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Pricing for Options in a Hull-White-Vasicek Volatility and Interest Rate Model

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Abstract

In this paper, by taking account into the no-arbitrage in a new stochastic process, we propose a pricing model for options in a Mixed Modified Fractional Hull-White-Vasicek volatility and interest rate model. By using the double Mellin transform approach, we derive closed-form pricing formulas for this pricing problem, which are the main contribution of this paper and expand the relevant literature's conclusions.

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Keywords: Bull spread option, Hull-White-Vasicek model, Double Mellin transform

1 Introduction

Among the models for pricing options in the financial market, the Black and Scholes (1973)[2] model is a basic model remains the most widely used on which all work on calculating the price of options. One of the weaknesses is that its volatility is constant. It is also widely known and many agents use it in trading rooms. It has a confidence interval which is correct and therefore gives a good indication to the

trader. It has a closed formula for the pricing of options, which is far from the case for the main competing models which are more faithful to the reality of the markets.

These competing models modeled the interest rate of the Black and Scholes model and authors such as Vasicek (1977)[12], Dothan (1978)[5], Cox-Ingersoll-Ross (1985)[4] and Black-Derman-Toy (1990)[3]have worked on these models.

The constant volatility assumption of the Black-Scholes model is unreliable, because in the market volatility is in the form of a smille. Hence the need to introduce in the market stochastic volatility models which give more realistic results compared to the Black-Scholes model. Pioneers in the field are: Hull-White (1987)[8], Stein-Stein (1991)[10], Heston (1993)[7], Bakshi, Cao-Chen (1997)[1] and Schobel-Zhu (1999)[11]. Each of these authors model the volatility of the underlying and propose models that present analytical solutions and closed formulas for option pricing.

Our paper looks at these two families of models (stochastic interest rate) and stochastic volatility). But our motivation comes from the fact that we model the two families of models by assuming that their random parts are described by mixed modified fractional Brownian motions in such a way that our market is without arbitrage. The combination of the stochastic interest rate model (Vasicek(1977)[12]) and the stochastic volatility model (Hull-White(1987)[8]) allows us to define a three-factors model. This new model has a double advantage in that, it can correctly price any bond unlike the one-factor models mentioned above and can allow any options to be valued. In this paper, we will therefore propose a closed formula for the pricing of the particular case of vulnerable options and the bull spread option using the double Mellin's transform method.

Named MMFHWV model for "Mixed-Modified-Fractional-Hull-White -Vasiseck", this new model is a combination of the Hull and White(1987) [8] and the Vasicek(1977) [12] model. In this model, the volatility process and asset model are not correlated, while the interest rate process and asset model, the volatility process and interst rates process are correlated, with each other and they are controlled by a distinct diffusion process. In MMFHWV model, the existence of the mean reversion process causes the adjustment of the volatility process and the interest rates behavior in the financial markets and it is a benefit of the MMFHWV model. The rest of this paper is organized as follows. In Sect. 2, we introduce the MMFHWV model framework. In Sect. 3, we study the pricing of European options under the Hull-White-Vasicek volatility and interest rate model.In Sect.4, we give the Option pricing formula and conclusions are presented in the last section.

2 The MMFHWV Model Framework

The Hull-White-Vasicek model is a combination of the Hull-White model and the Vasicek model which each model will be describe in Definition 2.2 and 2.1.

Definition 2.1 *[8] The Hull-White model can be considered as an alternative to Heston's model, proposing a stochastic process for the variance different from the root-square process. In this model, the volatility is stochastic process and it is*

determined under the risk-neutral probability measure, by the Stochastic Differential Equation (SDE) as follows:

$$\begin{cases} dS_t = rS_t dt + S_t \sigma_s dW_{t,s}, S_0 > 0, \\ dV_t = rV_t dt + \sigma_v V_t dW_{t,v}, V_0 > 0, \\ dW_{t,s} dW_{t,v} = 0. \end{cases} \tag{1}$$

where S_t denotes asset price at time t, $t \geq 0$ and κ is the mean reversion speed of the variance, θ is the long term variance, σ is the volatility of variance, r is interest rate, ρ is the correlation between $W_{t,S}$ and $W_{t,v}$, S_0 is the spot asset price, and V_0 is the spot variance. V_t denotes instantaneous variance and $W_{.,S}$, $W_{.,v}$ are wiener processes.

Definition 2.2 *[12] The Vasicek model is a stochastic interest rate model and the corresponding Stochastic Differential Equation (SDE) as follows,*

$$dr_t = \kappa(\theta - r_t)dt + r_t \sigma_r dW_{t,r}, r_0 > 0. \tag{2}$$

β, σ_r are positive constant and $W_{.,r}$ are wiener processes.

Definition 2.3 *[6]A Mixed Modified Fractional Brownian Motion process whose parameters a, b, ε and H is a linear combination of Brownian Motion B_t and independent semimartingale process $B_t^{H,\varepsilon}$, defined on a probability space $(\Omega, F, \mathbb{P})$ by:*

$$M_t^{H,a,b,\varepsilon} = M_t^{H,\varepsilon} = aB_t + bB_t^{H,\varepsilon}, \ \forall (a,b) \in \mathbb{R}_+^* \times \mathbb{R}_+, \ t \in [0,T]. \tag{3}$$

Where $H \in]\frac{1}{2}, 1[$ is the Hurst parameter.

Definition 2.4 *(The MMFHWV model) In the Hull-White model Eq.(1), if we select the parameter r as a stochastic process(see Vasicek model Eq.(2)), and substituting the Wierner process $W_{t,s}, W_{t,v}$, and $W_{t,r}$ by $M_{t,s}^{H,\varepsilon}, M_{t,v}^{H,\varepsilon}$, and $M_{t,r}^{H,\varepsilon}$ respectively, then we will obtain the new model named Mixed-Modified-Fractional-Hull-White-Vasicek (MMFHWV) model and then, under the martingale measure, the dynamics of S_t, V_t and r_t are given by the SDEs:*

$$\begin{cases} dS_t = r_t S_t dt + \sigma_s S_t dM_{t,s}^{H,\varepsilon}, \ S_0 > 0 \\ dV_t = r_t V_t dt + \sigma_v V_t dM_{t,v}^{H,\varepsilon}, \ V_0 > 0 \\ dr_t = \kappa(\theta - r_t)dt + r_t \sigma_r dM_{t,r}^{H,\varepsilon}, \ r_0 > 0 \\ dM_{t,S}^{H,\varepsilon} dM_{t,v}^{H,\varepsilon} = 0, dM_{t,s}^{H,\varepsilon} dM_{t,r}^{H,\varepsilon} = \chi^{H,\varepsilon} \rho_{sr}, dM_v^{H,\varepsilon} dM_{t,r}^{H,\varepsilon} = \chi^{H,\varepsilon} \rho_{vr} \end{cases} \tag{4}$$

3 PDE of Bull Spread Option under MMFHWV model

To solve Eq.(4),we recall the price formula of zero-coupon bond maturing at time T. The no-arbitrage price at time t of the bond with the nominal value 1 is given by

$$P(t,r,T) = \mathbb{E}\left[e^{-\int_t^T r_s ds} | r_t = r)\right], t < T. \tag{5}$$

with the final condition $P(T,r,T)=1$. As aforementioned specific case for specific vulnerable bull spread option, expiring at time T, with the strike price K_1 and K_2 ,the payoff function is given by

$$h(S_T,V_T)) = (S_T-K_1)^+\mathbb{1}_{\{V_T\geq\gamma^*\}} - (S_T-K_2)^+\mathbb{1}_{\{V_T\geq\gamma^*\}}, \tag{6}$$

with γ^* and the no-arbitrage price of the option is given by

$$U(t,s,v,r) = \mathbb{E}\left[e^{-\int_t^T r_s ds} h(S_T,V_T)|S_t=s, V_t=v, r_t=r)\right]. \tag{7}$$

By the non-arbitrage assumptions, the corresponding MMFHWV PDE of $U(t,s,v,r)$ according to the Feynman-Kac formula of european option, we have

$$\frac{\partial U}{\partial t} + \chi^{H,\varepsilon}\left[\frac{1}{2}\sigma_s^2 s^2\frac{\partial^2 U}{\partial s^2} + \frac{1}{2}\sigma_v^2 v_t^2\frac{\partial^2 U}{\partial v^2} + \frac{1}{2}\sigma_r^2\frac{\partial^2 U}{\partial r^2} + \rho_{sr}\sigma_s\sigma_r s\frac{\partial^2 U}{\partial s\partial r} + \rho_{vr}\sigma_v\sigma_r v\frac{\partial^2 U}{\partial v\partial r}\right]$$
$$+ r\left(s\frac{\partial U}{\partial s} + v\frac{\partial U}{\partial v}\right) + \kappa(\theta - r)\frac{\partial U}{\partial r} - rU = 0 \tag{8}$$

with the terminal condition $U(T,s,v,r)=h(s,v)$ and $\chi^{H,\varepsilon}=(a+b\varepsilon^{H-\frac{1}{2}})^2$.

4 Option price formula

In this section, we use the appoach of [9]to derive a price formula of a bull Spread option under the Mixed-Modified-Fractional-Hull-White-Vasicek model by using the double mellin transform. Setting $U_n(T,s,v,r)$ by

$$U_n(T,s,v,r) = \mathbb{E}\left[e^{-\int_t^T r_s ds} h(S_T,V_T)|S_t=s, V_t=v, r_t=r)\right] \tag{9}$$

we observe that, $U_n(t,s,v,r)$ satisfies Eq.(8) with the terminal condition $U_n(T,s,v,r)=h_n(s,v)$. The price $U(t,s,v,r)$ is given by taking the limit $U(t,s,v,r)=\lim_{n\longrightarrow\infty} U_n(t,s,v,r)$ Let $\widehat{U}_n(t,w_1,w_2,r)$ be the double Mellin transform of $U_n(t,s,v,r)$,.The expression of the inverse double Mellin transform of $U_n(t,s,v,r)$ is given by

$$U_n(t,s,v,r) = \frac{1}{(2\pi)^2}\int_{a_1-i\infty}^{a_1+i\infty}\int_{a_2-i\infty}^{a_2+i\infty}\widehat{U}_n(t,w_1,w_2,r)s^{-w_1}v^{-w_2}dw_1dw_2 \tag{10}$$

By applying Eq.(10) into Eq.(8), we obtain

$$-\frac{\partial\widehat{U}_n}{\partial\tau} + \left[\frac{1}{2}\chi^{H,\varepsilon}\sigma_s^2 w_1(w_1+1) + \frac{1}{2}\chi^{H,\varepsilon}\sigma_v^2 w_2(w_2+1) - r(w_1+w_2+1)\right]\widehat{U}_n$$
$$+\frac{1}{2}\chi^{H,\varepsilon}\sigma_r^2\frac{\partial^2\widehat{U}_n}{\partial r^2} + \left(\kappa\theta - \kappa r - \rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_r w_1 - \rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_r w_2\right)\frac{\partial\widehat{U}_n}{\partial r} = 0 \tag{11}$$

with $\tau=T-t$ and $\widehat{U}_n(0,w_1,w_2,r)=\widehat{h}(w_1,w_2)$ which is the Mellin transform of $h_n(s,v)$. Let

$$\widehat{U}_n(\tau,w_1,w_2,r) = \exp\left\{\left(\frac{1}{2}\chi^{H,\varepsilon}\sigma_s^2 w_1(w_1+1) + \frac{1}{2}\chi^{H,\varepsilon}\sigma_v^2 w_2(w_2+1)\right)\tau\right\}\widehat{f}_n(\tau,w_1,w_2,r) \tag{12}$$

Then, Eq.(11) is transformed into the following PDE for $\widehat{f}_n$.

$$-\frac{\partial \widehat{f}_n}{\partial \tau}+\frac{1}{2}\chi^{H,\varepsilon}\sigma_r^2\frac{\partial^2 \widehat{f}_n}{\partial r^2}+\left(\kappa\theta-\kappa r-\rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_r w_1-\rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_r w_2\right)\frac{\partial \widehat{f}_n}{\partial r}$$
$$-r(w_1+w_2+1)\widehat{f}_n=0 \tag{13}$$

with $\widehat{f}_n(0,w_1,w_2,r)=\widehat{U}_n(0,w_1,w_2,r)=\widehat{h}(w_1,w_2)$ To solve Eq.(13), we let

$$\widehat{f}_n(\tau,w_1,w_2,r)=\widehat{h}(w_1,w_2)Q(\tau,w_1,w_2)e-L(\tau)(w_1+w_2+1)r. \tag{14}$$

We have the terminal condition $\widehat{f}_n(0,w_1,w_2,r)=\widehat{h}_n(w_1,w_2)$ Substituting Eq.(14) into Eq.(13), we obtain the following ODEs:

$$\frac{\partial Q}{\partial \tau}+\left(\left(\kappa\theta(T-\tau)-\rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_r w_1-\rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_r w_2\right)L(\tau)(w_1+w_2+1)\right)Q$$
$$-\left(\frac{1}{2}\chi^{H,\varepsilon}\sigma_r^2L^2(\tau)(w_1+w_2+1)^2\right)Q=0 \tag{15}$$
$$\frac{\partial L}{\partial \tau}+\kappa L-1=0 \tag{16}$$

where $Q(0,w_1,w_2)=1$ and $L(0)=0$. Then, by solving Eq.(15), we have

$$Q(\tau,w_1,w_2)=\exp\left\{\frac{\chi^{H,\varepsilon}\sigma_r^2}{2\kappa^2}\left(\tau+\frac{2}{\kappa}\left(e^{-\kappa\tau}-1\right)-\frac{1}{2\kappa}\left(e^{-2\kappa\tau}-1\right)\right)(w_1+w_2+1)^2\right\}$$
$$+\exp\left\{\left(\frac{\rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_r}{\kappa}w_1(w_1+w_2+1)+\frac{\rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_r}{\kappa}w_2(w_1+w_2+1)\right)(\tau-L(\tau))\right\}$$
$$-\exp\left\{\int_0^\tau(w_1+w_2+1)\kappa\theta(T-\nu)L(\nu)d\nu\right\}$$

and

$$L(\tau)=\frac{1-e^{-\kappa\tau}}{\kappa}. \tag{17}$$

Then, from Eq.(12), we obtain

$$\widehat{U}_n(\tau,w_1,w_2,r)=\widehat{h}_n(w_1,w_2)\exp\left\{\left(N(\tau)(w_1+w_2+1)^2\right)\right\}$$
$$+\exp\left\{\left(\frac{1}{2}\chi^{H,\varepsilon}\sigma_s^2w_1(w_1+1)+\frac{1}{2}\chi^{H,\varepsilon}\sigma_v^2w_2(w_2+1)\right)\right\}$$
$$+\exp\left\{\left(\frac{\rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_r}{\kappa}w_1(w_1+w_2+1)+\frac{\rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_r}{\kappa}w_2(w_1+w_2+1)\right)(\tau-L(\tau))\right\}$$
$$-\exp\left\{M(\tau)+L(\tau)r(w_1+w_2+1)\right\} \tag{18}$$

with $N(\tau)$ and $M(\tau)$ are given by

$$\begin{cases} N(\tau)=\frac{\chi^{H,\varepsilon}\sigma_r^2}{2\kappa^2}\left(\tau+\frac{2}{\kappa}\left(e^{-\kappa\tau}-1\right)-\frac{1}{2\kappa}\left(e^{-2\kappa\tau}-1\right)\right)\\ M(\tau)=\int_0^\tau\kappa\theta(T-\nu)L(\nu)d\nu \end{cases} \tag{19}$$

We have

$$U_n(\tau,s,v,r)=\frac{1}{(2\pi)^2}\int_{a_1-i\infty}^{a_1+i\infty}\int_{a_2-i\infty}^{a_2+i\infty}\widehat{h}_n(w_1,w_2)e^{\widehat{J}(\tau,w_1,w_2,r)}s^{-w_1}v^{-w_2}dw_1dw_2 \tag{20}$$

where $\widehat{J}(\tau,w_1,w_2,r)$ is given by

$$\begin{aligned}\widehat{J}(\tau,w_1,w_2,r)&=N(\tau)(w_1+w_2+1)^2+\left(\frac{1}{2}\chi^{H,\varepsilon}\sigma_s^2w_1(w_1+1)+\frac{1}{2}\chi^{H,\varepsilon}\sigma_v^2w_2(w_2+1)\right)\tau\\&+\left(\frac{\rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_r}{\kappa}w_1(w_1+w_2+1)+\frac{\rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_r}{\kappa}w_2(w_1+w_2+1)\right)(\tau-L(\tau))\\&-(M(\tau)+L(\tau)r)(w_1+w_2+1).\end{aligned} \tag{21}$$

By setting

$$\begin{cases}\widehat{\sigma}_s^2(t)=\chi^{H,\varepsilon}\sigma_s^2+2\rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_rL(t)+\chi^{H,\varepsilon}\sigma_r^2L^2(t)\\ \widehat{\sigma}_v^2(t)=\chi^{H,\varepsilon}\sigma_v^2+2\rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_rL(t)+\chi^{H,\varepsilon}\sigma_r^2L^2(t).\end{cases} \tag{22}$$

We deduce the following analytic closed form formula of the price of a particular case European vulnerable Bull Spread option.

Theorem 4.1 *The price of specific vulnerable Bull Spread option with Mixed-Modified-Fractional-Hull-White-Vasicek model,defined by Eq.(7), is given by*

$$\begin{aligned}U(t,s,v,r)&=s\mathcal{N}(\widehat{a}_1,\widehat{a}_2,\xi)-K_1B(t,r,T))\mathcal{N}(\widehat{b}_1,\widehat{b}_2,\xi)\\&-s\mathcal{N}(\widehat{a}_3,\widehat{a}_4,\xi)+K_2B(t,r,T))\mathcal{N}(\widehat{b}_3,\widehat{b}_4,\xi).\end{aligned}$$

with $\xi=\frac{\overline{B}(\tau)}{2\sqrt{\overline{A}(\tau)B_1(\tau)}}$ *where*

$$\hat{a}_1=\frac{\ln\left(\frac{s}{K_1}\right)-\ln P(t,r,T)+\frac{1}{2}\int_0^\tau\left(\chi^{H,\varepsilon}\sigma_s^2+2\rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_rL(\nu)+\chi^{H,\varepsilon}\sigma_r^2L^2(\nu)\right)d\nu}{\sqrt{\int_0^\tau\left(\chi^{H,\varepsilon}\sigma_s^2+2\rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_rL(\nu)+\chi^{H,\varepsilon}\sigma_r^2L^2(\nu)\right)d\nu}} \tag{23}$$

$$\begin{aligned}\hat{b}_1&=\frac{\ln\left(\frac{v}{\gamma^*}\right)-\ln P(t,r,T)-\frac{1}{2}\int_0^\tau\left(\chi^{H,\varepsilon}\sigma_v^2+2\rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_rL(\nu)+\chi^{H,\varepsilon}\sigma_r^2L^2(\nu)\right)d\nu}{\sqrt{\int_0^\tau\left(\chi^{H,\varepsilon}\sigma_v^2+2\rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_rL(\nu)+\chi^{H,\varepsilon}\sigma_r^2L^2(\nu)\right)d\nu}}\\&+\frac{2\xi\int_0^\tau\widehat{\sigma}_s^2(\nu)d\nu\int_0^\tau\left(\sigma_s^2+2\rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_rL(\nu)+\chi^{H,\varepsilon}\sigma_r^2L^2(\nu)\right)d\nu}{\sqrt{\int_0^\tau\left(\sigma_v^2+2\rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_rL(\nu)+\chi^{H,\varepsilon}\sigma_r^2L^2(\nu)\right)d\nu}}\end{aligned} \tag{24}$$

$$\hat{a}_2=\frac{\ln\left(\frac{s}{K_1}\right)-\ln P(t,r,T)-\frac{1}{2}\int_0^\tau\left(\chi^{H,\varepsilon}\sigma_s^2+2\rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_rL(\nu)+\chi^{H,\varepsilon}\sigma_r^2L^2(\nu)\right)d\nu}{\sqrt{\int_0^\tau\left(\chi^{H,\varepsilon}\sigma_s^2+2\rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_rL(\nu)+\chi^{H,\varepsilon}\sigma_r^2L^2(\nu)\right)d\nu}} \tag{25}$$

$$\hat{b}_2 = \frac{\ln\left(\frac{v}{\gamma^*}\right) - \ln P(t,r,T) - \frac{1}{2}\int_0^\tau \left(\chi^{H,\varepsilon}\sigma_v^2 + 2\rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_r L(\nu) + \chi^{H,\varepsilon}\sigma_r^2 L^2(\nu)\right) d\nu}{\sqrt{\int_0^\tau \left(\chi^{H,\varepsilon}\sigma_v^2 + 2\rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_r L(\nu) + \chi^{H,\varepsilon}\sigma_r^2 L^2(\nu)\right) d\nu}} \tag{26}$$

and

$$\hat{a}_3 = \frac{\ln\left(\frac{s}{K_2}\right) - \ln P(t,r,T) + \frac{1}{2}\int_0^\tau \left(\chi^{H,\varepsilon}\sigma_s^2 + 2\rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_r L(\nu) + \chi^{H,\varepsilon}\sigma_r^2 L^2(\nu)\right) d\nu}{\sqrt{\int_0^\tau \left(\chi^{H,\varepsilon}\sigma_s^2 + 2\rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_r L(\nu) + \chi^{H,\varepsilon}\sigma_r^2 L^2(\nu)\right) d\nu}} \tag{27}$$

$$\hat{b}_3 = \frac{\ln\left(\frac{v}{\gamma^*}\right) - \ln P(t,r,T) - \frac{1}{2}\int_0^\tau \left(\chi^{H,\varepsilon}\sigma_v^2 + 2\rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_r L(\nu) + \chi^{H,\varepsilon}\sigma_r^2 L^2(\nu)\right) d\nu}{\sqrt{\int_0^\tau \left(\chi^{H,\varepsilon}\sigma_v^2 + 2\rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_r L(\nu) + \chi^{H,\varepsilon}\sigma_r^2 L^2(\nu)\right) d\nu}}$$
$$+ \frac{2\xi\int_0^\tau \widehat{\sigma}_s^2(\nu)d\nu \int_0^\tau \left(\sigma_s^2 + 2\rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_r L(\nu) + \chi^{H,\varepsilon}\sigma_r^2 L^2(\nu)\right) d\nu}{\sqrt{\int_0^\tau \left(\sigma_v^2 + 2\rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_r L(\nu) + \chi^{H,\varepsilon}\sigma_r^2 L^2(\nu)\right) d\nu}} \tag{28}$$

$$\hat{a}_4 = \frac{\ln\left(\frac{s}{K_2}\right) - \ln P(t,r,T) - \frac{1}{2}\int_0^\tau \left(\chi^{H,\varepsilon}\sigma_s^2 + 2\rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_r L(\nu) + \chi^{H,\varepsilon}\sigma_r^2 L^2(\nu)\right) d\nu}{\sqrt{\int_0^\tau \left(\chi^{H,\varepsilon}\sigma_s^2 + 2\rho_{sr}\chi^{H,\varepsilon}\sigma_s\sigma_r L(\nu) + \chi^{H,\varepsilon}\sigma_r^2 L^2(\nu)\right) d\nu}} \tag{29}$$

$$\hat{b}_4 = \frac{\ln\left(\frac{v}{\gamma^*}\right) - \ln P(t,r,T) - \frac{1}{2}\int_0^\tau \left(\chi^{H,\varepsilon}\sigma_v^2 + 2\rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_r L(\nu) + \chi^{H,\varepsilon}\sigma_r^2 L^2(\nu)\right) d\nu}{\sqrt{\int_0^\tau \left(\chi^{H,\varepsilon}\sigma_v^2 + 2\rho_{vr}\chi^{H,\varepsilon}\sigma_v\sigma_r L(\nu) + \chi^{H,\varepsilon}\sigma_r^2 L^2(\nu)\right) d\nu}} \tag{30}$$

5 Conclusion

In this paper, we have presented a pricing model for Bull Spread options in a Mixed Modified Fractional Hull-White-Vasicek stochastic volatility and stochastic interest rate model. We have discretize the stochastic process with Milstein discretiation scheme and we have priced bull spread option by using Monte Carlo algorithm. We have used the double Mellin transform to study bull spread specific case of vulnerable bull spread options under stochastic volatility (Hull-white) and stochastic interest rates (Vasicek).

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