Pricing Mortgage-backed Securities

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- The buyer of a mortgage security is short a call option which the homeowner can exercise during the life of the loan using the prepayment option. Just like in the case of stock options, the correct pricing of this prepayment option involves generating different scenarios and calculating the corresponding payoffs in each of these scenarios. The option-adjusted spread (OAS) analysis takes into account several possible paths the interest rates can take in the future and generates corresponding cash flows and discount rates. The OAS is the constant spread over all these paths such that the average price equals market price.
- Mortgage valuations are significantly more complicated than those of non-callable and simple callable bonds in other fixed-income markets because there is no one-to-one correspondence between interest rate levels and prepayment speeds. Prepayment speeds on a mortgage pool usually depend on several variables including the actual path of interest rates, seasoning of the pool, home price appreciation and slope of the yield curve among other factors. The dependence of prepayment speeds on the actual path of interest rates makes the prepayment option path-dependent.
- In this paper, we discuss the shortcomings of yield and yield spread as indicators of relative value for mortgages. Next, we discuss various relative value and risk measures that are frequently used in the mortgage market. We then go through the details of an OAS calculation including a discussion of the theoretical foundations for the OAS measure, how it is calculated in practice, and why it provides a more robust framework than static measures for mortgage valuations. We also look at the contributions of different factors to MBS pass-through price changes. Finally, we wrap up this paper with a discussion on the differences between empirical and model durations.

I. Introduction

Mortgage-backed securities (MBS) are debt obligations that represent claims to the cash flows from a pool of mortgages. There are a variety of structures in the MBS market but the most common types are pass-through certificates, which entitle the holder to a pro-rata share of all principal and interest payments made on a pool of mortgages. More complicated MBSs like collateralized mortgage obligations (CMOs) and mortgage derivatives allocate the risks embedded in pass-through securities disproportionately among different classes. Although the importance of mortgage securities with embedded credit risk has been growing, in this primer we focus on the interest rate risk exposure of MBS and ignore the credit risk component.

In the U.S., residential mortgages can be prepaid in part or whole at any time. The homeowner's ability to prepay their mortgage makes pricing even simple pass-through MBS much more complex than valuing non-callable Treasury bonds. Pricing MBS is also significantly more complicated than simple callable bonds in other fixed-income markets because there is no one-to-one correspondence between interest rate levels and prepayment speeds. Prepayment speeds on a mortgage pool usually depend on several variables including the actual path of interest rates, seasoning of the pool, home price appreciation and slope of the yield curve among other factors which means that the prepayment option is path-dependent.[1] Before we delve into the mechanics of valuing a typical MBS, let us look at a few important characteristics of MBS investments.

Variability of Cash Flows from a Mortgage Pool

To highlight the impact of the prepayment optionality embedded in the cash flows of mortgage securities, we plot the projected monthly principal payments on FNCL 6s in three interest rate scenarios in Figure 1 (as of 9/13/2006). We assume that the FNCL 6s have the following collateral characteristics: 1 WALA, 6.62% GWAC, and $230K average loan size. We consider the following three interest-rate scenarios: a) A base case scenario with interest rate levels as of 9/13/2006 closes; b) An instantaneous shock which has interest rates 250 bps lower; and, c) An instantaneous shock which has interest rates 250bps higher.

Note that the timing of principal cash flows changes drastically as interest rates change. When rates rally, borrowers are more likely to prepay their mortgage early which means that an investor receives principal earlier. On the other hand, if rates sell off, borrowers are less likely to prepay which means that the investor receives principal later than expected. In order to quantify the cash flow variability, let us look at the weighted average life (WAL), which is defined as the average time period it takes to receive a dollar of principal, of the mortgage pool in the three scenarios. The WAL is 5.7 yrs in the base case while it shortens to 1.1 yrs in case of the 250 bps rally and lengthens to 11.3 yrs in case of the 250 bps backup.

Significance of Reinvestment Risk for MBS Analysis

Reinvestment risk refers to the possibility that the realized yield on an investment is not same as the yield expected by the investor at the time of their investment even if the investor holds the security to maturity. This risk arises because of fluctuations in interest rates during the holding period of the security which forces the investor to reinvest any intermediate cash flows at yields that are different from the yield on their initial investment. To explain the

[1] Please see *Residential Mortgages: Prepayments and Prepayment Modeling* for more details.

concept of reinvestment risk more precisely, we'll work through an example. Consider the following three risk-free (i.e., no default risk) bonds:

- *Bond A* is a zero-coupon 30-year bond
- *Bond B* is a non-callable 30-year bond with a coupon of 5%
- *Bond C* is a callable (at par) 30-year bond with a coupon of 5%

Assume these bonds will be held to maturity by the investor and the coupons are paid semi-annually. Let us also assume that the face value of each bond is $1000, the initial yield curve is flat at 5.5% and that each one of these bonds is priced based on their market yields (the actual prices of these securities are not relevant for our discussion). We consider the following three scenarios: a) A base case scenario in which interest rates remain unchanged over the next 30 years; b) Interest rates rally by 250 bps immediately after the investment is made; and c) Interest rates backup by 250 bps immediately after the investment is made. The investor reinvests cash flows received prior to the 30-year maturity period in Treasury strips at yields prevailing at the time of the receipt of each cash flow. i.e., the investor reinvests any cash received prior to the 30-year maturity period in 5.5% yielding Treasury strips in the base case, 3.0% yielding strips in the 250 bps rally case and 8.0% yielding strips in the 250 bps backup case (recall that we assumed a flat yield curve). How much cash will the investor have in hand in each one of the three scenarios at the end of the 30-year period (Figure 2)?

- With *Bond A* (a zero coupon risk-free bond), the investor receives $1,000 at the end of the 30-yr period in each one of the three interest rate scenarios and realizes the 5.5% return on his investment. i.e., the investor knows exactly how much cash he would own and what the return on this investment is going to be at the end of 30 years.
- With *Bond B* (a 5% coupon paying risk-free non-callable bond), the investor receives $1000 principal back at the end of the 30-year period and also receives $25 coupon payments at the end of each 6 month period (including the last 6 month period). The investor will have a total cash of $4720 in hand at the end of the 30-year period in the base case, $3405 in 250 bps rally case and $6950 in the 250 bps backup case.[2] This variation in returns is attributable to the different yield levels at which the investor is forced to reinvest semi-annual coupon payments. Thus, the investor bears substantial reinvestment risk by holding this security. The significance of reinvestment risk for a coupon paying non-callable bond depends on the time-to-maturity and the actual coupon of the underlying security.
- With *Bond C* (a 5% coupon paying risk-free callable bond), cash flows received during the 30-year investment period will be dependent on the interest rate scenario which is unlike the case with *Bond A* and *Bond B*. In the 250 bps rally case, the issuer will call the bond and the investor receives $1000 face value immediately after they buy the bond.[3] He will have to reinvest the proceeds at 3% yield and assuming that he reinvests in a non-callable 3% yielding bond, the investor will have a total cash of $2443 at the end of the 30-year period. On the other hand, the

[2] The future value of coupon payments may be calculated using the following formula: $S=(CP/y/2)*((1+y/2)^N-1)$, where CP is the semi-annual coupon payment, y is the reinvestment rate and N is the number of coupon payments.
[3] There may be some scenarios in which *Bond C* is not callable after 250 bps rally, (e.g. when volatility is so high that the callable bond is not worth par even after the 250 bps rally), but they are very unlikely.

total cash in hand at the end of the 30-year period in the base case and the 250 bps backup case is the same as for *Bond B*.

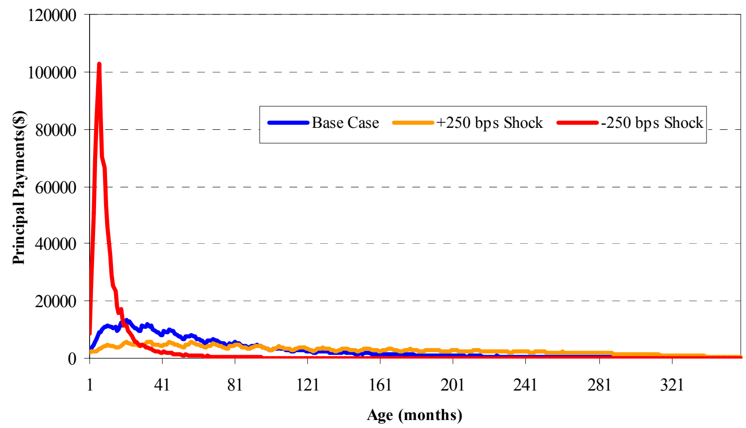
Thus, the coupon paying non-callable bond (*Bond B*) has more reinvestment risk than a zero-coupon bond (*Bond A*). Similarly, the callable bond (*Bond C*) has more reinvestment risk than the coupon paying non-callable bond (*Bond B*). Mortgage backed securities are essentially callable bonds and the realized return on the investment could be very different from the initial yield-to-maturity (YTM). We would like to point out that there are several additional complexities associated with MBS pass-throughs that are not captured by the above example. For instance: a) When interest rates back-up, mortgage prepayments slow down and cash flows are delayed which was not a factor in the above example; b) MBS pass-through is an amortizing security. i.e., face value of a security gradually changes over the life of the security and the pace of amortization itself changes with interest rate levels; and, c) Cash flows on an MBS security in any month will depend on the realized path of interest rates until that month during the life of the security.

These additional complexities mean that pricing MBS entails being able to capture complex evolution of interest rates and dependence of cash flows on the future evolution of interest rates. The goal of this primer is to present different relative value frameworks and discuss their strengths and shortcomings when used for mortgage securities. Special attention has been given to the Option Adjusted Spread (OAS) analysis owing to its popularity.

Outline of the Paper

The structure of this paper is as follows. We start by discussing the shortcomings of yield as an indicator of relative value for mortgages. Next, we discuss various relative value and risk measures used in the MBS market. We then go through the details of an OAS calculation including a discussion of the theoretical foundations for the OAS measure, how it is calculated in practice, and why it provides a more robust framework than static measures for mortgage valuations. We also look at the contributions of different factors to MBS pass-through price changes. Finally, we wrap up this paper with a discussion on the differences between empirical and model durations.

Figure 1: Projected Monthly Principal Cash Flows on FNCL 6s in Different Scenarios



Source: Banc of America Securities

Figure 2: Future Value of Cash Flows on Different 30-year Securities at Maturity

	250 bp rally	Base Case	250 bp Backup
Bond A	1,000	1,000	1,000
Bond B	3,405	4,720	6,950
Bond C	2,443	4,720	6,950

Source: Banc of America Securities

II. Understanding Key Valuation and Risk Metrics Relevant for MBS

Price vs. Yield Relationship for MBS Pass-throughs

To illustrate the differences between the characteristics of a non-callable bond and that of an MBS pass-through, we plot the price versus yield relationship for FNCL 6s and the 10-year Treasury in Figure 3. We assume that the Treasury yield curve is flat at 5% and the current coupon on the mortgage security is 6% which results in both the securities trading at par in the base-case scenario (assuming that the coupon on the 10-year Treasury is 5%).

Notice how the shape of the price versus yield relationship for the MBS (FNCL 6s) differs from that of the 10-year Treasury. While the price versus yield curve for the 10-yr Treasury looks almost like a straight line (actually slightly convex shape), the same curve for the MBS looks like an inverted convex shape. This divergence is caused by the optionality in the mortgage pass-through which prevents the price from appreciating at lower yield levels at the same pace as that of a non-callable bond. The prices of the callable and non-callable bonds (with the only difference between the two bonds coming from the optionality embedded in the callable bond) are related by the following equation:

$$P_{Callable} = P_{Non-Callable} - P_{Option}$$

As rates rally, the value of the prepayment option[4] keeps increasing as the bond is more likely to be called in such situations. This proves to be a risk for the investor since the borrower is more likely to prepay when rates are low, forcing the investor to re-invest the principal pay-downs and interest payments into a lower yielding security. On the other hand, if rates sell off, the borrower is less likely to prepay and the investor is unable to reinvest the principal and interest in higher yielding securities.

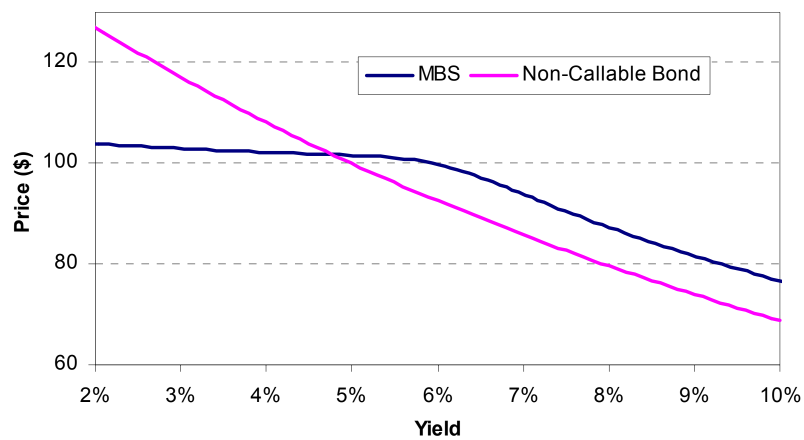
Another interesting observation from Figure 3 is that the price of FNCL 6s almost completely flattens out once yields rally substantially from the initial levels. To understand the reason behind this behaviour of price versus yield, let's consider a simple example. Assume that FNCL 6s are currently priced at 100-00 (total price) for October TBA settlement (October 12). In general, if rates rally from these levels, an investor would be willing to pay more for FNCL 6s. However, let us consider the extreme case in which interest rates rally by 300 bps and that there will be some 30-year FNMA 6s pools that will prepay completely in October (to be more precise, let's assume that all mortgages prepay on October 31). How much would you be willing to pay for FNCL 6s after the 300 bps rally in this case?

Essentially, an investor is going to receive a payment of \$100.5 (\$100 principal and 16 ticks of interest payment for October) on November 25 for an investment in \$100 face value of FNCL 6s for October settlement. An investor could also receive the same \$100.5 on November 25 by investing \$99.87 in Fed Funds on October 12 (The Fed Funds rate between 10/12 and 11/25 is assumed to be 5.25%). Thus the maximum an investor would pay for FNCL 6s for settlement on October 12 is only \$99.87 (full-price).[5] The price an investor would be willing to pay for FNCL 6s in this scenario is actually less than \$100 because of the time value of money (the investor is paying in October and receiving cash in November), but this is not the point we are trying to make here.

[4] This optionality arises in turn due to the ability of the borrower to prepay the mortgage before its maturity.
[5] We ignore changes in prices arising due to other risk characteristics of MBS.

The price of FNCL 6s did not appreciate at all in this example because of the callability feature embedded in underlying mortgages. Clearly, interest rates rallying by 300 bps immediately after an investor bought FNCL 6s and the entire FNMA 6s investment prepaying 100% in one month are very low probability scenarios. The important point here is that even though the price will appreciate in a rally, the increased prepayment risk reduces the value of the callable bond relative to that of a non-callable bond in this scenario.

Figure 3: Price versus Yield Relationship for MBS and Non-callable bonds



Source: Banc of America Securities

Yield to Maturity (YTM) and Nominal Spread

Almost all non-callable investment grade bonds in the U.S. are quoted in terms of a spread to Treasury securities. The spread between the yield of the mortgage security and the yield of a corresponding treasury security is called the **nominal spread** over Treasuries. The Treasury considered for comparison is the one whose time to maturity is close to the weighted average life of the MBS. Another commonly used nominal spread measure in the MBS market is the spread offered by an MBS over the swap rate of tenor equal to the WAL of the mortgage security and is called nominal spread over swaps. The difference between the nominal spreads over Treasuries and swaps is equal to the swap spread of the swap whose tenor is equal to the WAL of the MBS.

The major shortcoming of nominal spread as a measure of relative value comes from two sources: its failure to incorporate the effect of the shape of the yield curve and that of volatility levels in the market on MBS valuations. Let’s look at these two factors separately.

ZVOAS (Zero Volatility Option Adjusted Spread)

First, the nominal spread is calculated as the spread offered by MBS over a single point on the Treasury yield curve. Since the cash flows of a mortgage pass-through are typically distributed over several years, mortgage securities have risk exposure with respect to rate levels across the entire yield curve and a measure that gives cash flow spread over the entire curve will be a better measure of relative value than nominal spread. Thus, the MBS market uses what is called **zero volatility option-adjusted spread (ZVOAS)** defined below as a measure of cash flow spread of MBS over the yield curve.

Mathematically, ZVOAS is defined as follows:

$$\text{MBS Price} = \sum_{t=1}^{T} \frac{CF(t)}{(1+r(1)+z)*(1+r(2)+z)*......(1+r(t)+z)}$$

where z is the ZVOAS and *r(t)* is the short-term interest rate corresponding to the time period *t*. The ZVOAS gives us a better measure of incremental return than nominal spread since it calculates the spread over the whole zero curve rather than using just one point on the curve to discount MBS cash flows. The magnitude of the difference between the ZVOAS and nominal spread of a security depends on both the slope of the yield curve and the width of the cash flow window. All else being equal, this difference will be higher when the yield curve is steeper and cash flows are distributed over longer period of time.

We note that another spread measure called **Static Spread or Yield Curve Spread** (YCS) is used sometimes. The YCS and ZVOAS are distinguished from each other by noting that ZVOAS calculation accounts for the impact of interest rate evolution along the forward curve on projected cash flows while the YCS ignores it (i.e., while calculating YCS, prepayments are projected assuming that interest rates remain unchanged over the life of the security but discount rates account for the interest rate evolution along the forward curve). With ZVOAS, we account for the impact of interest rate evolution on both the projected cash flows and discount factors.

OAS (Option Adjusted Spread)

Now, let's look at the impact of volatility on MBS valuations. Although ZVOAS takes into account the shape of the yield curve and is an improvement over nominal spread, it is still a static measure since it assumes that interest rates follow the forward curve and ignores the day to day fluctuations in interest rates. The buyer of a mortgage security is short a call option, which the homeowner can exercise during the life of the loan using the prepayment option. Just like in the case of stock options, the correct pricing of this prepayment option involves generating different scenarios and calculating the corresponding payoffs in each of these scenarios. The OAS analysis takes into account the several possible paths that interest rates can take in the future and generates corresponding cash flows and discount rates. The OAS is the constant spread over all these paths such that the average present value of the cash flows discounted at this spread equals the market price.

To understand the impact of volatility on mortgage valuations, let us assume that current coupon mortgages offer yield spread of 100bps over the Treasury curve. In other words, all the cash flows of the mortgage security are discounted by adding 100bps to the corresponding yield on the Treasury curve (valued @100 bps nominal spread). We will now consider the following three interest rate scenarios:

- Rates remain unchanged with a probability of 50%
- Rates drop by 50bps right after we buy the MBS with a probability of 25%
- Rates rise by 50bps right after we buy the MBS with a probability of 25%

The Long-Term prepay rates and mortgage prices corresponding to each of these scenarios have been calculated in the following table:

Change in Interest Rates	-50bps	0bps	+50bps
Long-term Prepay Speed	17% CPR	10% CPR	7% CPR
Present Value of MBS@100bps over Curve	101.8	100	97.1
Scenario Probability	25%	50%	25%
Average Price of MBS@100bps over Curve		99.7	
Spread over Curve in each scenario such that Average Price of MBS = Market Price = 100		90bps	

If we keep a constant nominal spread of 100bps to price the mortgage security in the three scenarios, the expected price comes out to be $99.7, but the actual price of the security is $100. If we reduce the spread to the curve to 90 bps instead of 100bps, the expected price of the MBS comes out to be $100. Thus, although the investor expected to earn 100 bps of spread over the Treasury curve if interest rates remain unchanged, the spread drops to 90 bps if they consider the three scenarios mentioned above. The difference between this adjusted spread and the original spread represents the cost of changing prepayments speeds (because of the moves in interest rates) to the MBS holder. In our example, the original spread is 100bps and the adjusted spread is 90bps resulting in a 10bps cost from changes in prepayment speeds. This loss of spread itself will change with the change in volatility of interest rates.

For example, if we anticipate more volatility in interest rates, we can use +/- 100bps scenarios instead of the +/- 50bps scenarios used in the previous example. We will then get an adjusted spread of 80bps and the compensation for the prepayment risk changes to 20bps as shown in the table below:

Change in Interest Rates	-100bps	0bps	+100bps
Long-term Prepay Speed	25% CPR	10% CPR	6% CPR
Present Value of MBS@100bps over Curve	102.2	100	94.1
Scenario Probability	25%	50%	25%
Average Price of MBS@100bps over Curve		99	
Spread over Curve in each scenario such that Average Price of MBS = Market Price = 100		80bps	

The conclusion we draw from these examples is that the yield and spreads offered by pass-through securities could differ significantly depending upon the sensitivity of mortgage cash flows to interest rate volatilities. The OAS attempts to account for the potentially negative impact of mortgage prepayments on MBS when interest rates fluctuate.

Mathematically, OAS is defined as the constant spread over a variety of interest rate paths such that the average present value of the underlying mortgage cash flows (for future interest rate scenarios) equals the market price of the security:

$$\text{MBSPrice} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\frac{CF(i,t)}{(1+r(i,1)+s)*(1+r(i,2)+s)*......(1+r(i,t)+s)}$$

where s is the OAS, N is the total number of interest rate paths, T is the term of the MBS, CF(i,t) is the cash-flow at time t in scenario number i and $r(i,j)$ is the corresponding forward rate for interest rate scenario i and time period j. Note that OAS is the average spread over several likely interest rate paths while ZVOAS is the spread over one interest rate path (along the forward curve). A detailed description of the procedure used for calculating OAS is given in the following section.

Mortgage Price Sensitivity to Interest Rates and Volatility

Effective Duration

As the price versus yield relationship shown in Figure 3 demonstrates, prices of mortgage pass-throughs increase when interest rates rally and decrease when interest rates backup.[6] This is no different from a typical non-callable Treasury, agency or corporate bond. What really is different is that the most commonly used duration measure called the modified duration is not of much use in MBS analysis. The duration measure relevant for the MBS analysis is called **effective duration** and is defined as the percentage change in the price of a security for a 100 bps parallel shift in interest rates.

$$Duration\,(D) = -\frac{1}{P}\frac{dP}{dy} \approx -\frac{P^+ - P^-}{2 * P * \Delta y}$$

where P^+ is the price of the security for a $+\Delta y$ shock to the input curve, P^- is the price for a $-\Delta y$ shock and P is the price for the baseline case (P^+ and P^- are calculated at the same OAS as the OAS in the baseline case). For a non-callable security, cash flows remain unchanged with interest rate levels in the above calculation, but for a mortgage security future cash flows in the $y+\Delta y$ and $y-\Delta y$ yield scenarios will be different from each other and from the cash flows in the base case scenario. Thus, the fair prices at higher and lower yield levels need to consider not only the changes in discount factors but also the changes in cash flows. A related measure of effective duration is called **effective dv01** which is simply the change in the price of a mortgage security for 1 bp change in interest rates.

Effective Convexity

Let's suppose that an investor tries to estimate the price of a mortgage security at different interest rate levels using effective duration. Figure 4 compares the actual and the predicted prices using effective duration through the following formula:

$$P_{new} = P_{old} - D * P_{old} * \Delta y$$

where P_{old} is the price at the initial yield level (y_o=6%), D is the effective duration at y_o and P_{new} is the price at the new yield level ($y_{new}=y_o +/- \Delta y$). As shown in Figure 4, when rates rally (y_{new}=4.5%), the price of the MBS doesn't increase by as much as indicated by the duration measure. Similarly, when rates increase (y_{new}=7.5%), the price of the MBS declines by more than what is implied by the effective duration.[7] Consequently, the average of the MBS prices at two different yields (4.5% and 7.5%) is less than the price of the MBS at the average of the two yields (6%). This brings us to the all important concept of negative convexity in MBS analysis.

Convexity of a security refers to the curvature in its price versus yield relationship. Mathematically, it is defined as follows:

[6] There are some mortgage securities whose prices actually decline when interest rates rally and rise when interest rates backup. In general, mortgage pass-throughs almost always have positive durations.

[7] We recognize that effective duration of a mortgage security is a local measure of interest rate sensitivity. It gives good approximation of price changes for only small changes in interest rates and is not a good approximation for a 150 bps move. This difference does not impact our conclusions here.

$$Convexity\ (C) = \frac{1}{P}\frac{d^2P}{dy^2} \approx -\frac{D^+ - D^-}{2 * \Delta y}$$

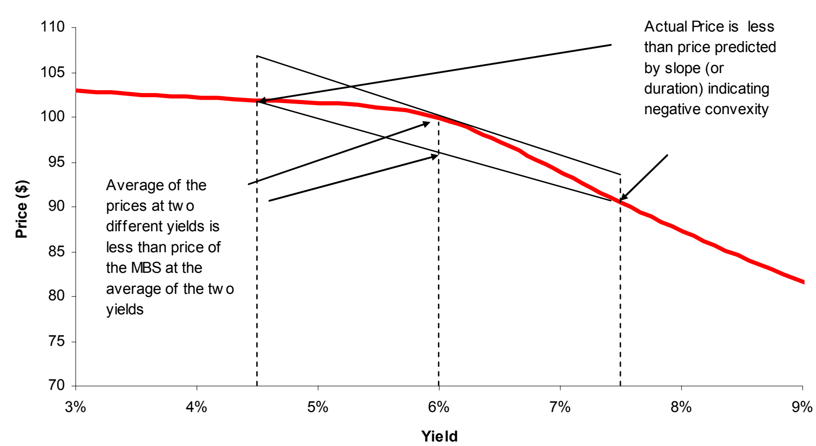
where D^+ and D^- are durations when rates move up and down by Δy, respectively. The second order approximation of the price of mortgage bond using duration (D) and convexity (C) is given by the following formula:

$$P_{new} = P_{old} - D * P_{old} * \Delta y + \frac{1}{2} C * P_{old} * (\Delta y)^2$$

Non-callable securities like Treasury bonds have positive convexity, which results in more price appreciation during rallies and less price depreciation during sell offs (Figure 5). On the other hand, a mortgage pass-through bond usually does not benefit as much when rates rally and gets hurt more when rates sell off. Consequently, as rates rally, the price of a mortgage pass-through bond appreciates less than the price appreciation implied by its effective duration, whereas for the non-callable bonds the price appreciates more than that implied by its duration. The opposite happens when rates sell off, i.e., the price of the mortgage bond decreases by more than that implied by duration while the price of the non-callable bond decreases by less than that implied by its duration. This phenomenon occurs because of the negative convexity of mortgage pass-through securities.

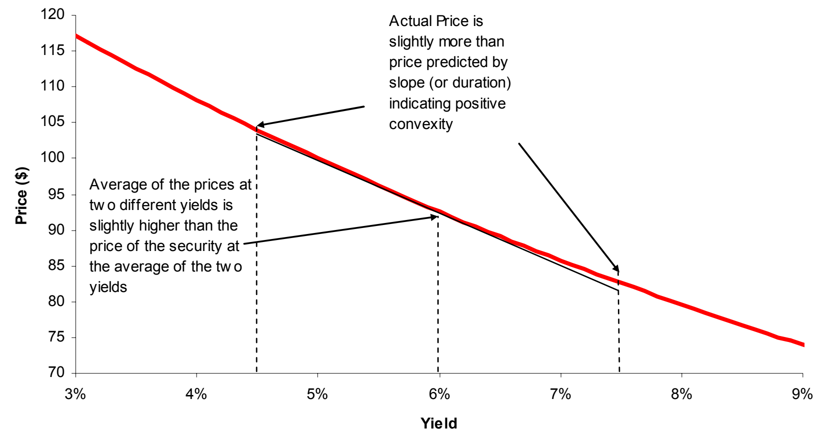
The convexity measure also gives an indication of how the duration of a bond changes with the level of rates. Graphically, duration is the slope of the tangent at the point (y, P(y)) on the price versus yield curve. The limitation of duration for hedging a mortgage bond should be clear by noticing the "convexity" or curvature of the price-yield curve (Figure 4) because of which a linear approximation (using the slope of the curve at a point) does not suffice. This means that hedging a mortgage security solely by durations would leave substantial residual exposure leading to unexpected profits/losses.

Figure 4: Actual and Estimated Prices of MBS at Different Yield Levels



Source: Banc of America Securities

Figure 5: Actual and Estimated Prices of a Non-callable Bond at Different Yield Levels



Source: Banc of America Securities

Key Rate Durations (Partial Durations)

So far, the discussion about duration and convexity proceeded as if there is only one interest rate of relevance for our analysis. Of course, mortgage cash flows are distributed over a long period of time and prepayment speeds actually depend on the shape of the yield curve. Thus mortgage valuations are exposed to interest rate levels across the entire yield curve. In addition, the relative exposure of a security to different parts of the yield curve keeps changing as interest rates move. The definitions used for effective duration and effective convexities can also be used for calculating durations and convexities with respect to different key rates to estimate the exposure of MBS to different parts of the yield curve.[8]

For instance, Figure 6 shows the duration exposures of different 30-year and 15-year Fannie Mae MBS as of 9/15/2006 to four key rates (2-yr, 5-yr, 10-yr and 30-yr rates). In general, higher coupons have more exposure to the shorter end of the curve relative to lower coupon securities (in terms of the percentage of total duration exposure). Similarly, 15-year pass-throughs have relatively higher exposure to the shorter end of the curve relative to 30-year pass-throughs.

Figure 6: Partial Durations of 30-yr and 15-yr Passthroughs as of 9/15/2006

30-yr FNMA Passthroughs				
	2yr	5yr	10yr	30yr
FNCL 4.5s	0.5	1.2	3.0	1.2
FNCL 5.0s	0.6	1.1	2.5	0.9
FNCL 5.5s	0.8	1.1	1.9	0.6
FNCL 6.0s	0.9	0.9	1.1	0.2
FNCL 6.5s	1.0	0.6	0.3	-0.1

15-yr FNMA Passthroughs				
	2yr	5yr	10yr	30yr
FNCI 4.0s	0.5	1.6	2.4	0.2
FNCI 4.5s	0.7	1.5	1.8	0.1
FNCI 5.0s	0.8	1.4	1.5	0.0
FNCI 5.5s	0.9	1.2	1.0	-0.1
FNCI 6.0s	1.0	1.0	0.4	-0.2

Source: Banc of America Securities

[8] Please see "*Fixed Income Securities: Tools for Today's Markets*" by Bruce Tuckman for more details on calculating key rate durations.

Current Coupon Spread Duration

The **current coupon spread** is the excess spread offered by the current coupon mortgage over the Treasury/swap curve. The intuition behind the current coupon spread duration of a mortgage security is as follows: A change in current coupon mortgage spread is a proxy for the change in the mortgage rate a borrower sees in the primary market. Thus, all else being equal, when the current coupon spread widens, mortgage rates have effectively increased even if reference interest rates in the market didn't change. The opposite occurs when the current coupon spread tightens. The change in mortgage rate impacts prepayment speeds and hence cash flows of mortgage securities which in turn could move their prices even with unchanged Treasury/swap rates. The Current Coupon Spread duration measures the price impact resulting from a change in the current coupon spread.

Volatility Duration

The **volatility duration** measures the sensitivity of the price of a security to changes in implied volatility. The procedure used for calculating volatility duration is similar to the one used for effective duration where P^+ and P^- are calculated using the price of the security for a +/- Δy bp shock to the implied volatility level. While we will move forward without getting into the intricate details of the volatility exposure of MBS, a few points are worth noting:

- First, there is no single implied volatility level in the market. Market participants usually talk of a volatility surface which gives implied volatilities of swaptions/straddles with a range of expirations and maturities. Implied volatilities in the caps market are also important for some MBS (e.g inverse floaters and IOs).
- Second, the relative exposure of mortgage securities to different parts of the volatility surface varies from one security to another and a rigorous analysis needs to account for this variation. For instance, the volatility exposure of a current coupon 5/1 hybrid ARM security is closer to that of a 2yr*5yr straddle than a 3yr*7yr straddle while the opposite will be the case with 30-year mortgages.
- Last, several term structure models use at-the-money (ATM) straddles for daily calibration, but the volatility exposure of some mortgage pools may be more closely linked to that of deep in-the-money and deep out-of-the-money straddles.

OAS Duration

The **OAS duration** measures the sensitivity of the price of a security to changes in the OAS and is defined as follows:

$$OAS\ Duration \approx -\frac{P^+ - P^-}{2 * P * \Delta y}$$

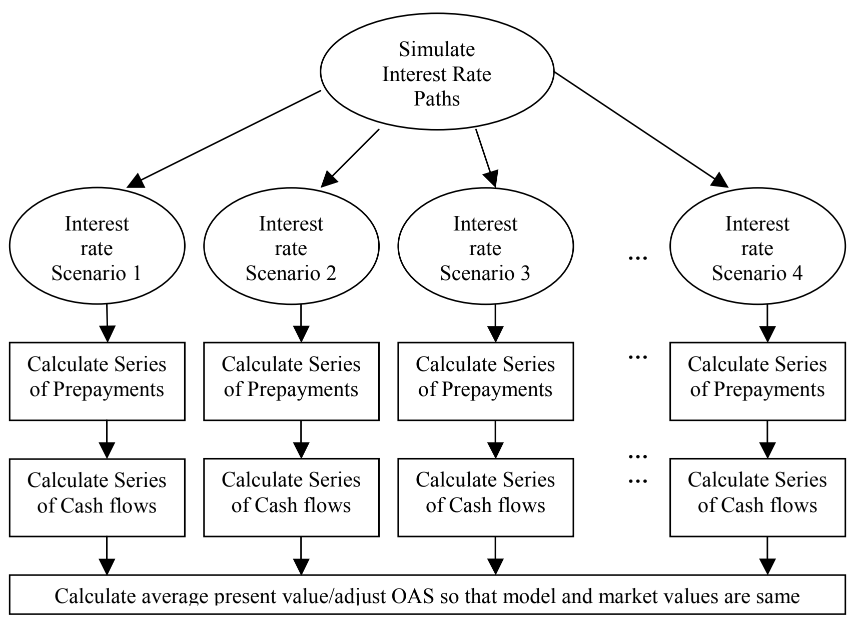
where P^+ is the price of the security for a $+\Delta y$ change in the OAS, P^- is the price Δy change in the OAS and P is the price in the baseline case. Note that this definition of OAS Duration is very similar to the definition of effective duration given in the prior section. The main difference comes from the fact that actual interest rate levels are shifted while calculating effective duration (which changes interest rate paths and projected cash flows in the up and down scenarios) but only OAS is changed while calculating OAS duration (which changes only discount factors and keeps cash flows along each path unchanged from the base case).[9]

[9] Some market participants refer to Effective Duration as OAS Duration.

III. Description of OAS Calculations

A consistent theme in the discussion so far is that cash flows from mortgage pass-throughs are strongly dependent on the level of interest rates. Cash flows are also dependent on the path of interest rates because prepayment speeds in any month depend on the set of interest rates that a mortgage pool has been exposed to up until that month. The OAS methodology tries to account for the potentially negative impact of prepayments on the value of a mortgage security. An OAS model for valuing MBS typically accounts for the level and shape of the yield curve, correlations between yields of different maturities and volatility levels in the market on the value of mortgage cash flows. Figure 7 outlines the procedure for calculating the OAS and we will look at the each step of the OAS calculation procedure in detail below.

Figure 7: OAS Calculation Process



Source: Banc of America Securities

The OAS valuation of mortgage securities is usually done using the Monte-Carlo approach. The Monte-Carlo approach generates several interest rate paths and averages the prices of mortgage securities along these paths. The OAS is the constant spread over all these paths such that the average price equals market price. There are three important components of an OAS model:

- **Interest Rate Model:** A term structure model that generates interest rate paths such that there are no arbitrage opportunities in the interest rates and volatility markets.
- **Current Coupon Model:** A model that projects current coupon mortgage rates along each interest rate path. The current coupon model essentially links the interest rates market with the mortgage market.
- **Prepayment Model**: A prepayment model that could capture the variation of

prepayments on a given mortgage pool in different interest rate scenarios.

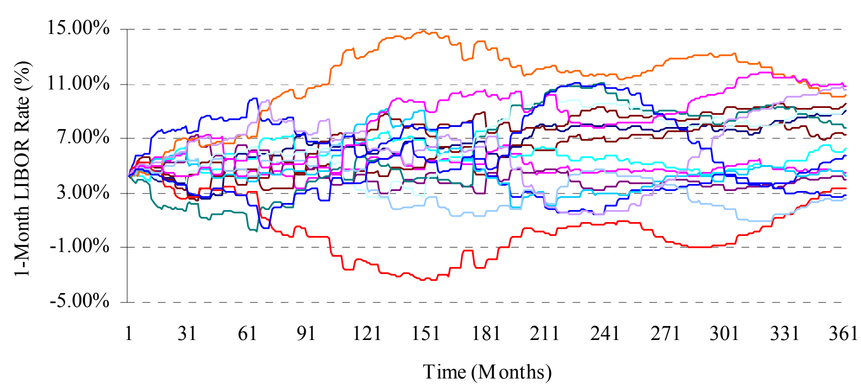
Step#1:Simulate Future Interest Rate Scenarios

The price of a mortgage security is heavily influenced by future interest rate predictions and the MBS valuation process relies on the interest rate paths generated by the term structure model. Apart from being realistic, these simulated paths need to be consistent with the current and future market expectations. The term structure model used in MBS valuations provides a framework to connect the interest rate sensitivity of mortgages to other liquid products like treasuries, swaps and swaptions.

Our RMBS desk uses the Brace-Gatarek-Musiela (BGM) interest rate model to simulate future interest rate scenarios for the pricing and valuation of mortgage securities. The model takes into account the current yield curve along with the correlations between various forward rates to simulate future rates. Another important feature of the interest rate model is its ability to capture the volatility surface. The "volatility surface" refers to the dependence of yield volatility on maturity and the option expiration period. Various interest rate derivative products like caps and swaptions, the pricing of which are contingent on market expectations of interest rate movements, give us the implied volatility of forward rates. The three-factor BGM model used by our desk is calibrated daily to 225 Swaption and 6 Cap volatilities.

Our BGM model generates 512 interest rate paths daily for the pricing of mortgage-backed securities. Figure 8 shows a set of 16 sample interest rate paths generated by our model.

Figure 8: Sample Simulation of 1-month LIBOR Rates



Source: Banc of America Securities

Step#2: Calculate a Series of Prepayments for Each One of These Scenarios

As discussed above, our term structure model generates 512 interest rate paths. These are paths of 1-month LIBOR rates, but mortgage cash flows depend on mortgage rates rather than on 1-month LIBOR rates. Thus we need a mortgage rate model ("current coupon model") to obtain a set of mortgage rates along each interest rate path. The current coupon mortgage rate so generated will be the mortgage rate in the secondary market. This secondary market rate in turn needs to be converted into the primary mortgage market rate that borrowers are likely to see. The primary mortgage rate is usually calculated by adding a constant spread to secondary mortgage rates. Note that this spread itself may change with time – when refinancing volumes are very high and/or hedging costs are high, originators tend to keep this spread high and the opposite occurs when originator pipelines are thin.

We use the simulated mortgage rates along each path to calculate the refinancing incentive for all future months and use the refinancing incentive along with several other variables to project prepayment speeds along each path of interest rates. For a detailed discussion of prepayments and prepayment models, please refer to our primer titled "*Residential Mortgages: Prepayments and Prepayment Modeling*".

Step#3: Calculate a Series of Cash-flows for Each Scenario

Using the prepayment speeds projected along each interest rate path, our analytics system estimates the total principal and interest payments in each month. For structured products, the cash flow calculation also takes into account the allocation of prepayments across the different tranches in the structure.

Step#4: Calculate Average Present Value and Adjust OAS Such that Market Price Equals Model Price

Following Steps 1-3, we have the cash flows and corresponding short-term interest rates for each path. The next step is to calculate the discounted price of the mortgage security along each one of the 512 interest rate paths. The average of these prices will give us the model price and the model price will only co-incidentally be equal to the market price. To adjust the model price so that it is equal to the market price, we introduce an incremental spread of δ to all the simulated interest rates. In other words, instead of using the simulated 1-month LIBOR rates, we use (1-month LIBOR+ δ) to discount all the cash flows along each interest rate path. The value of δ such that the market price equals the model price will give us the OAS.

What Does OAS Mean?

Conceptually, OAS is the spread an investor could earn versus the benchmark curve after hedging the prepayment risk involved in MBS. Figure 9 shows LOASs (OAS with respect to the Libor/swap curve) of 30-year and 15-year FNMA MBS as of 9/15/2006. Notice that all the MBS pass-throughs shown in this table have negative OASs. This does not necessarily imply that these securities are rich or cheap. Since the OAS is heavily model-dependent, the negative Libor OAS numbers might be occurring purely because of the model assumptions. However, when the current OAS for a particular coupon is compared to historical values, it usually gives a good estimate of how relatively rich or cheap the coupon has become.

There are a few other arguments for why LOAS of mortgage pass-throughs could be negative. First, dollar rolls offer funding levels that are usually lower than 1-month LIBORs by a few basis points which compensates the investor for the negative OAS numbers. Second, LOAS is calculated with respect to the LIBOR/swap curve. It is reasonable to think that agency MBS pass-throughs have less credit risk than the risk involved in swap transactions (although MBS have negative convexity risk unlike swaps) and hence MBS could trade tighter than swaps when marginal buyers expect interest rates to remain range-bound.

The Option Cost

ZVOAS is essentially the OAS calculated by plugging a volatility of zero in the Monte Carlo simulations. The interest rate path then becomes deterministic and the ZVOAS just becomes the spread earned by the MBS if forward rates were to be actually realized. The difference between the ZVOAS and the OAS gives us the *Option Cost*. It is defined as follows:

OC = ZVOAS – OAS

The option cost represents how much of the ZVOAS is a compensation for the impact of the future variability of interest rates. Like OAS, it is measured in basis points over the reference yield curve. The greater the option cost, the cheaper a bond would generally look using classical yield analysis (static yield, yield to WAL).

Figure 9: Valuation Metrics for Agency MBS as of 9/15/2006

30-yr FNMA Passthroughs							
	Yield	WAL	N Spread	OAS	ZVOAS	Duration	Convexity
FNCL 4.5s	5.7	8.7	38.4	-1.3	31.2	5.8	-0.6
FNCL 5.0s	5.8	7.9	49.7	-5.2	42.9	5.1	-1.5
FNCL 5.5s	5.9	7.5	58.8	-8.5	52.5	4.2	-2.0
FNCL 6.0s	6.0	6.0	72.9	-9.5	66.5	3.1	-2.4
FNCL 6.5s	5.9	3.6	69.4	-10.4	63.9	1.7	-2.4

15-yr FNMA Passthroughs							
	Yield	WAL	N Spread	OAS	ZVOAS	Duration	Convexity
FNCI 4.0s	5.3	5.9	4.5	-20.3	-1.7	4.6	-0.3
FNCI 4.5s	5.4	5.4	16.5	-13.4	10.3	4.0	-0.7
FNCI 5.0s	5.5	5.5	23.5	-14.7	17.9	3.6	-1.2
FNCI 5.5s	5.6	5.3	33.7	-17.1	28.2	2.9	-1.7
FNCI 6.0s	5.7	4.6	42.9	-15.8	37.5	2.2	-2.0

Source: Banc of America Securities

Treasury OAS versus Libor OAS

The OAS is usually calculated off the Treasury or the LIBOR/swap curves (sometimes using the agency curve too). Using a LIBOR curve instead of a Treasury curve in the OAS calculation allows us to net out the effect of swap spreads on mortgages directly instead of studying the correlation between swap spreads and Treasury OASs.

IV. Understanding Mortgage Price Movements

Mortgage Price Change Attribution to Different Factors

The market price of a mortgage security will change on a day-to-day basis depending on market conditions. These price changes can be attributed to various inputs that our analytics system uses for pricing MBS, i.e., the actual change in the price of a mortgage security can be captured using the sensitivity of the mortgage bond to the following factors:

- Changes in the level of interest rates and carry (includes curve shape also)
- Convexity losses
- Changes in Current Coupon spreads
- Changes in implied volatility levels
- OAS Changes

Ideally, changes in the price of a mortgage security due to these factors should add up to the actual change in the market price of the security. We work through an example to elaborate on this idea. The FNCL 6s were trading at 99-21+ on August 8th 2006 (for August TBA settlement) and at 100-04+ on September 8th 2006 (for September TBA settlement). Thus, there was a price appreciation of 15 ticks for this security over this one month period. Let us look at the contributions of different factors to this price change.

Figure 10 shows a screen snapshot of our analytics system SoFIA and lists various risk/sensitivity measures for FNCL 6s as of August 8th 2006. Figure 11 lists the sensitivity of FNCL 6s to interest rates, volatility[10], current coupon spread, OAS and their respective contributions to the price change over this one month period. The price change due to each factor is given by:

$$\Delta P_{factor} = -(DV01)_{factor} * \Delta(factor)$$

The price change due to the convexity of the security is given by:

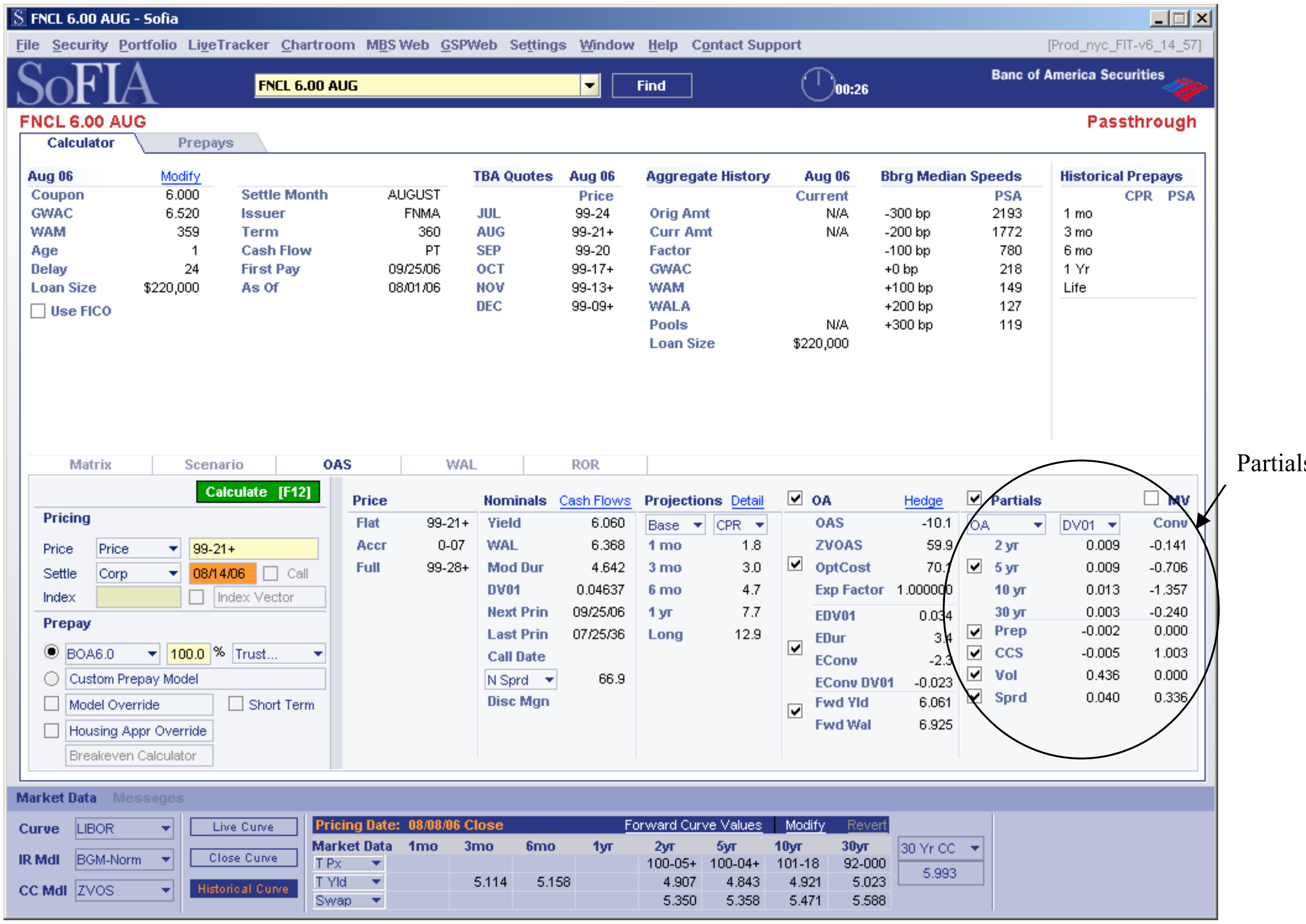
$$\Delta P_{convexity} = \frac{1}{2} * C * P * (\Delta y)^2$$

We add up the convexity losses on the security with respect to each individual key rate (i.e. the 2-year, 5-year, 10-year and 30-year rates) to estimate total change in price due to convexity. The contribution of different factors to price changes adds up to 14.4 ticks which is very close to the actual price change of 15 ticks.[11] Note that the relative contribution of different factors to the price change varies from one time period to the other. In the current example, current coupon spread, volatility and OAS changes did not make a noticeable contribution to the price change, but in some other periods the same factors may explain almost all the price movement.

[10] Note that SoFIA computes volatility DV01 as the change in the price of a security for a 10bps change in implied volatility of the 3yr*7yr swaption. We divide this by 10bps to get the actual volatility DV01 (which is used in Figure 11)

[11] In this example, carry and roll-down contributions to price changes are ignored as they are extremely small in the current flat yield curve scenario.

Figure 10: Obtaining Partial DV01s from SoFIA



Source: Banc of America Securities

Figure 11: Mortgage Price Attribution for FNCL 6s

Attribute	DV01	8-Aug	8-Sep	Change (bps)	Price Attribution (ticks)
2yr	0.009	5.35	5.21	-13.6	3.9
5yr	0.009	5.36	5.19	-16.5	4.8
10yr	0.013	5.47	5.30	-17.0	7.1
30r	0.003	5.59	5.42	-16.6	1.6
CCS	-0.005	59.7	62.7	3.0	0.5
Volatility	0.0436	86.9	87.3	0.3	-0.5
OAS	0.04	-10.1	-10.8	-0.7	0.9
Eff. Conv.	-2.3				-3.7
				Total	14.4

Source: Banc of America Securities

Empirical versus Model Durations

As discussed previously, mortgage prices move with changes in various market factors like interest rates, volatility, current coupon spread and OAS. We have already discussed mortgage price sensitivity measures that allow us to isolate the effect of the aforementioned factors on mortgage prices. Using these measures, an investor can choose to limit their exposure to OAS only by hedging exposure to all the other risk factors or they may choose to hedge only the interest rate exposure.

There are two ways of measuring the sensitivity of mortgage prices to different factors in the market – Model Durations and Empirical Durations. Model Durations are estimated by shocking individual factors and calculating changes in the value of mortgage securities at a constant OAS (see section II for details). On the other hand, empirical durations are calculated by regressing market prices of a mortgage security against a particular factor (for example the 10-year Treasury yield). Since empirical durations with respect to yield levels (specifically, the 10-year Treasury yield) are the most commonly talked about empirical duration measures, we focus on this measure here.

If ΔP and Δy are the changes in the actual MBS prices and the 10-year Treasury yields respectively, the regression equation for the empirical duration is given by:

$$\frac{\Delta P}{P} = a - b * \Delta y$$

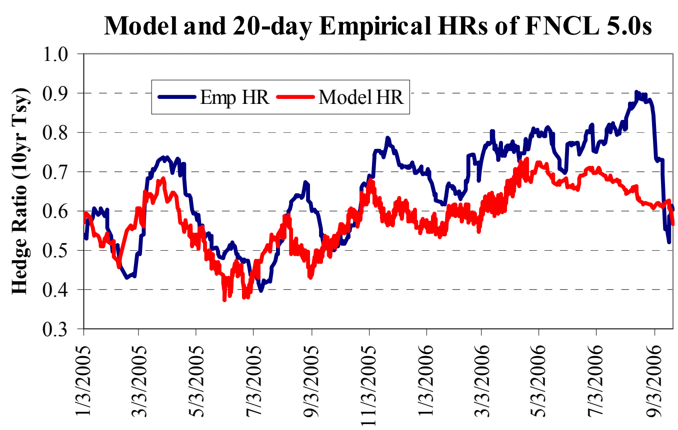
where b is the empirical duration.

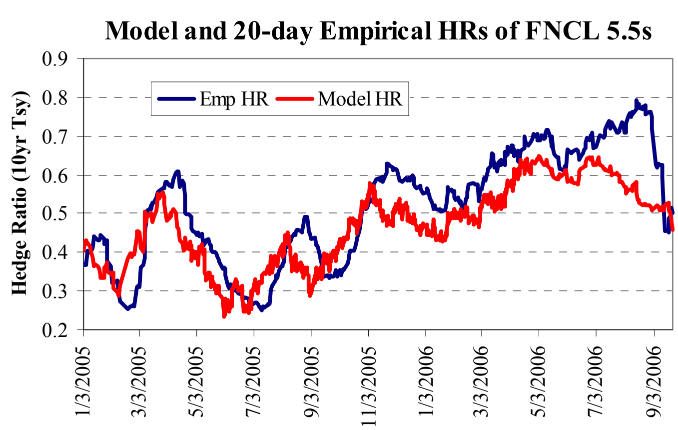
There are three important differences between model and empirical durations.

- Model durations are forward looking, but empirical durations are "backward looking" since we are looking at historical price and yield movements to estimate empirical durations. Thus, whenever the current level of yields is substantially different from the yield levels prevailing over the regression period, empirical duration may not be a good measures of the actual duration of the security.
- For calculating model durations, we change only one factor at a time and keep all the other variables the same in our simulations. On the other hand, we can't impose this restriction strictly while estimating empirical durations. For instance, if we are trying to estimate duration of a security with respect to the 10-year Treasury yield, we want to change only the 10-year Treasury yield and keep all variables in the model unchanged. This can be easily accomplished while estimating model durations, but it could be a very difficult condition to impose while estimating empirical durations. This difficulty arises because the shape of the curve, implied volatility levels and OAS may be correlated with the direction of the 10-year Treasury yield in the market. (These correlations can be different over different time periods.) Because all of these additional factors also impact the price of an MBS, the relationship between mortgage price changes and changes in 10-year Treasury yields derived in the regression model may not be because of changes in 10-year Treasury yields alone.
- Whether it is more appropriate to use model durations or empirical durations for hedging purposes depends on the holding period of the investor and investors' belief about the continuation of past relationships between different factors into the future.

Figure 12 provides a comparison between model and empirical hedge ratios since the beginning of 2005. Empirical hedge ratios can sometimes differ substantially from model hedge ratios either because of technical factors or because the market's expectations of prepayments differ from those of the model used for estimating model durations. For instance, 30-year discount MBS traded 10%-40% longer than our model hedge ratios over several months starting in November 2005. We attribute this to the market's expectations for a substantial slowdown in home price appreciation and housing turnover speeds.[12] This relationship reversed since the beginning of September (i.e., mortgages have been trading shorter than our model durations). We believe this is because the market is concerned about the potential for bank selling of MBS in a scenario where rates continue to rally. Consequently, mortgages have been underperforming when rates rally and outperforming when rates back-up. The important point here is that empirical durations price-in both the fundamental factors like market's expectations of prepayment speeds and the technical factors arising from supply and demand technicals.

Figure 12: Model versus 20-day Empirical Hedge Ratios





Source: Banc of America Securities

[12] Please see the *RMBS Trading Desk Strategy Report* published on 3/24/2006 for more details.

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