PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM

Today's Objectives:

Students will be able to:

1. Calculate the linear momentum of a particle and linear impulse of a force.
2. Apply the principle of linear impulse and momentum.

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Linear Momentum And Impulse
- Principle of Linear Impulse And Momentum
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. The linear impulse and momentum equation is obtained by integrating the ______ with respect to time.

   A) friction force
   B) equation of motion
   C) kinetic energy
   D) potential energy

2. Which parameter is not involved in the linear impulse and momentum equation?

   A) Velocity
   B) Force
   C) Time
   D) Acceleration

APPLICATIONS

A dent in an automotive fender can be removed using an impulse tool, which delivers a force over a very short time interval.

How can we determine the magnitude of the linear impulse applied to the fender?

Could you analyze a carpenter's hammer striking a nail in the same fashion?

APPLICATIONS
(continued)

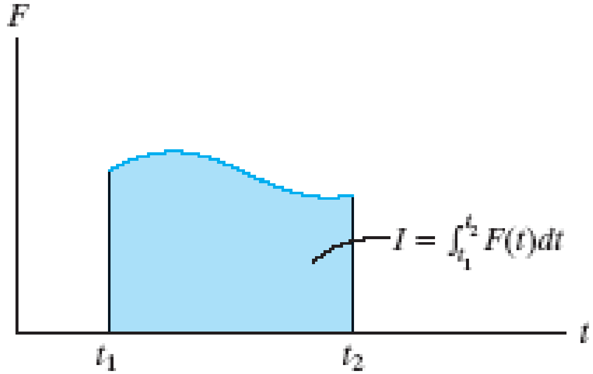
Sure! When a stake is struck by a sledgehammer, a large impulsive force is delivered to the stake and drives it into the ground.

If we know the initial speed of the sledgehammer and the duration of impact, how can we determine the magnitude of the impulsive force delivered to the stake?

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM (Section 15.1)

Linear momentum: The vector m***v*** is called the linear momentum, denoted as ***L***. This vector has the same direction as ***v***. The linear momentum vector has units of (kg·m)/s or (slug·ft)/s.

Linear impulse: The integral ∫***F*** dt is the linear impulse, denoted ***I***. It is a vector quantity measuring the effect of a force during its time interval of action. ***I*** acts in the same direction as ***F*** and has units of N·s or lb·s.



Variable Force

The impulse may be determined by direct integration. Graphically, it can be represented by the area under the force versus time curve. If ***F*** is constant, then

$$\boldsymbol{I} = \boldsymbol{F}\,(t_2 - t_1)\,.$$

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM
(continued)

The next method we will consider for solving particle kinetics problems is obtained by integrating the equation of motion with respect to time.

The result is referred to as the principle of impulse and momentum. It can be applied to problems involving both linear and angular motion.

This principle is useful for solving problems that involve force, velocity, and time. It can also be used to analyze the mechanics of impact (taken up in a later section).

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM
(continued)

The principle of linear impulse and momentum is obtained by integrating the equation of motion with respect to time. The equation of motion can be written

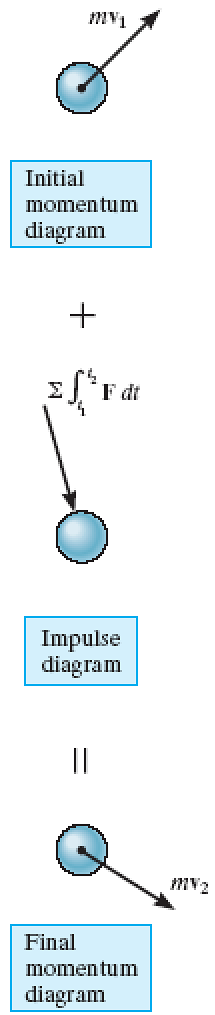
$$\sum \boldsymbol{F} = m\,\boldsymbol{a} = m\,(d\boldsymbol{v}/dt)$$

Separating variables and integrating between the limits $\boldsymbol{v} = \boldsymbol{v}_1$ at $t = t_1$ and $\boldsymbol{v} = \boldsymbol{v}_2$ at $t = t_2$ results in

$$\sum \int_{t_1}^{t_2} \boldsymbol{F}\,dt = m \int_{\boldsymbol{v}_1}^{\boldsymbol{v}_2} d\boldsymbol{v} = m\boldsymbol{v}_2 - m\boldsymbol{v}_1$$

This equation represents the principle of linear impulse and momentum. It relates the particle's final velocity, $\boldsymbol{v}_2$, and initial velocity ($\boldsymbol{v}_1$) and the forces acting on the particle as a function of time.

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM (continued)



The principle of linear impulse and momentum in vector form is written as

$$m\boldsymbol{v}_1 + \sum \int_{t_1}^{t_2} \boldsymbol{F}\, dt = m\boldsymbol{v}_2$$

The particle's initial momentum plus the sum of all the impulses applied from t_1 to t_2 is equal to the particle's final momentum.

The two momentum diagrams indicate direction and magnitude of the particle's initial and final momentum, $m\boldsymbol{v}_1$ and $m\boldsymbol{v}_2$. The impulse diagram is similar to a free body diagram, but includes the time duration of the forces acting on the particle.

IMPULSE AND MOMENTUM: SCALAR EQUATIONS

Since the principle of linear impulse and momentum is a vector equation, it can be resolved into its x, y, z component scalar equations:

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

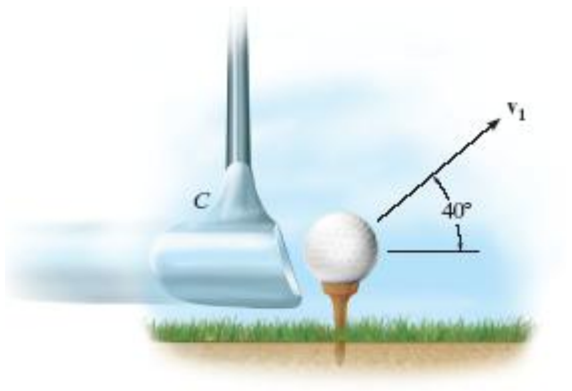
$$m(v_z)_1 + \sum \int_{t_1}^{t_2} F_z \, dt = m(v_z)_2$$

The scalar equations provide a convenient means for applying the principle of linear impulse and momentum once the velocity and force vectors have been resolved into x, y, z components.

PROBLEM SOLVING

- Establish the x, y, z coordinate system.
- Draw the particle's free body diagram and establish the direction of the particle's initial and final velocities, drawing the impulse and momentum diagrams for the particle. Show the linear momenta and force impulse vectors.
- Resolve the force and velocity (or impulse and momentum) vectors into their x, y, z components, and apply the principle of linear impulse and momentum using its scalar form.
- Forces as functions of time must be integrated to obtain impulses. If a force is constant, its impulse is the product of the force's magnitude and time interval over which it acts.

EXAMPLE



Given: A 40 g golf ball is hit over a time interval of 3 ms by a driver. The ball leaves with a velocity of 35 m/s, at an angle of 40 . Neglect the ball’s weight while it is struck.

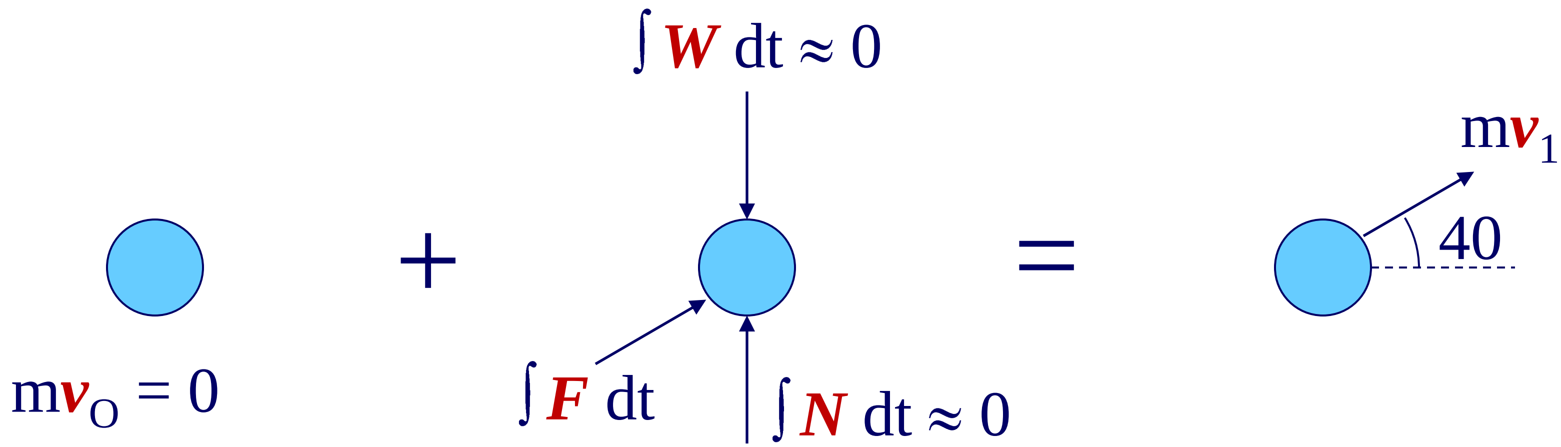
Find: The average impulsive force exerted on the ball and the momentum of the ball 1 s after it leaves the club face.

Plan:
1) Draw the momentum and impulsive diagrams of the ball as it is struck.
2) Apply the principle of impulse and momentum to determine the average impulsive force.
3) Use kinematic relations to determine the velocity of the ball after 1 s. Then calculate the linear momentum.

EXAMPLE
(continued)

Solution:

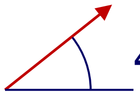
1) The impulse and momentum diagrams can be drawn:



The impulse caused by the ball's weight and the normal force $\boldsymbol{N}$ can be neglected because their magnitudes are very small as compared to the impulse of the club. Since the initial velocity ($\boldsymbol{v}_O$) is zero, the impulse from the driver must be in the direction of the final velocity ($\boldsymbol{v}_1$).

EXAMPLE
(continued)

2) The principle of impulse and momentum can be applied along the direction of motion:

 40

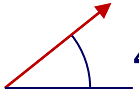
$$mv_O + \sum \int_{t_0}^{t_1} F\, dt = mv_1$$

The average impulsive force can be treated as a constant value over the duration of impact. Using $v_O = 0$,

$$0 + \int_{0}^{0.003} F_{avg}\, dt = mv_1$$

$$F_{avg}(0.003 - 0) = mv_1$$

$$(0.003)\, F_{avg} = (0.04)(35)$$

$$F_{avg} = 467 \text{ N}$$  40

EXAMPLE
(continued)

3) After impact, the ball acts as a projectile undergoing free-flight motion. Using the constant acceleration equations for projectile motion:

$$v_{2x} = v_{1x} = v_1 \cos 40° = 35 \cos 40° = 26.81 \text{ m/s}$$

$$v_{2y} = v_{1y} - gt = 35 \sin 40° - (9.81)(1) = 12.69 \text{ m/s}$$

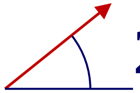
$$=> \quad \boldsymbol{v}_2 = (26.81\, \boldsymbol{i} + 12.69\, \boldsymbol{j}) \text{ m/s}$$

The linear momentum is calculated as $\boldsymbol{L} = m\, \boldsymbol{v}$.

$$\boldsymbol{L}_2 = m\boldsymbol{v}_2 = (0.04)(26.81\, \boldsymbol{i} + 12.69\, \boldsymbol{j}) \text{ (kg·m)/s}$$

$$\boldsymbol{L}_2 = (1.07\, \boldsymbol{i} + 0.508\, \boldsymbol{j}) \text{ (kg·m)/s}$$

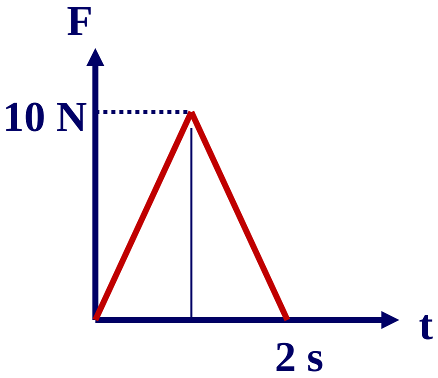
$$\boldsymbol{L}_2 = 1.18 \text{ (kg·m)/s}$$



25.4

CONCEPT QUIZ

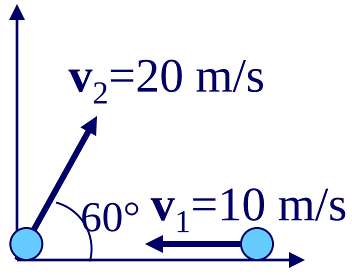
1. Calculate the impulse due to the force.



A) 20 kg·m/s   B) 10 kg·m/s

C) 5 N·s   D) 15 N·s

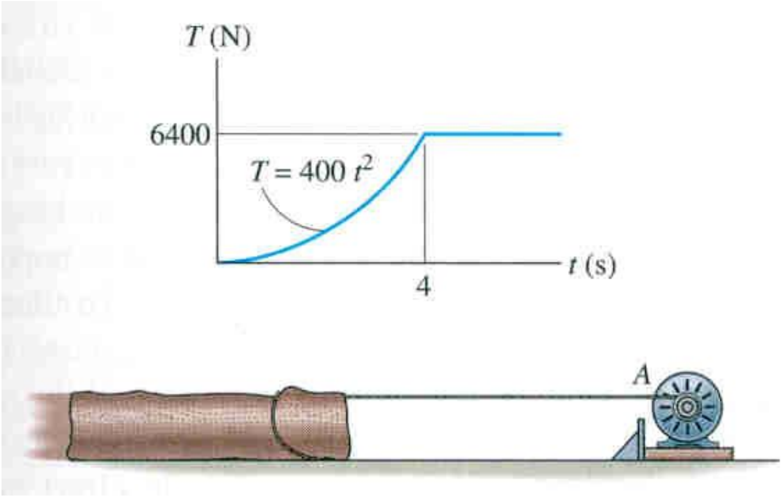
2. A constant force ***F*** is applied for 2 s to change the particle's velocity from $\mathbf{v}_1$ to $\mathbf{v}_2$. Determine the force ***F*** if the particle's mass is 2 kg.



A) (17.3 ***j***) N   B) (–10 ***i*** +17.3 ***j***) N

C) (20 ***i*** +17.3 ***j***) N   D) (10 ***i*** +17.3 ***j***) N

GROUP PROBLEM SOLVING



Given: The 500 kg log rests on the ground (coefficients of static and kinetic friction are μ_s = 0.5 and μ_k = 0.4). The winch delivers a towing force ***T*** to its cable at A as shown.

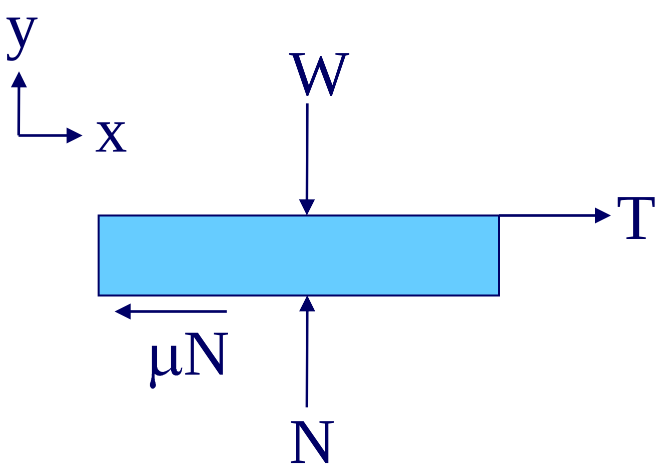
Find: The speed of the log when t = 5 s.

Plan:
1) Draw the FBD of the log.
2) Determine the force needed to begin moving the log, and the time to generate this force.
3) After the log starts moving, apply the principle of impulse and momentum to determine the speed of the log at t = 5 s.

GROUP PROBLEM SOLVING
(continued)

Solution:

1) Draw the FBD of the log:



$\sum F_y = 0$ leads to the result that $N = W = mg = (500)(9.81) = 4905$ N. Before the log starts moving, use μ_s. After the log is moving, use μ_k.

2) The log begins moving when the towing force T exceeds the friction force $\mu_s N$. Solve for the force, then the time.

$$T = \mu_s N = (0.5)(4905) = 2452.5 \text{ N}$$
$$T = 400\, t^2 = 2452.5 \text{ N}$$
$$t = 2.476 \text{ s}$$

Since $t < 4$ s, the log starts moving before the towing force reaches its maximum value.

GROUP PROBLEM SOLVING
(continued)

3) Apply the principle of impulse and momentum in the x-direction from the time the log starts moving at $t_1 = 2.476$ s to $t_2 = 5$ s.

$$\xrightarrow{+} \quad mv_1 + \sum \int_{t_1}^{t_2} F\,dt = mv_2 \quad \text{where } v_1 = 0 \text{ at } t_1 = 2.476 \text{ s}$$

$$0 + \int_{2.476}^{5} T\,dt - \int_{2.476}^{5} \mu_k N\,dt = mv_2$$

$$\int_{2.476}^{4} 400t^2\,dt + \int_{4}^{5} 6400\,dt - \int_{2.476}^{5} (0.4)(4905)\,dt = (500)v_2$$

$$(400/3)t^3 \Big|_{2.476}^{4} + (6400)(5 - 4) - (0.4)(4905)(5 - 2.476) = (500)v_2$$

$$\Rightarrow \quad v_2 = 15.9 \text{ m/s}$$

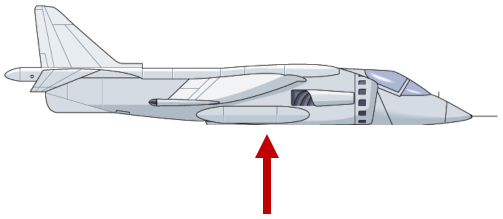
The kinetic coefficient of friction was used since the log is moving.

ATTENTION QUIZ

1. Jet engines on the 100 Mg VTOL aircraft exert a constant vertical force of 981 kN as it hovers. Determine the net impulse on the aircraft over t = 10 s.

   A) -981 kN·s B) 0 kN·s

   C) 981 kN·s D) 9810 kN·s



2. A 100 lb cabinet is placed on a smooth surface. If a force of a 100 lb is applied for 2 s, determine the impulse from the force on the cabinet.

   A) 0 lb·s ↗ B) 100 lb·s ↗

   C) 200 lb·s ↗ D) 300 lb·s ↗