Session 443

Principles to Actions:

Defining Core Practices of Teaching Mathematics

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NCTM National Council of Teachers of Mathematics

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Session Overview

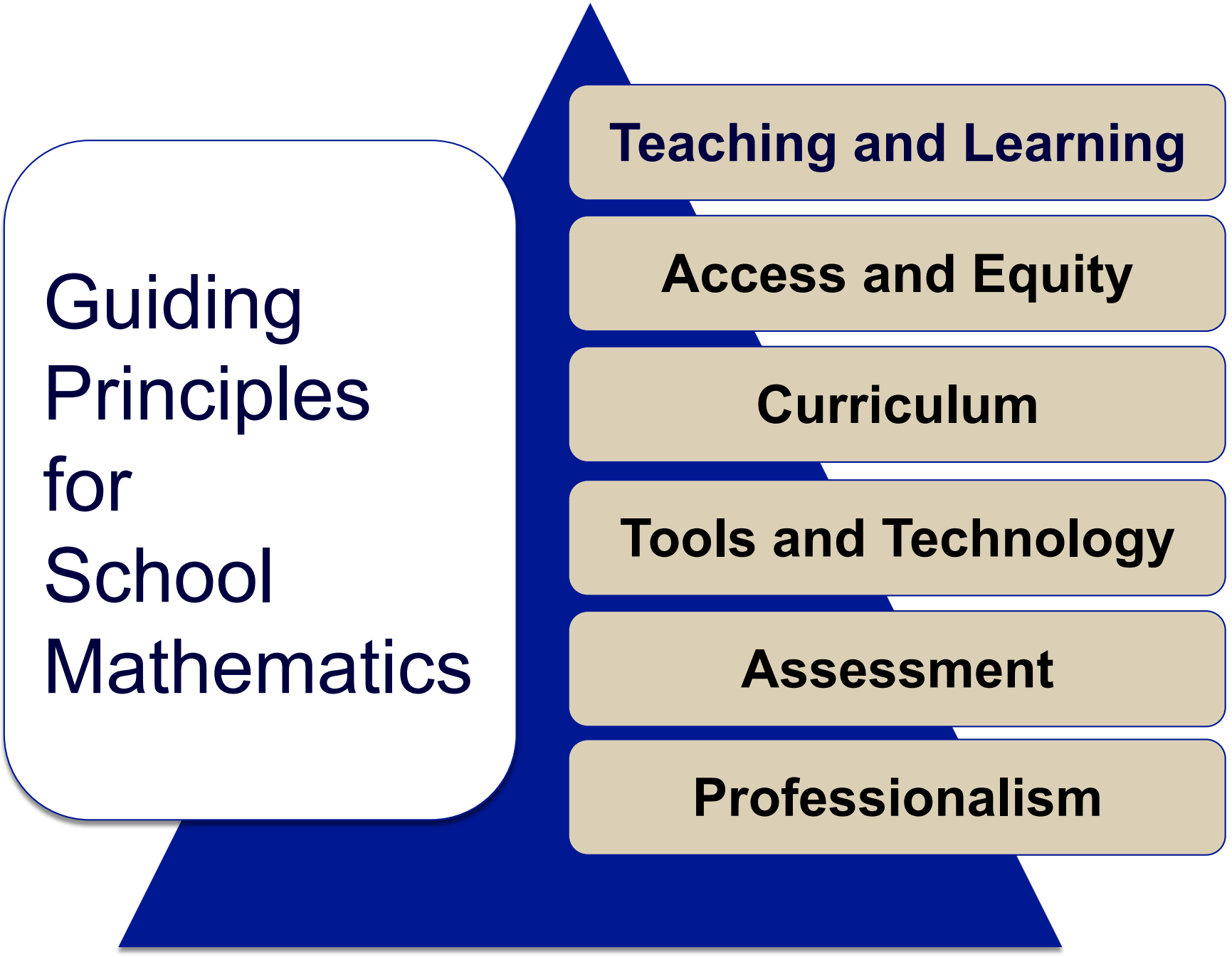
- Discuss the purpose of identifying a core set of effective teaching practices for mathematics.
- Examine each of these core mathematics teaching practices.
- Consider next professional steps and actions.

Session Learning Goal

We are learning to:

Recognize those aspects of our instructional practice that provide high leverage in strengthening and furthering students' success in learning of mathematics.

Principles to Actions


Guiding Principles for School Mathematics
Teaching and Learning
Access and Equity
Curriculum
Tools and Technology
Assessment
Professionalism

Overarching Message

Effective teaching is the non-negotiable core that ensures that all students learn mathematics at high levels.

Effective teaching requires a range of professional structures, supports, and actions at the state or provincial, district, school, and classroom levels.

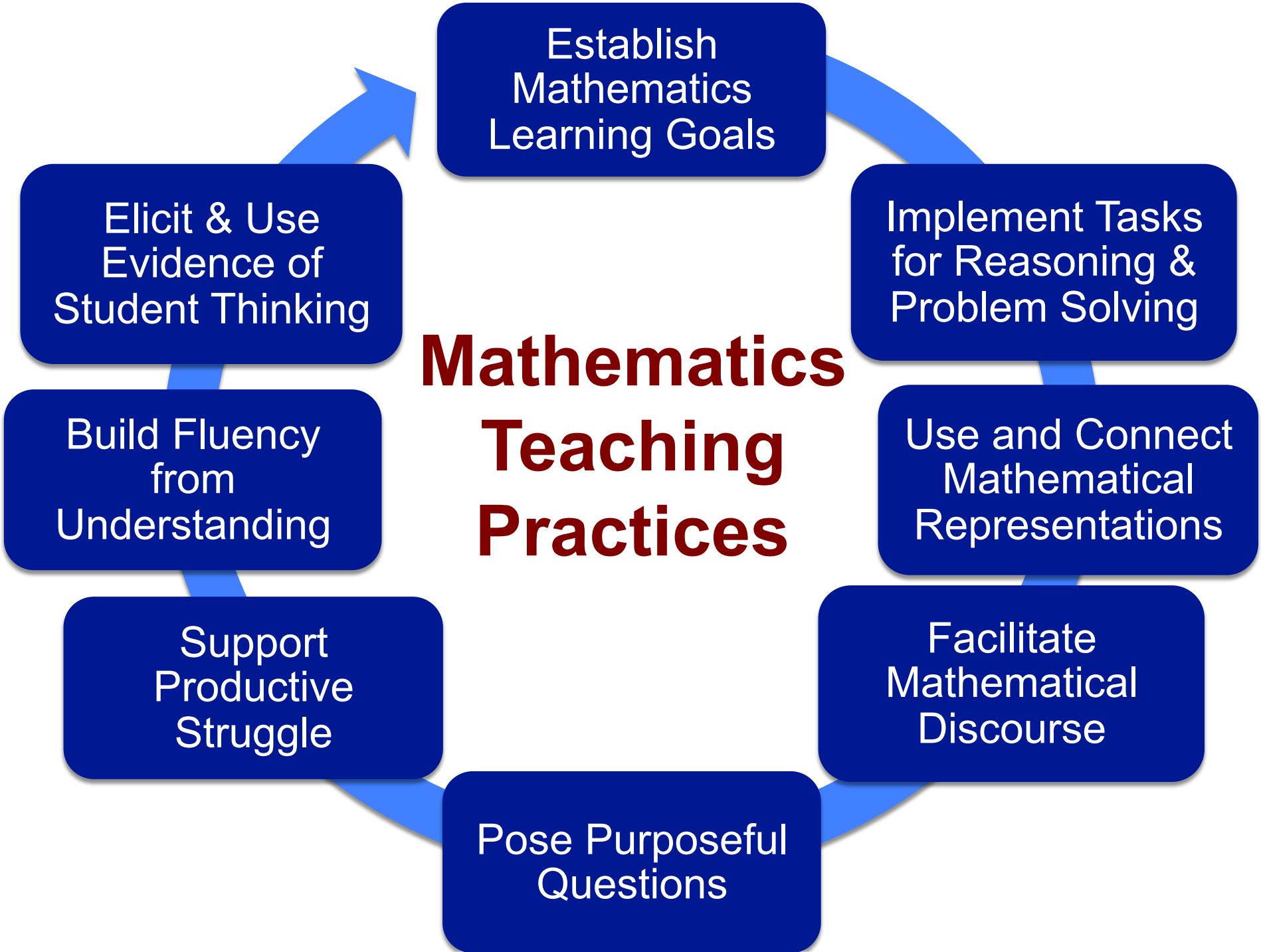
Mathematics Teaching Practices

“Best Lesson Ever”

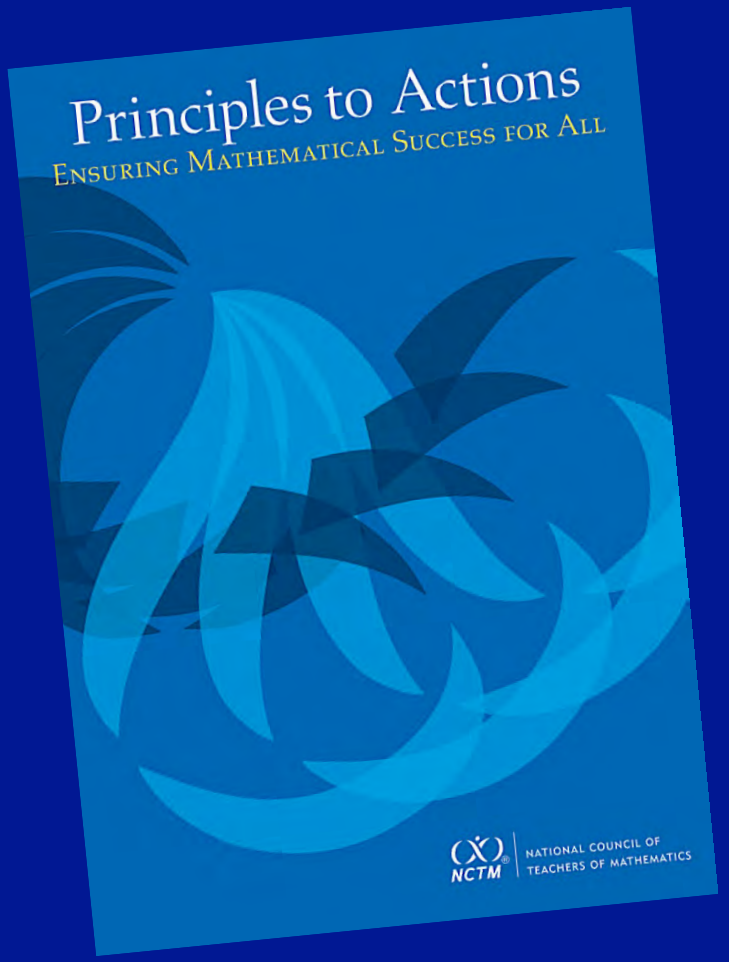
Visualize for a moment, the best math lesson you ever taught.

The best math lesson you ever taught.

- What made it so successful?
- What were you the teacher doing?
- What were students doing?
- What mathematics was being learned?


Establish Mathematics Learning Goals
Implement Tasks for Reasoning & Problem Solving
Use and Connect Mathematical Representations
Facilitate Mathematical Discourse
Pose Purposeful Questions
Support Productive Struggle
Build Fluency from Understanding
Elicit & Use Evidence of Student Thinking
Mathematics Teaching Practices

NCTM's Core Set of Effective Mathematics Teaching Practices



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The Band Concert

Make a quick sketch or diagram of the situation, and consider how Grade 3 students might solve it.

The third-grade class is responsible for setting up the chairs for their spring band concert. In preparation, they need to determine the total number of chairs that will be needed and ask the school's engineer to retrieve that many chairs from the central storage area.

The class needs to set up 7 rows of chairs with 20 chairs in each row, leaving space for a center aisle.

How many chairs does the school's engineer need to retrieve from the central storage area?

Math Goal	• What might be the math learning goal?
Task and Representations	• What diagrams or other representations might students use in solving the problem?
Fluency from Understanding	• How might we use student understanding to build toward aspects of procedural fluency?
Discourse & Questions	• How might we structure class discourse of student strategies to focus on key math ideas?
Struggles & Evidence	• How might we check in on student thinking and struggles and use it to adjust instruction?

MTP 1

Establish mathematics goals to focus learning.

Formulating clear, explicit learning goals sets the stage for everything else.

(Hiebert, Morris, Berk, & Janssen, 2007, p. 57)

#1. Establish math learning goals

Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Daro, Mosher, & Corcoran, 2011; Hattie, 2009;
Hiebert, Morris, Berk, & Jensen., 2007; Wiliam, 2011

#1. Establish math learning goals

Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

What math learning goal might the teacher have had in mind in selecting the Band Concert task?

Daro, Mosher, & Corcoran, 2011; Hattie, 2009;
Hiebert, Morris, Berk, & Jensen., 2007; Wiliam, 2011

Common Core State Standards

Grade 3.OA

Cluster: Represent and solve problems involving multiplication and division.

Standard 3.OA.3

Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Establish Math Goal

Band Concert

Set up 7 rows of chairs with 20 chairs in each row, with a center aisle.

Students will recognize the structure of multiplication within and among different representations and make connections to multiplication equations (i.e., identify the number of equal groups and the size of each group within collections or arrays).

Student-friendly version ...

We are learning to represent and solve word problems and explain how the representations match the story situation.

MTP 2

Implement tasks that promote reasoning and problem solving.

#2. Implement tasks to promote reasoning and problem solving

Effective teaching of mathematics engages students in solving and discussing tasks that promote reasoning and problem solving and allow multiple entry points and varied solution strategies.

Boaler & Staples, 2008; Hiebert et al., 1997;
Stein, Smith, Henningsen, & Silver, 2009

#2. Implement tasks to promote reasoning and problem solving

Effective teaching of mathematics engages students in solving and discussing tasks that promote reasoning and problem solving and allow multiple entry points and varied solution strategies.

In what ways does the Band Concert task provide opportunities for students to engage in reasoning and problem solving?

Boaler & Staples, 2008; Hiebert et al., 1997;
Stein, Smith, Henningsen, & Silver, 2009

MTP 3

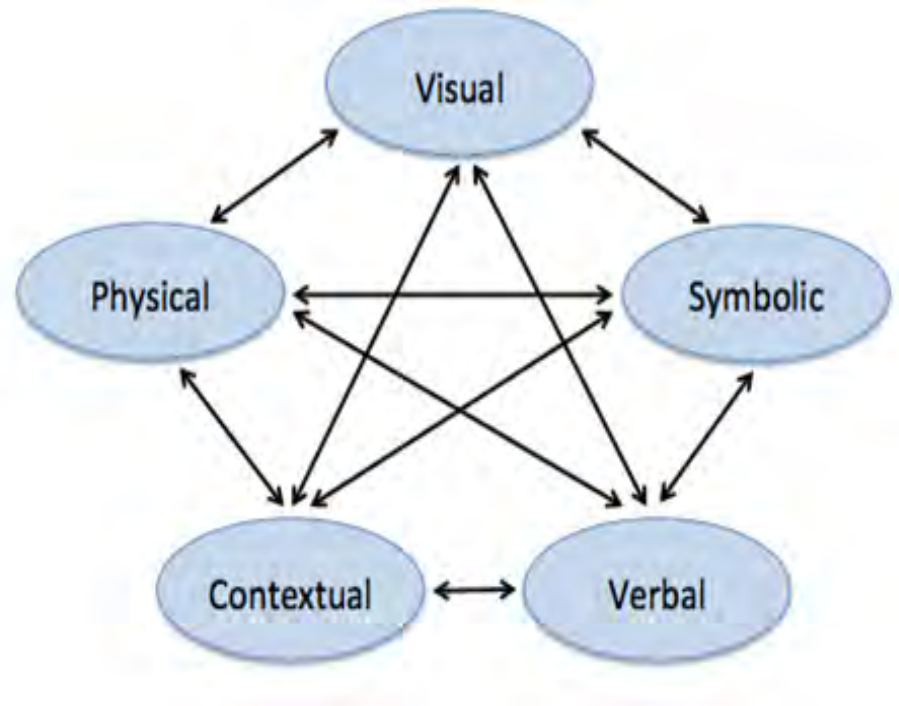
Use and connect mathematical representations.

Because of the abstract nature of mathematics, people have access to mathematical ideas only through the representations of those ideas.

(National Research Council, 2001, p. 94)

#3. Use and connect representations

Effective teaching of mathematics engages students in making connections among representations to deepen understanding of concepts and procedures and as tools for problem solving.



Lesh, Post, & Behr, 1987; Marshall, Superfine, & Canty, 2010; Tripathi, 2008; Webb, Boswinkel, & Dekker, 2008

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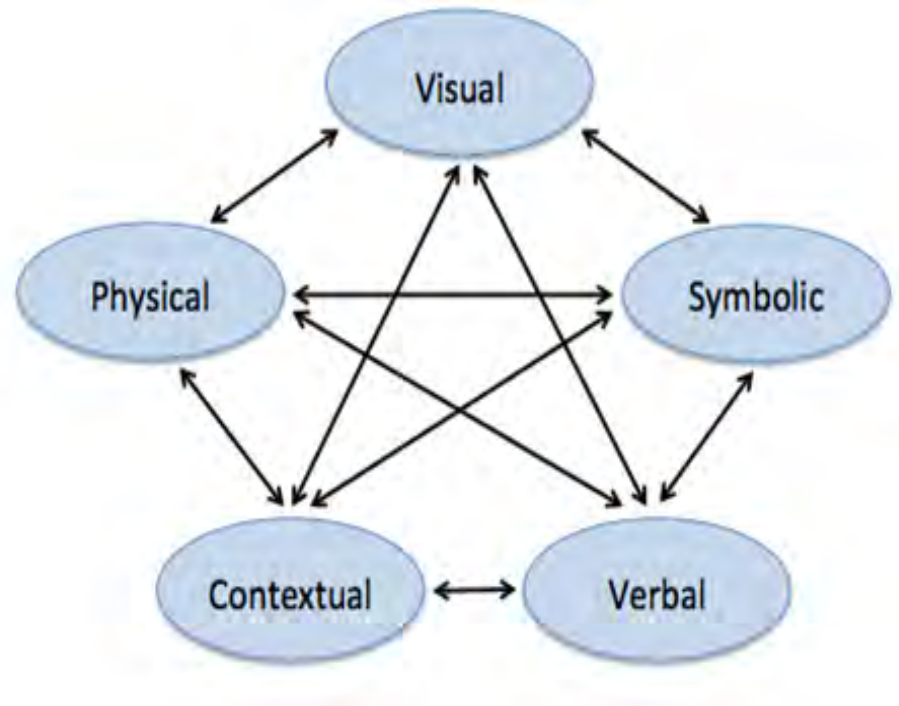


Lesh, Post, & Behr, 1987; Marshall, Superfine, & Canty, 2010; Tripathi, 2008; Webb, Boswinkel, & Dekker, 2008

Scenario: Band Concert Task

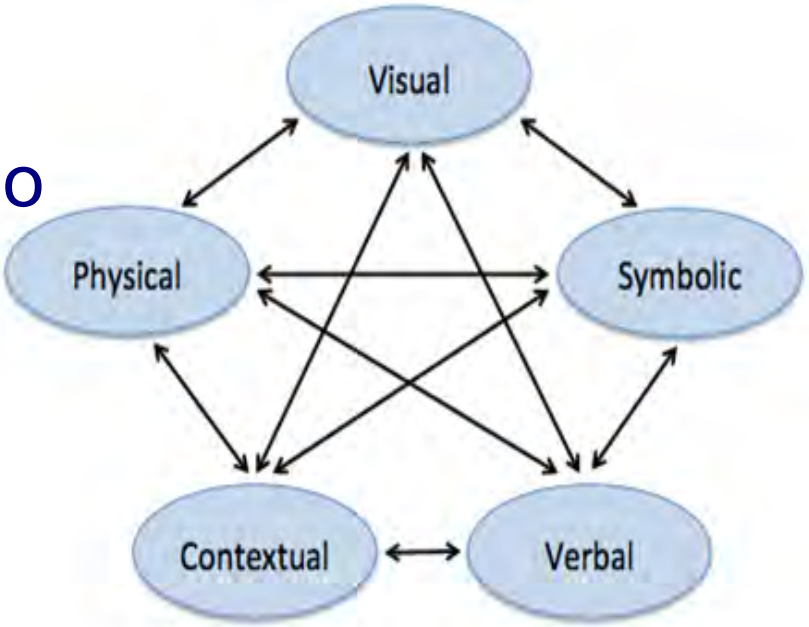
After introducing the task, Mr. Harris stated, "Before you begin working on the task, think about a representation you might want to use and why, and share your ideas with a partner."

Then Mr. Harris asked students to share some of their ideas in a brief whole group discussion.

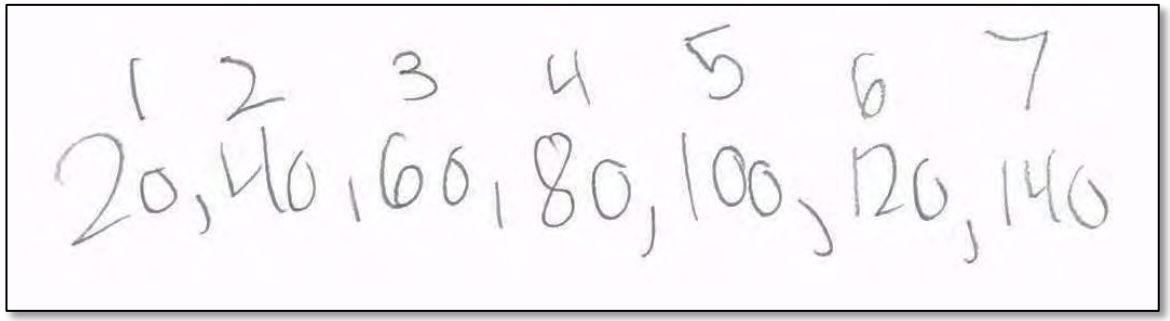
Next the students worked individually on the task.

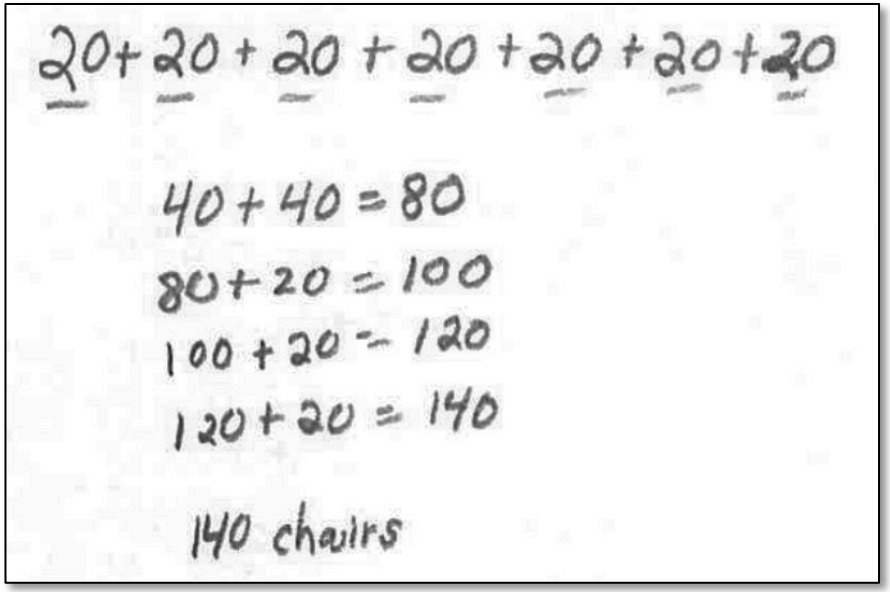
Most students sketched equal groups. Some decomposed area models. Two students cut an array out of grid paper. A few students made a table or T-chart. Some students wrote equations.

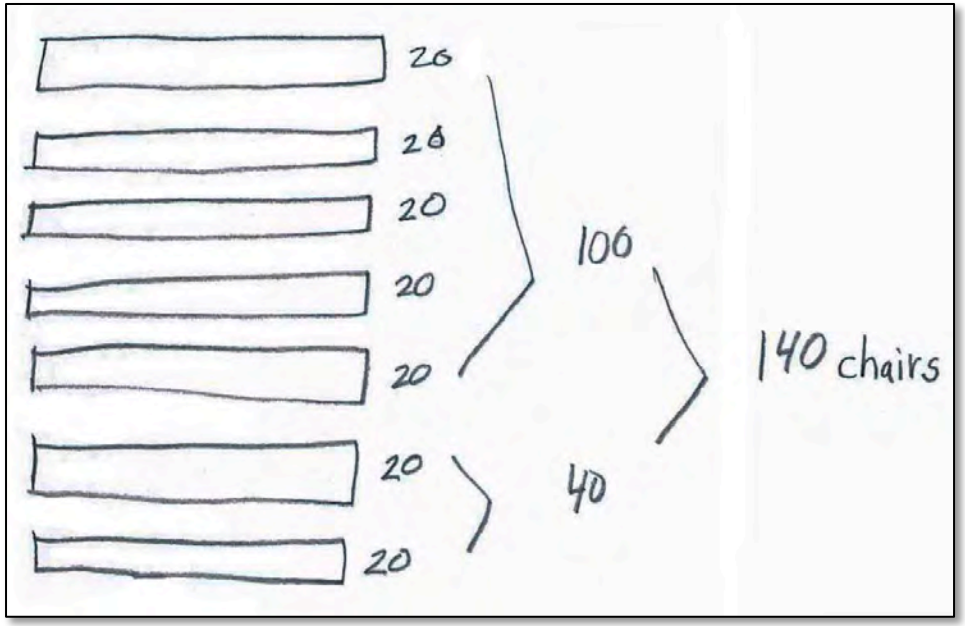
Later in the lesson, the students were told to find a classmate who used a different representation and to take turns explaining and comparing their work as well as their solutions.



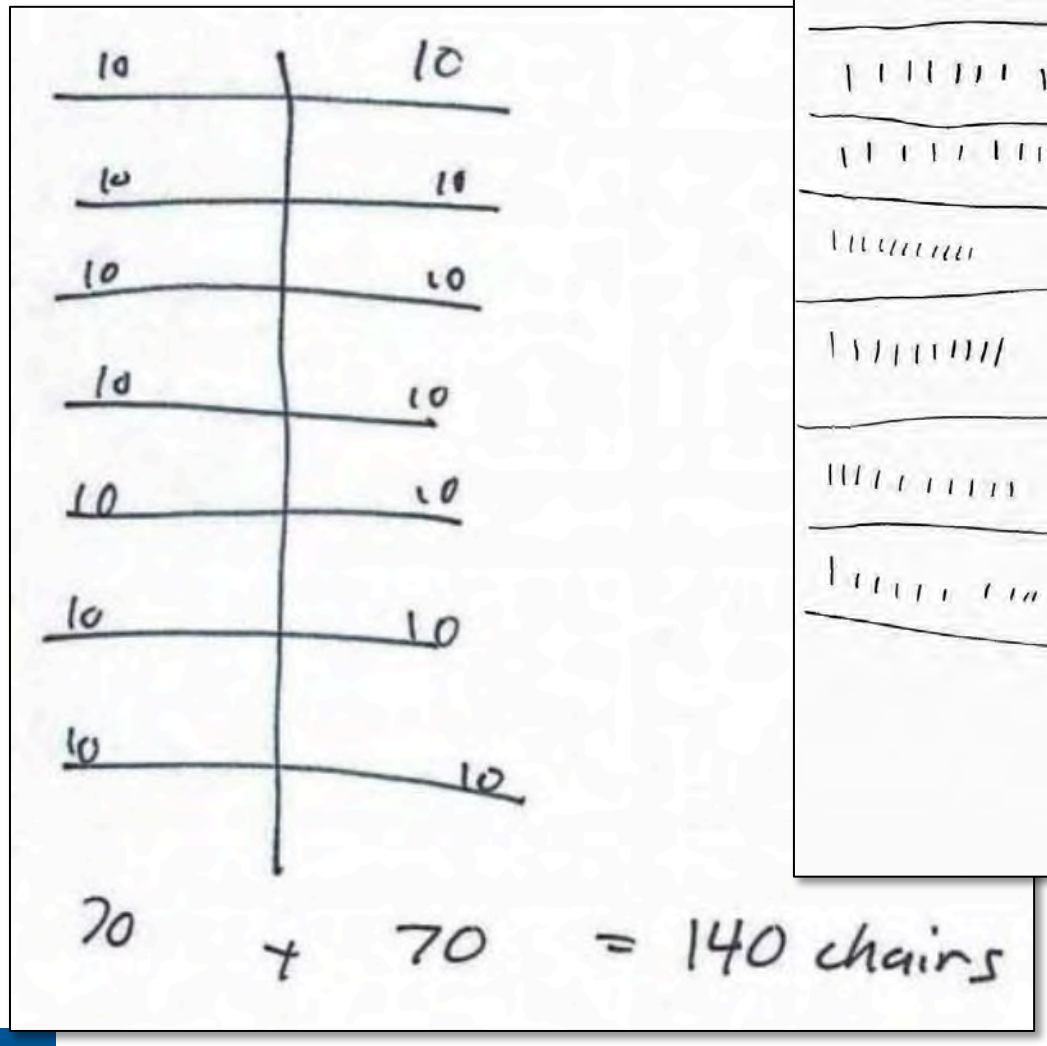
What might students notice as being similar or different among these representations?

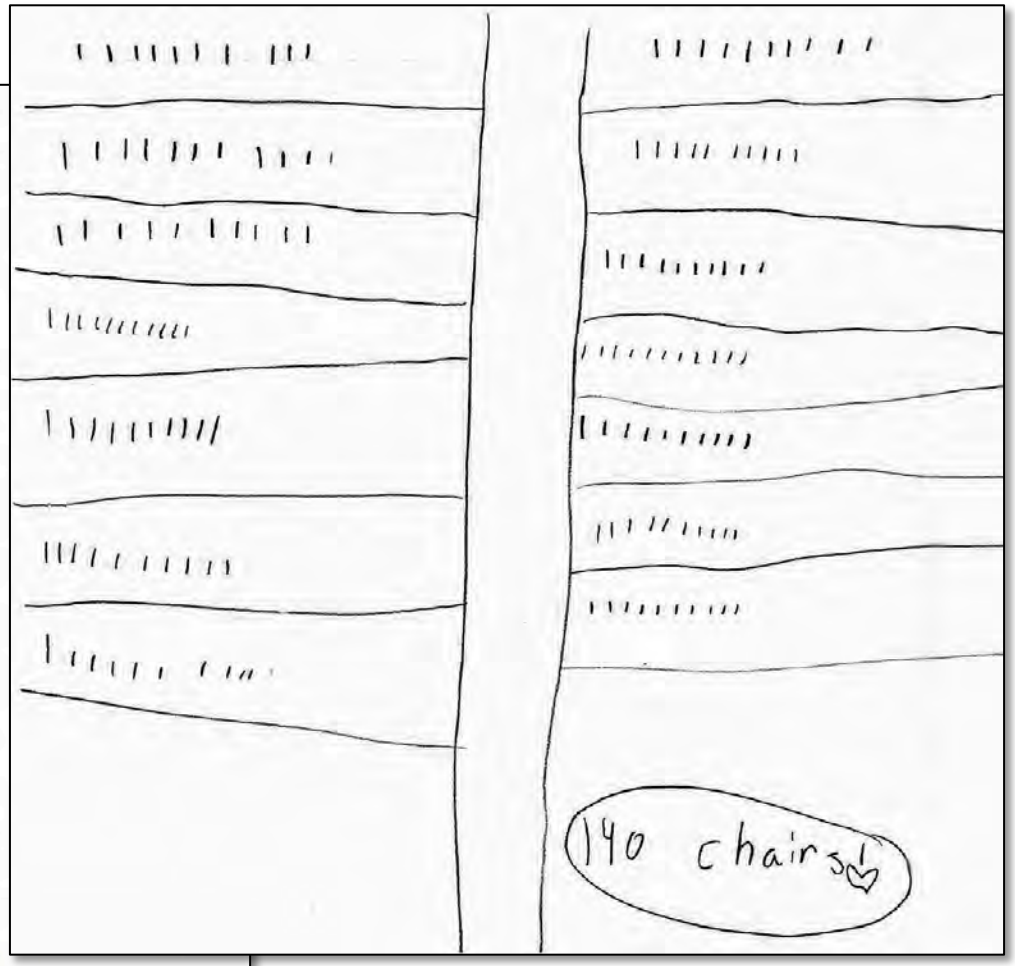






How do these representations compare and what do you notice about student thinking?





MTP 4

Facilitate meaningful mathematical discourse.

#4. Facilitate mathematical discourse

Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Carpenter, Franke, & Levi, 2003;
Fuson & Sherin, 2014; Smith & Stein, 2011

#4. Facilitate mathematical discourse

Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

How might the teacher have structured the whole class discussion of the various student approaches to move students toward the intended learning goal?

Carpenter, Franke, & Levi, 2003;
Fuson & Sherin, 2014; Smith & Stein, 2011

Scenario: Band Concert Task

In structuring the whole class discourse of the task, Mr. Harris was strategic in:

- Selecting specific student representations and strategies for discussion and analysis.
- Sequencing the various student approaches for analysis and comparison.
- Connecting student approaches to key math ideas and relationships.

Smith & Stein, 2011

MTP 5

Pose purposeful questions.

#5. Pose purposeful questions

Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Boaler & Brodie, 2004; Chapin & O'Connor, 2007;
Herbel-Eisenmann & Breyfogle, 2005

#5. Pose purposeful questions

Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

What "key" questions might the teacher have anticipated using during the lesson related to the math learning goal?

Boaler & Brodie, 2004; Chapin & O'Connor, 2007;
Herbel-Eisenmann & Breyfogle, 2005

Scenario: Band Concert Task

Math Goal: Students will recognize the structure of multiplication within and among different representations and make connections to multiplication equations.

As students worked individually, the teacher asked:

"How does your drawing show 7 groups?"

"How many twenties are you adding, and why?"

"Why are you adding tens? How does that relate to 20?

During the whole class discussion, the teacher wrote "7 x 20 = 140" and asked: Some of you wrote this equation on your paper, so I'm wondering how this equation does or doesn't match the representations that each of you used?"

MTP 6

Build procedural fluency from conceptual understanding.

#6. Build fluency from understanding

Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

Baroody, 2006; Fuson & Beckmann, 2012/2013;
Fuson, Kalchman, & Bransford, 2005; Russell, 2006

#6. Build fluency from understanding

Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

How might this lesson establish a meaningful foundation for and build toward fluency?

Baroody, 2006; Fuson & Beckmann, 2012/2013;
Fuson, Kalchman, & Bransford, 2005; Russell, 2006

Band Concert

Set up 7 rows of chairs with 20 chairs in each row, with a center aisle. How many chairs are needed?

What "fluency" expectations might the students be working toward?

How might this lesson establish a foundation for fluency?

Standard. 3.NBT.3.
Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

MTP 7

Support productive struggle in learning mathematics.

#7. Support productive struggle

Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

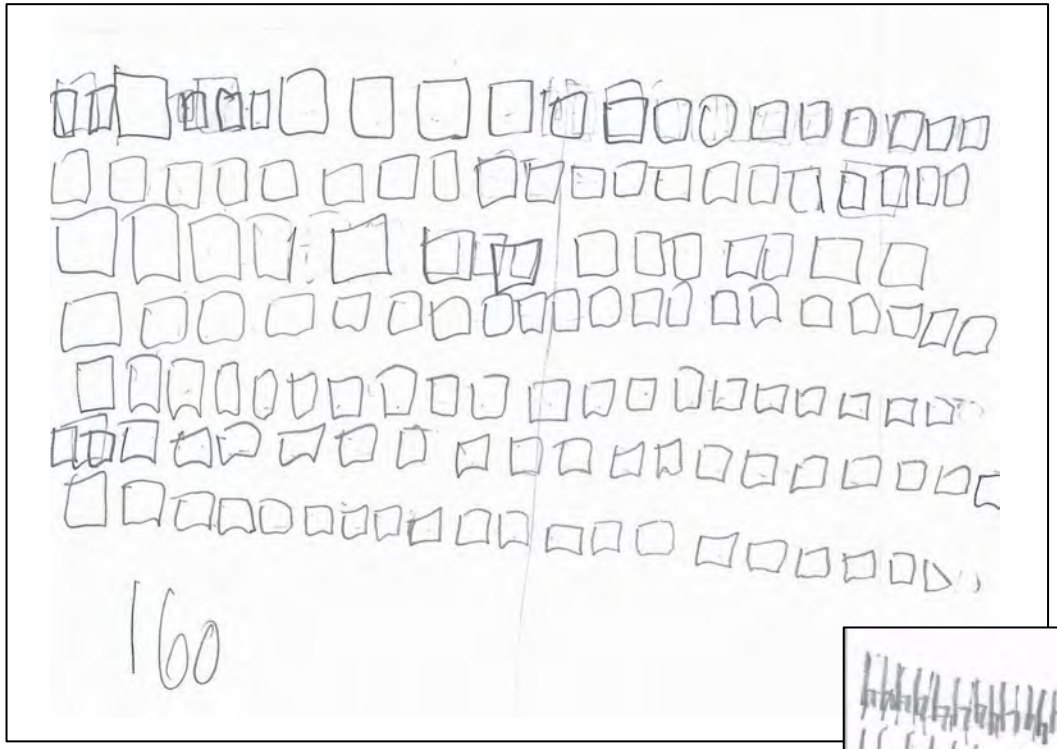
Black, Trzesniewski, & Dweck, 2007; Dweck, 2008;
Hiebert & Grouws, 2007; Kapur, 2010; Warshauer, 2011

#7. Support productive struggle

Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

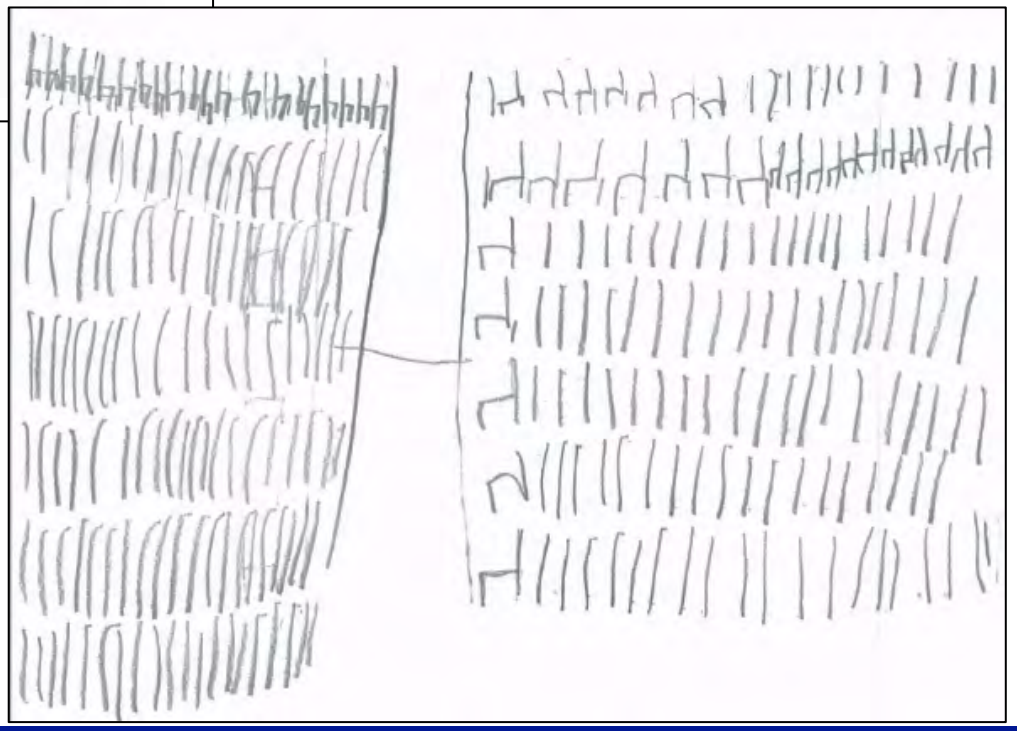
How might the teacher support students' effort while moving then toward the learning goal of the lesson . . . without taking over the thinking for them?

Black, Trzesniewski, & Dweck, 2007; Dweck, 2008; Hiebert & Grouws, 2007; Kapur, 2010; Warshauer, 2011



Support effort...

Allow grapple...



Build toward understanding and fluency...

MTP 8

Elicit and use evidence of student thinking.

#8. Use evidence of student thinking

Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Jacobs, Lamb, & Philipp, 2010;
Sleep & Boerst, 2010; van Es, 2010

#8. Use evidence of student thinking

Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

What might be some indicators of student thinking that the teacher would want to elicit and use during the lesson and to prepare for subsequent lessons?

Jacobs, Lamb, & Philipp, 2010;
Sleep & Boerst, 2010; van Es, 2010

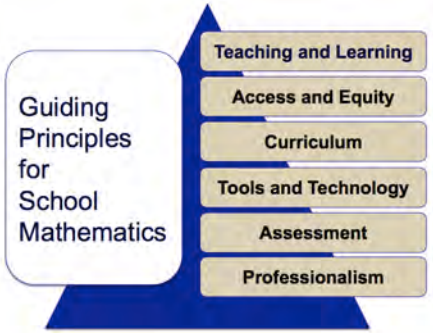
Scenario: Band Concert Task

Mr. Harris created a list of key indicators of student thinking to notice. He planned to look and listen for:

- Representations that showed 7 groups or rows.
- Strategies that decomposed groups or informally used the distributive property.
- Used concept-based language, such as making equal groups or rows of tens or twenties, in their explanations and discussions.

Next Steps and Actions...

Guiding Principle: Teaching and Learning



An excellent mathematics program requires effective teaching that engages students in ***meaningful learning*** through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.

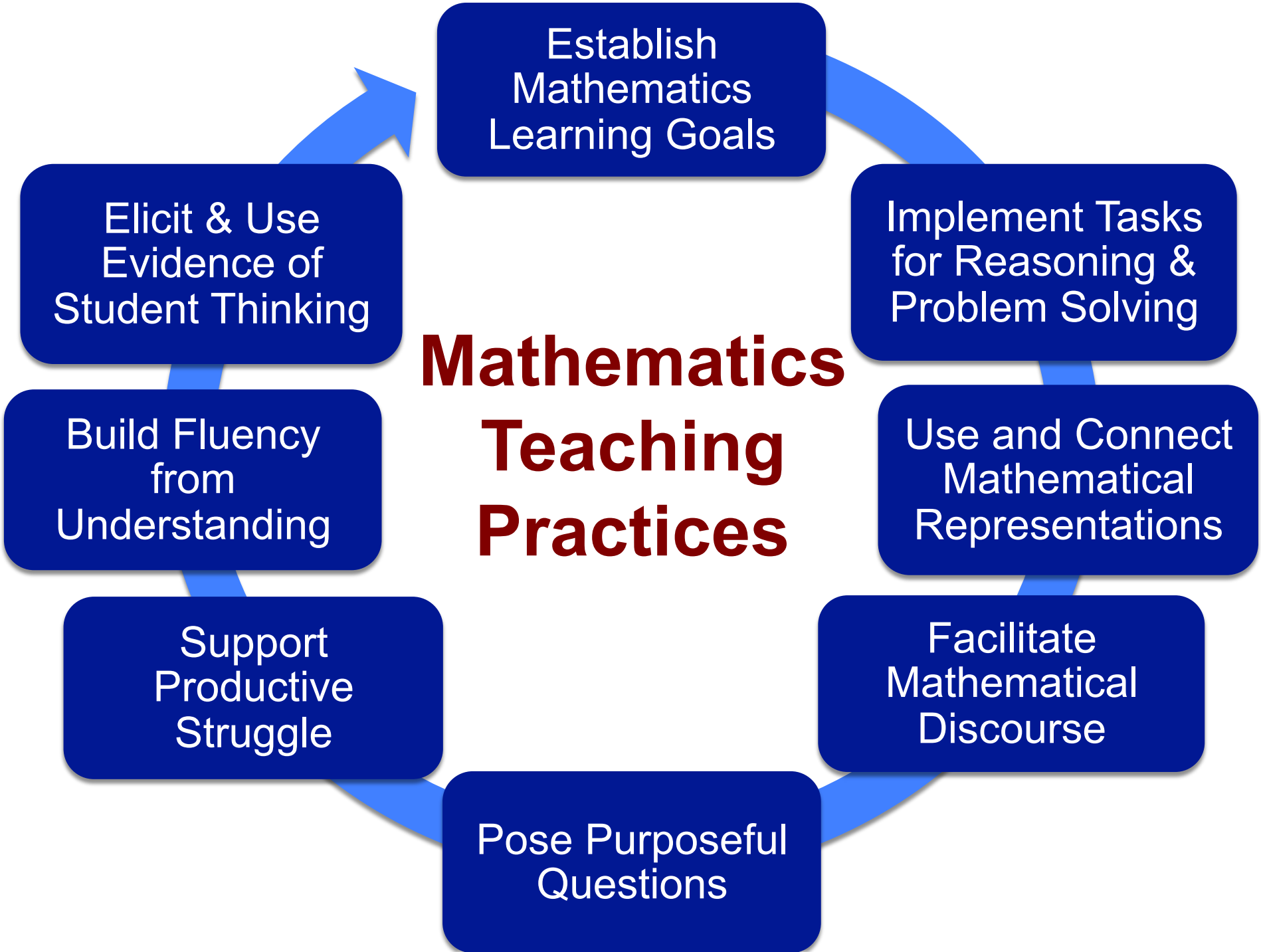
Effective Teaching Practices

Student learning of mathematics "depends fundamentally on what happens inside the classroom as teachers and learners interact over the curriculum."

(Ball & Forzani, 2011, p. 17)

Action: To work together as a profession toward implementation of a common set of high-leverage practices that underlie effective teaching .....
"those practices at the heart of the work of teaching that are most likely to affect student learning."

(Ball & Forzani, 2010, p. 45)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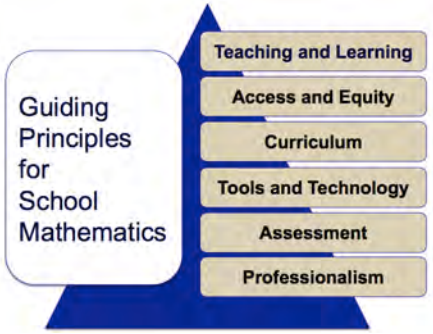
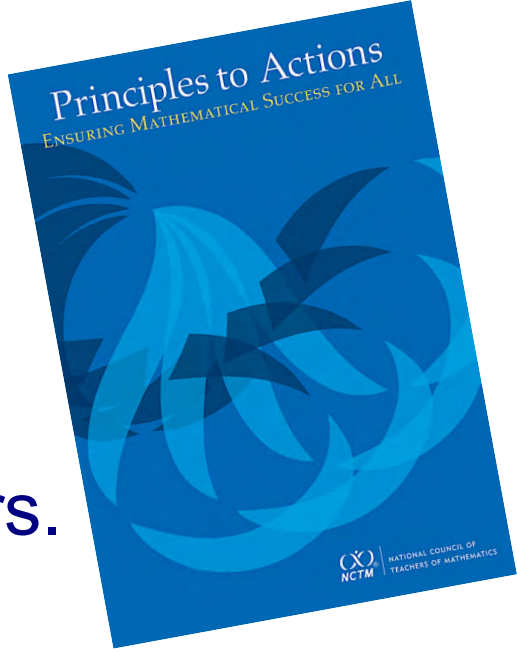
Mathematics Teaching Practices

Guiding Principle: Professionalism



In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for their personal and **collective professional growth** toward effective teaching and learning of mathematics.

Next Steps



- Read it. Read it again.
- Have conversations with your colleagues and with your administrators.
- Build a plan and work together toward common and consistent implementation of these core, high-leverage ***Mathematics Teaching Practices*** across classrooms in your school.
- Support deep, meaningful learning of mathematics and ensure mathematical success for all students.

Thank You!

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