L / B$ ratio is much greater than 1 or where an overturning moment is present.

One other special consideration for rectangular footings is in the placement of the reinforcement. The reinforcement in the long direction is computed in the same manner as for a square footing, using $d$ to the center of gravity (c.g.) of that steel. Steel in the short direction is computed similarly using the $d$ to the c.g. of the steel, which is usually placed on top of the longitudinal steel for some savings in mass and placing. Additionally, since the footing zone in the column area is more effective in resisting bending, a specified percentage of the total short-side steel is placed in this zone as shown on Fig. 8-12.

Example 8-8. Design a rectangular reinforced concrete footing for the following design data:
Loads: $\quad D=1110 \mathrm{kN} \quad L=1022 \mathrm{kN}\left(P_{u}=3291.4 \mathrm{kN}\right.$ computed $)$
Column: $\quad f_{c}^{\prime}=35 \mathrm{MPa} \quad$ Square $\mathrm{w} /$ side $=450 \mathrm{~mm}$
Column steel: eight No. 25 bars $\quad f_{y}=400 \mathrm{MPa}$
Footing: $\quad f_{c}^{\prime}=21 \mathrm{MPa} \quad q_{a}=240 \mathrm{kPa} \quad f_{y}=400 \mathrm{MPa}$

$$
B=2.20 \mathrm{~m} \text { (given) }
$$

## Solution.

Step 1. Find footing dimension $L$. Note that if $B$ is not given, then a number of combinations are possible:

$$
\begin{aligned}
B L q_{a} & =P=1110+1022 \\
L & =\frac{2132}{2.20 \times 240}=4.04 \mathrm{~m} \quad \text { Use } L=4.1 \mathrm{~m}
\end{aligned}
$$

The "ultimate" soil pressure is

$$
q_{\mathrm{ult}}=\frac{P_{u}}{B L}=\frac{3291.4}{2.20 \times 4.10}=365 \mathrm{kPa}
$$



Trial footing
Figure E8-8a


Figure E8-8b

As a check,

$$
q=\frac{2132}{(2.20)(4.10)}=236<240 \quad \text { O.K. }
$$

Step 2. Find the footing depth for shear. Check wide-beam value first. For a strip 1 m wide as shown in Fig. E8-8 $a$ and distance $d$ from the column we have
$\sum F_{v}=0$ on a 1 -m-wide section on right end of footing of length $L_{o}$ gives

$$
d(1.0) v_{c}-\left(\frac{4.10-0.45}{2}-d\right) q_{\mathrm{ult}}=0
$$

Inserting values of $v_{c}=0.65 \mathrm{MPa}$ from Table $8-2$ and $q_{\mathrm{ult}}$ from the foregoing (in MPa), we obtain

$$
\begin{aligned}
0.65 d+0.365 d & =0.666 \\
d & =\frac{0.666}{1.015}=0.66 \mathrm{~m}
\end{aligned}
$$

For this value of $d$ let us check the two-way action (approximately by neglecting upward soil pressure on the two-way action block) to obtain

$$
\begin{gathered}
\text { Perimeter of two-way action block }=(0.45+0.66) 4=4.4 \mathrm{~m} \\
P_{s}=\text { perimeter } \times d \times v_{c}=4.4(0.66)(1.30 \times 1000)=3775 \mathrm{kN}>3291.4
\end{gathered}
$$

A more refined analysis is not required nor do we need to check ACI Eq. (11-37) since $w<d$. Thus $d=0.66 \mathrm{~m}$ for longitudinal steel.
Step 3. Find steel required in long direction (longitudinal steel):

$$
L^{\prime}=\frac{4.10-0.45}{2}=1.825 \mathrm{~m} \quad(\text { see Fig. E8-8b) }
$$

and

$$
M_{u}=\frac{q_{\mathrm{ult}} L^{\prime 2}}{2}=\frac{365 \times 1.825^{2}}{2}=608 \mathrm{kN} \cdot \mathrm{~m} \quad a=\frac{400 A_{s}}{0.85 \times 21 \times 1}=22.41 A_{s}
$$

Using Eq. (8-2), we have

$$
A_{s}\left(0.66-\frac{22.41 A_{s}}{2}\right)=\frac{608}{0.9(400)(1000)}
$$

Cleaning up, we obtain

$$
\begin{aligned}
A_{s}^{2}-0.058 A_{s} & =-0.00015 \\
A_{s} & =0.0027 \mathrm{~m}^{2} / \mathrm{m}
\end{aligned}
$$

Checking the percentage of steel, we find

$$
\begin{aligned}
p=\frac{0.0027}{1(0.65)}=0.004 & >0.0018 \quad\left(\mathrm{~T} \text { and S of Art. } 7.12 .2 \text { for } f_{y}=400 \mathrm{MPa}\right) \\
& <0.016 \quad(\text { Table } 8-2)
\end{aligned}
$$

The total is

$$
\begin{aligned}
A_{s} & =27 \times 10^{-4} \times 2.20=5.94 \times 10^{-3} \mathrm{~m}^{2} \\
& =0.00594 \times 1000^{2}=5940 \mathrm{~mm}^{2}
\end{aligned}
$$

From the bar table (inside back cover), 12 No. 25 bars to furnish:

$$
A_{s, \text { furn }}=12 \times 500=6000 \mathrm{~mm}^{2}>5940 \quad \text { O.K. }
$$

Check required development (Art. 12.2.1) length against $L^{\prime}-0.07 \mathrm{~m}$ end cover

$$
L_{d}=\frac{0.02 A_{b} f_{y}}{\sqrt{f_{c}^{\prime}}}=\frac{0.02(500) 400}{\sqrt{21}}=873 \mathrm{~mm}<1.825-0.07 \text { end cover }
$$

Space the longitudinal bars at 11 spaces +0.07 m side clearance +1 bar:

$$
\begin{aligned}
11 s+2(0.07)+0.025 & =2.20 \mathrm{~m} \\
& =\mathbf{0 . 1 8 5} \mathrm{m}
\end{aligned}
$$

Step 4. Find steel in short direction (Fig. E8-8b). Place steel on top of longitudinal steel so $d^{\prime}=$ $0.66-0.025 / 2-0.025 / 2=0.635 \mathrm{~m}$ (assuming the short bars are also No. 25):

$$
\begin{aligned}
L^{\prime \prime} & =\frac{2.20-0.45}{2}=0.875 \mathrm{~m} \\
M_{u} & =\frac{365 \times 0.875^{2}}{2}=140 \mathrm{kN} \cdot \mathrm{~m}
\end{aligned}
$$

And $A_{s}$ is found ( $a=$ same as for longitudinal steel):

$$
\begin{aligned}
A_{s}\left[0.635-\frac{22.41 A_{s}}{2}\right] & =\frac{140}{0.9(400)(1000)} \\
A_{s}^{2}-0.057 A_{s} & =-0.000035 \\
A_{s} & =0.00062 \mathrm{~m}^{2} / \mathrm{m} \text { of width }
\end{aligned}
$$

Checking percent steel furnished, we find

$$
p=\frac{0.00062}{1(0.635)}=0.0009<0.0018
$$

Checking against ACI Art 10.5.2, we see

$$
A_{s}=1.33(0.00062)=0.000825 \rightarrow 825 \mathrm{~mm}^{2} / \mathrm{m}
$$

Since $p$ is less than required for temperature and shrinkage, use $A_{s}$ based on 0.0018:

$$
A_{s}=0.0018(1000)(635)=1143 \mathrm{~mm}^{2} / \mathrm{m}>825
$$

We will use 1143 but probably could use $825 \mathrm{~mm}^{2} / \mathrm{m}$ :

$$
A_{s, \text { total }}=1143(4.1)=4686 \mathrm{~mm}^{2}
$$

The minimum spacing of bars is $5 t$ or 0.457 m for $\mathrm{T} \& \mathrm{~S}$ steel. It is not necessary to check $L_{d}$ or use additional rebars in the zone $B$ centered on the column based on the equation shown on Fig. 8-12.

Let us use sixteen $20-\mathrm{mm}$ bars:

$$
A_{s}=16(300)=4800 \mathrm{~mm}^{2}>4686 \quad \text { and spacing } O . K .
$$

Step 5. Check bearing and design dowels:

$$
A_{1}=0.45^{2}=0.2025 \mathrm{~m}^{2} \quad A_{2}=(0.45+4 \times 0.65)^{2}=9.30 \mathrm{~m}^{2}
$$



Figure E8-8c

$$
\begin{aligned}
\dot{\psi} & =\frac{\sqrt{A_{2}}}{A_{1}}=\frac{\sqrt{9.30}}{0.2025}=6.8 \gg \quad \text { Use } 2 \\
f_{c} & =0.85(0.70)(21)(2)(1000)=24990 \mathrm{kPa} \\
f_{a} & =\frac{P_{u}}{A_{1}}=\frac{3291}{0.2025}=16254 \mathrm{kPa}<24990 \quad \text { O.K. for bearing }
\end{aligned}
$$

The minimum of $0.005 A_{\text {col }}$ (Art. 15.8.2.1) will be used

$$
A_{s}=0.005(0.2025) \times 1000^{2}=1012 \mathrm{~mm}^{2}
$$

Use four No. 25 bars (same as column):

$$
A_{s}=4(500 \mathrm{~mm})=2000>1013 \mathrm{~mm}^{2} \quad \text { O.K. }
$$

The depth of embedment per Art. 12.3 does not have to be checked since dowels are only for a code requirement to ensure column-to-base anchorage. We will run the dowels using ACI standard $90^{\circ}$ bends (for wiring) to the top of the reinforcing bars in the bottom of the footing and wire them in place for alignment.

Step 6. Develop the design sketch (Fig. E8-5c). Obtain overall $D$ as

$$
D_{c} \geq 0.66+\frac{0.025}{2}+0.070=0.7425 \mathrm{~m}
$$

Use $D_{c}=0.743 \mathrm{~m}=743.0 \mathrm{~mm}$.

## 8-10 ECCENTRICALLY LOADED SPREAD FOOTINGS

When footings have overturning moments as well as axial loads, the resultant soil pressure does not coincide with the centroid of the footing. If we assume the footing is somewhat less than rigid (and most are), the application of the statics equation of

$$
\begin{equation*}
\frac{P}{A} \pm \frac{M c}{I} \tag{8-14}
\end{equation*}
$$

gives a triangular soil pressure and displacement zone $a b 1$ as shown in Fig. 8-13. If $q_{\max }>$ $q_{\mathrm{ult}}$ as shown along the toe as line $1 b$, the soil pressure reduces to its ultimate value and stress is transferred to point 2 . When this $q_{\text {max, } 2}>q_{\mathrm{ult}}$ the pressure again reduces to $q_{\mathrm{ult}}$, and the process of load redistribution (similar to concrete beam analysis in Strength Design as given by Fig. 8-3) continues until equilibrium (or failure) is obtained.


Figure 8-13 Soil yielding under $P / A+M c / I$ toe stresses to produce an approximate rectangular pressure zone to resist $P$ and to satisfy statics (see also Fig. 4-4). For overturning stability always take a $\sum M$ check about point 1 at toe.

The displacements also initially have a somewhat linear shape as shown by line $a b$. This observation is consistent with concrete design where the compression zone continues to have an approximately linear variation of strain to some "ultimate" value but at the same time the assumed rectangular pressure block of depth $a$ shown in Fig. 8-3b is being produced [see also Fig. 4-1 of ACI Committee 336 (1988)]-at least for compressive stresses that are at or somewhat below the ultimate stress of the concrete. The equivalent of depth $a$ of the concrete beam is the length $L^{\prime}$ of the footing as shown in Fig. 8-13.

Meyerhof at least as early as 1953 [see Meyerhof (1953, 1963)], Hansen in the later 1950s [see Hansen (1961, 1970)], and Vesić (1975b) have all suggested computing the bearing capacity of an eccentrically loaded footing using Fig. 4-4 (see also Fig. 8-14). The soil analogy is almost identical to the Strength Design method of concrete.

After careful consideration it appears that the base should be designed consistent with the procedure for obtaining the bearing capacity. That is, use dimension $B^{\prime}, L^{\prime}$ for the design also.

This procedure ensures four items of considerable concern:

1. The resultant soil $R$ is never out of the middle one-third of the base so that overturning stability is always satisfied (taking moments about point 1 of Fig. 8-13). This $R$ always gives

$$
\mathrm{SF}=\frac{M_{\text {resist }}}{M_{\text {overturn }}}=\frac{P L}{2 M}
$$

2. The toe pressure will always be such that $q_{10 e} \leq q_{a}$.
3. The design is more easily done when a uniform soil pressure is used to compute design moments.
4. Approximately the same amount of steel is required as in the design using Eq. (8-14). One can never obtain a good comparison since a footing with overturning is heavily dependent

(a) General case of a spread footing with overturning - either about $y$ or $x$ axis (or both).

(b) Column is offset from centerline. Use a strap footing if column has no edge distance $a b$.

Prove that point $O$ is the center of area $B^{\prime} L^{\prime}$ where $B^{\prime} L^{\prime} q_{a} \geq P$ :
$L / 2=e_{x}+x_{2}$ (1)
$L^{\prime}=L-2 e_{x}$
Substitute (1) into (2): $L^{\prime}=2 e_{x}+2 x_{2}-2 e_{x}$

$$
L^{\prime}=2 x_{2} \leftarrow \leftarrow
$$



$$
\begin{aligned}
& P e-M-H \mathrm{D}_{c}=0 \\
& e=\frac{M+H \mathrm{D}_{c}}{P}
\end{aligned}
$$

(c) Offset column at an eccentricity so that a uniform soil pressure results.

Figure 8-14 General case of footings with overturning.
upon the assumptions used by the structural engineer. I was able to achieve some fairly reasonable agreements from the availability of two computer programs-one using Eq. ( $8-14$ ) and the other using the recommended procedure.

In order to satisfy the ACI 318 Building Code it is necessary to place restrictions on the values of $B^{\prime}, L^{\prime}$. These were stated in Chap. 4 and are repeated here for convenience:

$$
\begin{array}{ll}
B_{\min }=4 e_{y}+w_{y} & B^{\prime}=2 e_{y}+w_{y} \\
L_{\min }=4 e_{x}+w_{x} & L^{\prime}=2 e_{x}+w_{x}
\end{array}
$$

where the appropriate dimensions are defined on Fig. 8-14a. Note in Fig. 8-14 $a$ that the center of the resultant uniform soil pressure is at the centroid of the $B^{\prime}, L^{\prime}$ rectangle and is also at the eccentric distance(s) $e_{x}$ or $e_{y}$ computed as

$$
e_{x}=\frac{M_{y}}{P} \quad e_{y}=\frac{M_{x}}{P}
$$

from the column center.
By using dimensions of at least $B_{\text {min }}$ and $L_{\text {min }}$ the rectangular pressure zone will always include the column. This allows us to take the moment arms for tension steel on the pressed side, giving for the minimum values of $B^{\prime}$ and $L^{\prime}$ moment arms of length

$$
L_{y}=B^{\prime}-w_{y} \quad L_{x}=L^{\prime}-w_{x}
$$

## The amount of steel computed for a unit width is used across the full base dimensions of $B$ and $L$.

For two-way shear we have two options:

1. Compute an "average" $q=\frac{P}{B L}$ and use this $q$ value in Eq. (8-6).
2. Use the approximate Eq. (8-8), which does not use the upward soil pressure in the punchout zone around the column. The author recommends using approximate Eq. (8-8) both to achieve some small steel economy and to increase the base depth slightly for a somewhat more conservative design.

## 8-10.1 Can a Spread Footing Carry a Moment?

It should be evident that a column can transmit a moment to the footing only if it is rigidly attached. Nearly all concrete columns satisfy this criterion. Adequate anchorage of the base plate to the footing must be done to transfer a moment when steel columns are used.

The question of whether a spread footing (unless very large in plan) can sustain an applied column moment without undergoing at least some rotation according to Fig. 5-9 is a very important one. From elementary structural analysis, if the footing rotates an amount $\theta$, this results in moments in the opposite direction to that being applied by the column to develop:

Near end: $\quad M_{r}=\frac{4 E I_{c} \theta}{L_{c}} \quad$ Far end: $\quad M_{r}^{\prime}=\frac{2 E I_{c} \theta}{L_{c}}$
The resultant column end moments are
Near end: $\quad M_{f}=M_{o}-M_{r} \geq 0$
Far end: $\quad M_{f}^{\prime}=M_{o}^{\prime}-M_{r}^{\prime} \quad$ (with a sign on $M_{o}^{\prime}$ )

Thus, any footing rotation reduces the moment $M_{f}$ applied to the footing with a corresponding change to the far-end moment $M_{f}^{\prime}$ on the column. Obviously a sufficiently large rotation can reduce the footing moment to zero (but not less than zero). ${ }^{9}$ How much rotation is required to reduce the moment to zero depends on the $E I_{c} / L_{c}$ of the column. How much rotation actually occurs is somewhat speculative; however, Fig. 5-9 gives a quantitative estimate (see also Example 5-8).

If the structural designer opts to make a rigid base analysis, it is usually done using the following form of Eq. (8-14):

$$
\begin{equation*}
q=\frac{P}{B L}\left(1 \pm \frac{6 e}{L}\right) \tag{8-14a}
\end{equation*}
$$

where terms are identified on Figs. 8-13 and 8-14. Strictly, when using this type of equation one should include the moment from the footing weight (and any overlying soil) on the resisting side of the footing axis. Doing this will reduce the maximum (toe) pressure slightly and increase the minimum pressure. This is seldom if ever done in practice.

When the eccentricity $e$ of Eq. (8-14) is sufficiently large, the minimum $q$ becomes negative, indicating base-soil separation. Much effort has been expended in developing curves and other design aids to identify the line of zero pressure for those cases where Eq. (8-14a) produces a negative $q$. Clearly, one method is to use some kind of finite element/grid computer program (such as B-6 or B-19; see your diskette) and plot a line through those grid points that after several iterations have either negative or zero displacements (and the next adjacent nodes have positive displacements).

The principal use of Eq. (8-14a) in this text is, by rearranging the equation, to solve for the situation where the minimum $q=0$. When we set $q=0$ and solve for the eccentricity we find

$$
e=L / 6
$$

Since the author is recommending that the base design proceed according to the same procedure used to obtain the allowable bearing pressure, the following two examples (using computer output from the program identified on your diskette as FOOTDES) are included. Note that select data in these examples are hand-checked, and the procedure is exactly that of Example $8-2$. You can readily see this similarity from the sketches accompanying the computer output, giving selected dimensions and showing some of the computed quantities checked by hand. The only difference between Example 8-2 and Examples 8-9 and 8-10 following is that approximate Eq. (8-8) is used to obtain the effective depth $d$. As shown earlier, this approximation gives a conservative depth $d$ and results in a slight reduction in the mass of footing steel required, thus making the footing slightly more economical at no loss of design safety.

## Example 8-9.

Given. The following load, column, footing, and soil data:

$$
\begin{aligned}
& P_{\text {des }}=800 D+800 L=1600 \mathrm{kN} \\
& P_{\text {ult }}=1.4(800)+1.7(800)=2480 \mathrm{kN}
\end{aligned}
$$

[^1]\[

$$
\begin{aligned}
& M_{y}=300 D+500 L=800 \mathrm{kN} \cdot \mathrm{~m} \\
& M_{y, \text { ult }}=1.4(300)+1.7(500)=1270 \mathrm{kN} \cdot \mathrm{~m} \\
& \text { Column dimensions: } \quad A X=0.50 \mathrm{~m} \\
& \\
& \quad A Y=0.40 \mathrm{~m} \\
& f_{c}^{\prime}=21 \mathrm{MPa} \quad F_{y}=400 \text { (grade } 400 \text { rebars) } \\
& s_{u}=c=200 \mathrm{kPa} \quad(\phi=0) \quad \text { Use } \mathrm{SF}=3 \text { for } q_{a} \\
& \text { Depth of footing } D=1 \mathrm{~m} \quad \gamma_{\text {soil }}=17.50 \mathrm{kN} / \mathrm{m}^{3}
\end{aligned}
$$
\]

The groundwater table is not a consideration for the design.
Required. Design a spread footing using the Hansen bearing-capacity equations and the effective base area $B^{\prime} \times L^{\prime}$.

Solution. A computer program FOOTDES is used with the foregoing data as input. Note (Fig. E89) that the data file used for the execution was printed (EXAM89.DTA), so if a parametric study is made a copy of the data set can be made, renamed, and edited.

After the program found a base of dimensions $3.23 \mathrm{~m} \times 2.23 \mathrm{~m}$, a screen request was made if I was satisfied with these dimensions. I decided to round the dimensions to $B X=3.25 ; B Y=2.25$ m as shown on Fig. E8-9a. Using these fixed dimensions the program checked for adequacy and output data indicating the base dimensions are acceptable.

Note that the computer program allowed me to modify the computed dimensions $B, L$ so that reasonable multiples of meters or feet (or inches) result. The program also gave me the opportunity to limit the allowable bearing pressure.

Next the program asked if I wanted a steel design and I responded that I did. The program then computed the allowable concrete two-way shear stress and used Eq. (8-8) to find a depth $D E(d)$. In this type of design wide-beam shear often controls and this was checked. In fact depth $d$ for wide-beam does control at 595 mm (versus 501.73 mm for two-way).

A check for overturning stability (or safety factor SF) is routinely made by taking moments about the appropriate pressed edge. The resisting moment $M_{r}$ is always

$$
M_{r}=\frac{P L}{2}
$$

where $\quad L=$ footing dimension perpendicular to the pressed edge
$P=$ either the working or ultimate column axial load
The overturning moment $M_{\text {o.t. }}$ equals either the working design or ultimate moment producing the edge compression. This check is shown with the output sheets.

The computer program does an internal check and always produces dimensions such that application of Eq. (8-14a) places the eccentricity within the middle third of the base. This ensures that the overturning stability will always be adequate, however, always make a routine check to be sure the input is correct.

The program does not design dowels when there is overturning moment. What would normally be done here is to use the column steel as dowels, as this would ensure an adequate column-tofooting interface. Short column bars would either be used and later spliced above the footing at a convenient location or, preferably, be extended the full column height. They would be bent using a standard ACI Code $90^{\circ}$ bend at the lower end and set onto the footing steel and wired securely. A bend substantially increases the bar pull-out capacity if the depth $d$ does not provide sufficient length for a straight bar-and it often does not. The portion above the footing would be encased in the column form or, if not convenient to do so, would be held in place using a temporary support until the footing is poured and the concrete hardened.

Figure E8-9a


Figure E8-9b
ECCENTRICALLY LOADED FOOTING FOR EXAMPLE 8-9 FOR FAD 5/E
++++++++++++++++++ THIS OUTPUT FOR DATA FILE: EXAM89A.DTA
++++ HANSEN BEARING CAPACITY METHOD USED--ITYPE $=1$
FOOTING DIMENSIONS AND BEARING PRESSURES FOR
DEPTH OF FTG $=1.00 \mathrm{M} \quad$ UNIT WT OF SOIL $=17.500 \mathrm{KN} / \mathrm{M}^{*} 3$ INTTIAL INPUT $S E=3.0$

PHI-ANGLE $=.000$ DEG SOIL COHES $=200.00 \mathrm{KPA}$
VERT LOAD $=1600.0 \mathrm{KN}$ (DRSIGN VALUB)
MOM ABOUT X-AXIS $=\quad .00 \mathrm{KN}-\mathrm{M}$ ECCENTRICITY, $\mathrm{BCCY}=.000 \mathrm{M}$
MOM ABOUT Y-AXIS $=800.00 \mathrm{KN}-\mathrm{M}$ ECCENTRICITY, ECCX $=.500 \mathrm{M}$ (1)
THE HANSEN N-FACTORS: $N C=5.14 \quad N Q=1.0 \quad N G=10$
ALL SHAPE, DEPTH AND INCLINATION FACTORS FOR HANSEN

|  | B-DIR | L-DIR |
| :--- | :--- | :--- |
| SCB, SCL $=$ | 1.195 | 1.195 |
| SQB, SQL $=$ | 1.000 | 1.000 |
| SGB, SGL $=$ | .600 | .600 |
| DCB, DCL $=$ | 1.123 | 1.178 |
| DQB, DQL $=$ | 1.000 | 1.000 |
| ICB, ICL $=1.000$ | 1.000 |  |
| IQB, IQL $=1.000$ | 1.000 |  |
| IGB, IGL $=$ | 1.000 | 1.000 |

Figure E8-9b (continued)

*FOOTING DIMENSIONS: $\quad \mathrm{BX}=3.250 \quad \mathrm{BY}=2.250 \mathrm{M} \quad$ FOR $\mathrm{SF}=3.000$ ALLOW SOIL PRESS $=457.34$ KPA COMPUTED ALLOW FTG LOAD AS $P=B^{\prime} L^{\prime} Q A L L=2315.29 \mathrm{KN} \quad(1600.00)$ (4) * = FOOTING DIMENSIONS INPUT OR REVISED--NOT COMPUTED ++++++

based on the vltimate load and ultimate moment
ECCENTRICITIES, ECCX $=.512 \checkmark$ ECCY $=.000 \mathrm{M}$ (5)
REDUCED FOOTING DIMENSIONS BX' $=2.226$ BY' $=2.250 \mathrm{M}$
ULTIMATE COLUMN LOAD,PCOL $=2480.0 \mathrm{KN}$
ULTIMATE BEARING PRESSURE, PCOL $/\left(\mathrm{BX}^{\prime *} \mathrm{BY}^{\prime}\right)=495.20 \mathrm{KPA}$


+++ RECTANGULAR FOOTING DESIGN BY ACI 318-89++++
+++++++++++ HAVE SQUARE OR RECTANGULAR COLUMN +++++++++++++++
GIVEN DESIGN DATA:
COL COLX $=500.000 \mathrm{MM}$
COL COLY $=400.000 \mathrm{MM}$
YIELD STR OF STEEL FY $=400.0 \mathrm{MPA}$
28-DAY CONCRETE STR $=21.0 \mathrm{MPA}$
ALLOWABLE SOIL PRESS $=457.34 \mathrm{KPA}$
FOOTING DIMENSIONS USED FOR DESIGN: $B X=3.25 \quad B Y=2.25 M$
FOR FACTORED COLUNS LOAD, PCOL $=2480.00 \mathrm{KN}$
ALLOW TWO-WAY CONC STRESS VC $=1298.40 \mathrm{KPA}$
EFF DEPTH FOR TWO-WAY SHEAR, DE $=501.73 \mathrm{MM}$
FOR SOIL PRESSURE, QULT $=495.20 \mathrm{KPA} \quad$ PCOL $=2480.00 \mathrm{KN}$
THIS SOIL PRBSS FOR WIDE-BEAM AND IS BASBD ON PCOL/(BX'*BY')
ALLOW WIDE-BEAM CONC STRESS, VCW = 649.20 KPA
EFF DEPTH FOR WIDE-BEAM SHEAR, DWB $=594.99 \mathrm{MM}$ (8)
EFFECTIVE FOOTING DEPTH USED FOR DESIGN $=594.99 \mathrm{MM}$
DESIGN BFFECTIVE FOOT DEPTH $=594.99 \mathrm{MM}$
AREA STEEL REQD: LENGTH DIR $=2283.6970 \checkmark \quad$ B DIR $=1189.9720$ MM*2/M
ACT \% STEEL L DIR $=.0038 \%$
ACT \% STEEL B DIR $=.0020$ \%
MAX ALLOWABLE \% STEEL EITHER DIR $=.0171$ \%
REINFORCING BARS FOR BENDING--DIR PARALLEL TO L:
BAR EMBEDMENT LENGTH PROVIDED, LD $=1305.00 \mathrm{MM}$


Figure E8-9b (continued)


OBTAIN TOTAL FOOTING DEPTH BASED ON BARS YOU USE TO ALLOW FOR 3-IN OR 70-MM COVER
NOTE THAT REQ'D LD FOR TENSION REBARS IS REDUCED FOR BAR SPACING AND RATIO ASFURN/ASREQD ACCORDING TO ACI 318, ART 12.2

DOWELS NOT COMPUTED SINCE A MOMENT IS ON COLUMN HAND COMPUTE DOWELS FOR MOMENT TRANSEER

The following are selected computations to verify the computer generated output for Example 8-9. Refer to key code and $\boldsymbol{\sim}$ marks on output sheets.
(1): $e_{x}=\frac{M}{P}=\frac{800}{1600}=0.50 \mathrm{~m}$
$s_{c}^{\prime}=1.195-1.000=0.195$
(2): $d_{c}^{\prime}=1.123-1.000=0.123$
$i_{c}^{\prime}=1.000-1.000=0.000$
(3): Check bearing capacity

$$
\begin{aligned}
q_{u l t} & =c N_{c}\left(1+s_{c}^{\prime}+d_{c}^{\prime}-i_{c}^{\prime}\right)+\gamma D \\
& =200(5.14)(1+0.195+0.123-0.000)+17.5 \times 1.0 \\
& =1372.4 \mathrm{kPa} \\
q_{a} & =q_{u t t} / S F=1372.4 / 3=\mathbf{4 5 7 . 4 6} \mathrm{kPa}
\end{aligned}
$$

(4): $B^{\prime}, L^{\prime}$ and $P_{\text {max }}$

$$
\begin{aligned}
B^{\prime}=B & =2.25 \mathrm{~m} ; \quad L^{\prime}=3.25-2 \times 0.50=2.25 \mathrm{~m} \\
P_{\max } & =B^{\prime} \times L^{\prime} q_{\mathrm{ult}} \\
& =2.25 \times 2.25 \times 457.34=\mathbf{2 3 1 5 . 2 8} \mathrm{kN}
\end{aligned}
$$

The actual base contact pressure

$$
q=P / B^{\prime} L^{\prime}=1600 /(2.25 \times 2.25)=316.0, \mathrm{KPa} \ll 457.5
$$

(5): Now new eccentricity $e_{x}=\frac{M_{u}}{P_{u}}=\frac{1270}{2480}=0.512 \mathrm{~m}$

New $B^{\prime}=3.25-2(0.512)=2.226 \mathrm{~m}$

## Computation Check Continued

(6): New $q_{\mathrm{ult}}=\frac{P_{u}}{B^{\prime} \times L^{\prime}}=\frac{2480}{.226 \times 2.225}=\mathbf{5 0 0 . 7}$ (vs. 495.2 computer)
(7): Depth for two-way beam shear and using Eq. (8-8):

$$
\begin{aligned}
4 d^{2}+2\left(w_{x}+w_{y}\right) d & =\frac{P_{u}}{v_{c}} \\
4 d^{2}+2(0.50+0.40) d & =\frac{2480}{1298.4} \\
d^{2}+0.45 d & =0.4775 \\
d & =\mathbf{0 . 5 0 1 7} \mathrm{m}(501.7 \mathrm{~mm})
\end{aligned}
$$

(8): Depth for wide beam shear in long direction (controls)

$$
\begin{aligned}
(L-d) q_{\mathrm{ult}} & =b d v_{c, w b} \rightarrow L=\frac{3.25-0.50}{2}=1.375 \mathrm{~m} \\
1.375-d & =\frac{(1.0)(d)(649.2)}{495.2} \\
d & =1.375 / 2.3014=\mathbf{0 . 5 9 4 9} \mathrm{m}(=594.9 \mathrm{~mm})
\end{aligned}
$$

(9): ACI 318 Code requirements:

Across footing width of $2.25 \mathrm{~m}:=\mathrm{L}$ dir $=0.0038$ percent
Across footing length of $3.25 \mathrm{~m}:=\mathrm{B}$ dir $=0.0020$ percent
(this is the Art. 7-12 minimum for grade 300-we are using grade 400 so could actually have used 0.0018-this percent also complies with ACI Art. 10-5.3)

$$
\begin{aligned}
& a=\frac{f_{y} A_{s}}{0.85 f_{c}^{\prime} b}=\frac{400 A_{s}}{0.85(21)(1)}=22.41 A_{s} \\
& M_{u}=q_{u t t} L^{2} / 2=495.2\left(1.375^{2}\right) / 2=468.1 \mathrm{kN} \cdot \mathrm{~m} \\
& \phi f_{y} A_{s}(d \quad-\quad a / 2) \quad=\quad M_{u} \\
& A_{2}\left(0.595-22.41 A_{s} / 2\right)=468.1 /(0.9 \times 400 \times 1000) \\
& 0.595 A_{s}-11.2 A_{s}^{2}=0.0013 \\
& A_{s}=\frac{0.0531 \pm \sqrt{0.0531^{2}-4(0.00016)}}{2} \\
&=0.002283 \mathrm{~m}^{2} / \mathrm{m} \\
&=\mathbf{2 2 8 3} \mathrm{mm}^{2} / \mathrm{m}
\end{aligned}
$$

$L_{d}=(1.375-0.70) \times 1000=1305 \mathrm{~mm}$ (compare to required $\left.L_{d}\right)$
$A_{s}($ req'd for sale $)=3.25 \times 2283.69=5138.32 \mathrm{~mm}^{2}$

## For short side:

$$
L=(2.25-0.400) / 2=0.925 \mathrm{~m}
$$

Estimate $d \approx 595-1$ bar $=595-25=570 \mathrm{~mm}$.

$$
M_{u}=495.2\left(0.925^{2} / 2=211.9 \mathrm{kN} \cdot \mathrm{~m}\right.
$$

$$
\begin{aligned}
0.570 A_{s}-11.2 A_{s}^{2} & =211.9 /(0.9 \times 400 \times 1000)=0.0005886 \\
A_{s}^{2}-0.05089 A_{s} & =0.0000526 \\
A_{s} & =0.001055 \mathrm{~m}^{2} / \mathrm{m} \\
& =1055 \mathrm{~mm}^{2} / \mathrm{m}
\end{aligned}
$$

Check against $T \& S$ requirements of ACI Art 7.12 (and use $d=0.595 \mathrm{~m}$ not 0.575 m and 0.002 not 0.0018 as conservative)

$$
\begin{aligned}
A_{s} & =0.002 b d=0.002(1000 \times 595) \\
& =1190 \mathrm{~mm}^{2} / \mathrm{m}>1055 \rightarrow \text { and controls } \\
L_{d} & =\left(0.925-0.070 \times 1000=\mathbf{8 5 5} \mathrm{mm}\left(\text { compare to required } L_{d}\right)\right. \\
A_{s}(\text { req'd for long side }) & =3.25 \times 1089.972=\mathbf{3 8 6 7 . 4 1} \mathrm{mm}^{2}
\end{aligned}
$$

Finally you must check overturning stability (check about toe of compressed edge)

$$
\begin{aligned}
& M_{\text {o.t. }}=800 \text { or } 1270 \mathrm{kN} \cdot \mathrm{~m} \\
& \begin{aligned}
M_{r} & =(1600 \text { or } 2480) L / 2=) 1600 \text { or } 2480) 3.25 / 2=2600 \\
& =2600
\end{aligned} \\
& \begin{array}{rlc}
\text { S.F. }=M_{r} / M_{\text {o.t. }} & =2600 / 800=\mathbf{3 . 2 5} & \text { (O.K.) } \\
& =4030 / 1270=\mathbf{3 . 1 7} & \text { (O.K.) }
\end{array}
\end{aligned}
$$

## Example 8-10.

Given. Same data as Example 8-9 except we have a moment about both axes (see Figure E8-10b). Also revise the column dimensions to

$$
A X=A Y=0.50 \mathrm{~m}
$$

Required. Design the footing.
Solution. Again computer program FOOTDES is used. You should be aware that with equal moments about both axes the optimum footing shape will be a square.

The remainder of the design is almost identical to that of Example 8-9. Note that depth for twoway shear again uses Eq. (8-8). The principal difference is that a square column is now used whereas a rectangular one was used in Example 8-9, so the effective depth computes slightly less (484.86 vs. 501.73 mm ).

Refer to the computer output sheets (Figure E8-10a) for select computations or user checks.
For dowels refer to the comments made in Example 8-9.

```
    TWO-WAY ECCENTRICALLY LOADED FOOTING FOR EXAMPLE 8-10 FOR FAD 5/E
    t++++++t++++++++++++ THIS OUTPUT FOR DATA FILE: EXAMB9B.DTA
++++ HANSEN BEARING CAPACITY METHOD USED--ITYPE = 1
FOOTING DIMRNSIONS AND BEARING PRESSURES FOR
    DEPTH OF FTG = 1.00 M UNIT WT OF SOIL = 17.500 KN/M*3
    INITIAL INPUT SF = 3.0
    PHI-ANGLE = .000 DEG SOIL COHES = 200.00 KPA
```

Figure E8-10a


Figure E8-10 $a$ (continued)

```
    VERT LOAD = 1600.0 KN (DESIGN VALUE)
MOM ABOUT X-AXIS = 800.00 KN-M ECCENTRICITY, ECCY = .500 M
MOM ABOUT Y-AXIS = 800.00 KN-M ECCENTRICITY, ECCX = .500 M
THE HANSEN N-FACTORS: NC = 5.14 NQ = 1.0 NG = .0
ALLOWABLE BEARING PRESSURE FOR JCOUN = 1
IGB OR IGL (USED VALUE) = 1.000 FOOTING WIDTH USED = 2.00
QULT COMPONENTS: FCOH = .00
    FQBAR = .00
    FGAMM = .00 KPA OR KSF
    QA(JCOUN) = 460.86
*FOOTING DIMENSIONS: BX = 3.000 BY = 3.000 M FOR SF = 3.000
    ALLOW SOIL PRESS = 460.86 KPA
    COMPUTED ALLOW FTG LOAD AS P = B'L'QALL = 1843.42 KN\checkmark ( 1600.00)
* = FOOTING DIMENSIONS INPUT OR REVISED--NOT COMPUTED +++++++
+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
```

BASED ON THE ULTIMATE LOAD AND ULTIMATE MOMENT
ECCENTRICITIES, ECCX $=.512$ ECCY $=.512 \mathrm{M}$
REDUCED FOOTING DIMBNSIONS BX' $=1.976 \mathrm{BY}^{\prime}=1.976 \mathrm{M}$
ULTIMATE COLUMN LOAD, PCOL $=2480.0 \mathrm{KN}$
ULTIMATE BEARING PRESSURE, PCOL/(BX'*BY') $=635.28$ KPA $\downarrow$

```
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
    +++++SQUARE FOOTING DESIGN BY ACI 318-89-++++
++++++++++++ HIAVE SQUARE OR RBCTANGULAR COLUMN ++++++++++++++++++
GIVEN DESIGN DATA:
                            COL COL.X = 500.000 MM
                    COL COLY =500.000 MM
    YIELD STR OF STEEL FY = 400.0 MPA
    28-DAY CONCRETE STR = 21.0 MPA
    ALLOWABLE SOIL PRESS = 460.86 KPA
            COL LOADS: AXIAL = 2480.0 KN (EACTORED)
                    MOMENT = 1270.00 KN-M ABOUT THE X-AXIS
                    MOMENT = 1270.00 KN-M ABOUT THE Y-AXIS
FOOTING DIMENSIONS USED FOR DESIGN: BX = 3.00 BY = 3.00 M
                                    FOR FACTORED COLUMN LOAD, PCOL = 2480.00 KN
                                    ALLOW TWO-WAY CONC STRESS VC = 1298.40 KPA
                                    EFF DEPTH FOR TWO-WAY SHEAR, DE = 484.86 MM
            FOR SOIL PRESSURE, QULT = 635.28' KPA PCOL = 2480.00 KN
    THIS 8OIL PRESS FOR WIDE-BEAM AND IS BABED ON PCOL/{BX'*BY')
            ALLOW WIDE-BEAM CONC STRESS, VCW = 649.20 KPA \checkmark /
                        BFF DBPTH FOR WIDE-BEAM SHEAR, DWB = 618.23 MM
                BFPECTIVE FOOTING DEPTH USED FOR DESIGN = 618.23 MM
                DESIGN EFFECTIVB FOOT DEPTH = 618.23 MM
            AREA STEEL REQD: LENGTH DIR = 2328.2330 \checkmark
                                    B DIR = 2328.2330 MM*2/M
            ACT % STPEEL L DIR = .0038 % 
            ACT & STEEL B DIR = .0038%
MAX ALLOWABLE % STEEL EITHER DIR = .0171%
REINE BARS FOR BITHER DIR BENDING (8Q FTG)
            BAR EMBEDMENT LENGTH PROVIDED, LD = 1180.00 MM
\begin{tabular}{ccccccc} 
BAR & BARS REQD \& SPAC CEN-TO-CEN & AB FURN & AB RBQD & LD REQD \\
MM & NO & MM & MM*2 & MM*2 & MM \\
19.5 & 24 & 123.5 & 7167.56 & 6984.70 & 406.4 \\
25.2 & 25 & 202.5 & 7481.41 & 6984.70 & 650.3
\end{tabular}
```

OBTAIN TOTAL FOOTING DEPTH BASED ON BARS YOU USE TO ALLOW FOR 3-IN OR 70-MM COVER
NOTE THAT REQ'D LD FOR TENSION RBBARS IS REDUCED FOR BAR SPACING AND RATIO ASFURN/ASREQD ACCORDING TO ACI 318, ART 12.2

DOWELS NOT COMPUTED SINCE A MOMENT IS ON COLUMN HAND COMPUTE DOWELS FOR MOMENT TRANSFER

Check overturning stability:

$$
M_{\text {o.t }}=800 \text { or } 1270 \mathrm{kN} \cdot \mathrm{~m} \text { (about either axis) }
$$

$$
\text { The resisting moment } \begin{aligned}
M_{r} & =(1600 \text { or } 2480) L / 2 \\
& =1600(3.0 / 2)=2400 \\
& =2480(3.0 / 2)=3700 \mathrm{kN} \cdot \mathrm{~m}
\end{aligned}
$$

$$
\begin{aligned}
S F=\text { Stability Number } & =M_{r} / M_{0 . t} \\
& =2400 / 800=3.0 \\
& =3720 / 1270=2.93 \quad \text { (both O.K.) }
\end{aligned}
$$

Example 8-11. Design a footing $B \times L$ such that the soil pressure will be approximately uniform for the following conditions (see Fig. 8.14c):

$$
\begin{array}{rlrl}
D & =419.6 \mathrm{kN} & L & =535.4 \mathrm{kN} \\
M_{d} & =228 \mathrm{kN} \cdot \mathrm{~m} & M_{L} & =249.5 \mathrm{kN} \cdot \mathrm{~m} \\
H_{D} & =42.1 \mathrm{kN} & H_{L} & =53.2 \mathrm{kN}
\end{array}
$$

Column: $\quad$ Square $500 \times 500 \mathrm{~mm}$ with eight No. 25 bars

$$
\begin{array}{lll} 
& f_{c}^{\prime}=28 \mathrm{MPa} & f_{y}=400 \mathrm{MPa} \\
\text { Footing: } & f_{c}^{\prime}=21 \mathrm{MPa} & f_{y}=400 \mathrm{MPa} \\
& q_{a}=150 \mathrm{kPa} &
\end{array}
$$

## Solution.

Step 1. Find footing dimensions $B \times L$ :

$$
\begin{aligned}
P & =955 \mathrm{kN} \\
M & =477.5 \mathrm{kN} \cdot \mathrm{~m} \\
e & =\frac{M}{P}=\frac{477.5}{955}=\mathbf{0 . 5 0} \mathrm{m}
\end{aligned}
$$

Note this $e$ should be increased slightly owing to the additional overturning moment from $H_{d}+H_{L}$. Use $e=0.55 \mathrm{~m}$ (see Fig. E8-11a).

For $e=0.55 \mathrm{~m}$ the edge of the column is 0.30 m from the footing centerline. The required footing area for $q=q_{a}=150 \mathrm{kPa}$ is $955 / 150=6.4 \mathrm{~m}^{2} . B$ for a square footing is 2.52 m . Try $B \times L=2.60 \times 2.60 \mathrm{~m}$ :

$$
q_{\mathrm{ult}}=\frac{1.4(419.6)+1.7(535.4)}{2.60^{2}}=221.5 \mathrm{kPa}
$$

Step 2. Find the footing depth (refer to Fig. E8-11b).

## Figure E8-11b

Figure E8-11a

(0.55)


For two-way action (note edge distance limits $d / 2 \leq 0.50$ ) use approximate Eq. (8-8):

$$
\begin{gathered}
4 d^{2}+2(b+c) d=\frac{B L q}{v_{c}} \\
v_{c}=1.30 \mathrm{MPa} \\
4 d^{2}+2(0.5+0.5) d=\frac{2.60 \times 2.60 \times 221.5}{1300} \\
d^{2}+0.5 d=0.2880 \\
d=0.34 \mathrm{~m}<0.50 \quad \text { O.K. }
\end{gathered}
$$

Do not check ACI Eq. (11-37) yet.
Find the depth for wide-beam shear at $d$ from the column for a strip 1 m wide. We could, of course, check $d=0.34 \mathrm{~m}$, but it is about as easy to compute the required $d$ :

$$
\begin{gathered}
d(1)\left(v_{c}\right)=1(1.3+0.30-d) q_{\mathrm{utt}} \\
v_{c}=0.649 \mathrm{MPa}=649 \mathrm{kPa} \quad \text { (Table 8-2) }
\end{gathered}
$$

Inserting values, we obtain

$$
\begin{aligned}
649 d & =1.60(221.5)-221.5 d \\
d & =0.407 \mathrm{~m}>0.34 \quad \text { (therefore, wide-beam shear controls) }
\end{aligned}
$$

Use

$$
d=0.410 \mathrm{~m} \quad\left(D_{c} \simeq 0.50 \mathrm{~m}\right)
$$

Since wide-beam shear depth controls, it is not necessary to check ACI Eq. (11-37).
Step 3. Check $\sum M$ about centerline using $D_{c}=0.50 \mathrm{~m}$ (Fig. 8-14c)

$$
\begin{aligned}
& 0.50(62.1+53.2)+228+249.5-955(0.55)=? \\
& 47.7+477.5-525.3=-0.1 \quad \text { (should be } 0.0 \text { ) }
\end{aligned}
$$

This small unbalance may be neglected, or $e$ may be reduced and the problem recycled until $\sum M=$ 0 . Alternatively, since a value of $e \simeq 0.55$ is feasible, directly solve the moment equation for $e$ to obtain $e=0.5499 \mathrm{~m}$.

With footing dimensions tentatively established, the sliding stability should be investigated as

$$
\frac{H_{\text {resisting }}}{\mathrm{SF}} \geq H_{d}+H_{L}
$$

Generally,

$$
H_{\text {resisting }}=P \tan \delta+c^{\prime} A_{\text {footing }}+\text { passive pressure }
$$

For $c^{\prime}, \delta$ see Table 4-5. For passive pressure see Chap. 11.
Step 4. Find the required reinforcing bars for bending. This step is the same as for spread footings. That is, find bars required for bending for the long dimension $(b c+0.3 \mathrm{~m}=1.3+0.3=1.6 \mathrm{~m})$ and for the short dimension $[(2.60-0.50) / 2=1.05 \mathrm{~m}]$. For the long direction (length $a b c$ ) run the required bars the full length of $2.60-2(0.07)=2.46 \mathrm{~m}$ long bars (the 0.07 m is the clear-cover requirement).

For the short direction also run the bars the full distance of 2.46 m (the footing is square). You might here, however, consider placing about 60 percent of the total bars required in the distance $a b$ and the other 40 percent in the distance $b c$.

Use final footing dimensions

$$
2.60 \times 2.60 \times D_{c} \text { with } d \geq 0.41 \mathrm{~m}
$$

## 8-10.2 Eccentricity Out of the Middle $\mathbf{1 / 3}$ of a Footing

In Sec. 8-10.1 we noted that the eccentrically loaded footings were forced to have the eccentricity

$$
e=\frac{M}{P} \leq \frac{L}{6} \quad\left(\text { we have } \pm \frac{L}{6}, \text { so } 2 \times \frac{L}{6}=\frac{L}{3}\right)
$$

This ensures that the overturning stability is adequate.
There are occasions where it is impossible to have the eccentricity $e \leq L / 6$. In these cases one has two options:

1. Increase the base dimension(s) until the eccentricity is

$$
e=L / 6
$$

2. Make an analysis similar to the base plate design that had a large column moment. Refer to Fig. 8-11 and replace the rectangular pressure diagram for the allowable concrete stress with the allowable soil pressure $q_{a}$. Replace the tension bolt(s) with tension piles. Solve the resulting quadratic equation for $k d$ and use that as $B^{\prime}$ in the bearing-capacity equations and iterate back and forth until the assumed and required soil pressures are $q \leq q_{a}$.

A method used by many structural designers assumes a triangular pressure distribution as shown in Fig. 8-15. The necessary equation for this is derived as follows:

$$
L / 2=e+L^{\prime} / 3 \quad \text { and } \quad P=\frac{q}{2}\left(B L^{\prime}\right)
$$

Substituting $L^{\prime}$ into the expression for $P$ and solving for the soil pressure $q$ at the toe, we obtain

$$
\begin{equation*}
q=\frac{2 P}{3 B(L / 2-e)} \leq q_{a}^{\prime} \tag{8-15}
\end{equation*}
$$

In this equation $P=$ fixed footing load, and the allowable soil pressure for this type of pressure distribution $q_{a}^{\prime}$ is somehow estimated by the geotechnical engineer. With these values set the structural designer solves Eq. (8-15) by trial until a set of dimensions $B, L$ is found. Note carefully that $q_{a}^{\prime}$ is estimated by the geotechnical engineer. There is no current method to compute the allowable bearing pressure for a linear variable (triangular) pressure diagram.

The solution of Eq. (8-15) is particularly difficult when there is two-way eccentricity, i.e., both $e_{x}$ and $e_{y}>L / 6$. This is also true when Fig. 8-11 is used for base plates since there are two values of $k d_{i}$, where now $k d_{1}=B^{\prime}, k d_{2}=L^{\prime}$. Because of these difficultiles in using Eq. (8-15), the method used in Examples 8-9 and 8-10 based on Fig. 8-14 is particularly attractive.

A finite-difference program or program B-6 (Finite Grid on your diskette) might be used in this type of example. Steps include the following:

1. Grid the proposed base into rectangles or squares.
2. Assume or compute or obtain from the geotechnical engineer a value of modulus of subgrade reaction and the allowable bearing pressure $q_{a}$.
3. Activate the nonlinear program option and set the maximum soil displacement XMAX such that

$$
q_{a} \leq k_{s} \cdot \mathrm{XMAX}
$$



Figure 8-15 An alternate soil pressure profile for footings with large eccentricities.
This step ensures that, regardless of displacement (which may well be linear), the soil will become plastic at displacement XMAX so the ultimate soil pressure is restricted. Of course $q_{a}$ is less than ultimate for a safety margin.
4. If the foregoing are carefully done, you can then inspect the final computer output and locate the line of zero soil pressure, and at least a part of the pressure diagram will be rectangular. Alternatively, $q_{a} B^{\prime} L^{\prime}=P$, so solve by trial for $B^{\prime}, L^{\prime}$.
5. Use the computer output to check statics. Use the base dimensions and given loads to check overturning stability.

## 8-11 UNSYMMETRICAL FOOTINGS

There are occasions where it is necessary to use a T, L, or other unsymmetrical shape for the foundation. It may also be necessary to cut a notch or hole in an existing footing for some purpose. In these cases it is necessary to estimate the base pressure for $q \leq q_{a}$ for the design. For the cut base it is also necessary to check that the resulting base pressure $q \leq q_{a}$.

The current recommendation for solution of this class of problems is to use your mat program B6 using the finite grid method that is presented in Chap. 10. A notched base example there considers both a pinned and fixed column. As with the footing with overturning, the soil, base thickness, and column fixity are significant parameters that are not considered in
the conventional "rigid" unsymmetrical base design still used by some foudation designers. This footing design is very computationally intensive and not recommended by the author.

## 8-12 WALL FOOTINGS AND FOOTINGS FOR RESIDENTIAL CONSTRUCTION

Load-bearing walls are supported by continuous-strip footings. Sometimes they are corbeled out to accommodate columns integral with the wall. In these cases the columns support a major portion of the interior floor loads; the walls carry self-weight and perimeter floor loads. Figure 8-16 illustrates typical wall footings.

Design of a wall footing consists in providing a depth adequate for wide-beam shear (which will control as long as $d \leq \frac{2}{3} \times$ footing projection). The remainder of the design consists in providing sufficient reinforcing steel for bending requirements of the footing projection. Longitudinal steel is required to satisfy shrinkage requirements. Longitudinal steel will, in general, be more effective in the top of the footing than in the bottom, as shown in Fig. 8-17. Note that as settlement occurs in Fig. 8-17c the wall should increase the effective footing $I$ somewhat to resist "dishing."

Wall footings for residential construction are usually of dimensions to satisfy local building codes or Federal Housing Administration (FHA) requirements or to allow placing foundation walls. The contact pressure is usually on the order of 17 to 25 kPa including the wall weight. The FHA requirements are shown in Fig. 8-18. Again longitudinal steel, if used, should be placed in the top rather than the bottom for maximum effectiveness in crack control when the foundation settles.

Interior footings for residential construction are usually nonreinforced and sized to carry not over 20 to 45 kN , resulting in square or rectangular foundations on the order of 0.5 to 1.5 m . Often these footings are concrete-filled predrilled auger holes to a depth below seasonal volume change. Additional information on foundations for residential construction can be found in Bowles (1974b).

Figure 8-16 Footings for residential construction.


(a) The dashed lines represent either varying location of existing ground before filling or the resulting qualitative shape of the ground settlement caused by fill and/or building.

(b) Interior settlements may be resisted by larger moment of interia based on wall contribution so that tension stresses in footing are minimal.

Figure 8-17 Settlements of residences.


Figure 8-18 Federal Housing Administration (FHA) minimum wall-footing dimensions. Recommend use of at least two No. 10 reinforcing bars ( $11.3-\mathrm{mm}$ diameter) (author's, not FHA). Always use an outside perimeter drain with a basement. [Further details in Bowles (1974b).]

Example 8-12. Design the wall footing for an industrial building for the following data. Wall load consists in $70.1 \mathrm{kN} / \mathrm{m}(D=50, L=20.1 \mathrm{kN} / \mathrm{m})$ including wall, floor, and roof contribution.

$$
\begin{array}{ll}
f_{c}^{\prime}=21 \mathrm{MPa} & f_{y}=400 \mathrm{MPa} \\
q_{a}=200 \mathrm{kPa} & \text { Wall of concrete block } 200 \times 300 \times 400 \mathrm{~mm}(8 \times 12 \times 16 \mathrm{in} .)
\end{array}
$$

Solution. From Table 8-2, wide beam $v_{c}=649 \mathrm{kPa}$ (no two-way action).
Step 1. Find footing width:

$$
B=\frac{70.1}{200}=0.35 \mathrm{~m}
$$

Since this is only 50 mm wider than the $300-\mathrm{mm}$ concrete block, we will arbitrarily make the footing project 150 mm on each side of the wall, or

$$
B=300+150+150=\mathbf{6 0 0} \mathrm{mm}
$$

We will arbitrarily make the total footing depth $D_{c}=400 \mathrm{~mm}(d=400-80=320 \mathrm{~mm})$. The "pseudo" ultimate soil pressure (neglecting any weight increase from displacing the lighter soil with heavier concrete) is

$$
q_{\mathrm{ult}}=\frac{P_{\mathrm{ult}}}{B}=\frac{1.4(50)+1.7(20.1)}{600 / 1000}=174 \mathrm{kPa} / \mathrm{m}<200 \text { allowable }
$$

Step 2. Check wide-beam shear for the trial depth $d=320 \mathrm{~mm}$. This will be done at the face of the wall (not $d$ out) for the most severe condition: the soil pressure equals 174 kPa and the projected length $L^{\prime}$ equals 150 mm , giving $V_{u}=q L^{\prime}=174 \times 150 / 1000=26.1 \mathrm{kN} / \mathrm{m}$. The allowable concrete shear stress is 649 kPa (from Table 8-2), and the actual shear stress for a wall length of $L=1 \mathrm{~m}$ is

$$
v_{a}=\frac{V_{u}}{L d}=\frac{26.1}{1 \times 320 / 1000}=82 \mathrm{kPa} \ll 649 \quad \text { depth is } \mathrm{OK}
$$

Step 3. Find required steel for transverse bending. Using meters rather than millimeters for computations to avoid huge numbers, we obtain

Moment arm $L^{\prime}=$ overhang $+\frac{1}{4}$ concrete block width

$$
L^{\prime}=0.150+\frac{1}{4} 0.300=0.225 \mathrm{~m}
$$

(See Fig. 8-5b for masonry columns.) The resulting "ultimate" moment $M_{u}$ is

$$
M_{u}=q \frac{L^{2}}{2}=\frac{174 \times 0.225^{2}}{2}=4.04 \mathrm{kN} \cdot \mathrm{~m} / \mathrm{m}
$$

Since the concrete $f_{c}^{\prime}=21$ and $f_{y}=400 \mathrm{MPa}$ are the same as used in Example 8-2, we have from that example $a=22.4 A_{S}$ (see Step 6). Now making substitutions into the previously used rearrangement of Eq. (8-2), we obtain (using $d=320 / 1000, B=1 \mathrm{~m}$, and $\phi=0.9$ )

$$
A_{s}\left(0.32-\frac{22.4 A_{s}}{2}\right)=\frac{4.04}{0.9 \times 400 \times 1000}
$$

and simplifying obtain $A_{s}^{2}-0.0286 A_{s}=1.0 \times 10^{-6}$. Solving the quadratic, we obtain

$$
A_{s}=0.035 \times 10^{-3} \mathrm{~m}^{2} / \mathrm{m}=350 \mathrm{~mm}^{2} / \mathrm{m} .
$$

Figure E8-12


Final sketch

Step 4. Check for temperature and shrinkage ( T and S ) based on ACI Art. 7.12, and grade 400 bars so that the percentage $p=0.0018$. The steel area $A_{S}$ per meter of wall length is

$$
\begin{aligned}
A_{S}=p d B=0.0018 \times 0.320 \times 1.0 & =0.576 \times 10^{-3} \mathrm{~m}^{2} / \mathrm{m} \\
& =576 \mathrm{~mm}^{2}>350 \text { and controls }
\end{aligned}
$$

Use six No. 10 bars $/$ meter, giving $A_{S}=6 \times 100=600 \mathrm{~mm}^{2} / \mathrm{m}>576$, OK. The spacing will be $1000 / 6=167 \mathrm{~mm}<5 \times D_{c}<500 \mathrm{~mm}$ of ACI Art. 7.12.2.2.
Step 5. Select longitudinal steel. Since there is no moment arm, we will make an arbitrary selection based on a minimum of $T$ and $S$ of $576 \mathrm{~mm}^{2}$. Let us use eight No. 10 bars as follows:

$$
\text { Furnished } A_{s}=8 \times 100=800 \mathrm{~mm}^{2}>576 \text { for } T \text { and } S
$$

Two bars in bottom of wall footing and 80 mm ( 75 mm clear) above soil. These two bars will provide support for the transverse bars being used at six bars/meter of wall.

Six bars in top part across the width of 600 mm and with 40 mm of clear cover. We are using all the bars of same size to minimize the number of bar sizes on site and reduce any chance of installing an incorrect bar size. The reader should check that we can get six No. 10 bars into the width of 600 mm and not violate any ACI Code spacing requirements.

Step 6. Make a design sketch like Fig. E8-12.

## PROBLEMS

8-1. Design the assigned problem of Table P8-1 (refer to Fig. P8-1). For this exercise take both columns and footings as square.

TABLE P8-1

|  | Column data |  |  |  |  |  | Footing data |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\underset{\mathrm{mm}}{\mathrm{~m}}$ | $\begin{gathered} \boldsymbol{f}_{y}, \\ \mathbf{M P a} \end{gathered}$ | $\begin{gathered} \boldsymbol{f}_{c}, \\ \mathbf{M P a} \end{gathered}$ | Number of bars, type | $\underset{\mathbf{k N}}{\mathrm{DL}}$ | $\underset{\mathbf{k N}}{\boldsymbol{L L},}$ | $\begin{gathered} \boldsymbol{f}_{y}, \\ \mathbf{M P a} \end{gathered}$ | $\begin{gathered} \boldsymbol{f}_{c}^{\prime}, \\ \mathbf{M P a} \end{gathered}$ | $\begin{gathered} q_{a}, \\ \mathbf{k P a} \end{gathered}$ |
| $a$ | 460 | 400 | 28 | 10 \#35 | 1,300 | 1,300 | 400 | 21 | 210 |
| $b$ | 530 | 400 | 24 | 6 \#30 | 900 | 1,250 | 400 | 21 | 170 |
| c | 360 | 400 | 24 | 4 \#35 | 620 | 600 | 400 | 24 | 120 |
| $d$ | 360 | 300 | 24 | 4 \#30 | 450 | 580 | 400 | 28 | 200 |
| $e$ | 460 | 300 | 28 | 6 \#35 | 800 | 670 | 300 | 28 | 150 |

## Partial answer:

|  | $B, \mathbf{m}$ | $d, \mathrm{~mm}$ | $A_{s}, \mathrm{~mm}^{2} / \mathrm{m}$ |
| :--- | :---: | :---: | :---: |
| $a$ | 3.60 | 640 | $1716(0.0027)^{*}$ |
| $b$ | 3.60 | 550 | $1608(0.0029)^{*}$ |
| $c$ | 2.30 | 346 | $1189(0.0034)^{*}$ |
| ${ }^{*}()=$ |  |  |  |
|  |  |  |  |



Figure P8-1

8-2. Use the data of Table P8-1 to design a rectangular footing using in all cases $L=2.75 \mathrm{~m}$ (input one dimension of footing).
Partial answers:

|  | $\mathbf{2 . 7 5} \times \boldsymbol{L}$, | $\mathbf{d}, \mathbf{m m}$ | $\boldsymbol{A}_{\mathbf{s}, \text { long }}, \mathrm{mm}^{2} / \mathrm{m}$ | $\boldsymbol{A}_{\mathbf{s}, \text { short }}, \mathrm{mm}^{2} / \mathrm{m}$ |
| :--- | :---: | :---: | :---: | :---: |
| $a$ | 4.60 | 682 | 2923 | 1363 |
| $c$ | 3.70 | 414 | 1816 | 909 |
| $d$ | 1.90 | $350(\mathrm{w}-\mathrm{b})$ | 1839 | 743 |

8-3. Use the data of Table P8-1 and design a footing if $w=$ diameter.
Partial answers:

|  | $\boldsymbol{w}, \mathbf{m m}$ | $\boldsymbol{B}, \mathbf{m}$ | $\boldsymbol{d}, \mathbf{\mathrm { mm }}$ | $\boldsymbol{A}_{\boldsymbol{s}}, \mathrm{mm}^{\mathbf{2} / \mathbf{m}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $a$ | 460 | 3.60 | 747 | 1508 |
| $b$ | 530 | 3.60 | 647 | 1409 |
| $e$ | 460 | 3.20 | 478 | 1707 |

8-4. Design the base plate and pedestal for a $W 310 \times 86$ column carrying $D=450$ and $L=490 \mathrm{kN}$. Use $f_{y}=250 \mathrm{MPa}$ steel for column and base plate, $f_{c}^{\prime}=24 \mathrm{MPa}$ psi for footing and pedestal, and Grade 400 reinforcing bars. The allowable soil pressure is 150 kPa . The pedestal is 1.40 m from the underside of the floor slab. Select two anchor bolts and draw a neat sketch of the column, plate, and anchor bolt locations.
8-5. Using the data of Problem 8-4, design a base plate for the column if it interfaces directly with the footing. Assume the footing is sufficiently large that the allowable concrete bearing stress $f_{c}=0.7 f_{c}^{\prime}$.
8-6. Design the base plate and pedestal for a $\mathbf{W} 360 \times 196$ column carrying $D=1400$ and $L=$ 1200 kN . Use $f_{y}=345 \mathrm{MPa}$ for column and base plate, and Grade 300 reinforcing bars. Use $f_{c}^{\prime}=28 \mathrm{MPa}$ for footing and pedestal and an allowable soil pressure $q_{a}=200 \mathrm{kPa}$. The pedestal is 1.90 m from underside of floor slab.
8-7. Design the footing and a column base plate for the $\mathbf{W} 360 \times 196$ column (no pedestal) data of Prob. 8-6.

8-8. Refer to Example 8-6. Redesign the base plate with the column axial load $P=600 \mathrm{kN}$ and the moment $M=120 \mathrm{kN} \cdot \mathrm{m}$ (instead of the 500 kN and $100 \mathrm{kN} \cdot \mathrm{m}$ of the example).
8-9. Refer to Example 8-7. Take the pedestal dimensions at $700 \times 700 \mathrm{~mm}$ and rework the example if four bolts that are centered on the heel (tension flange) are used. In the example the bolts are all put into the heel projection of the base plate. Be sure to check the bolt tension to see if it controls the plate thickness.
8-10. Rework Example $8-7$ if the column moment $M=190 \mathrm{kN} \cdot \mathrm{m}$ and all the other data are the same.
8-11. Rework Example $8-9$ if the load and moment are increased 10 percent $(800 \mathrm{D} \times 1.1=880$; $800 L \times 1.1=880 \mathrm{kN}$, etc.). Assume the column dimensions are $A X=0.60 \mathrm{~m}$ and $A Y=$ 0.40 m . All other data such as $f_{c}^{\prime}, f_{y}, q_{a}$, etc. are the same.

8-12. Rework Example $8-10$ if the allowable soil pressure $q_{a}=450 \mathrm{kPa}$ (Remember: It was set back from about $490^{+}$to 400 ). As in Prob. 8-11, assume a 10 percent increase in all loads and moments. Hint: The footing will continue to be square and $B^{\prime}=L^{\prime}$.
8-13. Design a wall footing for a concrete-block-wall building. The building has a $5-\mathrm{m}$-high wall; the footing is 1.2 m in the ground and has a plan area of $12 \times 36 \mathrm{~m}$. The roof will weigh about 0.9 kPa , and snow load is 1.5 kPa . The allowable soil pressure is 100 kPa , and about one-half of the building length ( 36 m ) is on a fill of varying depth from 0 to 1.2 m .
8-14. Design a wall footing for a two-story office building of concrete block and brick veneer. The building is $16 \times 30 \mathrm{~m}$ in plan. The footing is 1 m below ground. The first floor slab rests directly on the ground. Assume the floor dead load averages 2.0 kPa and live load 4.4 kPa . The roof is about 0.75 kPa , and snow is 1.0 kPa . Concrete blocks are $200 \times 300 \times 400 \mathrm{~mm}$ and weigh 4.2 kPa (wall surface). Brick ( $100 \times 200 \times 90 \mathrm{~mm}$ ) will weigh 1.9 kPa (wall surface). The undrained shear strength $s_{u}$ may be taken as 60 kPa . Hint: estimate wall height.
Partial answer: $B \approx 1.6 \mathrm{~m}$
8-15. Design the foundation for a residence with approximately $135 \mathrm{~m}^{2}$ of floor area. A perimeter wall will be used and a single interior post-on-pad. Assume wood frame, aluminum siding, and brick trim. Take snow load at 1.5 kPa . The floor plan is $9.80 \times 13.8 \mathrm{~m}$. Draw a building plan and place the post at a convenient location. Comment on the design as appropriate. You must assume or specify any missing data needed for your design.


[^0]:    ${ }^{8}$ The ACI Code specifies gross column area-that is, no area reduction for column reinforcing. The symbol often used is $A_{g}$, but this text uses $A_{\text {col }}$.

[^1]:    ${ }^{9}$ As the moment rotates the footing, the rotation reduces the moment, which in turn reduces rotation, etc., until some rotation equilibrium is reached consistent with moment and stiffness of column, footing, and soil.

