Principles of lasers

A. Conditions for oscillation
   - Amplifier with feedback
   - Condition on gain: threshold
   - Condition on phase:
     - longitudinal modes
   - Possible active modes

B. Laser gain
   - Laser cross section
   - Rate equations

C. Examples of laser media
   - 3 level systems
   - 4 level systems
   - semi-conductor lasers

D. Longitudinal modes
   - Possible modes
   - Single mode operation
   - Technical linewidth

E. Transverse modes
   - Diffraction losses
   - Transverse modes
   - Example: Hermite -Gauss

F. Laser : concentrated light
   - Concentration in space
   - Concentration in spectrum/time
   - Laser source: all photons in the same mode

Laser source (ex. ruby laser)

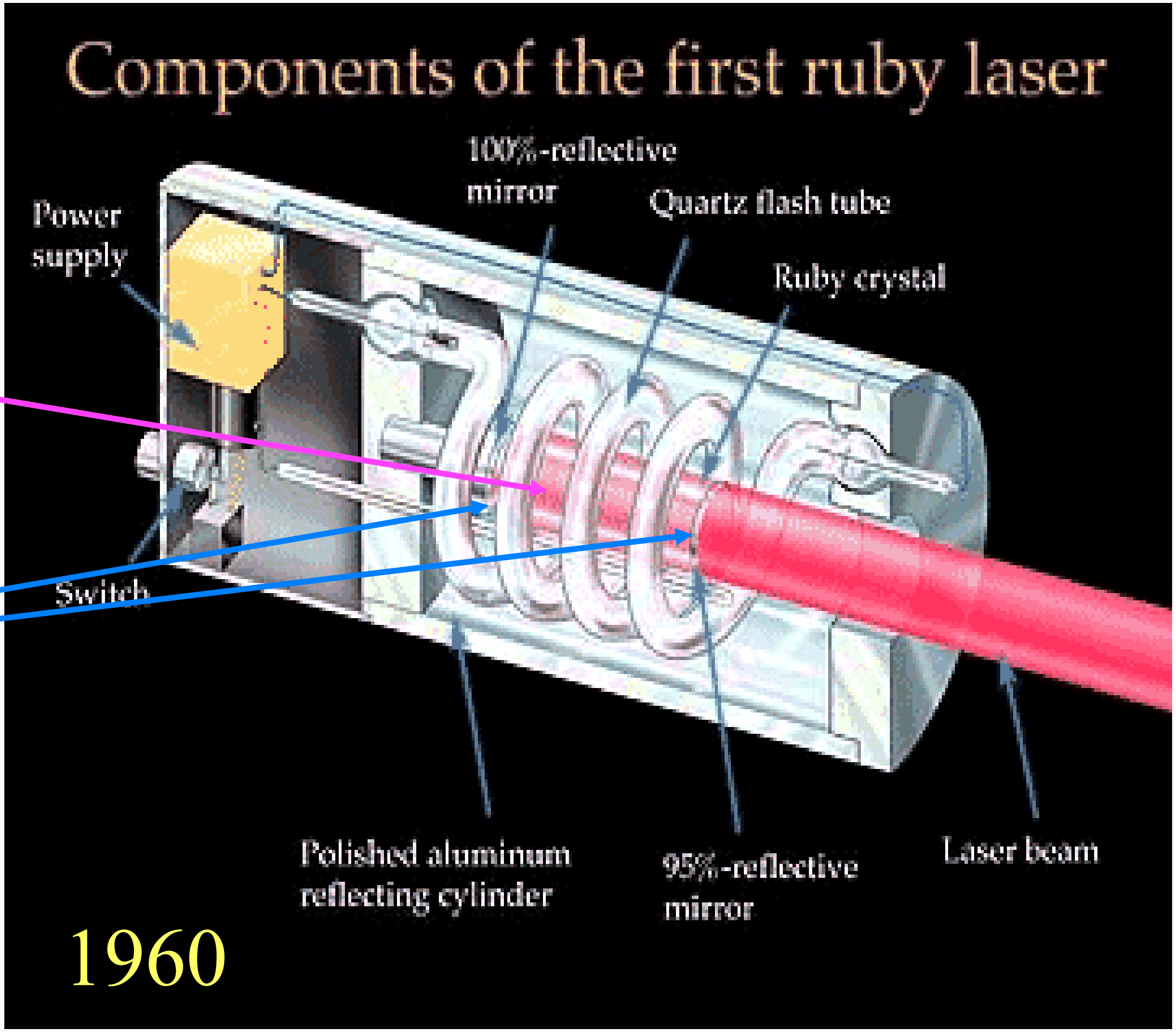
Light emitter

with

a laser amplifier

and

mirrors



Light Amplification by Stimulated Emission of Radiation

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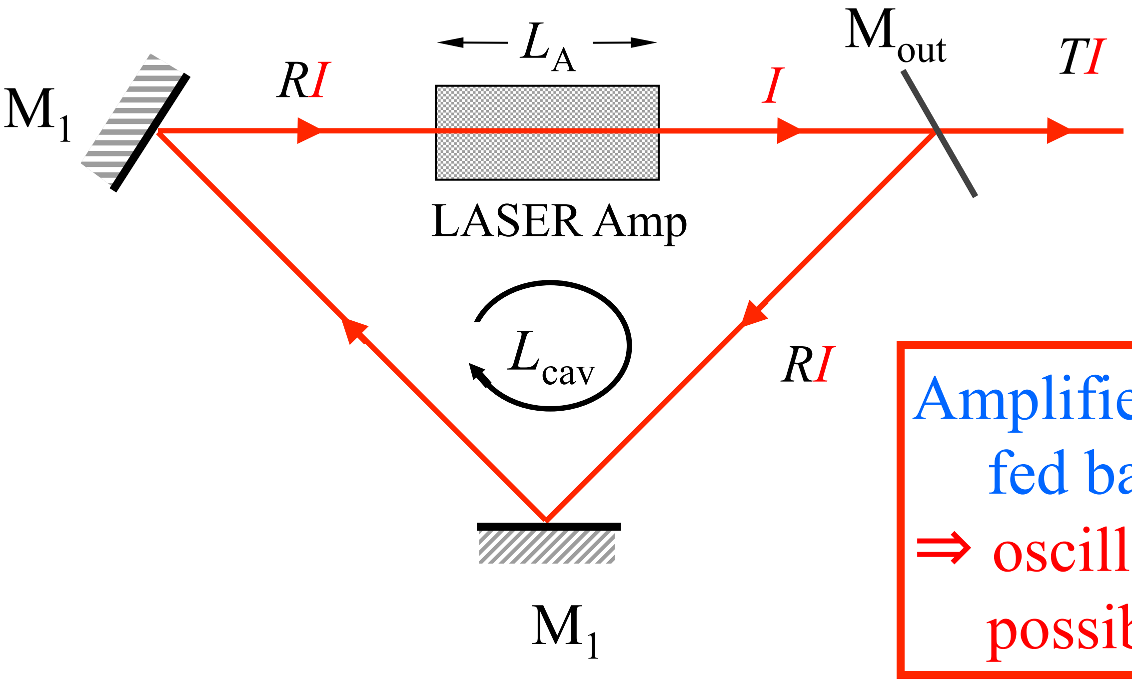
Laser oscillator: amplifier with feedback



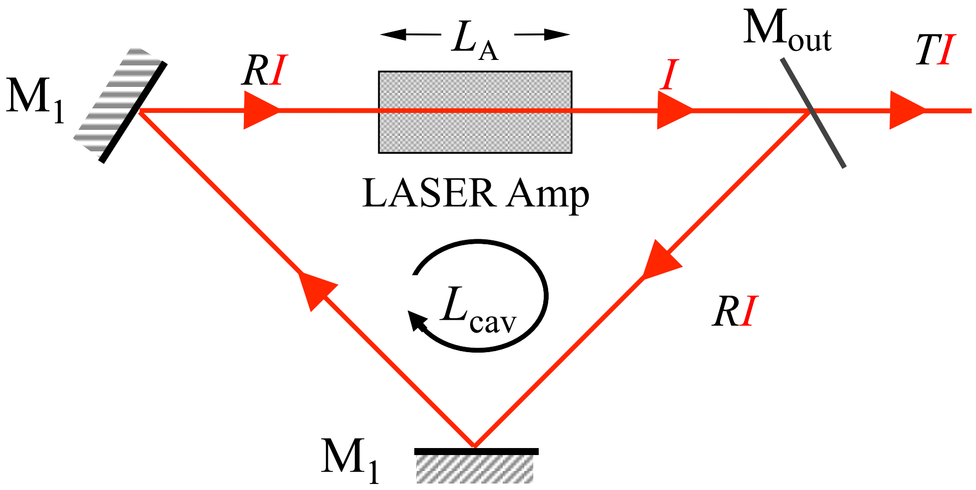
Laser medium:
amplifies the field
(keeping the phase)

Amplifier with output
fed back to input:
⇒ oscillation is
possible

Conditions for oscillation :

- the gain must be large enough to compensate for the losses
- the field must be fed back with the right phase

Condition on gain : threshold



Gain of the amplifier

When $I << I_{sat}$ non-saturated gain

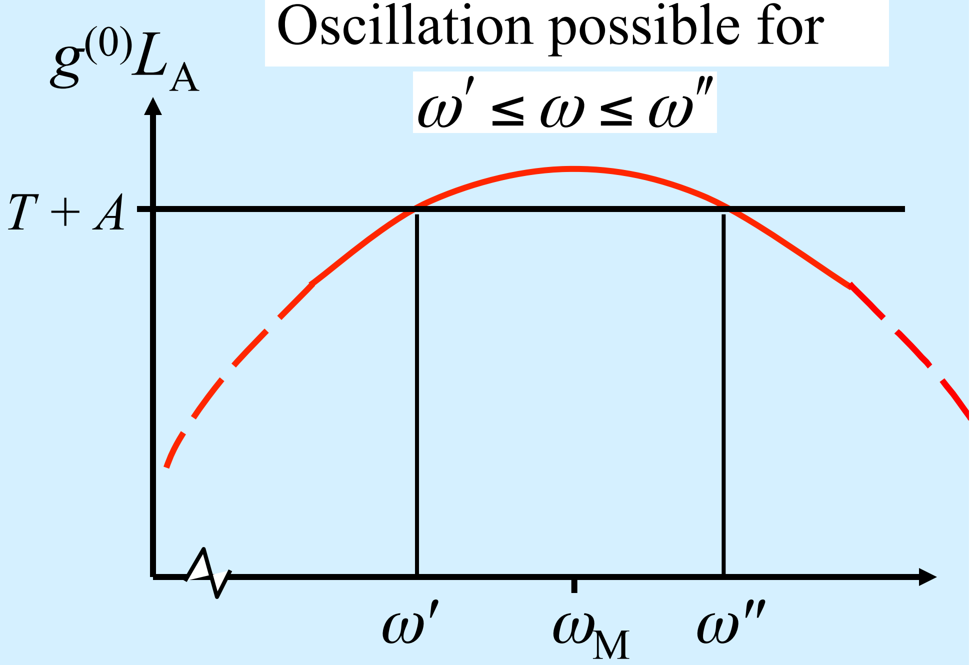
$$G^{(0)} = \frac{I_{out}}{I_{in}} \simeq \exp\left\{g^{(0)} L_A\right\} \approx 1 + g^{(0)} L_A$$

- resonant at $\omega_M = \omega_0 = \frac{|E_b - E_a|}{\hbar}$

Decreases with I (saturation) $g(I) = \frac{g^{(0)}}{1 + I/I_{sat}} \le g^{(0)}$

Oscillation possible for

$\omega' \le \omega \le \omega''$



Oscillation condition: threshold

$$G^{(0)}(1-T)(1-A) \ge 1$$

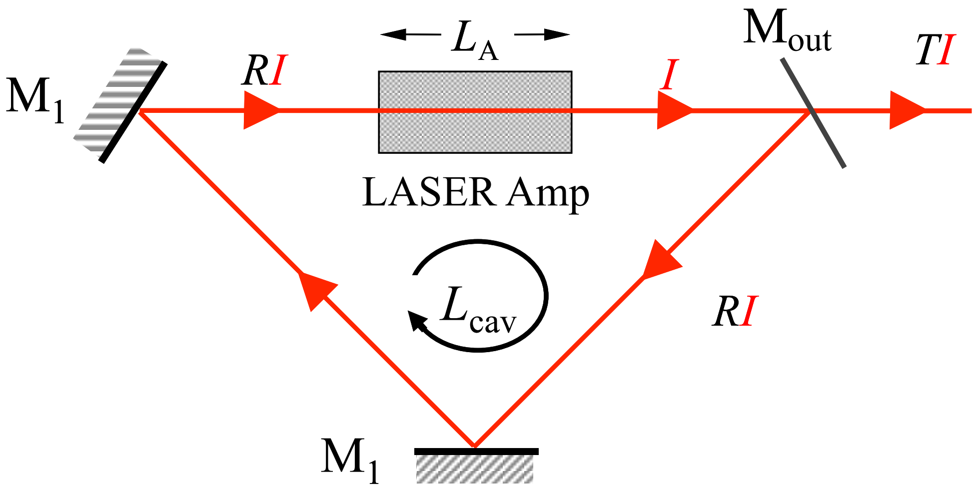
$$\boxed{g^{(0)} L_A \ge T + A}$$

non saturated gain

output coupling

absorption losses

Condition on phase : longitudinal modes



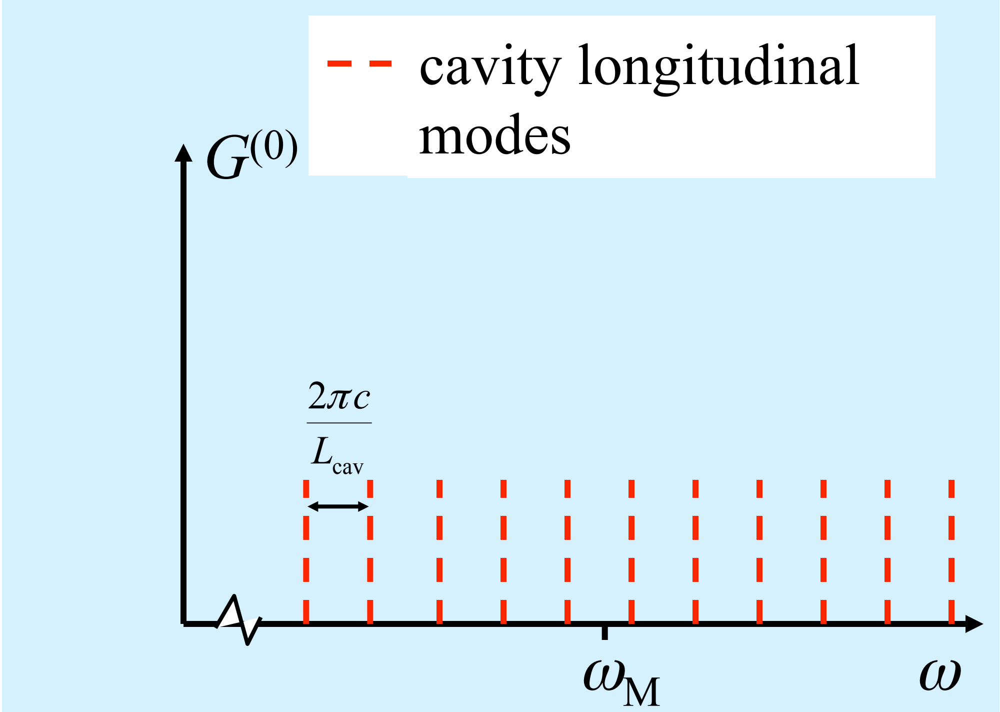
Cavity round trip optical length

$$L_{cav} = \oint n(r)\,dr$$

refractive index

Phase shift for 1 round trip

$$\phi = \frac{\omega}{c} L_{cav}$$



Feedback with right phase

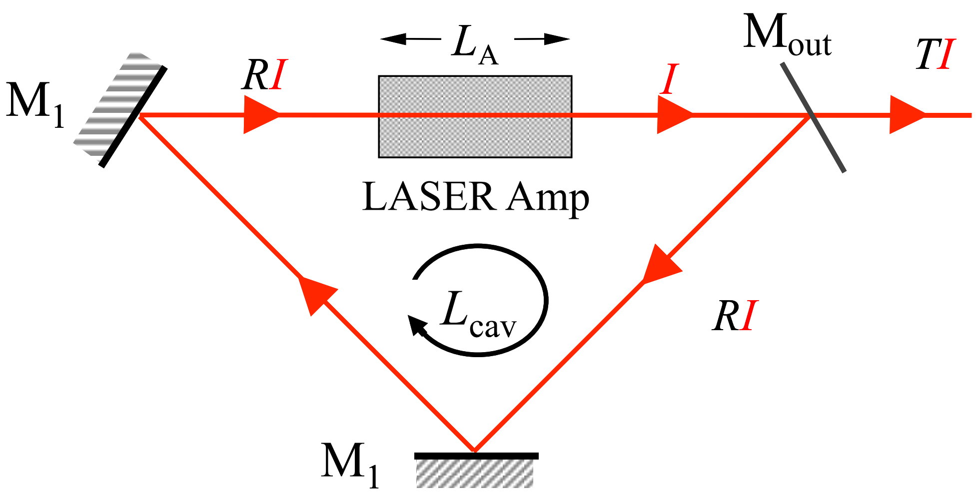
$$\phi = \frac{\omega}{c} L_{cav} = p\,2\pi$$

$$\boxed{\omega_p = p\,2\pi \frac{c}{L_{cav}}}$$

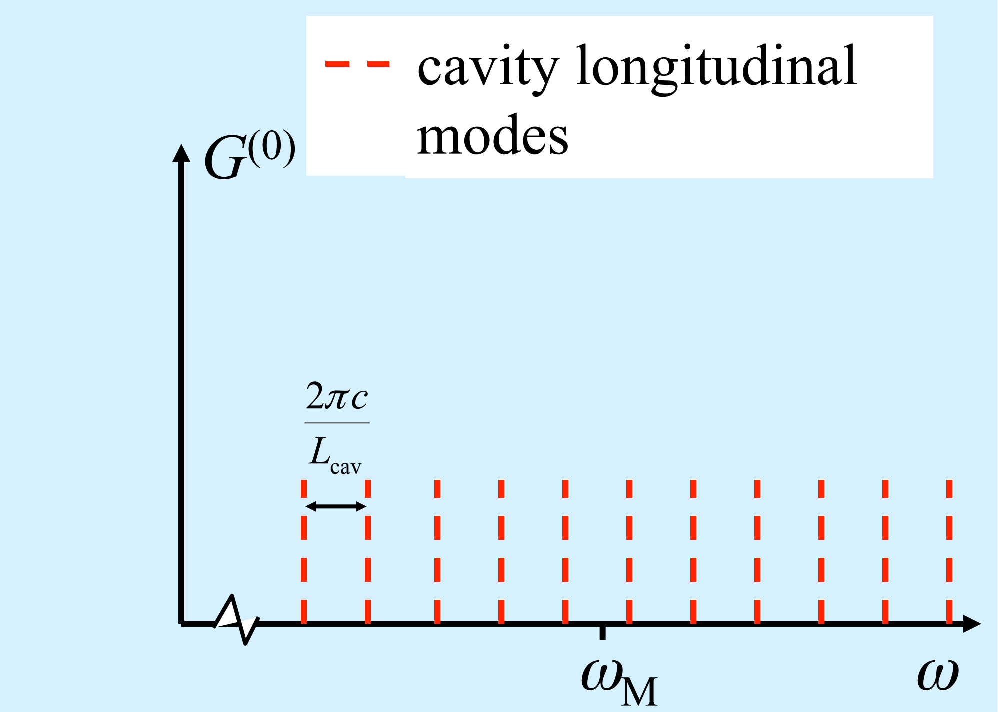
longitudinal mode frequency

integer

Condition on phase : longitudinal modes



Mode = stationary solution of propagation equations, with boundary conditions



Feedback in phase

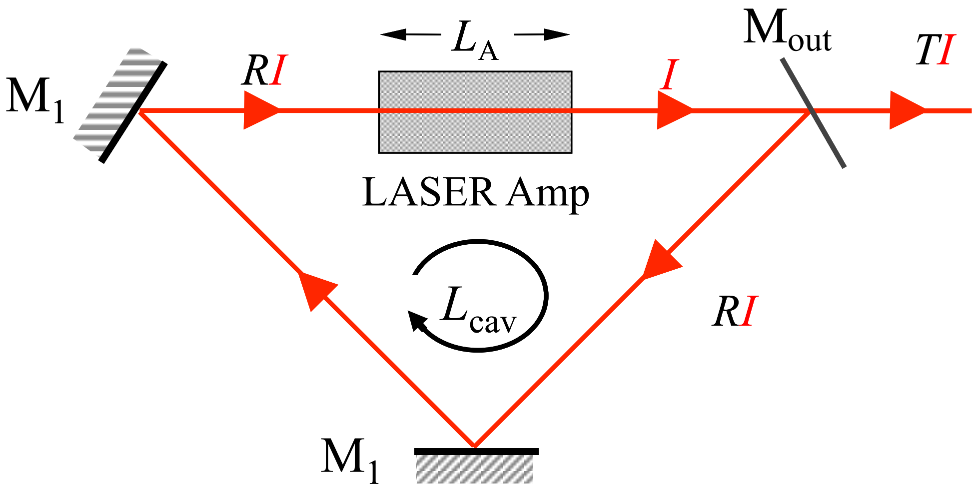
$$\phi = \frac{\omega}{c} L_{cav} = p\,2\pi$$

$$\boxed{\omega_p = p\,2\pi \frac{c}{L_{cav}}}$$

longitudinal mode frequency

integer

Both conditions: possible modes

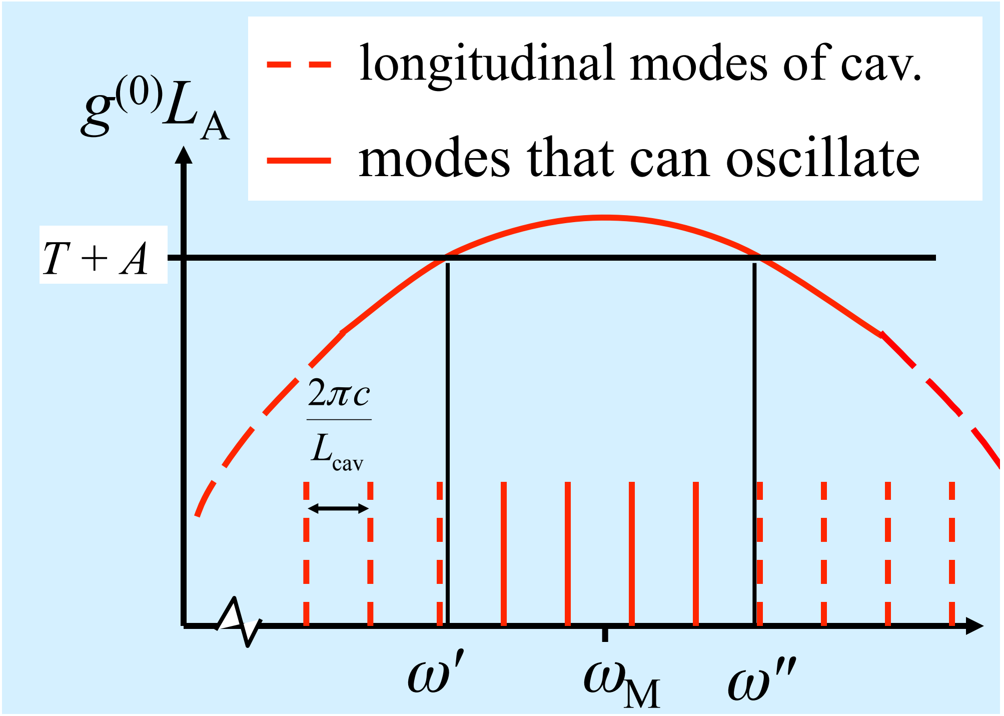


Condition on gain

$$\omega' \leq \omega \leq \omega''$$

In phase feedback

$$\omega_p = p\, 2\pi \frac{c}{L_{cav}} \quad , p \text{ integer}$$



Example
(Helium-Neon laser)

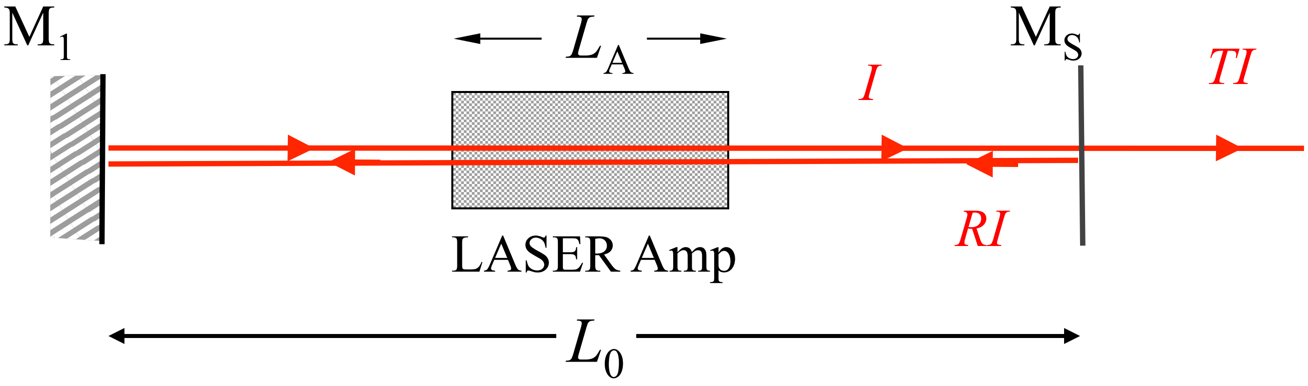
$$L_{cav} = 0.6\,\text{m} \Rightarrow \frac{c}{L_{cav}} = 5\times10^{8}\ \text{Hz}$$

$$(\omega' - \omega'')/2\pi \approx 2.5\text{GHz}$$

$\Rightarrow$ 4 to 5 active modes

$$\frac{\omega}{2\pi} \approx 5\times10^{14}\ \text{Hz} \Rightarrow p \approx 10^{6}$$

Linear cavity laser



Same principles as ring cavity laser

One can use ring laser results, with correspondance

$L_{\mathrm{cav}} = 2L_0$ cavity round trip optical length

$G = \left(G_{\mathrm{ampli}}\right)^2$ gain over 1 cavity round trip

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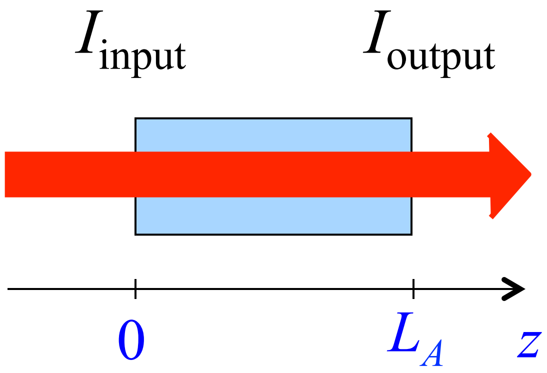
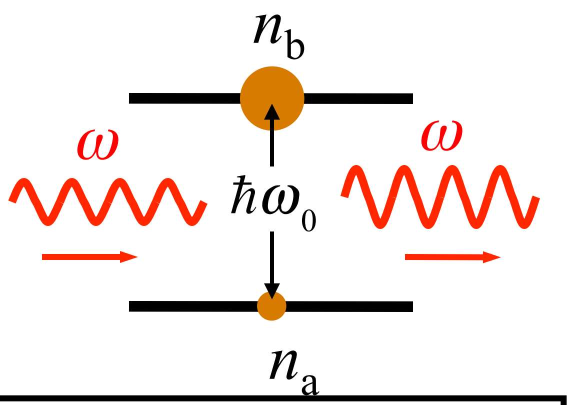
Laser gain (matter light interaction)



GAIN $\Leftrightarrow n_b > n_a$

population inversion



$$E(z,t) = E_0 \exp\{-k''z\}\cos(\omega t - k'z)$$

$$k'' = \frac{k'}{|k'|}\frac{\omega}{(1+\chi'/2)c}\frac{\chi''}{2}$$

$$\chi'' = (n_a - n_b)\frac{d^2}{\varepsilon_0 c}\boxed{\frac{\Gamma_D}{\Gamma_D{}^2+\delta^2}}$$

linear term

quantum term $\rightarrow$ $n_a - n_b = \dfrac{[n_a - n_b]^{(0)}}{1+s}$

Gain if population inversion

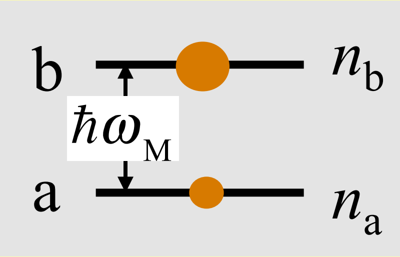
$$\boxed{g = \frac{1}{I}\frac{dI}{dz} = -2k'' \propto (n_b - n_a)}$$

$[L]^{-1}$ $\qquad$ $[L]^{-3}$

dimension equation $\left[\dfrac{g}{n_b - n_a}\right] = [L]^2$

surface : cross section

Laser cross section



Easy to use formula (dimensions)

$$g = \frac{1}{I}\frac{dI}{dz} = \sigma(\omega)\left(n_\mathrm{b} - n_\mathrm{a}\right)$$

laser cross section

$$\delta = \omega - \omega_\mathrm{M}$$

$$\boxed{\sigma(\omega) = \frac{\sigma(\omega_\mathrm{M})}{1+\delta^2/\Gamma_\mathrm{D}{}^2}}$$

Cross section: useful to write rate equations for photons and atoms

Lamb toy model

$$\sigma(\omega_\mathrm{M}) = \frac{\omega}{\Gamma_\mathrm{D}}\frac{d^2}{\varepsilon_0 c^2}$$

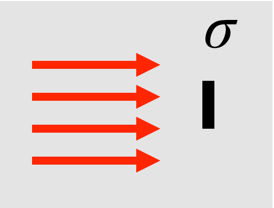
In practice σ (ω_M) and Γ_D measured data: can be found in tables for known laser lines (Ex.: 2 x 10^{-20} cm^2 for Cr^{3+} in ruby or Nd^{3+} in glass)

Rate equation for photons

Rate equation model : surface σ receives a photon flux

$$\frac{I}{\hbar\omega}$$

Number of photons per unit time and surface

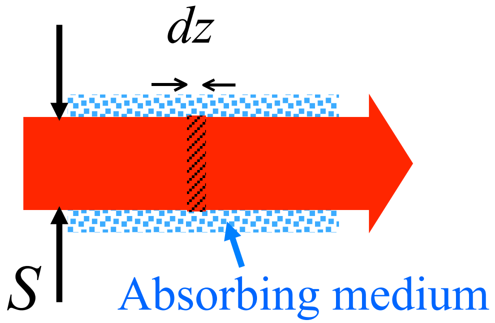


Number of “collisions” per atom per second : $\sigma \dfrac{I}{\hbar\omega}$

Absorption in slice S x dz :

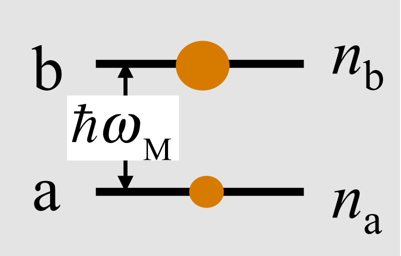
$$\boxed{d\left(S\frac{I}{\hbar\omega}\right) = -\sigma \frac{I}{\hbar\omega} n_{\mathrm{a}} S\, dz}$$

$$\boxed{\Rightarrow \frac{1}{I}\left[\frac{dI}{dz}\right]_{\mathrm{abs}} = -n_{\mathrm{a}}\sigma}$$

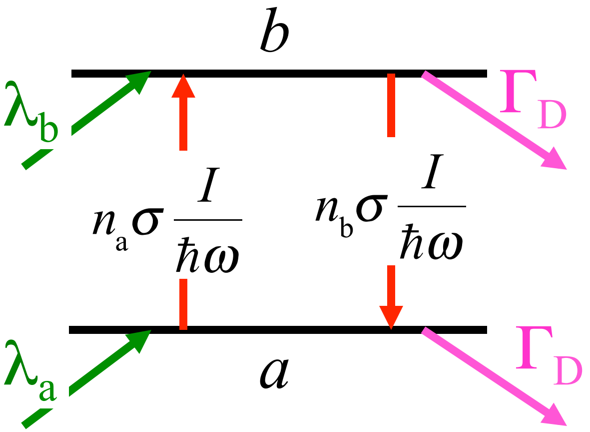


Generalization to stimulated emission, with assumption that stimulated photons add to the beam

$$\boxed{\frac{1}{I}\left[\frac{dI}{dz}\right]_{\mathrm{sti}} = n_{\mathrm{b}}\sigma} \Rightarrow \boxed{\frac{1}{I}\frac{dI}{dz} = (n_{\mathrm{b}} - n_{\mathrm{a}})\sigma}$$



Rate equations for atomic populations



Absorption and stimulated emission described with atomic populations transfer rates equal to rate for photons: plus relaxation and feeding terms

⇒ Rate equations for atomic populations:

$$\frac{dn_a}{dt} = \lambda_a - n_a \sigma \frac{I}{\hbar\omega} + n_b \sigma \frac{I}{\hbar\omega} - \Gamma_D n_a$$

$$\frac{dn_b}{dt} = \lambda_b + n_a \sigma \frac{I}{\hbar\omega} - n_b \sigma \frac{I}{\hbar\omega} - \Gamma_D n_b$$

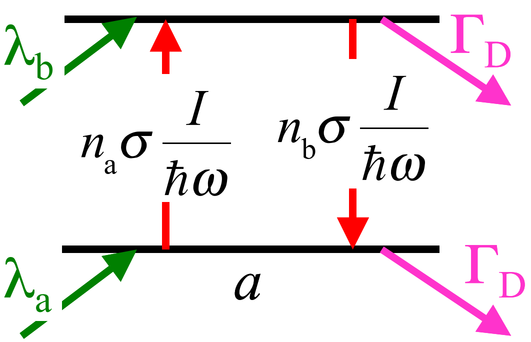
$$\frac{d(n_a + n_b)}{dt} = \lambda_a + \lambda_b - \Gamma_D (n_a + n_b)$$

$$\Rightarrow \quad \left[n_a + n_b \right]_{\text{stat}} = \frac{\lambda_a + \lambda_b}{\Gamma_D}$$

Steady state solution $\frac{d(n_a + n_b)}{dt} = 0$

$$\frac{d(n_b - n_a)}{dt} = \lambda_b - \lambda_a - \left(\Gamma_D + 2\sigma \frac{I}{\hbar\omega} \right) (n_b - n_a)$$

Stationnary population inversion



$$\frac{d(n_b - n_a)}{dt} = \lambda_b - \lambda_a - \left(\Gamma_D + 2\sigma \frac{I}{\hbar\omega}\right)(n_b - n_a)$$

⇒ Steady state:

$$\boxed{\left[n_b - n_a\right]_{\text{stat}} = \frac{\lambda_b - \lambda_a}{\Gamma_D + 2\sigma \dfrac{I}{\hbar\omega}}}$$

Result with already found form:

$$\boxed{\left[n_b - n_a\right]_{\text{stat}} = \frac{\left[n_b - n_a\right]^{(0)}}{1+s}}$$

non saturated

saturation

avec $\boxed{\left[n_b - n_a\right]^{(0)} = \frac{\lambda_b - \lambda_a}{\Gamma_D}}$ and $s = \frac{2\sigma I}{\hbar\omega \Gamma_D}$

Remembering $\sigma = \frac{\sigma(\omega_M)}{1+\delta^2/\Gamma_D{}^2}$ and defining $I_{\text{sat}} = \frac{\hbar\omega\Gamma_D}{2\sigma(\omega_M)}$

⇨ $\boxed{s = \frac{I/I_{\text{sat}}}{1+\delta^2/\Gamma_D{}^2}}$

Same form as found in lecture 2 with Lamb model

Description of laser amplification by rate equations

We have observed that it is possible to describe quantitatively interaction between the laser medium and light using rate equations for atomic populations and photons.

This result was definitely not obvious *a priori* : an atom is a quantum object, described by a state vector, a much richer description than probabilities to be in each level*: oscillating dipole associated with coherence between |a> and |b>. Derivation of rate equations difficult : relaxation must be taken into account: Optical Bloch Equations.

In most cases ($T_2 << T_1$), laser amplification can indeed be described by rate equations leading to formulae analogous to the ones found for Lamb toy model: very useful.

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* Similarly an electromagnetic wave is more than a flux of photons !

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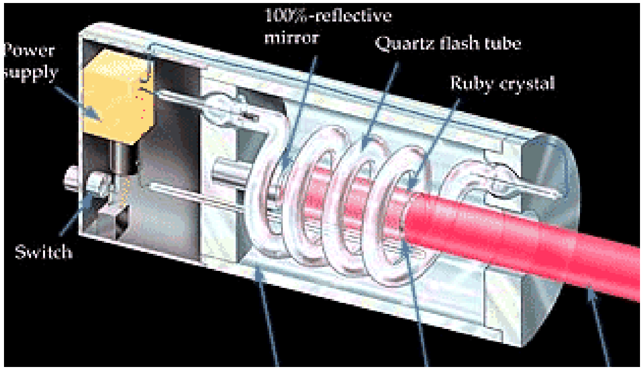
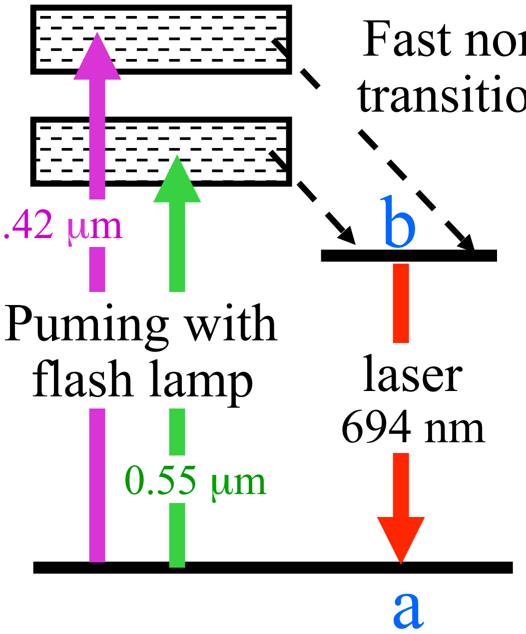
3-level amplification



Ruby laser (0.694 µm) ;
Erbium doped fiber laser (1.5 µm)

Ruby : Cr^{3+} ions substituting some Al^{3+} ions in alumine crystal



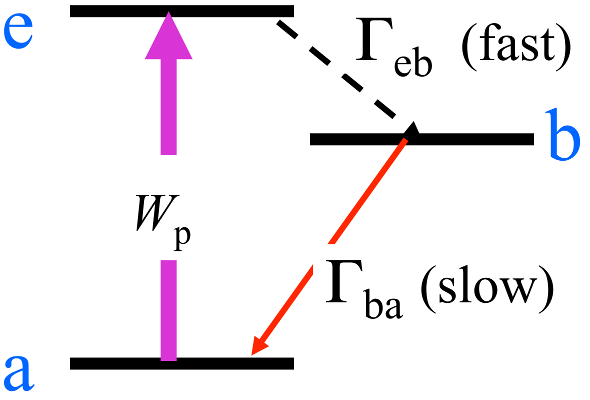
Fast non radiative transitions (10^{-7} s)

Metastable level (3×10^{-3} s)

Puming with flash lamp

laser 694 nm

3-level modeling



population inversion if

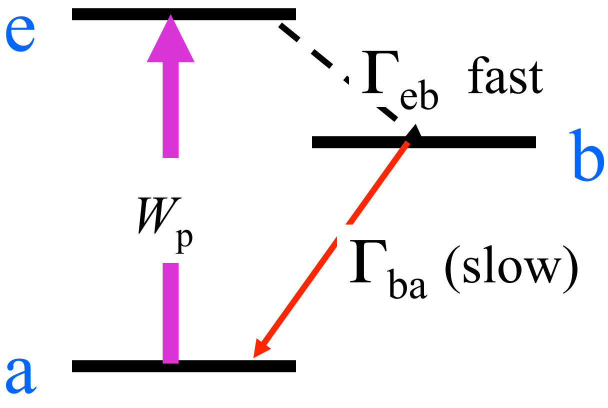
$$\boxed{W_p > \Gamma_{ba}}$$

pumping

Spontaneous de-excitation

3-level laser: population inversion
Rate equations modelling

3-levels system



Rate equations (no laser emission)

$$\frac{dn_e}{dt} = W_p(n_a - n_e) - \Gamma_{eb} n_e \quad = 0$$

$$\frac{dn_b}{dt} = \Gamma_{eb} n_e - \Gamma_{ba} n_b \quad = 0$$

$$n = n_a + n_b + n_e$$

Steady state

$$\Gamma_{eb} \gg W_p \quad \Rightarrow \quad n_e \simeq \frac{W_p}{\Gamma_{eb}} n_a$$

$$\Rightarrow \quad n_b = \frac{W_p}{\Gamma_{ba}} n_a$$

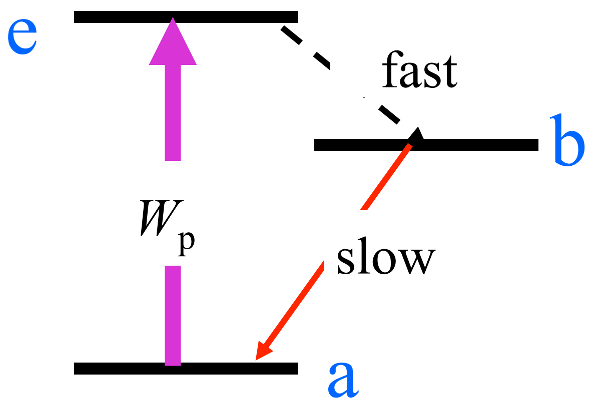
population inversion if

$$W_p > \Gamma_{ba}$$

3-level system: the medium must be bleached to be inverted

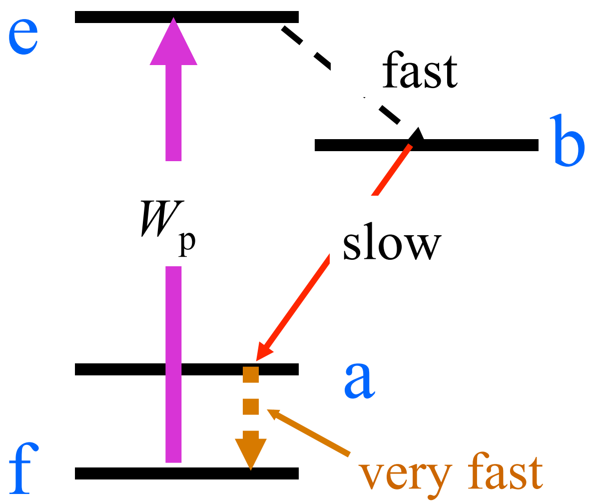
From 3- to 4-levels

3-level system



Inversion difficult because one must not only feed *b* but also empty *a*

4-level system

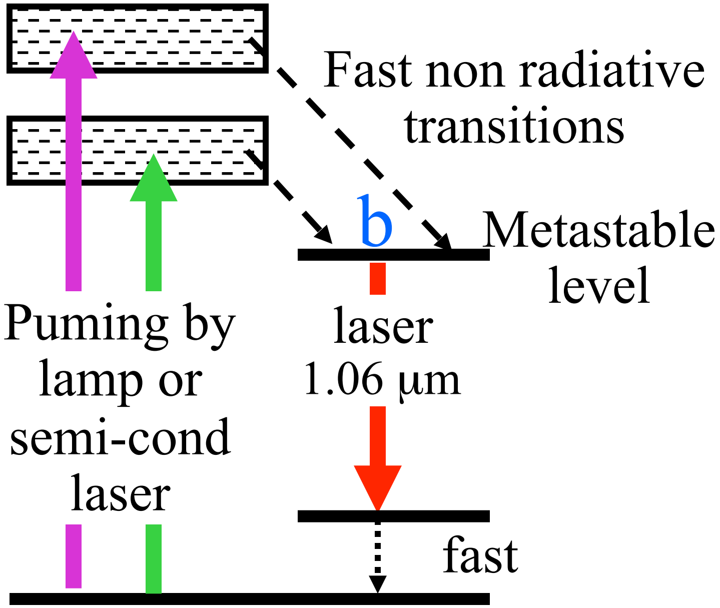


Inversion easy since *a* always almost empty. Fast relaxation: continuous (cw) laser possible.

Rate equations modeling

Examples of 4-level laser sytems

Nd:YAG (or glass)



“easy” to double

⇒ green at 530 nm

Electric discharge lasers

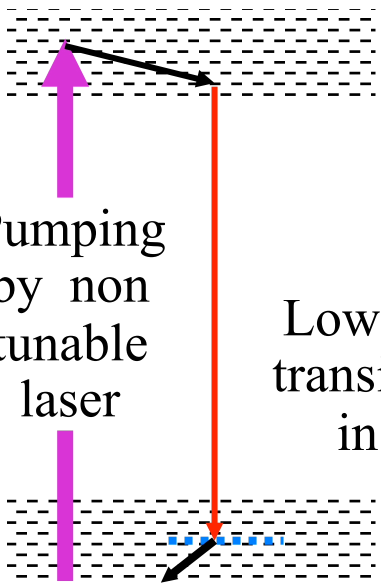
- Helium-Neon,
- Ionised Argon, Krypton …

Many different wavelengths, but fixed, not tunable

etc… many other systems

Tunable 4-level laser
(dye, Ti:sapphire)

Broadband laser amplif.



Fast relaxation to band bottom

Pumping by non tunable laser

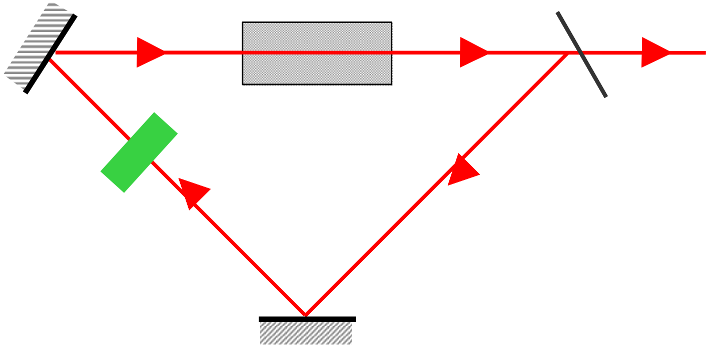
Lower end of laser transition anywhere in lower band

Fast relaxation to band bottom

Emission bandwidth equal to lower band width

- dye: [565 nm , 595 nm] (25 x 10^{12} Hz)
- Ti: Sapphire: [700 nm , 1100 nm]

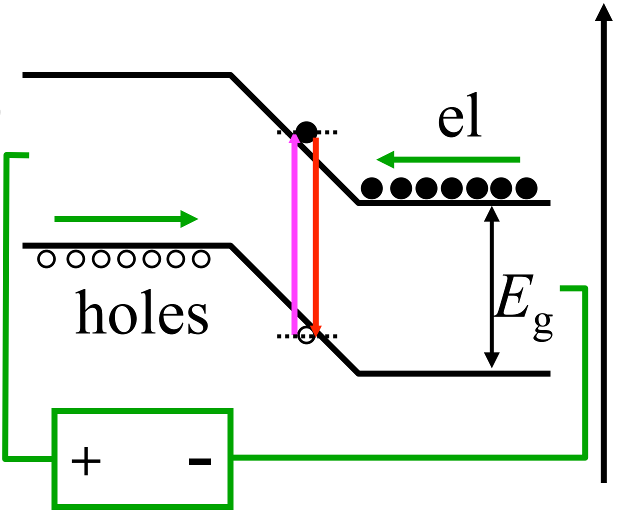
Wavelength selection by filter in the cavity



Semiconductor laser (diode laser)

p-n junction between 2 semi-conductors

- Lack of charge carriers (electrons and holes)
- A photon with energy larger than gap can be absorbed, with creation of an electron-hole pair: photodetection
- Conversely, an electron-hole pair can annihilate and emit a photon: photoemission



If injected current density large enough:

Stimulated emission dominates (4-levels)

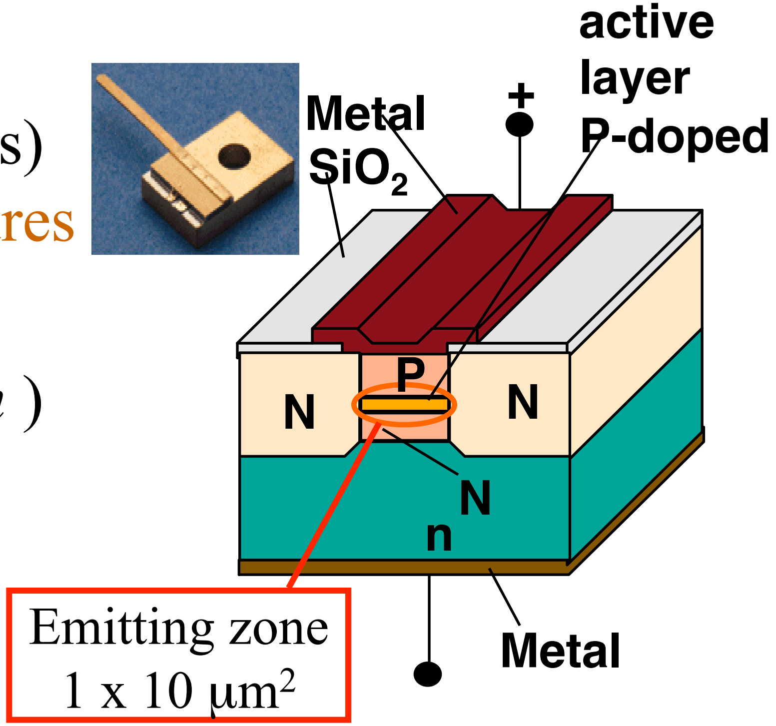
Current concentrated with heterostructures

Linear laser cavity

Cleaved faces, perfectly parallel (high *n*)

Many different wavelenths

- From 1.3 or 1.5 µm (telecom) down to 0.32 µm
- Massive investment, but mass production : low prices



Emitting zone
1 x 10 µm^2

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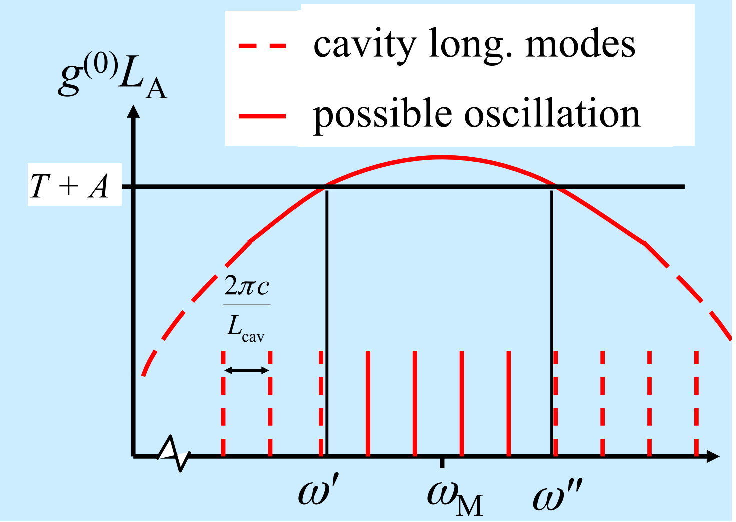
Longitudinal modes of a laser source



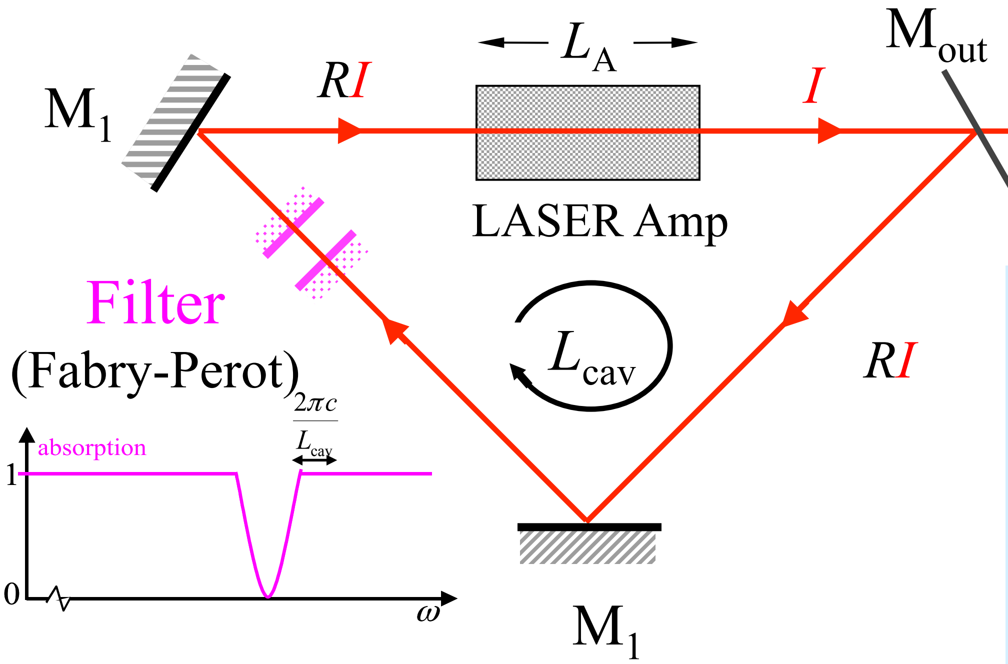
	Gain band-width	L_{cav}	c / L_{cav}	Number possible modes
He-Ne	10^9 Hz	0.6 m	0.5×10^9 Hz	3
Ti:Sapphire	10^{14} Hz	1.5 m	0.2×10^9 Hz	5×10^4
diode laser	10^{12} Hz	3 mm	10^{11} Hz	10

Narrow lines, separated by $\dfrac{\Delta\omega}{2\pi} = \dfrac{c}{L_{cav}}$

Number max $N = \dfrac{\text{gain bandwidth}}{\Delta\omega}$

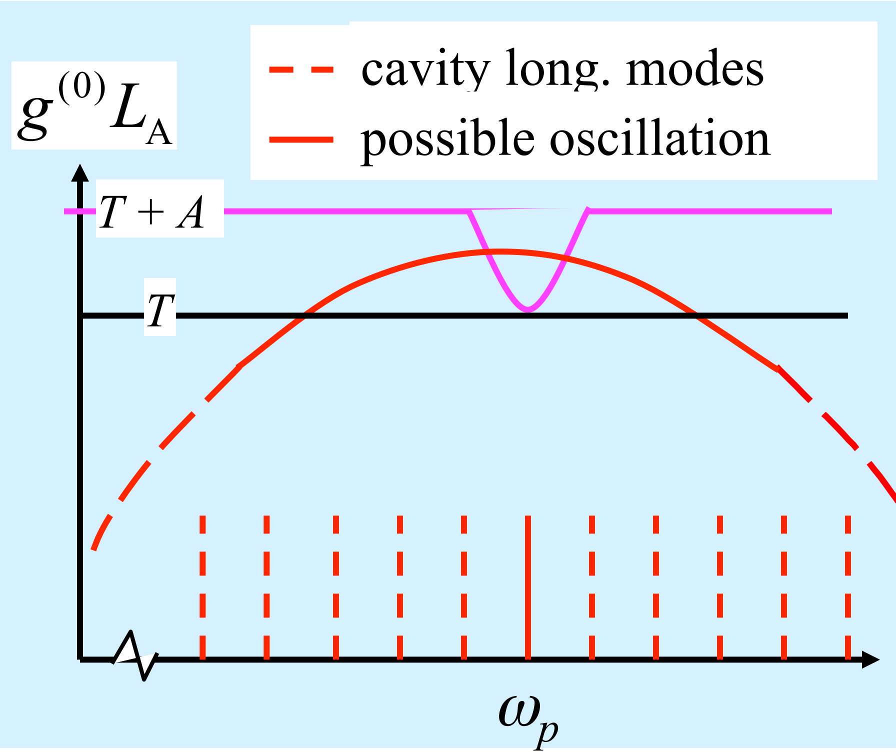
- Frequently, not all the modes oscillate simultaneously: mode competition (cf. lecture 4).
- One can force single mode operation

Single longitudinal mode operation



Oscillation if

$$g^{(0)} L_A \geq T + A$$



Filter makes losses at all wavelengths except a narrow band

Oscillation possible only in the filter bandpass

Demands a very high selectivity: cascaded filters; frequencies of the filters must be aligned (feedback loops). High tech devices

Technical linewidth (jitter)

Single longitudinal mode $\omega_p = p\,2\pi \frac{c}{L_\text{cav}}$; $p \in \mathbb{N}$ $p = \frac{L_\text{cav}}{\lambda_p} \approx 10^6$

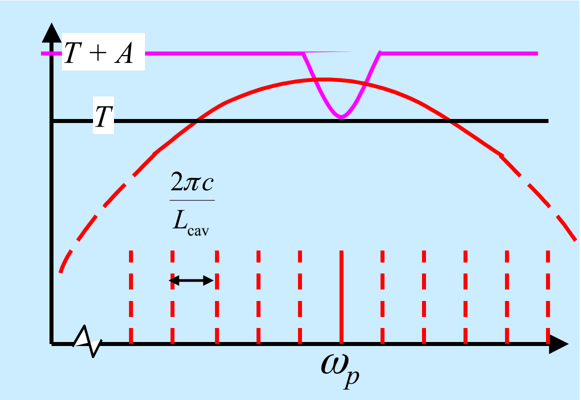
Fluctuations of L_cav : $\Rightarrow$ ω_p fluctuates : jitter

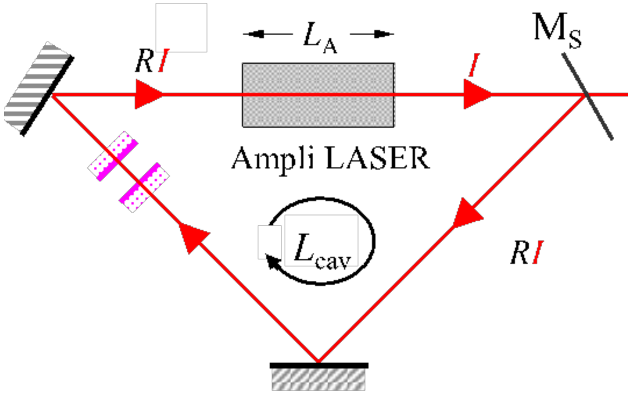
- Vibrations, temperature: length
- Pressure (refraction index)

$$\boxed{\delta L_\text{cav} = \lambda_p \Rightarrow \delta\omega_p = 2\pi c / L_\text{cav}}$$

It is enough to have L_cav varied by λ (less than 1 μm) to have one mode replacing its first neighbour

- Hard to do better with passive methods (temperature controlled within 10^{-4} °C, pressure within 10^{-5} atm)





Servo-controlled cavity length

- Mirror on PZT (position control)
- Error signal on frequency

⇨

$$\boxed{\begin{array}{l} \delta\omega_p / 2\pi < 10^3\,\text{Hz} \\ \leftrightarrow\ \delta \text{L}_\text{cav} < 10^{-2}\ \text{nm !!!} \end{array}}$$

Principles of laser sources

A. Conditions for oscillation
   - Amplifier with feedback
   - Condition on gain: threshold
   - Condition on phase:
     longitudinal modes
   - Possible active modes

B. Laser gain
   - Laser cross section
   - Rate equations

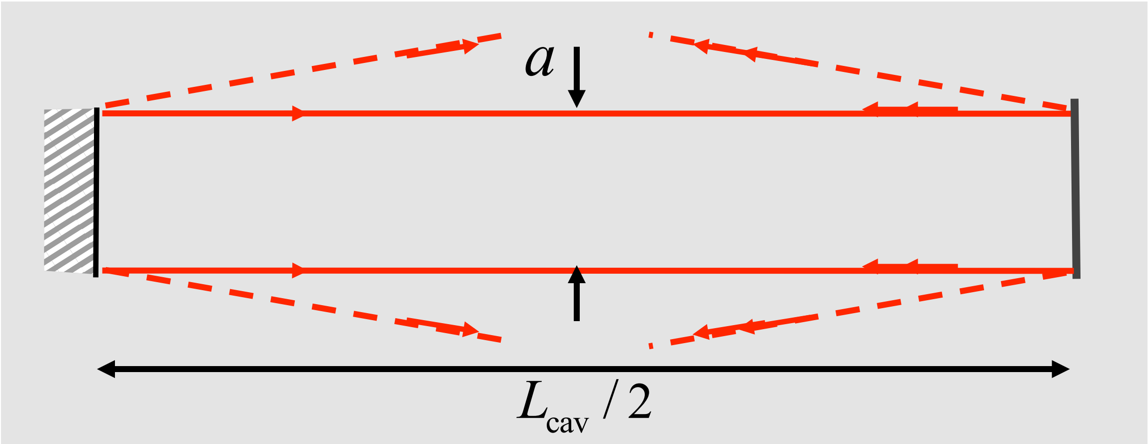
C. Examples of laser media
   - 3 level systems
   - 4 level systems
   - semi-conductor lasers

D. Longitudinal modes
   - Possible modes
   - Single mode operation
   - Technical linewidth

E. Transverse modes
   - Diffraction losses
   - Transverse modes
   - Example: Hermite-Gauss

F. Laser : concentrated light
   - Concentration in space
   - Concentration in spectrum/time
   - Laser source: all photons in the
     same mode

Diffraction losses in a laser cavity



Losses negligible if

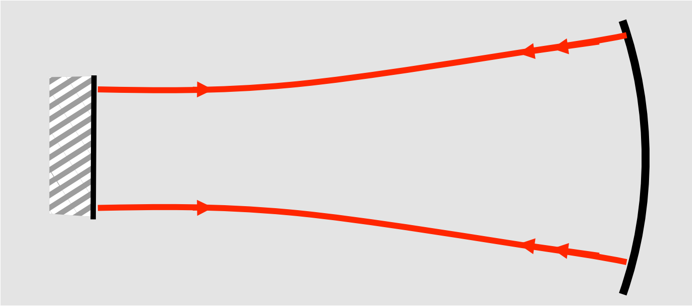
$$\frac{\lambda}{a} L_{\text{cav}} \ll a$$

Not the case, in general
(confined laser amplifiers)

$$a = 1\,\text{mm} \quad \Rightarrow \frac{a^2}{\lambda} = 1\ \text{m}$$

A solution : stable cavity with curved mirrors

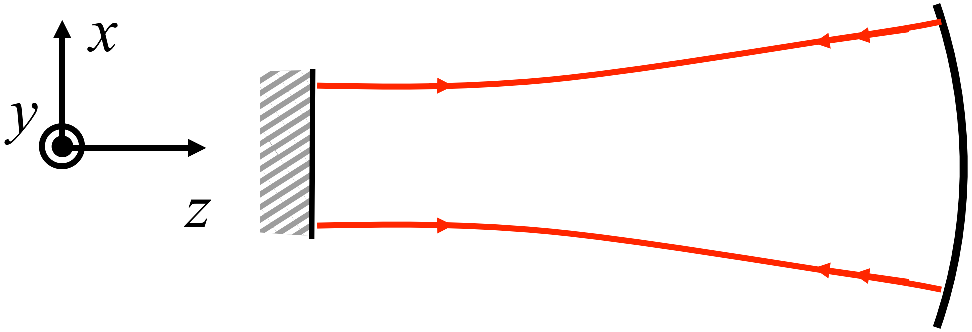
The mirrors impose their curvatures to the laser wave



NB. Semiconductor laser: guided propagation, plane wave

Transverse modes of a stable cavity
(cf. complement 3B)

Modes: stationary solution of 3D propagation equation, with boundary conditions (mirrors)



Series $u_{m,n,p}(x,y,z)\,\mathrm{e}^{-i\omega_p t}$ of solutions, depending on 3 integer numbers

p : longitudinal index $\omega_p = p\,2\pi\dfrac{c}{L_{\text{cav}}} + \varepsilon_{m,n,p}$

m, n : transverse indices : number of nodes (zeroes) in transverse profile

Example : Hermite-Gaussian modes

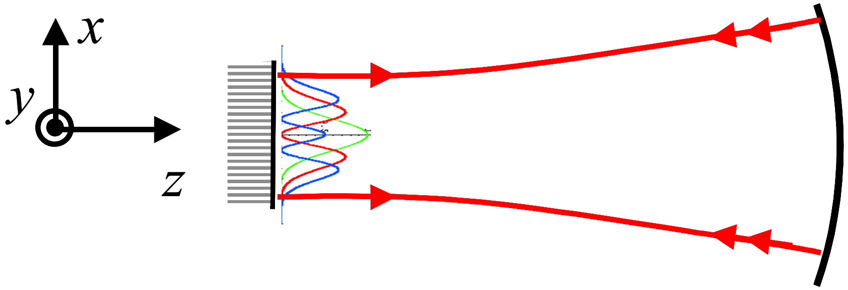
$$u_{m,n,p}(x,y,z) = A\exp\left\{-\frac{x^2+y^2}{w(z)^2}\right\}H_m\left(\frac{x\sqrt{2}}{w(z)}\right)H_n\left(\frac{y\sqrt{2}}{w(z)}\right)\cos\left(\frac{\omega_p}{c}z+\phi(z)\right)$$

$$H_0(u) = 1$$

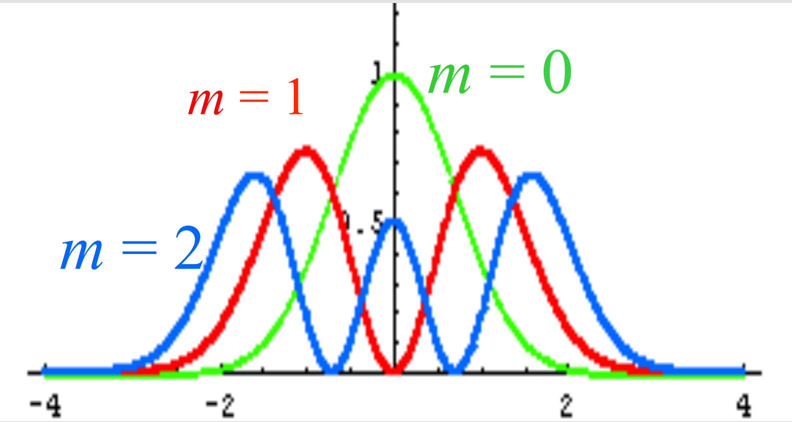
$$H_1(u) = 2u$$

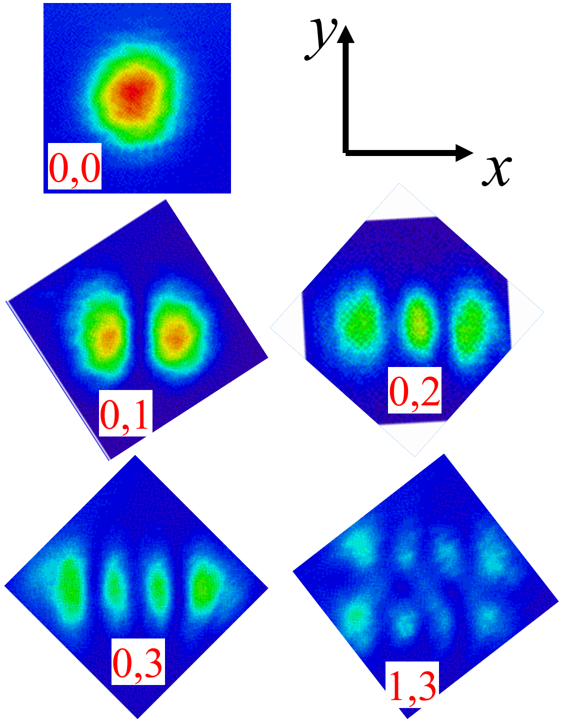
$$H_2(u) = 4u^2 - 2$$

Hermite polynomials

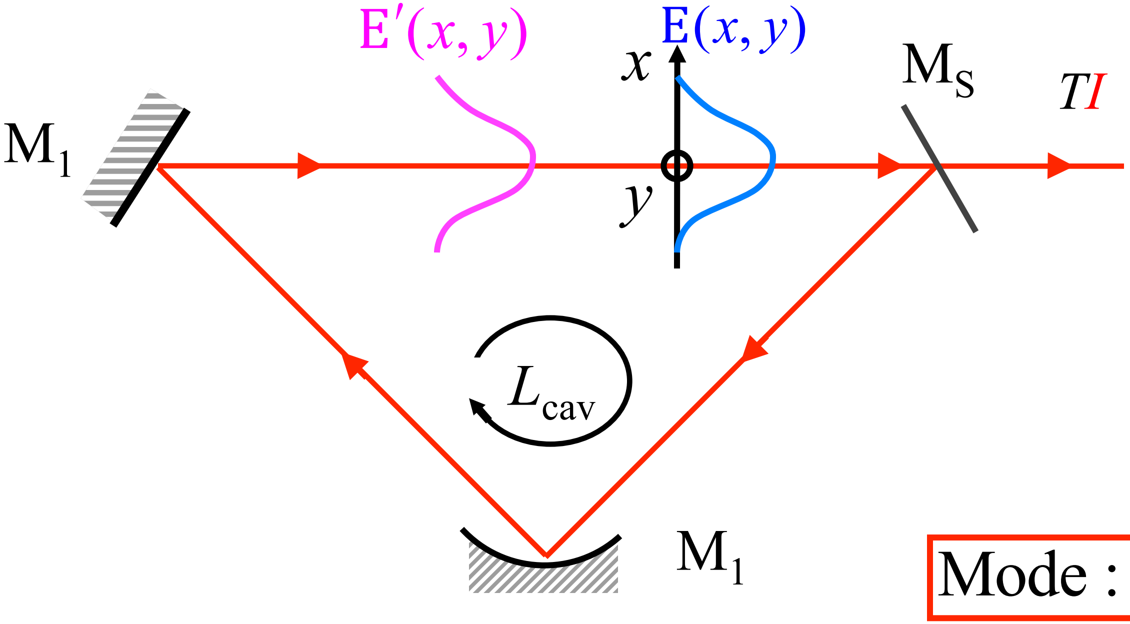


$$\frac{1}{m!2^m}\left[H_m(u)\right]^2 e^{-u^2}$$





Another point of view on transverse modes: self consistent propagation with diffraction



The field profile
$E(x, y)$

becomes,after a
round trip

$\rightarrow E'(x, y)$

$$\boxed{\text{Mode : } E'(x, y) = \alpha\, E(x, y)}$$

Kirchhoff integral
(diffraction)

$$E'(x, y) = \iint E(x_0, y_0)\, P\,(x - x_0, y - y_0)\; dx_0\, dy_0$$

Round trip propagator

Each point of the profile is coupled to all other points
(diffraction over a round trip) ⇒ locking of the phase of
the field over a transverse plane: transverse coherence

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  - Concentration in space
  - Concentration in spectrum/time
  - Laser source: all photons in the same mode

What is so wonderful about laser light?
Certainly not the price per Watt of light

Laser sources

He-Ne	5 mW	100 €	20 k€ / W
Ar^+	2 W	40 k€	20 k€ / W
CO_2	1 kW	150 k€	0.15 k€ / W
Diode laser	1 mW	1 €	1 k€ / W
Diode laser	500 mW	500 €	1 k€ / W

?

Standard sources

Light bulb	100 W	1 €	0.01 € / W
discharge	40 W (light)	4 €	0.1 € / W

Concentration in space: laser vs. standard source

Standard (incoherent) source

$$S' \cdot \sin^2 u' \geq S \cdot \sin^2 u \quad \Rightarrow S'_{\text{mini}} \approx S \cdot \sin^2 u$$

E = image irradiance (W m^{-2})

L = source luminance (W m^{-2} Sr^{-1})

Energy conservation:
Irradiance E (W / m^2) in
image cannot exceed π x L

$$\boxed{E \leq 500 \text{ W / cm}^2 \quad \text{(tungsten wire)}}$$

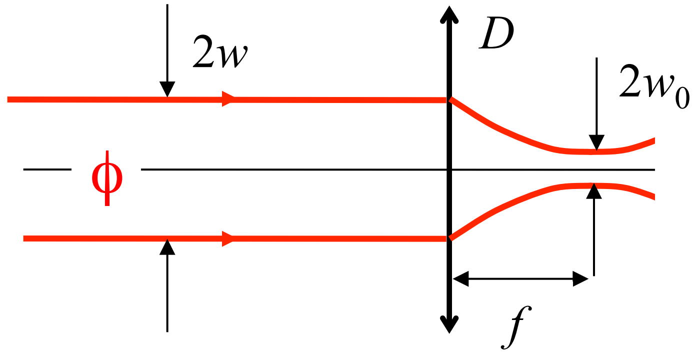
$$\boxed{E \leq 5 \text{ kW / cm}^2 \quad \text{(xenon arc)}}$$

Fundamental limit (2nd principle of thermodynamics)

Laser beam (coherent) : addition of amplitudes

$$S'_{\text{mini}} \approx \lambda^2 \qquad \Rightarrow \quad E_{\max} \approx \frac{\phi}{\lambda^2}$$

$$\boxed{\begin{matrix} \phi = 10 \text{ mW} \\ \lambda = 0.6\ \mu\text{m} \end{matrix} \quad \Rightarrow \quad E \simeq 3 \times 10^6 \text{ W / cm}^2}$$

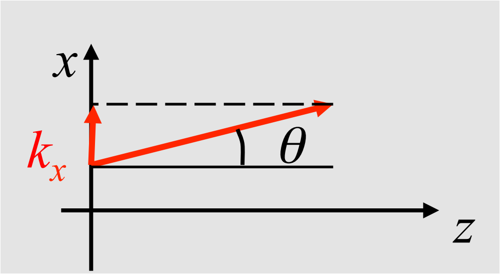


Concentration at diffraction limit

Concentration in direction of laser light

Position and angle: complementary variables (conjugate)

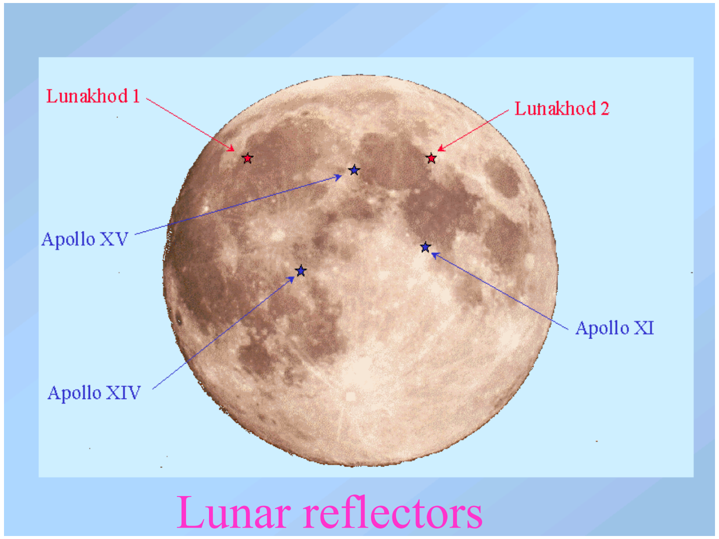
more precisely x and $k_x = \dfrac{2\pi}{\lambda}\sin\theta_x$



Concentration in direction possible if large size

Divergence of a laser beam of diameter D

$\theta \simeq \dfrac{\lambda}{D} = 10^{-6}$ for $D = 0.6$ m 300 m wide on moon



Lunar reflectors

Documents Observatoire Côte d'Azur

Concentration in spectrum (or time)

Bulb lamp (3500 K)

Power spread over all visible range (and IR)

$$\frac{\Delta\omega}{2\pi} \simeq 10^{14}\ \text{Hz}$$

Maximum irradiance per unit bandwidth

$$\boxed{\frac{dE}{d\nu} \approx 5\times10^{-12}\ \text{W}\ \ \text{cm}^{-2}\ \text{Hz}^{-1}}$$

Laser 10 mW

Linewidth : $\frac{\Delta\omega}{2\pi} \simeq 10^{6}\ \text{Hz}$

Maximum irradiance per unit bandwidth

$$\boxed{\frac{dE}{d\nu} \approx 3\ \text{W}\ \ \text{cm}^{-2}\ \text{Hz}^{-1}}$$

12 orders of magnitude more !

Conjugate variable : time. Ultra short lasers (< 10^{-15} s). Energy concentrated in time (giant peak power)

Laser light: concentrated light

- Concentration in space (position / direction)
- Concentration in spectrum (frequency / time)

Laser: energy concentrated in a single mode of radiation $\Delta x \cdot \Delta k_x = 1$

⇒ Incoherent source: energy diluted over many modes

Number of photons per mode

Laser : $\boxed{\mathcal{N} \approx 10^{10} \text{ to } 10^{20} \text{ photons / mode}}$

Thermal source (blackbody radiation)

0.1 photon / mode
@ $\lambda = 0.6\ \mu\mathrm{m}$ @ 3000 K

$$\mathcal{N} = \frac{1}{\exp\left\{\frac{\hbar\omega}{k_{\mathrm{B}}T}\right\} - 1} \simeq \exp\left\{-\frac{\hbar\omega}{k_{\mathrm{B}}T}\right\}$$

Laser beam: all photons in the same mode of the electromagnetic field

All photons in the same mode:

- Same direction
- Same frequency
- Same phase
- Same polarisation

indistinguishability: coherence

Photons are bosons : it is possible to accumulate as many as one wants in the same quantum state (actually they tend to accumulate by bosonic stimulation).

A laser beam can be considered as a kind of Bose-Eintein Condensate of photons (not in thermal equilibrium)