Principles of Modern Communications
Digital Communications

based on 2011 lecture series by Dr. S. Waharte.
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Outline

Digital Communications

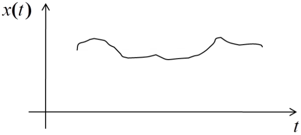
Transmission Fundamentals

Learning Objectives

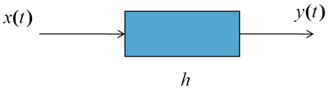
1. Familiarize with signals and systems
2. Understand the Fourier transform and frequency representation
3. Understand the structure and terminology of a digital communication system

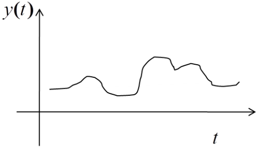
Transmission Fundamentals
Signals and Systems

- Signal is simply any quantity that varies with time (e.g., electric voltage, velocity, force, speech intensity).



- A system simply transforms a signal into another signal

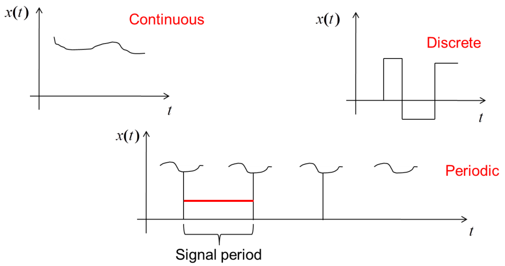




Transmission Fundamentals

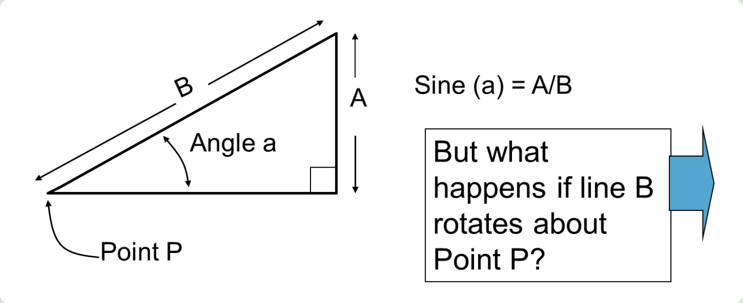
Signal Categories

- Continuous/Analog signal - Varies continuously with time
- Discrete/Digital signal - Maintains a constant level over some time duration and then switches into another level
- Periodic signal - Its pattern its repeated after a specific time duration (signal period).
- Non-periodic signal – The opposite of periodic, i.e., its pattern is not repeated.

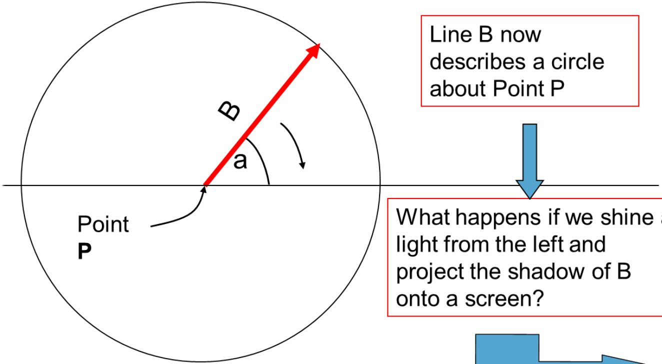


Sine Wave Review - 1

- We all know that the Sine of an angle is the opposite side divided by the hypotenuse, i.e.



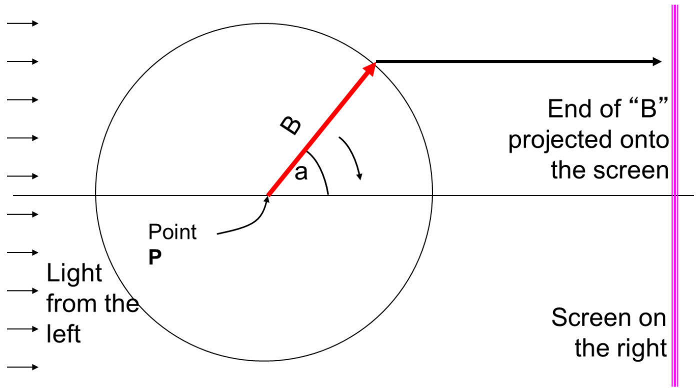
Sine Wave Review - 2



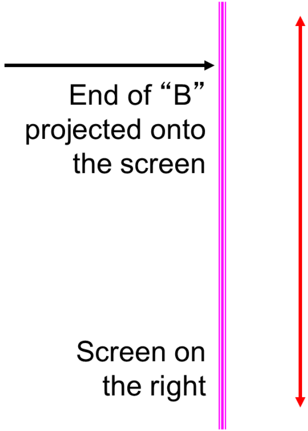
Line B now describes a circle about Point P

What happens if we shine a light from the left and project the shadow of B onto a screen?

Sine Wave Review - 3

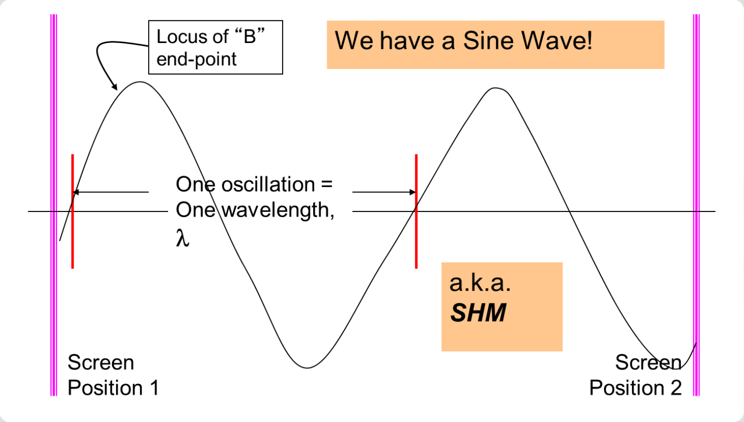


Sine Wave Review - 4

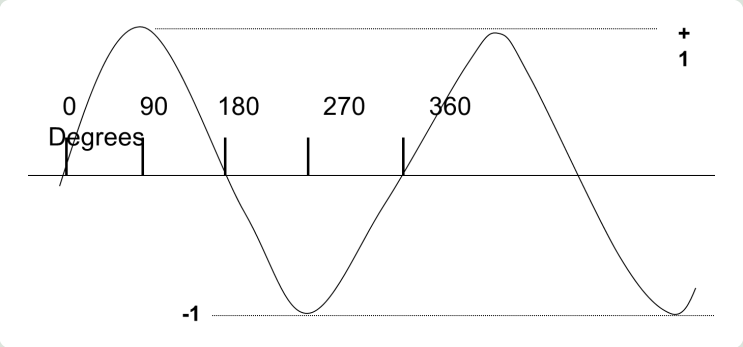


As line “B” rotates about the center point, P, the projected end of “B” oscillates up and down on the screen. What happens if we move the screen to the right and ‘remember’ where the projected end of “B” was?

Sine Wave Review - 5



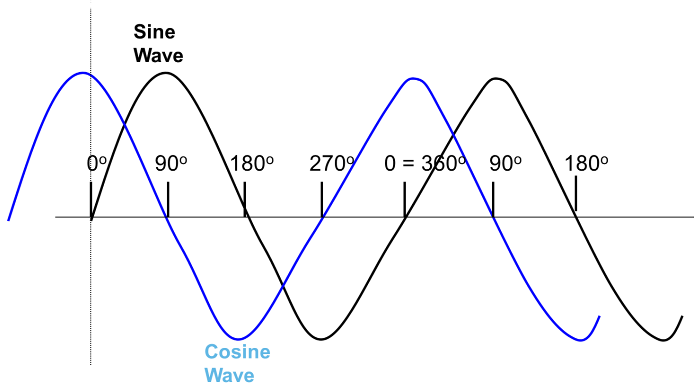
Sine Wave Review - 6



Remember:Sine 0 = 0; Sine 90 = 1; Sine 180 = 0; Sine 270 = -1; Sine 360 = Sine 0 = 0

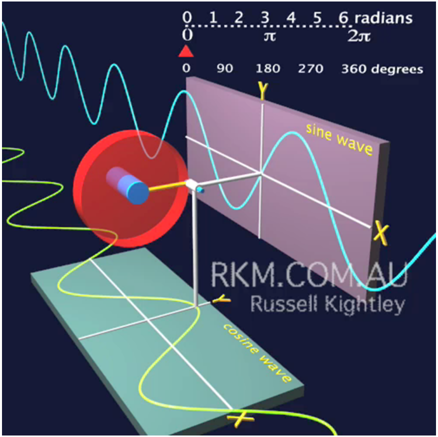
Sine and Cosine Waves – 1

- Sine Wave = Cosine Wave shifted by 90 degrees

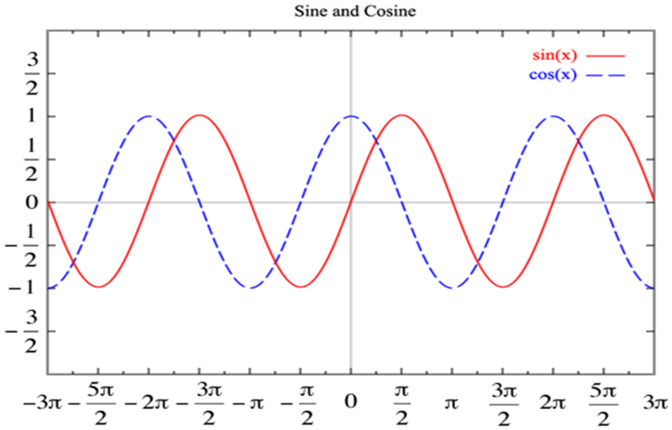


Sine and Cosine Waves – 2

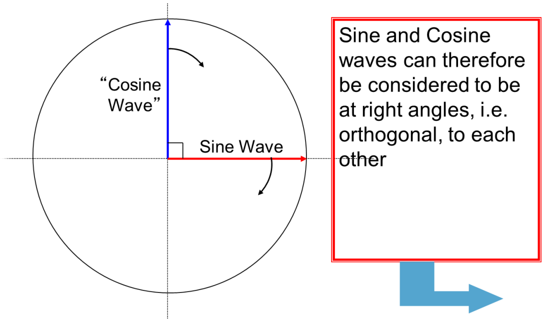
- There is a useful java applet that will show you a sine wave derived from circular motion (simple harmonic motion)



Sine and Cosine Waves – 3



Sine and Cosine Waves - 4

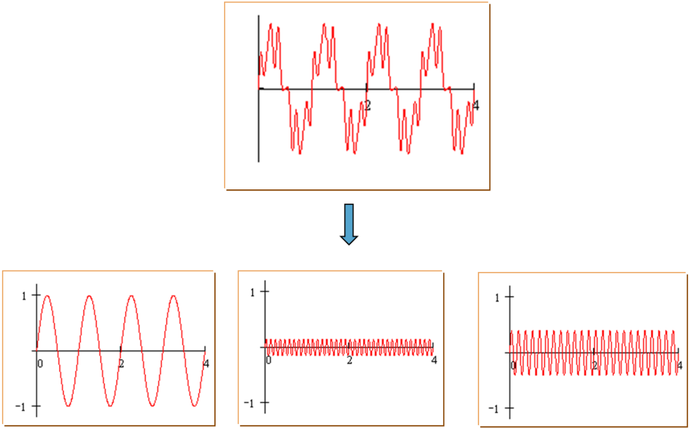


- Sine and Cosine waves can therefore be considered to be at right angles, i.e. orthogonal, to each other

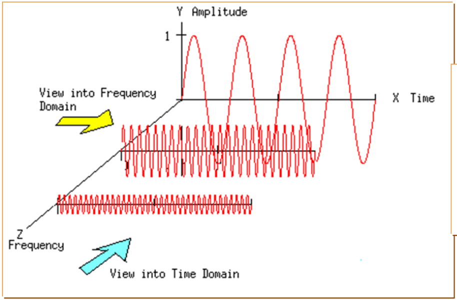
Sine and Cosine Waves – 6

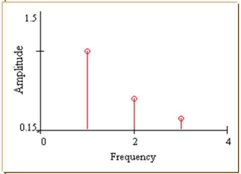
- Any wave that is periodic (i.e. it repeats itself exactly over succeeding intervals) can be resolved into a number of simple sine waves, each with its own frequency
- This analysis of complex waveforms is part of the Fourier Theorem
- You can build up a complex waveform with harmonics of the fundamental frequency

Example



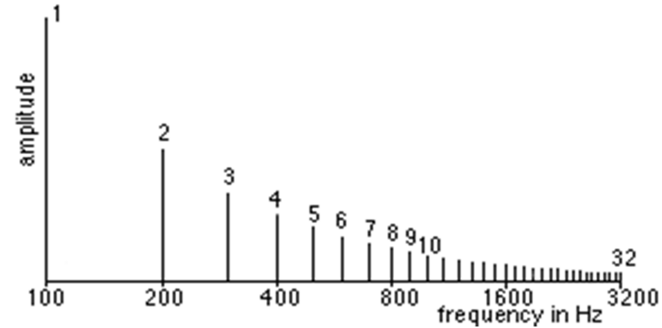
Example


Y Amplitude
1
View into Frequency Domain
X Time
Z Frequency
View into Time Domain


1.5
Amplitude
0.15
0
2
4
Frequency

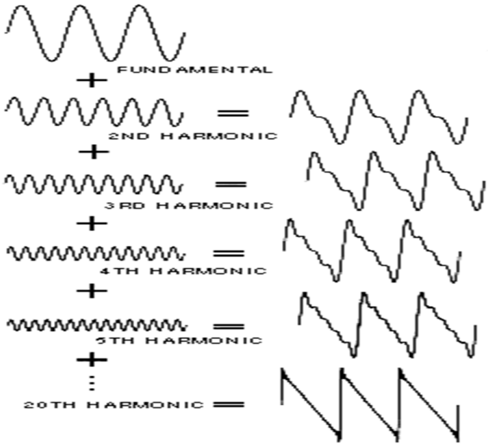
Harmonics – 1

- A harmonic is a multiple of a fundamental frequency. In the figure below, a fundamental frequency of 100 Hz is shown with 31 harmonics (total of 32 "lines").



Harmonics – 2

- In this example, 20 harmonics are mixed together to form a saw-tooth waveform



Fourier series expansion

- For periodic signal we use the Fourier series expansion as a simple way of frequency representation of such signals
- The Fourier series expansion of a general periodic signal x(t) with period T is defined as
  - Trigonometric expansion

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t)\right]$$

  - where

$$a_0 = \frac{1}{T}\int_0^T x(t)dt$$

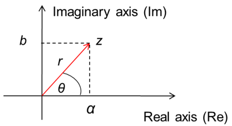
$$a_n = \frac{2}{T}\int_0^T x(t)\cos(2\pi n f_0 t)dt$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin(2\pi n f_0 t)dt$$

Transmission Fundamentals

Complex Plane-Complex Numbers

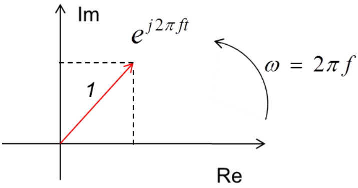
- Complex plane is a two dimensional representation of complex numbers and complex signals. The general definition of a complex number z is
- $z = a + jb$
- a is the real part and b the imaginary part of z, respectively and j is a non-real constant such that $j^2 = -1$ simply implies a phase difference of 90 degrees



- An alternative and very common representation of complex numbers is
- $z = a + jb = r\cos\theta + jr\sin\theta = re^{j\theta}$
- where $\cos\theta + j\sin\theta = e^{j\theta}$, r is the amplitude and θ is the phase of z
- $r = \sqrt{a^2 + b^2}$, $tan\theta = b/a$, $a = r\cos\theta$, $b = r\sin\theta$,

Complex exponential harmonic signal

- $x(t) = e^{j2\pi ft} = \cos(2\pi ft) + j\sin(2\pi ft)$
- It is a very useful signal used for frequency domain analysis. Its amplitude is equal to one whereas it rotates in the complex plane in the counter-clockwise direction with an angular frequency $\omega = 2\pi f$



Transmission Fundamentals

Frequency domain analysis-Fourier Series

- For periodic signal we use the Fourier series expansion as a simpler way of frequency representation of such signals
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  - Complex exponential expansion

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{-j2\pi n f_0 t}$$

  - where

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-j2\pi n f_0 t} dt$$

- Generally, the Fourier series expansion is a simpler way of representing periodic signals

Frequency domain analysis-Fourier Transform

- Definition of frequency content: Frequency content of a time domain signal is the weighed superposition of all the complex exponential harmonic signals that accurately reconstruct the initial time domain signal.
- The frequency content is defined through the well-known Fourier transform F

$$F[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi n f_0 t}dt$$

- The original signal can be reconstructed by its Fourier transform through the so-called Inverse Fourier transform F^{-1}

$$x(t) = F^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f)e^{j2\pi n f_0 t}dt$$

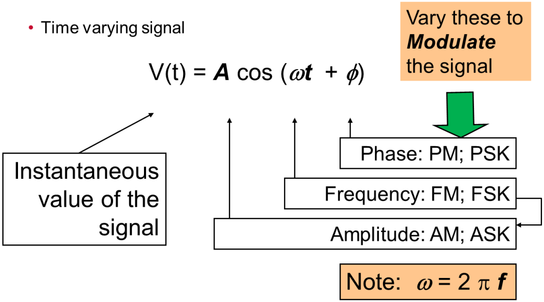
- Generally, the Fourier transform is a complex valued function having both amplitude and phase. In the above definitions, the functions x(t) and X(f) are recognized as Fourier transform pairs

Sine and Cosine Waves - 9

- Two concepts
- The signal may be thought of as a time varying voltage, V(t)
- The angle, θ, is made up of a time varying component, ωt, and a supplementary value, ϕ, which may be fixed or varying
- Thus we have a signal
- $V(t) = A\cos(\omega t + \phi)$

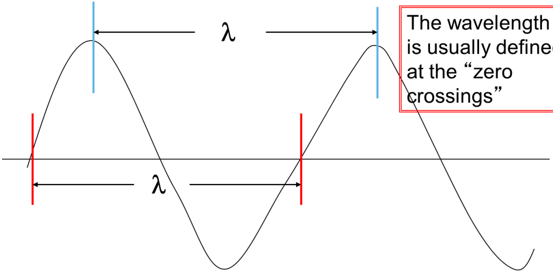
Sine and Cosine Waves - 10

- Time varying signal



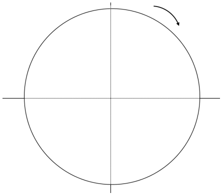
Back to our Sine Wave – 1

Defining the Wavelength



The wavelength is usually defined at the "zero crossings"

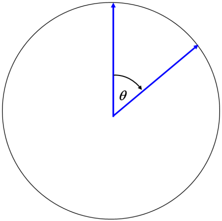
Back to our Sine Wave - 2



- One revolution = 360 degrees
- One revolution also completes one cycle (or wavelength) of the wave.
- So the "phase" of the wave has moved from 0 degrees to 360 degrees (i.e. back to 0 degrees) in one cycle.The faster the phase changes, the shorter the time one cycle (one wavelength) takes

Back to our Sine Wave – 3

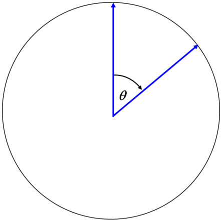
Two useful equations



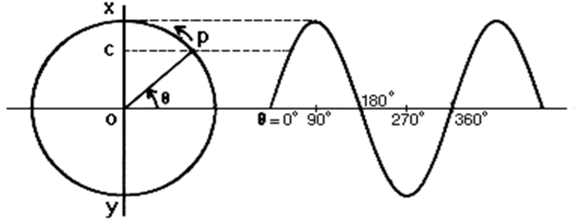
- The time taken to complete one cycle, or wavelength, is the period, T.
- Frequency is the reciprocal of the period, that is
  - $f = \frac{1}{T}$
- Phase has changed by θ
- The rate-of-change of the phase, $\frac{d\theta}{dt}$, is the frequency, f.

Sine Wave – 4

- What do we mean "Rate-of-change of phase is frequency"?
- One revolution = 360 degrees = 2π radians = 1 cycle
- One revolution/s = 1 cycle/s = 1 Hz
- Examples:
  - 720 degrees/s = 2 revolutions/s = 2 Hz
  - 18,000 degrees/s = 18,000/360 revs/s = 50 revs/s = 50 Hz



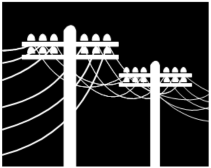
Simple Harmonic Motion

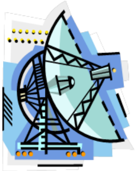


- Geometric derivation of simple harmonic motion. A point p moves at constant speed on the circumference of a circle in counter-clockwise motion. Its projection OC on the vertical axis XOY is shown at right as a function of the angle θ. The function described is that of a sine wave.

Sine Wave Continued

- Can think of a Sine Wave as a Carrier Signal, i.e. the signal onto which the information is loaded for sending to the end user
- A Carrier Signal is used as the basis for sending electromagnetic signals between a transmitter and a receiver, independently of the frequency





Carrier signals - 1

- A Carrier Signal may be considered to travel at the speed of light, c, whether it is in free space or in a metal wire
- Travels more slowly in most substances
- The velocity, frequency, and wavelength of the carrier signal are uniquely connected by

$$c = f\lambda$$

- where:
  - c = velocity of light (m/s)
  - f = frequency (1/s, Hz)
  - λ = wavelength (m)

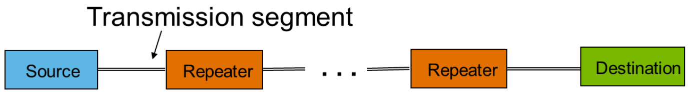
Carrier signals - 2

- Example
- WAMU (National Public Radio) transmits at a carrier frequency of 88.5 MHz
- What is the wavelength of the carrier signal?
  - Answer:
  - $c = (3 \times 10^8) m/s = f\lambda = (88.5 \times 10^6) \times (\lambda)$
  - Which gives $\lambda = 3.3898 = 3.4$ m
- Remember: Make sure you are using the correct units

Transmission Fundamentals

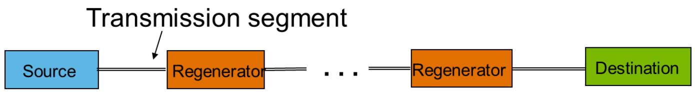
Why Digital Communications?

- Analog Transmission
  - Each repeater attempts to restore analog signal to its original form
  - Restoration is imperfect
    - Distortion is not completely eliminated
    - Noise & interference is only partially removed
  - Signal quality decreases with number of repeaters
  - Communications is distance-limited
  - Still used in analog cable TV systems



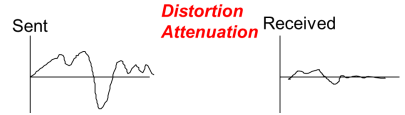
Transmission Fundamentals
Why Digital Communications?

- Digital Transmission
  - Regenerator recovers original data sequence and retransmits on next segment
  - Then each regeneration is like transmitting for first time!
  - Communications is possible over very long distances
  - Digital systems vs. analog systems
    - Less power, longer distances, lower system cost
    - Monitoring, multiplexing, coding, encryption, protocols. . .



Transmission Fundamentals
Why Digital Communications?

- Analog transmission: all details must be reproduced accurately

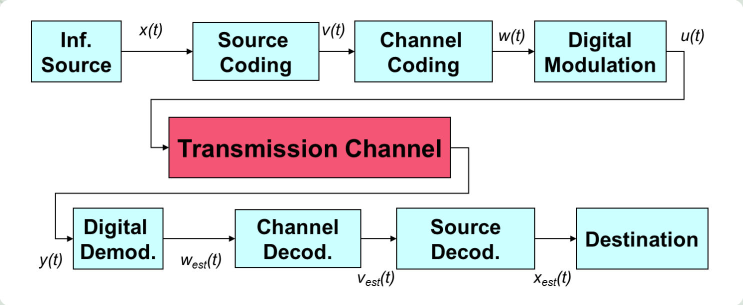


- Analog transmission: all details must be reproduced accurately

- Simpler Receiver: Was original pulse positive or negative?

Transmission Fundamentals

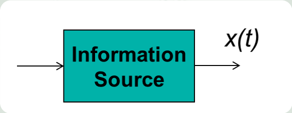
Structure of a Digital Communication System



- In a digital communication system the message produced by the source is converted into a digital signal, i.e., a sequence of binary digits.
- Source coding, channel coding and digital modulation are carried out at the transmitter's side
- Digital demodulation, channel decoding and source decoding are carried out at the receiver's side

Transmission Fundamentals
Information Source

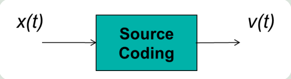
- The information source produces the message, i.e., the information, that is to be transmitted
- The output of the source can be either analog or digital data

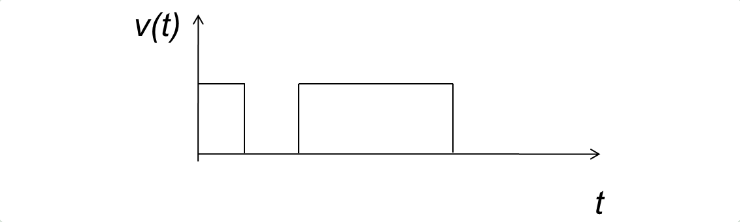


- Examples of messages can be the following
  - Earthquake wave: 0.01 10 Hz
  - Nuclear explosion signal: 0.01 10 Hz
  - Electrocardiogram (ECG): 0 100 Hz
  - Wind noise: 100 1000 Hz
  - Speech: 100 4000 Hz (4 KHz)
  - Audio: 20 20000 Hz (20 KHz)
  - NTSC TV: 6 MHz
  - HDTV: $>$ 10 MHz

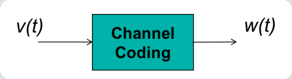
Transmission Fundamentals
Source Coding

- Source coding also called data compression is a process in which the output of the information source is converted into a digital signal, i.e., a sequence of binary digits
- The sequence of binary digits is called information sequence





- Channel coding is a process that adds redundancy (e.g, parity bits, repetition codes, etc.) in the information sequence to make the information more tollerant against channel impairments such as additive noise, interferences and so on

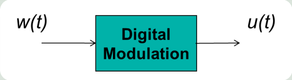


- For example, a trivial channel coding scheme is each binary digit of the information sequence to be repeated several times

Transmission Fundamentals

Digital Modulation

- Digital modulation acts as the interface between the digital section of the transmission system and the transmission medium (twisted pair, coaxial cable, radio channel and so on)
- It maps the binary information sequence into electrical signals



- When two signals are used to transmit the two digits (one for 0, one for 1), this is called binary modulation
- When $M = 2^b$ signals are used to transmitt b coded digits (a sequence of 0's and 1's), this is called M-ary modulation

Transmission Fundamentals

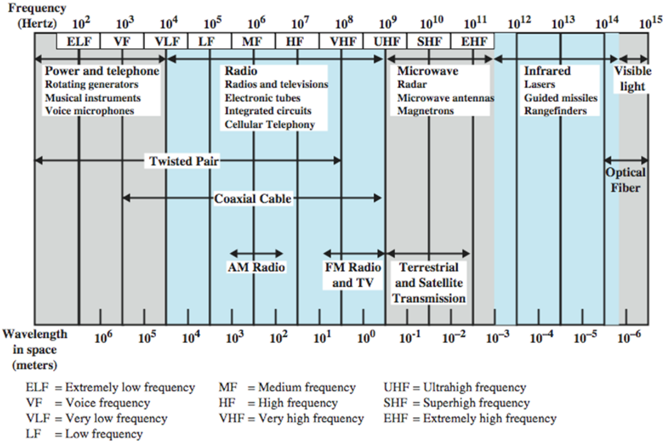
Transmission Channel

- The transmission channel is the medium that sends the signal from the transmitter to the receiver
- In the following, we see the main categories of channels transmitting electrical signals

- Wireless Electromagnetic Channel
  - LF (30 300KHz, Navigation)
  - MF/HF (300 3000KHz, AM/SW radio)
  - VHF (30 300MHz, TV & FM radio)
  - UHF (0.3 3GHz, TV, mobile phone)
  - SHF (3 30GHz, satellite, microwave)
  - EHF (30 300GHz, experimental com)
  - Infrared (no frequency allocation)
- Wired Media
  - Twisted pair (0 10MHz)
  - Coaxial cable (100K 500MHz)
  - Optical fiber (180 370THz)

Transmission Fundamentals

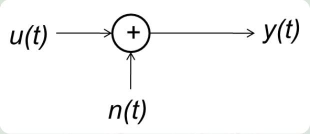
Transmission Channel: Electromagnetic Spectrum



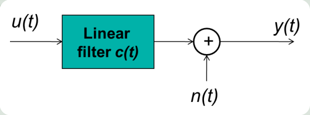
Transmission Fundamentals

Transmission Channel: Mathematical Models

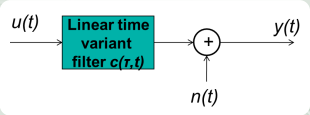
- The additive noise channel
- $y(t) = u(t) + n(t)$



- The linear filter channel
- $y(t) = \int_{-\infty}^{\infty} u(\tau)c(t-\tau)d\tau + n(t)$



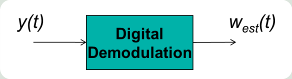
- The linear time variant filter channel
- $y(t) = \int_{-\infty}^{\infty} u(\tau)c(\tau, t-\tau)d\tau + n(t)$



Transmission Fundamentals

Digital Demodulation

- Digital demodulation transforms the transmitted signal into a sequence of binary digits that represents estimates od the transmitted binary symbols



Transmission Fundamentals

Channel Decoding

- Channel decoding attempts to reconstruct the original information sequence according to knowledge of the code used through the channel coding process

Transmission Fundamentals
Source Decoding

- Source decoding attempts to reconstruct the original signal from the source according to knowledge of the code used through the source coding process

Key Design Issues - 1

- S/N
  - Signal-to-Noise Ratio (Analog)
    - Need to be above user's threshold for Required QoS
- C/N
  - Carrier-to-Noise Ratio (Analog and Digital)
    - Need to be above demodulation threshold for useful transfer of information
- BER
  - Bit Error Rate (Sometimes Bit Error Ratio) $\rightarrow$ S/N
    - Need to satisfy the Performance and Availability Specifications

Signal-to-Noise Ratio - 1

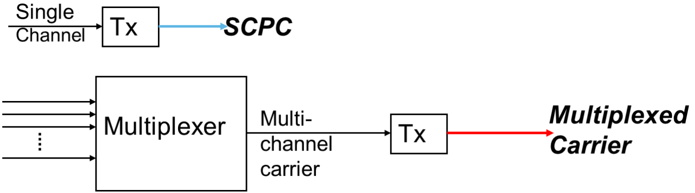
- Signal-to-Noise, written as S/N, is mainly used for Analog Systems
- S/N is specified at the Baseband of the Information Channel
    - Baseband is a range of frequencies close to zero
    - Information is what is sent to the user and the channel over which it is sent is the Information Channel

Digression - UNITS

- Standard units to use are MKS
  - M = meters written as m
  - K = kilograms written as kg
  - S = seconds written as s
- Hence
  - the velocity of light is in m/s
  - The wavelength is in m
  - And the frequency is in Hz = hertz

Carrier signals - 3

- A Carrier Signal can
  - carry just one channel of information (this is often called Single Channel Per Carrier = SCPC)
  - Or carry many channels of information at the same time, usually through a Multiplexer



- Note: The modulator has been omitted in these drawings

Decibel (dB) Notation - 1

- Historically the Bel, named after Alexander Graham Bell, is a unit of sound
- It was developed as a ratio measure: i.e., it compares the various sound levels
- The Bel was found to be too large a value and so a tenth of a Bel was used, i.e., the decibel
- A decibel, or 1 dB, was found to be the minimum change in sound level a human ear could detect

Decibel (dB) Notation - 1

- Question
- How do you get a dB value?
- Answer
- Take the $\log_{10}$ value and multiply it by 10
- Example
- One number is 7 times larger than another. The dB difference
- $= 10 \times log_{10} 7 = 10 \times 0.8451 = 8.5dB$
- NOTE: Never quote a dB number to more than one place of decimals

Decibel (dB) Notation - 2

- Some things to remember
- A dB value is always 10 $\log_{10}$; it is never, ever, 20 $\log_{10}$, however . . .
- $10log_{10}(x)^a = 10 \times a \times log_{10}(x)$
- e.g. $10log_{10}(x)^2 = 10 \times 2 \times log_{10}(x) = 20log_{10}(x)$
- The dB ratio may be referenced to a given level, for example
  - 1 W (unit would be dBW)
  - 1 mW (unit would be dBmW)

Decibel (dB) Notation - 3

- Question
- An amplifier increases power by a ratio of 17:1, what is the dB gain?
- Answer
- $10 \log_{10} 17 = 12.3$ dB
- Question
- The amplifier is fed with 1W, how many watts are output?
- Answer
- 17 Watts which is equivalent to 12.3 dBW

Decibel (dB) Notation - 4

- Examples of dB notations of power, etc.
  - 425 W $\rightarrow$ 26.3 dBW
  - 425 W = 425,000 mW $\rightarrow$56.3 dBm
  - 0.3 W $\rightarrow$ -5.2 dBW
  - 0.3W = 300 mW $\rightarrow$ 24.8 dBm
  - 24,500 K $\rightarrow$ 43.9 dBK
  - -273 K $\rightarrow$ Error – you cannot take a logarithm of a negative number

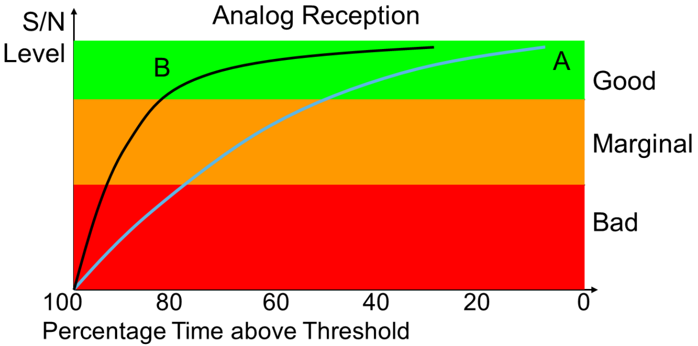
Decibel - Examples

Signal Levels of Common Sounds	
0 dB	Lowest level for hearing
10 dB	Sound of crumpling paper
20 dB	Sound of a whisper
65 dB	Sound of normal conversation
110 dB	Sound of a tractor trailer
120 dB	Sound that causes pain

Signal-to-Noise Ratio - 2

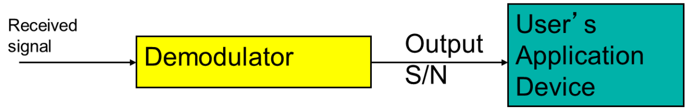
- What S/N value gives a good reception?
  - Telephone and TV channels require a minimum of 50 dB
  - 50 dB $\rightarrow$ ratio of 100,000
  - IE:the Signal power is 100,000 $>$ the Noise power
- Analog signals have "graceful degradation" characteristics

Signal-to-Noise Ratio - 3



Signal-to-Noise Ratio - 4

- The S/N is what the user perceives, but it is usually measured at the demodulator output

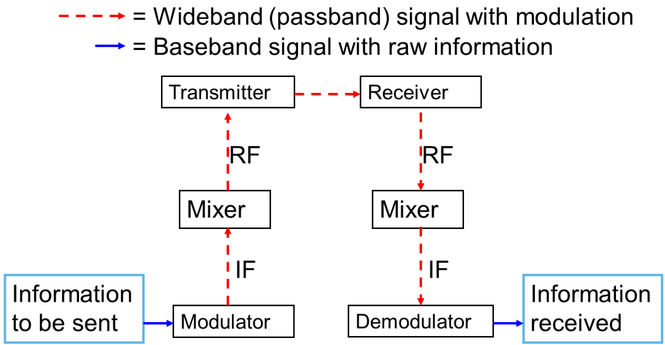


- The C/N at the demodulator input will determine the output S/N

Carrier-to-Noise Ratio - 1

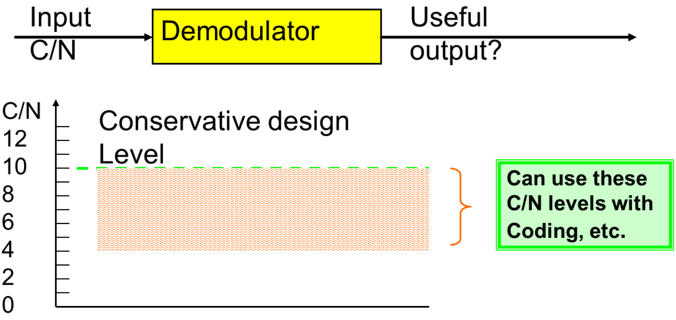
- Carrier-to-Noise, written as C/N, is used for both Analog and Digital Systems
- The Carrier signal has information from the sender impressed upon it, through modulation. The carrier, plus the modulated information, will pass through the wideband portion of transmitter and receiver, and also over the transmission path

Carrier-to-Noise Ratio - 2



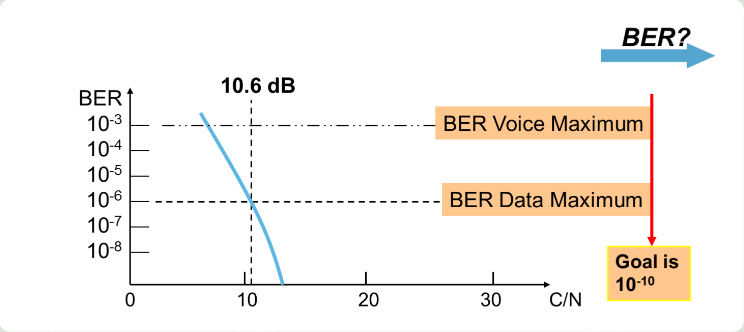
- The C/N at the input to the demodulator is the key design point in any communications system

Carrier-to-Noise Ratio - 3



Carrier-to-Noise Ratio - 4

- Useful design reference for uncoded QPSK
- BER $= 10^{-6}$ at 10.6 dB input C/N to Demodulator



BER - 1

- BER means Bit Error Rate, however some people refer to it as the Bit Error Ratio (i.e. the ratio of bad to good bits)
- Strictly speaking, it is the Probability that a single Bit Error will occur
- BER is usually given as a power exponent, e.g. 10^{-6}, which means one error in 10^{6} bits

BER - 2

- A BER of 10^{-6} means on the order of one error in a page of a FAX message
- To improve BER, channel coding is used
  - FEC codes
  - Interleaved codes
- Communications systems are specified in many ways, but the two most common are performance and availability

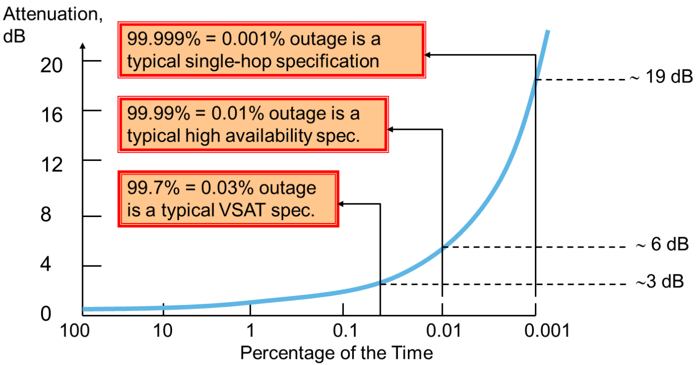
BER - 3

- Performance
  - Generally specified as a BER to be maintained for a very high percentage of the time (usually set between 98% and 99% of the time)
- Availability
  - Generally specified as a minimum BER below which no information can be transmitted successfully - i.e. an outage occurs

BER - 4

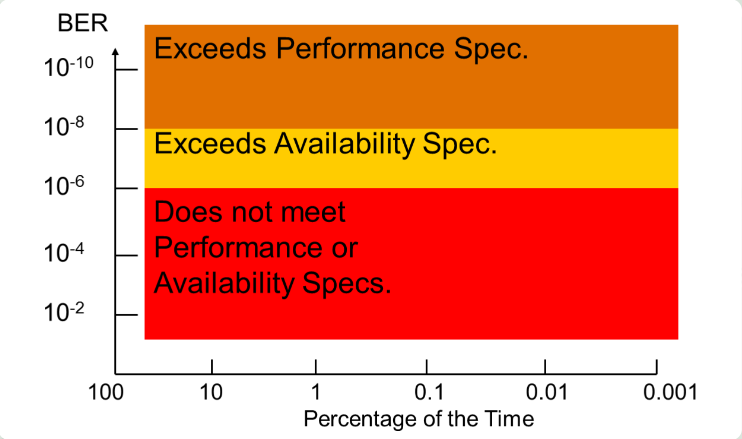
- What causes the change in BER?
- Since BER is determined by C/N, change in BER is caused either by
  - Changes in C (i.e. carrier power level)
    - Antenna loses track
    - Attenuation of signal
  - Changes in N (i.e. noise power level)
    - **Interference** (see next slide)
    - Enhanced noise input

BER - 5

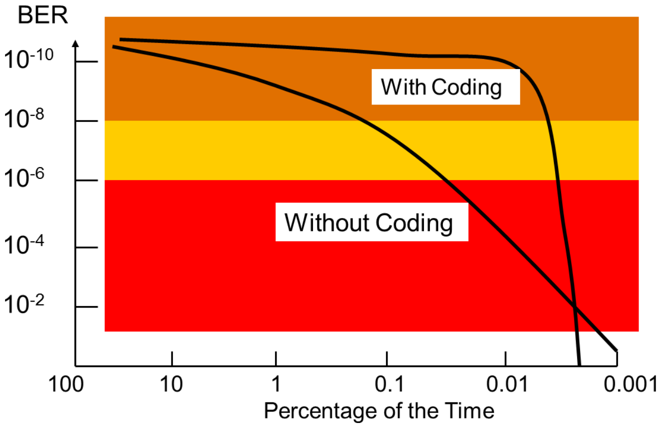

Attenuation, dB
20
16
12
8
4
0
99.999% = 0.001% outage is a typical single-hop specification
99.99% = 0.01% outage is a typical high availability spec.
99.7% = 0.03% outage is a typical VSAT spec.
~ 19 dB
~ 6 dB
~3 dB
100
10
1
0.1
0.01
0.001
Percentage of the Time

BER – 6

Performance & Availability



BER – 7
Performance & Availability



Amplitude Modulation

What is Amplitude Modulation?

- Amplitude modulation is the process of changing the amplitude of a relatively high frequency carrier signal in proportion with the instantaneous value of the modulating signal (information)

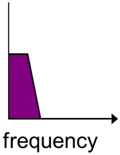
Amplitude modulation

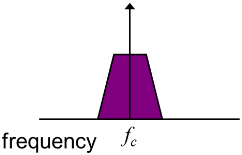
Information signal: $v_m(t)$

High frequency carrier: $v_c(t) = E_c \cos(\omega_c t + \theta_c) = E_c \cos(2\pi f_c t + \theta_c)$

f_c is much higher than the highest frequency in $M(f)$

$$v_c(t)v_m(t) = E_c v_m(t)\cos(\omega_c t + \theta_c) = E_c v_m(t)\cos(2\pi f_c t + \theta_c)$$





Linear modulation / Amplitude Modulation

$$v_m(t) = E_m \cos(2\pi f_m t)$$

E_m = peak modulating signal amplitude (Volt)
f_m = modulating signal frequency (Herz)

$$v_c(t) = E_c \cos(2\pi f_c t + \theta_c)$$

E_c = peak carrier amplitude (Volt)
f_c = carrier frequency (Herz) $f_c >>> f_m$

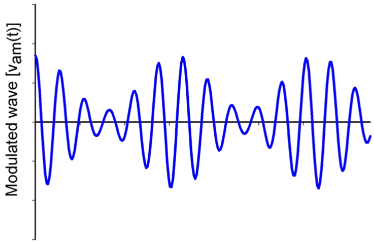
$$v_{am}(t) = \{E_c + E_m \cos(2\pi f_m t)\}\cos(2\pi f_c t + \theta_c)$$

$$v_{am}(t) = E_c\{1 + m\cos(2\pi f_m t)\}\cos(2\pi f_c t + \theta_c)$$

$$m = \frac{E_m}{E_c}$$, m is called “modulation coefficient (dimensionless)

Modulated signal

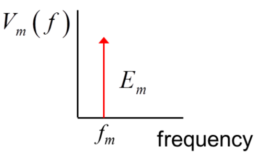
$$v_{am}(t) = E_c\left\{1 + m\cos\left(2\pi f_m t\right)\right\}\cos\left(2\pi f_c t\right)$$

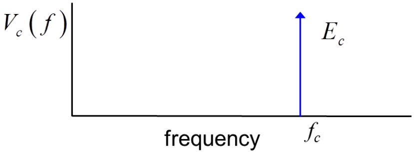


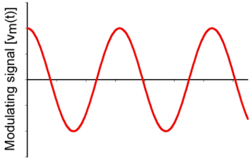
$$v_{am}(t) = E_c\left\{\cos\left(2\pi f_c t\right) + \frac{m}{2}\cos\left(2\pi\left[f_c + f_m\right]t\right) + \frac{m}{2}\cos\left(2\pi\left[f_c - f_m\right]t\right)\right\}$$

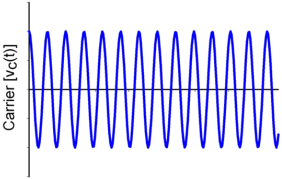
Amplitude spectra

$$v_{am}(t) = E_c\left\{\cos(2\pi f_c t) + \frac{m}{2}\cos(2\pi[f_c + f_m]t) + \frac{m}{2}\cos(2\pi[f_c - f_m]t)\right\}$$



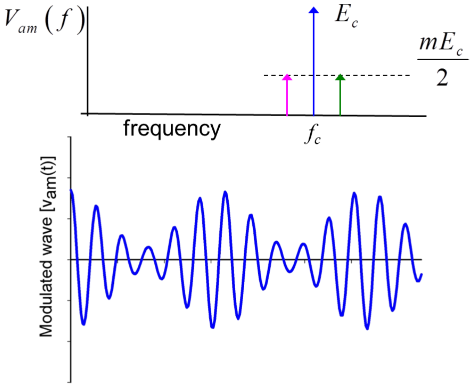






Amplitude spectra

$$v_{am}(t) = E_c\left\{\cos(2\pi f_c t) + {}^{m}\!/_{2}\cos(2\pi[f_c + f_m]t) + {}^{m}\!/_{2}\cos(2\pi[f_c - f_m]t)\right\}$$



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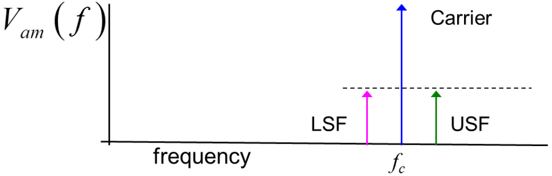

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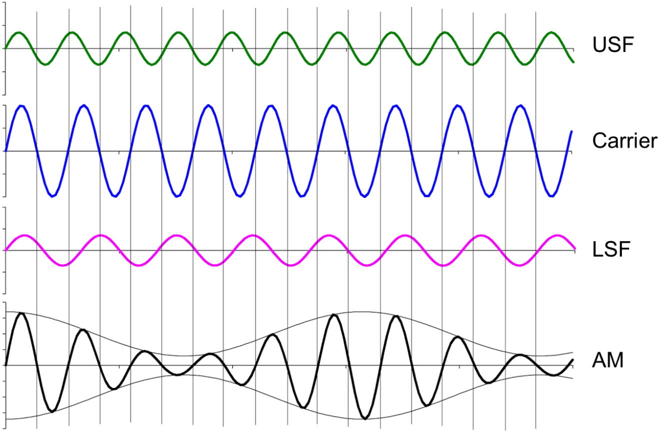
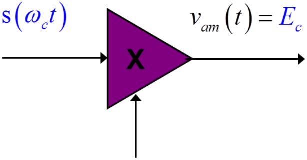


General idea of an AM modulator

$E_c \cos(\omega_c t)$

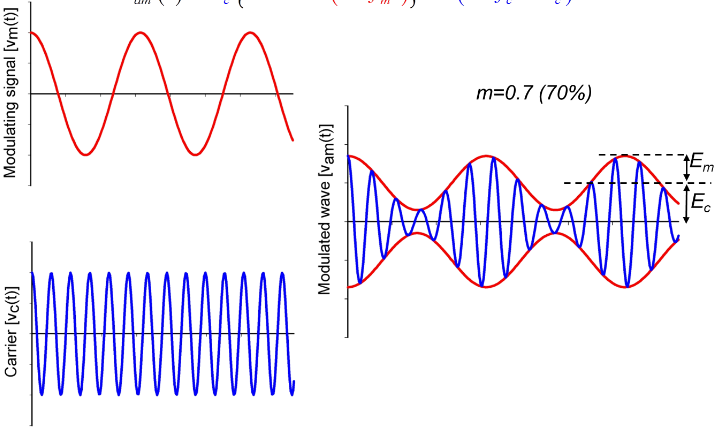
$v_{am}(t) = E_c\{1 + m\cos(2\pi f_m t)\}\cos(2\pi f_c t + \theta_c)$



$gain = 1 + m\cos(2\pi f_m t)$

Modulation coefficient

$$v_{am}(t) = E_c \{1 + m\cos(2\pi f_m t)\}\cos(2\pi f_c t + \theta_c)$$



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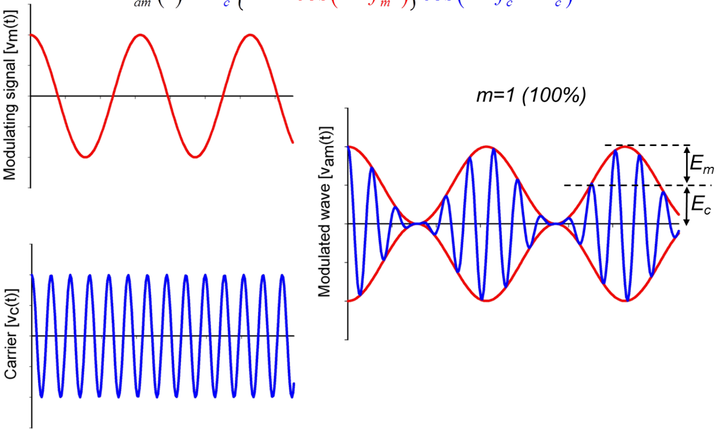

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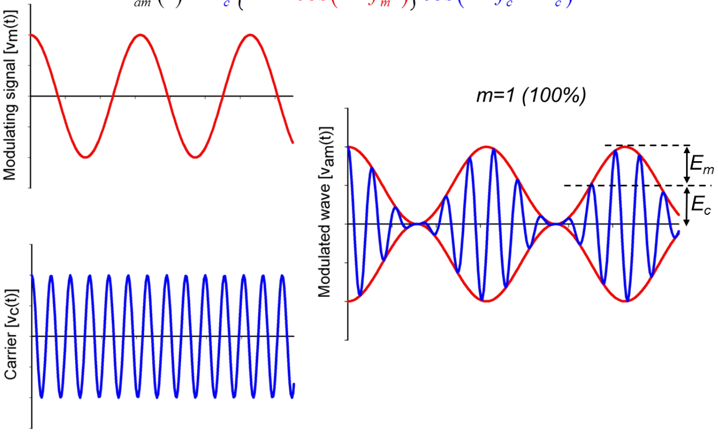


Modulation by a Complex Information Signal

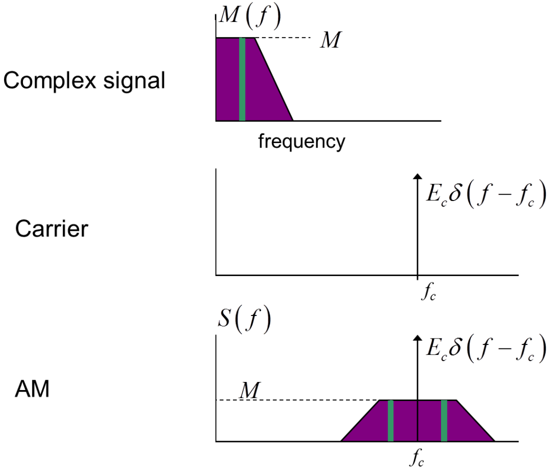
$$v_m(t) = \sum_n m_n \cos(2\pi f_n t)$$

$$v_{am}(t) = \{E_c + v_m(t)\}\cos(2\pi f_c t)$$

$$s(t) = E_c\left\{\cos(2\pi f_c t) + \sum_{n=1}^{N} {}^{m_n}\!/_2 \cos(2\pi[f_c + f_n]t) + {}^{m_n}\!/_2 \cos([f_c - f_n]t)\right\}$$

$$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + \ldots + m_N^2}$$

Modulation by a Complex Information Signal



AM Amplitude spectrum