IMPRINTS

20 Celebrating years of Mathematical Synergy

NUS National University of Singapore | IMS Institute for Mathematical Sciences

JANUARY– JUNE 2020 ISSUE 35

NEWSLETTER OF THE INSTITUTE FOR MATHEMATICAL SCIENCES, NATIONAL UNIVERSITY OF SINGAPORE

Modeling and Simulation for Quantum and Kinetic Problems

From 30 September 2019 to 31 March 2020, the Institute hosted a program on “Quantum and Kinetic Problems: Modeling, Analysis, Numerics and Applications”. The program organizers have contributed this invited article to Imprints.

BY WEIZHU BAO *(National University of Singapore)*

Modeling for quantum information
Weizhu Bao (First row, second from left) and Ionut Danaila (First row, first from left) planned great interactive training sessions for the program participants

Quantum and kinetic models have been widely used in the modeling and description for many problems arising in science and engineering with quantum effect (wave-particle duality and/or quantization) and/or particle interaction. Traditional examples include quantum physics and chemistry for particles (atoms, molecules and subatomic particles) at small scales and low temperatures based on first principle, semiconductor devices, nonlinear optics, superfluids of liquid Helium, laser beam propagation, protein folding, electronic structures in materials simulation and design, rarefied gas dynamics, etc. Over the last two decades, quantum and kinetic models have been adapted for the modeling of tremendous new experiments in physics, such as Bose-Einstein condensation, fermion condensation, quantum fluids of light, degenerate quantum gas, graphene and 2D materials, etc., and for the kinetic description of emerging applications in biology and social science, such as cell migration, collective dynamics of active matter, network formation and dynamics in social sciences, coherent structures in crowd and traffic dynamics, flocking, swarming, etc. These new surprising experiments and emerging applications generate a big wave in the study of challenging quantum and kinetic problems in terms of modeling, analysis and simulation. In fact, the new experiments and applications also call for greater participation of mathematicians and computational scientists to address some fundamental questions related to quantum and kinetic problems, to work together with applied scientists from the modeling to computational stages, to provide mathematical analysis for justifying different models, and to design efficient and accurate computational methods.

A semester-long program on Quantum and Kinetic Problems: Modeling, Analysis, Numerics and Applications was held at the IMS during 30 September 2019 to 31 March 2020. The organizing committee consisted of 4 co-chairs with Weizhu Bao (NUS), Peter A. Markowich (KAUST and University of Vienna), Benoît Perthame (Sorbonne Université) and Eitan Tadmor (University of Maryland), and 12 members with Jose Carrillo (Imperial College London), Antonio H. Castro-Neto (NUS), Ionut Danaila (University of Rouen Normandy), Yuan Ping Feng (NUS), Dieter Jaksch (University of Oxford), Shi Jin (Shanghai JiaoTong University), Henrik Jönsson (University of Cambridge), Choy Heng Lai (NUS), Mark Lewis (University of Alberta), Christian Lubich (University of Tübingen), Lorenzo Pareschi (University of Ferrara) and Zhouping Xin (Chinese University of Hong Kong). The main activities included 5 week-long workshops, 4 tutorials, 1 forum, 3 distinguished lectures, 1 public lecture and collaborating research during the program period.

The first workshop focused on Recent Progress and Challenge in Quantum and Kinetic Problems. It focused on recent new developments, mathematical and computational challenges, and multiscale modeling and simulation techniques for quantum and kinetic problems. The second workshop was on the topics of Modeling and Simulation for Quantum Condensation, Fluids and Information. Here the main themes were on new and non-standard mathematical models, multiscale and hybrid computational methods, asymptotic and perturbation analysis, and emerging applications. The third workshop was devoted to Emergent Phenomena — from Kinetic Models to Social Hydrodynamics. Participants reviewed and explored new kinetic models arising from emerging applications in biology, traffic flow, economics, network formation and collective dynamics in social sciences, and coherent structures in crowd and traffic dynamics, flocking, swarming, etc. The fourth workshop was on Mathematical Biology: Modeling, Analysis and Simulation, which focused on recent progress in mathematical modeling and analysis and numerical simulation for problems arising in biology. The emphasis of the fifth workshop were on the development of new mathematical theory and multiscale

Interacting particle systems for modeling complex real-world phenomena

methods for quantum and kinetic problems which can be modeled by dispersive PDEs and fluid equations. A forum on Nonlinear PDEs and Related Topics was organized during the program. More than 20 local speakers were invited to present their research works during these activities.

Interspersed among the workshops were four week-long tutorials designed to provide graduate students and non-experts with some background and introduction to various aspects of quantum and kinetic problems. Twelve international leading experts delivered lectures on mathematical models, applied and numerical analysis, computational methods and large-scale simulations for quantum and kinetic problems.

In a lecture on "Continuum Models of Transportation Networks with Differential Equations", Peter A. Markowich (KAUST and University of Vienna) described mathematical models and their theories for networks formation and transportation by partial differential equations. In his distinguished lecture entitled "Emergent Behavior in Collective Dynamics", Eitan Tadmor (University of Maryland) reported latest development on discrete and continuous models for collective dynamics arising from different applications and their mathematical theories. In another distinguished lecture on "PDEs for Neural Assemblies: Models, Analysis and Behavior", Benoît Perthame (Sorbonne Université) surveyed PDEs models and their analytical and numerical results for neural assemblies. Finally, in a public lecture entitled "Some Equations from Mathematical Biology", Benoît Perthame (Sorbonne Université) fascinated the audience with photographic illustrations of topics in biology that are modeled by means of partial differential equations. The chosen topics, drawn from physics, biology and social sciences, well illustrate the power of mathematics in the study of the natural world.

Last but not the least, due to the outbreak of COVID-19 in the end of January 2020, some activities which were scheduled in the last two months of the program were unfortunately size-reduced or even cancelled.

IMS Distinguished Visitors

BENOÎT PERTHAME

Benoît Perthame is professor at Sorbonne Université and member of the French Academy of Sciences. He has served for 7 years as director of Laboratory Jacques-Louis Lions. His awards include the Blaise Pascal Medal of European Academy of Sciences and the Inria Prize. He was invited speaker at the International Congress of Mathematicians in Zürich (1994), plenary speaker in Seoul (2014) and invited speaker at ICIAM 2015. Presently he is running the ERC Advanced Grant Adora. His research interests are in mathematical biology and include multiscale aspects of chemotaxis and cell populations self-organization, living tissues, neural networks and Darwinian evolution.

Professor Perthame visited IMS for the program on Quantum and Kinetic Problems: Modeling, Analysis, Numerics and Applications (30 September 2019–31 March 2020). He gave one talk on 22 January 2020 and four hours of tutorial lectures on 16 and 17 January 2020.

IMS arranges visits to the Institute by distinguished scientists who are prominent leaders in their communities. The program started in 2015. This initiative aims to enhance the diversity of people participating in our research programs, and provide mentoring/ inspire junior researchers and graduate students. Each distinguished visitor spends at least two weeks in Singapore, and participate in a variety of activities, including lecturing about their own research, give public talks, meet with faculty, and interact with program participants.

Under this program, the Institute has enjoyed visits from a stellar array of distinguished scientists. The list of distinguished visitors may be found on our website.

EITAN TADMOR

Eitan Tadmor is a Distinguished University Professor at the University of Maryland, College Park with a joint appointment at the Department of Mathematics, the Institute for Physical Sciences and Technology and the Center for Scientific Computation and Mathematical Modeling (CSCAMM). He served as the Director of CSCAMM 2002–2016, and of the NSF Research network "Kinetic Description of Emerging Challenges in Natural Sciences" (Ki-Net), 2012-2020.

Professor Tadmor received his Ph.D. in Mathematics from Tel Aviv University, and has held positions at Tel-Aviv University and at University of California, Los Angeles (UCLA). He was the founding co-director of the NSF Institute for Pure and Applied Math (IPAM) at UCLA. He serves on the editorial boards of international journals (Acta Numerica, SIAM Journal on Mathematical Analysis and Journal of Foundations of Computational Mathematics) has given lectures in the 2002 International Congress of Mathematicians (ICM) and 2019 ICIAM. Professor Tadmor was awarded the SIAM-ETH Peter Henrici Prize in 2015, and is a Fellow of the American Mathematical Society. He has published more than one hundred and eighty research papers, mostly in Numerical Analysis and Applied Partial Differential Equations.

Professor Tadmor visited IMS for the program on Quantum and Kinetic Problems: Modeling, Analysis, Numerics and Applications (30 September 2019–31 March 2020). He gave one talk on 16 December 2019 and four hours of tutorial lectures on 11 and 12 December 2019.

The Institute would like to congratulate Professor San LING, our current Management Board member, on becoming Nanyang Technological University's Provost and vice-president for academic matters from January 2020.

The Institute's housing officer Lee Jia Ling gave birth to a girl on 15 May 2020. The Institute takes the opportunity to thank Angela Aw, who left IMS on 23 June 2020, for her service and wishes her success in her future endeavors.

PAST ACTIVITIES

Quantum and Kinetic Problems: Modeling, Analysis, Numerics and Applications

30 SEPTEMBER 2019–31 MARCH 2020

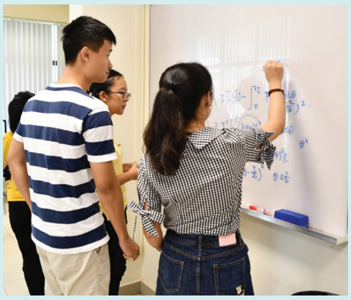
CO-CHAIRS:
Weizhu Bao | National University of Singapore
Peter A. Markowich | King Abdullah University of Science and Technology and University of Vienna
Benoit Perthame | Sorbonne Université
Eitan Tadmor | University of Maryland

Quantum and kinetic models have been widely used in the modeling and description for many problems arising in science and engineering with quantum effect (wave-particle duality) and particle interaction. Traditional examples include quantum physics and chemistry for particles (atoms, molecules and subatomic particles) at small scales and low temperatures based on first principle, semiconductor devices, nonlinear optics, superfluids of liquid Helium, laser beam propagation, protein folding, electronic structures in materials simulation and design, rarefied gas dynamics, etc. Over the last two decades, quantum and kinetic models have been adapted for the modeling of tremendous new experiments in physics, such as Bose-Einstein condensation, fermion condensation, quantum fluids of light, degenerate quantum gas, graphene and 2D materials, etc., and for the kinetic description of emerging applications in biology and social science, such as cell migration, collective motion of active matter, network formation and dynamics in social sciences, coherent structures in crowd and traffic dynamics, flocking, swarming, etc.

This six-month thematic program has gathered mathematicians, theoretical physicists, computational materials scientists and other applied scientists to come together and address some fundamental questions related to quantum and kinetic problems, provide mathematical analysis for justifying different models, and design efficient and accurate computational methods.

This program hosted a series of interconnected workshops and tutorials, offering participants updates on the latest research. Lectures in the workshops were arranged to have both applied mathematicians, physicists as well as materials scientists speak on the different aspects of the same subject. Panel discussions assembled panelists from a diverse research spectrum, to lead further dialogue. The lively debate from the interaction of participants from the various disciplines promoted cross fertilization of ideas, as they engaged in extended discussion sessions after the talks. Apart from the workshops and tutorial lectures, the organizers hosted a forum, steering conversations to more insight on a broad range of issues. What started off as a simple discussion around new ideas may evolve into valuable knowledge which could be used to the benefit of the whole community.

Facing mathematical and computational challenges in quantum and kinetic problems



Mathematical modeling and analysis

Peter A. Markowich: On WKB Asymptotics, Wigner and Bohmian measures and Schrödinger equations with relaxation-generation

Round table discussion (From left: José A. Carrillo, Eitan Tadmor, Lorenzo Pareschi and Gil Ariel)

There were four week-long tutorials by twelve international leading experts on the following topics; mathematical models, applied and numerical analysis, computational methods and large-scale simulations for quantum and kinetic problems. Thus, this program also achieved the goal of training junior researchers and graduate students by exposing them to introductory lectures and research seminars/workshops. There were six junior researchers who were recruited for the Institute's long-term junior visiting program, as they visited between one-three months.

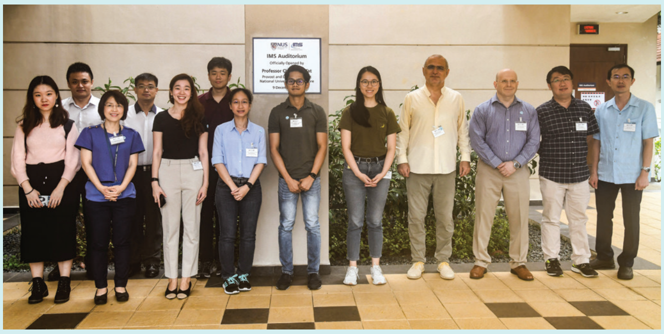
There were a total of 133 invited talks from the workshops/forum activities. The program had close to 200 participants. This program has developed and fostered local collaborations between mathematics and applied sciences within NUS, and between NUS and A*STAR (Institute of High Performance Computing and Institute of Materials Research and Engineering).

Computational Approaches to the Analysis of Biomolecular Sequences, Structures and Their Functions and Applications to Biotechnology and Clinical Data Studies

23–27 MARCH 2020

CO-CHAIRS:
Igor N. Berezovsky | Bioinformatics Institute, A*STAR
Frank Eisenhaber | Bioinformatics Institute, A*STAR
Lars Nordenskiöld | Nanyang Technological University

The workshop has brought together computational and mathematical biologists working in different areas of research, where participants discussed on cutting-edge experimental methods and computational approaches. There were 15 talks and four hours of tutorial lectures by three speakers from Bioinformatics Institute. The workshop had more than 50 attendees.



A dynamical relationship with biomolecular function

Frank Eisenhaber: Identification of the potential role of computational biology

Konstantin Pervushin: Abeta chaperones in Alzheimer's disease

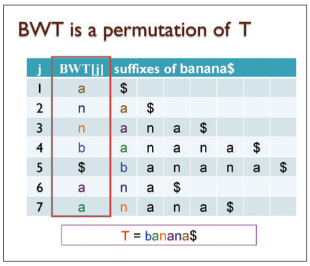
Combinatorial Problems for String and Graph and Their Applications in Bioinformatics

30 MARCH–2 APRIL 2020

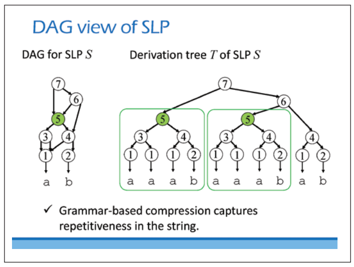
ORGANIZING COMMITTEE:
Kwok Pui Choi | National University of Singapore
Sohel Rahman | Bangladesh University of Engineering & Technology
Vaibhav Rajan | National University of Singapore
Kunihiko Sadakane | The University of Tokyo
Wing-Kin Sung, Ken | National University of Singapore

The program activities began with lectures by three speakers on 30 March 2020 who delivered tutorials on an introduction to Burrows-Wheeler transform, combinatorial algorithms for grammar-based text compression, and graph models and algorithms in genome assembly.

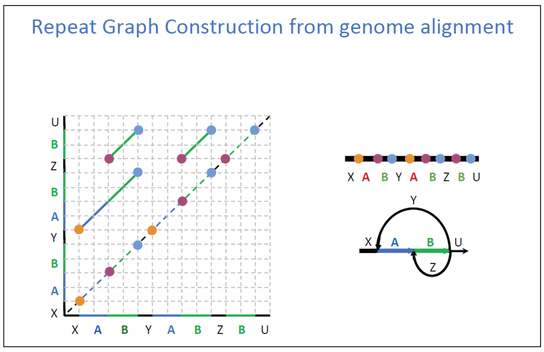
The annual International Conference and Workshop on Algorithm and Computation (WALCOM) continued from 31 March to 2 April 2020. The invited speakers were Osamu Watanabe (Tokyo Institute of Technology, Japan), Md. Saidur Rahman (Bangladesh University of Engineering and Technology, Bangladesh) and Zhang Louxin (National University of Singapore, Singapore). There were eight sessions with 23 presentations, which were conducted online in view of the travel restrictions due to the COVID-19 situation. There were more than 50 participants.



An introduction of Burrows-Wheeler transform



Tutorial on Combinatorial algorithms



Tutorial on Graph models and algorithms in genome assembly



Invited speakers of the 14th international conference and workshops on algorithms and computation (WALCOM)
[From left: Louxin Zhang, Md. Saidur Rahman and Osamu Watanabe]

OUTREACH

Ng Kong Beng Public Lecture Series

20 JANUARY 2020

Some Equations from Mathematical Biology

The institute thanks Prof. Perthame for giving a lecture

Many areas of physics are recognized by equations — mostly partial differential equations —that bear the name of the people who invented them. For instance, Newton's equation describes the fundamental principles of dynamics, the Navier-Stokes equation describes the flow of fluids, and the Boltzmann equation relates the previous two; and there are many more. Are there such equations in biology, or is mathematics even a good language to describe biology? Prof. Perthame gave an affirmative answer to this question by means of many interesting examples.

Probably the most famous partial differential equations are the Lotka-Volterra equations proposed in the early 20th century, and they describe a population of prey and a population of predators interacting with each other and giving rise to periodic behaviour. The name has become generic for a large class of equations in ecology. Even older are the Kermack–McKendrick equations from epidemics, having its roots in mathematical equations developed by Daniel Bernoulli in the mid-18th century.

A third example is the Hodgkin–Huxley model from the 1950s to describe how action potentials in neurons are initiated and propagated, which lead to a Nobel prize for the two inventors of the model. In the spirit of Boltzmann, who bridged the microscopic particle-based dynamics described by Newton's equation with the macroscopic dynamics described by the Navier-Stokes equation by means of intermediate probabilistic models, modern physicists have proposed equations that simplify the Hodgkin–Huxley model in order to describe the behaviour not of one neuron, but of millions of neurons connected in a network. Modelling of chemotaxis, bacterial motion and even living tissue are further areas in biology and medical research where partial differential equations have played an important role in understanding the dynamics governing those processes.

Prof. Perthame concluded by emphasising that mathematical equations have been important and successful in describing complex biological systems, but he also highlighted that the situation is more complicated than in physics. This is in large parts due to the fact that parameters in biological systems are not as universal as in physics and can vary substantially from system to system. While the speed of light is universal, the speed of cell division is not, since it depends on the tissue, the time of day, health of the individual and many other factors.

Prof. Perthame of Sorbonne Université, France delivered the public lecture at NUS on 20 January 2020. Over 92 people attended the lecture.

Adrian Röllin

Watch a recording of our public lectures

WWW

The Importance of Quantum Mechanics to Saving Our Planet
Kieron Burke, *University of California, Irvine, USA*

Can Every Mathematical Problem Be Solved?
Menachem Magidor, *The Hebrew University of Jerusalem, Israel*

Mobile Health Intervention Optimization
Susan A. Murphy, *Harvard University, USA*

Mathematics in the Solar System
William A. Casselman, *The University of British Columbia, Canada*

Available on our webpage and YouTube channel

YouTube

EMMANUEL ULLMO:

DIOPHANTINE GEOMETRY – FROM ERGODIC THEORY TO O-MINIMAL THEORY

Interview of Emmanuel Ullmo by Y.K. Leong

"Much more, I am pleading for the abolition of the word 'use', and for freeing of the human spirit."

– Abraham Flexner

Emmanuel Ullmo made important contributions to arithmetic geometry and ergodic theory.

Born in Paris of a French, Vietnamese and Eastern European heritage, Emmanuel Ullmo studied at *École Normale Supérieure de Cachan* before going to *Université Paris-Sud* in Orsay to do a PhD under the direction of Lucien Szpiro. After his PhD, he remained there as *Maître de Conférences* (Assistant Professor with tenure) for six years. He was subsequently an assistant professor at Princeton University for two years before moving back as full professor to his alma mater *Université Paris-Sud*, where he subsequently served as Director of the Department of Mathematics and as President of the Hiring Committee. He has been a member of the Scientific Council of the Centre Émile Borel. In 2013 he took over from Jean-Pierre Bourguignon as Director of *Institut des Hautes Études Scientifiques* (IHES, Institute of Advanced Scientific Studies).

He was awarded the *Prix Elie Cartan* of the French Academy of Sciences and was a member of the *Institut Universitaire de France* and an invited speaker at the International Congress of Mathematicians in Beijing in 2002. He was a member of the editorial board of the collection Panoramas et Synthèses of the Société Mathématique de France (Mathematical Society of France). He has been on the editorial board of *Inventiones Mathematicae* since 2006, being one of its Editor-in-Chiefs from 2008 to 2014.

Ullmo made important contributions to arithmetic geometry, notably in Arakelov geometry, and to ergodic theory. He proved the Bogomolov conjecture that the algebraic points of a curve form a discrete set in its Jacobian[1]. His recent research is in the fields of automorphic forms and functional transcendence theory.

Ullmo has been involved in the organization of the International Workshop on the Zilber-Pink Conjecture at CIRM (*Centre International de Rencontres Mathématiques*), Luminy, the Trimester Program (in 1999) on "Diophantine Geometry" at Centre Émile Borel in Paris and the International Conference in celebration of the 60th birthday of Lucien Szpiro.

Ullmo had close mathematical connections with some of the faculty of the Department of Mathematics in the National University of Singapore (NUS) before he became a member of the Scientific Advisory Board of the Institute for Mathematical Sciences (IMS) in 2017. He was invited to the IMS Program "Complex Geometry, Dynamical Systems and Foliation Theory (1–26 May

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[1] E. Ullmo, "Positivité et discrétion des Points Algébriques des Courbes", *Annals of Mathematics*, **147** (1), 167-179

INTERVIEW

2017)". On 24 May 2017 he also gave an invited talk on "Bi-algebraic Arithmetic and Bi-algebraic Geometry" at the Mathematics Department. It was during this visit to IMS that Y.K. Leong interviewed him on behalf of *Imprints* on 25 May 2017. The following is an edited and updated version of this interview in which Ullmo gave us a glimpse of the esoteric area of Diophantine geometry and an insight into *Institut des Hautes Études Scientifiques*, one of the world's foremost mathematical research institutes situated in Bures-sur-Yvette just south of Paris.

Acknowledgement Y.K. Leong would like to thank Von Bing Yap of NUS's Department of Statistics and Applied Probability for preparing a draft of the transcript of the interview.

IMPRINTS I What did attract you to do your doctoral thesis in number theory and algebraic geometry?

EMMANUEL ULLMO U I've always been attracted to mathematics. When I started my graduate studies, I followed a course by Michel Raynaud in algebraic geometry. I was quite fascinated by what he was teaching even though I thought it was quite difficult for me at the beginning. The course was very quick-paced; you had to learn a number of things in a very short amount of time. But it's something I kept in my mind and then, when I started looking for a PhD advisor, I decided to find one in that direction. With the help of Michel Raynaud, I found Lucien Szpiro[2]. When I started my PhD, he was a CNRS researcher at *École Normale Supérieure*. In t he middle of my PhD, I went to IMPA[3] in Brazil. So I finished my PhD there. When I came back from Brazil, I defended my thesis at *Université Paris-Sud* where Szpiro had just moved to from *École Normale Supérieure*.

I After your PhD from *Université Paris-Sud*, you were there as *Maître de Conférences* (Assistant Professor with tenure) from 1993-1999. I believe that during this period you also spent time at IMPA in Brazil (18 months) and Tsinghua University in China (6 months). How did this come about?

U It was not exactly the same period. Indeed, I went to Tsinghua in China for 6 months, but it was in 2005 when I was already professor at *Université Paris-Sud*, not between `93 and `99.At that time, Jean-Marc Fontaine who was professor also at Paris-Sud, working on *p*-adic theory, has launched a joint program with Tsinghua University. The modus operandi was that a professor from *Université Paris-Sud* would spend several months there and teach a PhD course to attract some of the best students from Tsinghua who would be willing to start a PhD in France at Paris-Sud. The program was quite successful. People like Luc Illusie, Michel Raynaud, Jean-Marci Fontaine and myself gave courses at Tsinghua. Several very good students were attracted to France and prepared a PhD in mathematics at Paris-Sud. Some of them stayed in France and some of them went back to China. For my time in IMPA it was part of my military service. In France, at that time, you had to do a military service. You could do it in a civilian form by going to a foreign country for a longer period of time than usual. I took the advantage of my civil service, to spend the 18 months at IMPA. I left in June 1992 when I was finishing my PhD.

I Your time at IMPA was counted as military service? That's a nice way to fulfill military service.

U Yes. This said, I was not the first French mathematician who did that. The Fields medalist Jean- Christophe Yoccoz[4], who passed away, did that too. He came back to Brazil quite often after that and eventually married a Brazilian. My first son was born in Brazil at the end of my first year there.

I Is there any specification of the place that you should spend your military service?

U There is no military service anymore in France, but at that time, you could do military service anywhere in the world, as long as the French government accepted your proposition. It was part of the cultural exchanges between France and other countries, like a diplomatic relation in education.

I In 1999 your PhD advisor Lucien Szpiro moved to City University of New York (CUNY) in the US, and coincidentally, you moved to Princeton University that year. Was it sheer coincidence?

U It was two independent moves. After my proof of the Bogomolov conjecture, around 1997, I was contacted by some professors from Princeton University asking me if I would be willing to spend some time there. For family reason, I couldn't accept at that time but I was able to do so in 1999.

I How much influence did Szpiro have on your research?

[2] Lucien Szpiro passed away in Paris on 18 April 2020 at the age of 78
[3] *Instituto de Matemàtica Pura e Aplicada*
[4] Jean-Christophe Yoccoz (1957-2016), Fields Medal 1994

U I think he had a lot of influence on my taste. His point of view was always that what you're really aiming at is to solve problems in Diophantine geometry. To achieve this goal, you may have to use a lot of different theories but you should always keep in mind that you shouldn't develop the theory for its own sake, but you should be guided by some applications in Diophantine geometry. So I tried to follow his path and I still think in this way. When I started my PhD, he suggested me to work in Arakelov theory. At that time the subject was quite new. I was the first to do a PhD on this subject, in France. It really was a topic which was moving fast and therefore was a very interesting area.

I This theory arose from a problem itself?

U Yes. Arakelov theory arose because you could prove the Mordell conjecture for function fields and you could try to prove it for number fields if you were able to have some more ingredients like an arithmetic intersection theory. Arakelov theory was developed with this goal in mind. The proof of the Mordell conjecture by Gerd Faltings (in 1983) is in the spirit of Arakelov theory.

I I believe that besides algebraic geometry and number theory, you are also interested in ergodic theory, which goes back to Henri Poincaré almost 120 years ago. Are there any significant developments at the interface of dynamical systems and your areas of research?

U Yes! A lot of things! The proof of the Bogomolov conjecture uses in an essential way some equidistribution properties of sequences of points with small heights. At that time, I started to study several other questions in equidistribution theory. With Laurent Clozel and Hee Oh, I proved some general theorems concerning the equidistribution of Hecke points on homogeneous varieties. I then became very interested in the equidistribution properties of CM points[5] on Shimura varieties, but this turned out to be very difficult. When I was at Princeton, I heard a lot about the work of Ratner[6], concerning ergodic properties of unipotent flows on homogeneous spaces. When I started to learn it, I realized that I could applied it to prove some equidistribution properties of special subvarieties of Shimura varieties.

I Did you use any probabilistic methods?

U I wouldn't say so. Though I'm using sequences of probability measures, I am not using deep results from probability theory. I would say it's more measure theory than probability theory.

I Probability is actually just analysis.

U In my work, it's just analysis.

I I recall that you proved the Bogomolov conjecture for the algebraic points of a curve in its jacobian some years ago. What problems are you working on now?

U I will start by recalling what is the Bogomolov conjecture. The Bogomolov conjecture concerns curves of genus at least two. By Faltings' proof of the Mordell conjecture, we know that algebraic curves of genus at least two over a number field, have only finitely many rational points. The question asked by Bogomolov was: if you look at the set of all algebraic points of an algebraic curve, which is an infinite set, you want to somehow say that the set is not too big. We expect to have few algebraic points on an algebraic curve of genus at least 2. The Bogomolov conjecture asserts that the algebraic points of such an algebraic curve form a discrete set in the Néron-Tate topology of its Jacobian. I proved that you can always find, around any algebraic point, a small ball for this topology for which there are only finitely many other algebraic points. If you wish, it's a little bit like **Z** inside **R**. It's a discrete set. So you have infinitely many algebraic points, but somehow not so many. Currently, I am working on some problems in unlikely intersections, which are generalizations of problems linked to the Manin-Mumford conjecture and the André–Oort conjecture.

I What nationalities are they?

U Yves André is French, Frans Oort is Dutch, Yuri Manin is of Russian origin, and David Mumford is American. Now I'm working on some generalizations of these two conjectures by Boris Zilber and Richard Pink. I think Zilber is of Russian origin and he works in England. Richard Pink is German and works in Switzerland. I am still using ideas from ergodic theory, but also ideas coming from mathematical logic, more precisely, o-minimal theory. This is a kind of "tamed geometry" which is more flexible than algebraic geometry, as you may use more functions as the real exponential, but still has some rigidity properties usual in algebraic geometry. The heart of what I'm doing is trying to prove some functional transcendence results that you can state purely in terms of complex geometry but, in fact, the proof is crucially using some o-minimal theory.

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[5] CM stands for "complex multiplication" and means maximal possible endomorphisms of an elliptic curve, an abelian variety or a Hodge structure

[6] Marina Eyseevna Ratner (1938-2017)

I **In 2001 you moved from Princeton back to your alma mater *Université Paris- Sud*, where you remained for 12 years until you became director of *Institut des Hautes Études Scientifiques* (IHES). Does Paris-Sud hold some special attachment for you?**

U I would just say "yes"! In fact, I'm somehow still a professor at *Université Paris-Sud*. I try to go there once a week and maintain relations with colleagues. The distance between the Institut des Hautes Études Scientifiques and the math department of Université Paris-Sud is a little bit like the one between IMS and the math department here. About 20 minutes' walk. It's really important for an Institute like IHES to have a lot of scientific links. I try to develop joint activities with the math department of Paris-Sud, which is one of the best math departments in the world. Four researchers from this department were awarded Fields Medals and a lot of colleagues are of very high quality. It's a department that I value a lot.

I **IHES is an independent body, isn't it?**

U Yes it is. IHES is a private foundation recognized as in the public interest. As a consequence, anyone making a gift to IHES may have a tax deduction. This plays an important role for us.

I **IHES was modeled after the Institute for Advanced Study (IAS) in Princeton. Please tell us some of the similarities and differences between IHES and IAS.**

U Indeed, the Institute for Advanced Study served as a model for IHES which was created almost thirty years later. The idea was really to take advantage of what had been done at the IAS – to attract the best people. From the start, Robert Oppenheimer[7], who was then the director of IAS, was in the IHES board of directors as well as in its scientific council. He helped Léon Motchane[8], the founder of IHES to establish the status for the newly formed Institute. We were fortunate enough to hire as our first permanent professor Alexandre Grothendieck[9]. He created a lot of activity around himself at the beginning of IHES. I should say the Institute is smaller in size than the Institute for Advanced Study. We are only focused on mathematics and theoretical physics whereas IAS has also humanities and life sciences. Nevertheless, we are operating for math and physics in a way comparable to the Institute for Advanced Study. We run a huge visitor programme: more than 200 researchers are coming every year for an average visit of two months and a half. At any time we have around 40 visitors. We also try to have permanent professors at the highest international level. Out of the 10 permanent professors we hired in math, seven were awarded the Fields Medal. In a way, we try to "bet" on our permanent professors, and we did pretty well so far. We hire our permanent professors when they are young. For example, the average age of a permanent professor in mathematics when hired by IHES is 31.

I **Did they get the Fields Medal after being hired or before?**

U Generally after.

I **So you had the foresight to . . .**

U Yes, the idea is to try to hire people before they are fully recognized.

I **That's interesting. How many permanent members are there?**

U At the moment, we have six; I should say five. A new permanent professor Slava Rychkov will arrive in October. He works in theoretical physics. He is of Russian origin. He works at CERN for the moment and will quit it to come at the Institute.

I **There seems to be a strong Russian connection in IHES.**

U We have a long tradition of Russian researchers at the IHES. Léon Motchane who was the founder of IHES, was of Russian origin. The first mathematician hired at IHES was Alexandre Grothendieck, who was also of Russian origin. Then IHES later hired Misha Gromov, who was really very influential and changed drastically the way of thinking about Riemannian geometry. Then, maybe he helped us to convince Maxim Kontsevich to come to the Institute.

I **Although IHES was established in 1958, it seems to be not so well known outside Europe. In 2008, IHES held a number of activities outside France, namely in the US, Japan and China, to celebrate its 50th anniversary. How has the visibility of IHES been enhanced during the past 10 years?**

U I disagree with your starting point. I think that in the small world of academics in mathematics, IHES is well-known in the USA as well as in Russia, in Japan and in Europe. We have a lot of connections with them and we host every year a huge number of researchers from all parts of the world. So, maybe your question is

[7] Julius Robert Oppenheimer (1904 - 1967)

[8] Léon Motchane (1900 – 1990)

[9] Alexandre Grothendieck (1924-2014), Fields Medal 1966

more about the level of the relations between IHES and Singapore. I think that for people working in pure math in first-class universities, IHES is somewhere at the top. I mean, probably we are the institution with the highest number of the Fields medalists in the world. For pure math, it's something significant. I think that there are maybe not yet too many links between IHES and Singapore. At the moment Wee Teck Gan, who is a professor at the math department here, is at IHES. Tien Cuong Dinh is quite related to me scientifically. In fact, that's why I'm here. He is originally from Vietnam and he knows IHES very well. He was supposed to come [to IHES] for a visit, but unfortunately he had to cancel it. I think links are starting to develop and I hope they will continue to do so.

I Could you let us have a glimpse of some of the future plans for IHES?

U I started my work at IHES in 2013. Since then, I tried several things. The first one was to ask the IHES permanent professors to teach courses every year like those in the Collège de France. The courses are attended by PhD students, postdocs or colleagues. They are advanced research courses, generally on the work of the permanent professor. We now have a program every year of six or seven courses, as we also invite very high-level professors to give such courses. I started also to organize summer schools on a more regular basis. So every year now we have a two-week summer school for more than 100 PhD or postdoctoral students. The topic is new every year and the scientific council selects the best program three years in advance. It is quite successful. I hope it's now on track. At the moment we have more than 250 lectures a year. I also try to develop the relations between IHES and the natural environment of mathematicians and physicists in the Paris area. People from Orsay or *École Polytechnique* can organize activities at IHES. Every year some professors from Orsay spend half of the year at IHES. We start to organize joint seminars. I try to multiply the links and activities between our institutions.

I That means you need to expand physically.

U I expand the number of events organized and the number of links between the people.

I What about the physical infrastructure?

U I'm forced to build a new building that will have 13 new offices. It's not a huge extension, but at the moment we have too many activities and it was starting to get annoying for people. Visitors are used to have very good conditions. As we developed significantly the postdoctoral programs and we hired new permanent professors, our activities are really increasing rapidly. We will have this new building by the end of 2018. This said, we want to keep the Institute at a reasonable size while expanding.

I A mega-university has been planned at the *Université Paris-Saclay* and has started its academic year in 2015. Since IHES itself is in the Saclay area, will this mega-university project have any effect on the development of IHES as a center of excellence?

U IHES is really a part of this Paris-Saclay concept. We are one of the 19 founding members of this university. The establishment of this university is quite difficult because you have to get 19 different institutions work together. So I hope we will see soon some very important progress in this direction. One of the missions of Paris-Saclay is to attract and to maintain exceptional talented researchers in the Paris-Saclay area, and this is also one natural mission of IHES. We can benefit from Paris-Saclay because we have the same goal and so we can help each other. For example, when we hired our last permanent professor in mathematics, Hugo Duminil-Copin who is doing probability theory, the *Université Paris-Saclay* gave us a very significant grant to set up a team with him when he started his position. It is a very useful tool for us to convince someone to move to IHES as we can tell him that he will be able to work in very good conditions from the start. I believe it's a very important step and I hope you will see it in the future that this university will be recognized as one of the main universities.

I Do you know if this mega-university is going to have one central governing body?

U Yes. Well, the idea is to have one central governing body and some components. The components could have a lot of autonomy.

I What about the name? Will it be called *Université Paris-Saclay*?

U Yes.

I Then, for the other components, would they have to give up their names?

U I don't think so. For example, *École Normale Supérieure de Cachan* is to be called *École Normale Supérieure Paris-Saclay* because they will move out from Cachan. But I would believe that other institutions like IHES will keep their names.

I This question is about the funding and viability of IHES. Of the six permanent members of IHES, three are in theoretical physics, and three in pure mathematics (four if we include yourself). It seems that the research interests of IHES may appear to be rather narrow to the wider

scientific community. Has this affected the funding of IHES, especially from the state, which often expects some foreseeable result or returns?

U We have a very important support from the French government. Almost half of the budget is directly coming from the French government. I think that a lot of people are convinced of its usefulness. Flexner[10], who was the first director of IAS, wrote a paper about "the usefulness of useless knowledge". It's a really very important concept that even if you are looking for applications, you have to support fundamental research. You won't be able to invent electricity by making candles better and better. You need to "discover" electromagnetism in order to develop modern communication. This is not done by developing homing pigeon companies. Crucial changes of paradigm can only be obtained by a free theoretical research in some theoretical center, and IHES is one of these theoretical centers. I think we have the confidence and the support of the French government. For the remaining part of the funding, I have to find donations from companies, individuals and international scientific organizations. This is somehow difficult because, as you said, some of them are expecting some foreseeable returns. Thankfully, there are some companies which understand that the model of IHES is something that can be of value. In France, we have the chance of having one of the best schools in mathematics. Several companies now know that to maintain a high level of mathematics in industry, it's really important to maintain a very high academic level in mathematics. IHES is one of the possible centers for this.

I **I think France is one of the few countries that has a high respect for intellectual tradition, more than some other countries.**

U Yes, I hope so. It may not be completely true, though. It may be more so than in some other countries.

I **Recently there was a proposed cut in funding by the president [Emmanuel Macron]. Did it have any effect [on IHES]?**

U The funding for universities is not very good in France, but I don't think we had real cuts. It's not increasing unlike the number of students. So for some universities, it's sometimes difficult.

I **A more personal question; Ullmo doesn't seem to be a very French-sounding name, isn't it?**

U No, it's not very French-sounding.

I **Is it more Italian?**

U It's more Eastern European. My grandmother was Vietnamese, and my mother, who is half-Vietnamese, half-French, was raised in Vietnam. My father was raised in Monaco, and his father came from Eastern Europe.

I **That's not too far from France.**

U No, it's not too far from France, and not too far from Italy.

[10] Abraham Flexner (1866-1959)

CALL FOR PROPOSALS

The Institute for Mathematical Sciences (IMS) of the National University of Singapore (NUS) invites submissions of proposals from researchers in academia and industry. The proposals are for organizing thematic programs or workshops to be held at IMS.

The IMS is particularly interested in receiving proposals of programs/workshops that focus on exciting new developments in the mathematical sciences. Proposals of interdisciplinary nature in areas that interface mathematics with science, social science or engineering are welcome.

A soft copy of the proposal, for the period of funding from April 2022 to March 2023, should be sent to the Director of the Institute at imsdir@nus.edu.sg by 31 May 2020.

The exposition of a proposal should be aimed at the non-specialist and will be evaluated by a scientific panel. Proposals of interdisciplinary programs/workshops should describe how the activity would benefit the intended audience with diverse backgrounds and facilitate research collaboration.

Information on the Institute and its activities, as well as a detailed format for the proposal are available on the IMS website ims.nus.edu.sg. Enquiries may be directed to imssec@nus.edu.sg.

INGRID DAUBECHIES

WAVELETS AT YOUR SERVICE

Interview of Ingrid Daubechies by Y.K. Leong

Ingrid Daubechies has made significant contributions to quantum physics and to time-frequency analysis and its applications to mathematics, engineering, computational science, biomedical sciences and art. She is best known for her groundbreaking discovery of the so-called "Daubechies wavelets" in the theory of wavelets.

Born in Houthalen, Belgium, Daubechies had already shown a precocious interest in numbers and geometric figures at an early age of six. Her mathematical talent was clearly recognised in school and she went on to obtain a bachelor's degree and PhD in physics from *Vrije Universiteit Brussel* (Free University of Brussels), where she was a research assistant and then a research professor. In 1987-1994 she was attached to the Mathematics Research Center, AT&T Bell Laboratories and was concurrently appointed professor in mathematics in Rutgers University from 1991 to 1993. From 1994 to 2010 she was at Princeton University where she became the first female full professor of mathematics. In 2011 she moved to Duke University as the first female tenured mathematics professor at Duke University. She is currently the James B. Duke University professor in the department of mathematics and electrical and computer engineering.

Daubechies had started her research work in quantum mechanics and, in fact, in 1984 she had won the Louis Empain prize[11] for extension of her dissertation work in quantum mechanics. From 1975 until 1987, her research was centered primarily on variational principles in quantum physics. In the 1980s, her PhD mentor Alex Grossmann, together with Jean Morlet, Yves Meyer and Stéphane Mallat, had been thinking about wavelets[12] and found a simple, though not efficient, way of computing them. The breakthrough came in 1986, when Daubechies was on leave from her Brussels appointment as a guest-researcher at the Courant Institute of Mathematical Sciences. She then discovered compactly supported continuous wavelets[13], linked to quadrature mirror filter-technology, that require only a finite amount of processing and henceforth completely changed the face of digital signal processing. These wavelets provide a more efficient means of storing and manipulating data (data compression, sub-band coding and noise reduction) insignal and data processing.

[11] Awarded every five years to a Belgian scientist for work done before the age of 29.

[12] Wavelets are elementary mathematical functions that can be used as fundamental building blocks for constructing more complicated functions.

[13] Ingrid Daubechies, "Orthonormal bases of compactly supported wavelets", *Communications on Pure and Applied Mathematics*, (**41**), (1988) 909-996

Further developments in wavelet theory have now been applied in mathematics (harmonic analysis, partial differential equations, fractals, turbulence), computational science and information technology. The impact of wavelets on knowledge and modern society is extensive and far-reaching: astronomy (such as the deconvolution of the Hubble telescope images and the recent LIGO detection of gravitational waves created by the collision of two black holes), nuclear engineering, biomedical science (biomedical imaging, human vision, neurophysiology), scientific technology (radar, earthquake prediction, digital cinema, archiving, optics, acoustics, musical recording, study of fossils, speech recognition, history of art paintings and detection of art forgeries).

The name of Daubechies is now attached to the orthogonal Daubechies wavelet and the biorthogonal CDF (Cohen-Daubechies-Fauveau) wavelet[14] (which is now used in the JPEG 2000 standard). At the non-mathematical (but astronomical) level, an asteroid has been named after her: (42609) Daubechies. The list of scientific prizes and awards, honorary doctorates, membership of national academies of scientific bodies worldwide is too long to be enumerated (at least 35 in number). In 2000 she is the first woman to receive the US National Academy of Sciences Award in Mathematics, presented every 4 years for excellence in published mathematical research. In 2011-2014 she became the first woman President of the International Mathematical Union. She was awarded the title of Baroness by King Philip of Belgium in 2017. Her latest award is Spain's Princess of Asturias Award for Technical and Scientific Research in 2020.

Daubechies has an enormous capacity in collaboration and management of projects in a wide range of disciplines ranging from scientific ones to human pursuits in the visual arts like photography and painting. She has written a highly cited joint paper[15] on framelets in multiresolution analysis with a team that includes Zuowei Shen, Vice-Provost (Graduate Education) of the National University of Singapore (NUS). She was at NUS' Institute for Mathematical Sciences (IMS) as Distinguished Visitor from 29 May-16 June 2017 for the Institute's program "Data Sciences: Bridging Mathematics, Physics and Biology". She gave a colloquium lecture "Mathematicians helping art historians and art conservators" at the Mathematics Department on 14 June 2017 and a public lecture "Biologically relevant distances between morphologicial surfaces representing teeth and bones" on 15 June 2017. She was interviewed on 16 June 2017 by Y.K. Leong on behalf of *Imprints*. The following is an edited version of the interview in which she talks about how she was led to her first groundbreaking discovery of wavelets and her views of what drives the applied mathematician.

Acknowledgment Y.K. Leong would like to thank Von Bing Yap of the Department of Statistics and Applied Probability of the National University of Singapore for preparing a raw version of the transcript of the interview.

IMPRINTS I Your mathematical talent and inclination were evident from early childhood and in school. Yet you chose to major in physics in university and continued to do a PhD in theoretical physics. Why did you not choose to do mathematics from the start?

INGRID DAUBECHIES D Well, I do like mathematics. But I also very much like to understand why things work, and so physics seemed very natural for me to study. I've never regretted studying physics because it has given me a lot of intuition and ways of of looking at things that I think have been very useful for me. I also think it's given me an unconventional path towards my field; having several ways of approaching a problem and coming in from a different angle than many other people do, is often of benefit. Why did I not choose mathematics from the start? I think it's because I saw mathematics as an interesting way, a tool to get to understanding things. And I still do. I mean, I enjoy the mathematics for its own sake, but I just take whatever mathematics I know or I can learn in order to understand more of problems that I see. I'm not really a big theory builder in mathematics, I think, because I build what I need in order to gain a better understanding of the questions I see.

I After your PhD, you were in the Department of Physics at the *Vrije Universiteit Brussel* up to 1987. Did your work in physics have any influence or bearing on your later research on wavelets?

D Well, on some of my work. So for my PhD thesis, I worked on integral transforms that are related to coherent states in quantum mechanics. And that definitely had a bearing [on my later research] because

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[14] A. Cohen, I. Daubechies, J-C. Fauveau, "Biorthogonal bases of compactly supported wavelets", *Communications on Pure and Applied Mathematics*, (**45**) (1992), pp 485-560

[15] Ingrid Daubechies, Bin Han, Amos Ron, Zuowei Shen, "Framelets: MRA-based construction of wavelet frames", *Applied and Computational Harmonic Analysis*, **14** (1), (2003), pp 1-46

that was an example of a square integrable representation of a group that is related to quantum mechanics and wavelets. I mean, that's how I got into wavelets. Wavelets can be viewed as the square integrable representation of a different group. Actually, I worked with Alex Grossmann on this for my PhD thesis and it's because of his early involvement with wavelets that I got into wavelets. So, absolutely, they were related.

I **The modern development of wavelet theory started from some serendipitous turn of events involving pioneers like Morlet, Grossmann and Meyer in the early 1980s. You published your ground-breaking paper on orthonormal bases of compactly supported wavelets in 1988, but you moved to the United States from Belgium in 1987. How did you come to work on wavelets?**

D Well, I had started working on wavelets already around the mid-eighties. I published my PhD work in 1980. I then went for a postdoc in the United States from 1981 to 1983 and I had come back to Europe in 1983. I soon met up with Alex Grossmann again and he had just started working on wavelets. And so I learned about wavelets from him. First I worked on redundant families of wavelets. There was a paper I wrote with Alex Grossmann and Yves Meyer, which was called "painless nonorthogonal expansions".[16] And that stimulated Yves Meyer into seeing whether he could prove, which is what we believed, that no nice orthonormal bases of wavelets existed, which we believed because for the windowed Fourier transforms of canonical coherent states, we knew that such nice bases didn't exist. And so there was a subliminal belief that they didn't exist for dilations and translations, and Yves Meyer constructed his basis, which was a real surprise. We had no idea that was possible. And so, trying to understand that basis, and then, trying to understand how you could apply it without having to truncate, led to my work [on orthonormal bases]. So that was all going on at that time. Although the paper[17] was published in 1988, I did the work really in 1986 and 1987.

I **When you were still in Europe?**

D I was in Europe and in the [United] States. Prior to moving to the States in 1987, I spent a year as a postdoc at the Courant Institute. So the whole year of 1986, I was at Courant. I was then working on these bases and I finalized that work after I'd gone back to Belgium in early 1987.

I **If I understand correctly, Meyer was in Paris, but you were not in Paris.**

D Yes, but he had met Alex Grossmann. I met Yves Meyer in Marseilles when he was visiting Alex Grossmann. But actually the paper that we wrote together, we wrote it prior to my having met him [Meyer] because I had worked with Alex Grossmann, and Alex Grossmann had discussed with Yves Meyer and the different elements came together. I mean, Alex Grossmann told me about Meyer's slant on things, and so then we wrote it up together.

I **The title is very interesting – "Painless ..."**

D "Painless Nonorthogonal Expansions". The idea being that you have an expansion that is like an orthonormal basis. So you take your functions and you take inner products with the elements of the expansion and then you use that as coefficients to reconstruct the function. So they were what we now call "tight frames".

I **There are some detractors of wavelet theory who think that wavelet analysis is just applied harmonic analysis. How has wavelet theory affected the development of harmonic analysis?**

D Well, as always when people say this is just that, there's a kernel of truth. It's not that they're wrong but they may miss an important insight. Actually the big merit of Yves Meyer in this whole business is that he did not say, "Oh, it's just something we already knew." Because whenever you say "This is something we already knew." you are putting yourself in a negative position of "This is nothing new." On the other hand, what he said is, "Oh yes, this dovetails nicely with what we already knew, but it can also be looked at it in a different way. So how can I bring everything I know to bear on this new different way and how can that lead to more?" And that's really, I think, what happened. So yes, certainly in the beginning I don't think there were any big theorems that were open and that were waiting for wavelets to come in order to be proved. But wavelets did simplify the proof of many theorems and then the next generation, of course, learned about the existence of orthonormal wavelet bases. And so now that the tool is there, they use the tool. So I think it probably has simplified some work of the younger generation of harmonic analysts. But it's not about trying to get bragging points. I mean, it's just about building more mathematics.

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[16] Ingrid Daubechies, A. Grossmann, Y. Meyer, "Painless nonorthogonal expansions", *Journal of Mathematical Physics* **27** (1271-1283), 1986

[17] Daubechies, *loc.cit.*

I **I think these days a lot of people take things for granted now.**

D Yeah. But it's good. I mean, that's when you have really understood it – when you take it for granted, when it's so internalized that you just use it.

I **Wavelet methods seem to be originally computational and deterministic in nature. Has there been any attempt to introduce a probabilistic element?**

D Well, you can certainly use wavelets in probabilistic arguments. I mean, you can try to use them for situations where you're studying stochastic PDEs and so on. At some point, somebody asked me whether they could possibly be used as a tool to study Brownian motion for instance, because you have these different scales in there. But I don't think much came out of that. Maybe it could. They have been used to characterize things like fractional Brownian motion and they make it very easy to study phenomena like that, but I don't know that they have led to big theoretical results there.

I **If I understand it correctly, wavelets are also fractal in nature.**

D Yes. Compactly supported wavelets, because of the way they are recursively defined, have fractal properties themselves, but they are also just very good tools to study fractals.

I **Does the effective implementation of wavelet methods depend on advances in hardware?**

D Ah, that I don't know. Whether hardware could render wavelets obsolete, I don't know. It might mean that some of the programs to decompose into wavelets would be obsolete if you build it in hardware. But that doesn't mean that the mathematics is obsolete. The theory is still there.

I **Of course, whenever there's progress, things seem to be trivialized.**

D But, in a sense, that is the great thing about mathematics. I give this example to my students about Cauchy sequences and so on. I say, for us that's entirely natural now because if you think about what students learn in school about the number line – that you can zoom in and that's what the decimal expansion means. We teach it to kids in elementary school, but to the ancient Greeks, it would have been miraculous, I mean, since they had not really understood this. And so I think we benefit when things become internalized and trivialized because then we can build further.

I **It seems that mathematical research is very much male-dominated and your achievements are exceptions to the rule. It was only in 2014 that a woman won the Fields Medal for the first time. Do you think that this disparity could be narrowed in the near future?**

D I very much hope so. And I was very, very glad that there was a woman who finally got the Fields Medal[18]. And it has nothing to do with the fact that she was woman. She did fantastic mathematics and that's why she got it. But I think the fact that women are not so present in mathematics is something that seems to be a cultural phenomenon more than anything else. If you look at the numbers in Europe, there are good statistics for the number of women in academia in different countries. It varies enormously from one country to the next, even though the genetics doesn't differ so much. So I think it is a question of culture and of how many examples young women see and how the career of a mathematician is built and how it's perceived by students who are thinking of going into different fields. In Italy, Spain and Portugal, there are many more women [mathematicians], for instance, than in Switzerland where there are almost none. So I do hope it will change because I think that having people come from a more diverse range of backgrounds leads to more ideas and so it helps the field progress. So I think it will be good not just for the women but also for mathematics.

I **Personally, have you encountered anything negative?**

D Well, I have, like everybody, encountered the occasional person who kind of assumed (much less once I became well-known, but certainly before) that I probably wasn't really good at mathematics. But, I mean, I don't think I've been impacted by it. I've been lucky in that it didn't impact my career.

I **Yves Meyer was recently awarded the 2017 Abel Prize for his pioneering work on wavelets. And you also were awarded in 2016 a huge grant by the Simons Foundation. Now that the wavelet revolution is well recognized and its impact felt, what do you think would be the next phase of development in this area of research?**

D Well, I think we already are seeing the next phase. With hindsight now, we can see that wavelets were a first example of the realization that it can be very powerful to have a good basis in which phenomena of interest have a sparse expansion. And once we had examples, people started thinking of other ways in which that could be done and in which you could

[18] Maryam Mirzakhani (1977-2017), Fields Medal 2014

achieve that. And so that's what's now called "compressive sensing". And so that has in turn, I think, a big impact in many fields and in applications. And then with the progress made in computational work on neural networks, people are finding such sparse representations empirically in many different ways. We haven't understood them yet. So now the big thing is understanding more complex situations than we had in wavelet analysis where we were helped a lot by the fact that there was all this harmonic analysis from which we could borrow theorems. We don't have the theory for computational neural networks, and so now that needs to be built. I think that's a big challenge for mathematicians and I think it will keep us busy for a while.

I **You are also in the engineering department. That's a very practical department to be in.**

D Yes. My major position is in the math department, but I have a part position in electrical engineering and I have been elected to both the Academy of Sciences and to the Academy of Engineering. I'm not an engineer but I very much appreciate that the work I've done is also appreciated by electrical engineers because I am motivated by applications. Talking with people who have real applications is something that is a real stimulus for me.

I **What about coding theory? Does it sort of connect with wavelet analysis?**

D Yes and no. I mean there are ways in which you can use wavelets to build effective ways of coding or characterizing or organizing computations. I haven't worked much on that. My husband [A.R. Calderbank] is in algebraic coding theory, so he has worked more on that. Actually we have a couple of joint papers; one of that has been very, very successful.[19]

I **I believe that your husband is an electrical engineer.**

D Well, he was trained as a mathematician. But he's in the electrical engineering department as well as the math department.

I **So very much like yourself.**

D Yes, but in a different field. He's an algebraist and I'm an analyst.

I **Can you tell us something about your projects on application of wavelet analysis to art restoration and forgery?**

D So what it really is – it's just interesting applications of image analysis, and wavelets are good for image analysis. But I've always been interested in art. So I think it is wonderful that specially tailored tools in image analysis can be used for art conservation or art study. It's much more than detecting art forgery. It's trying to give insight into some questions that are of interest to art historians. But what is really interesting is that it really appeals to students. These projects are driven partly because graduate students as well as undergraduates are interested in working on them. And sometimes we get research-type questions and then indeed we involve graduate students or post- docs. Sometimes the tools are more off-the-shelf tools, but that still have to be tailored somewhat to applications. And then I do it with undergraduates who like it very much. It's also great to make contact with a completely different direction of expertise in which people typically don't know much about mathematics and to work with them.

I **What is your advice to graduate students who are undecided, whether they do pure research or applied research?**

D Well, I don't like that question very much because I don't like to really make a big distinction. In my case, I am an applied mathematician because I'm interested in applications, but I'm not an applied mathematician because of the mathematics I do. Sometimes I have students who are working with me because they want to work on a particular application, but who want to – I mean, for instance, there might be somebody visiting and teaching a course on algebraic geometry, and they say, "I'd like to take that course. Is that okay?" I say [to them] "You should take every single course in every direction of mathematics that you have time for and an appetite for. As an applied mathematician, you need to learn, you need to know as much mathematics as you can because you never know."

It's the problem that decides what mathematical fields, what mathematical techniques [you may need]. It's not you, it's not because you're an expert. For instance you can't use wavelets for everything. I mean, some problems don't lend themselves to that. I am here for a workshop on applications of computational geometry in biology. And in that work, I've had to learn much more about differential geometry and computational geometry than wavelets. I'm not using wavelets there, or my students are not (at least, not yet). And so I think it's what you find stimulating, what drives you, that should guide whether you do more pure theory or more applications. Some of my papers have

[18] A.R. Calderbank, I. Daubechies, W. Sweldens and B-L. Yeo, "Wavelet transforms that map integers to integers", Applied and Computational Harmonic Analysis 5 (1998), 332-369.

been published in journals that are pure math journals. When I do the mathematics, the excitement I have at solving a problem is the same, whether it's proving a theorem that I was looking at or whether it was finally making something work for an application that wasn't working. The creativity and the intellectual excitement are the same. In making it all come out and work, it feels the same in both cases. So I don't think the experience is that different. I think it's what motivates you that's different.

Well, usually if students are wondering about one or the other, I try to find out why it is that they're wondering. If they're wondering only because they say, "Oh, I really want to do pure math but there's not going to be a job", then I think I encourage them to still do the pure math even if, after their thesis, they have to look for a different type of job. The fact that they did a thesis, a good thesis in math, regardless of how pure it was, will be a good credential for them wherever they go. And if they want to do applied math, then I encourage them to study a lot of pure math as well because they will need it.

I **I think generally applied mathematicians are more open-minded. They are prepared to learn . . . .**

D Actually, when I was still in Princeton we had a visiting committee, like what departments have sometimes, and Professor Calabi, I mean, the very famous Professor [Eugenio] Calabi, was on the committee for Princeton. And at dinner in the evening, he asked me whether I know the difference between a pure and applied mathematician. And I said that I had my own views, but I was very interested in hearing his different view. He said, (a pure mathematician) when they are studying a problem and they get stuck, they narrow down the problem a bit more. He said: an applied mathematician, studying a problem and getting stuck, says, "okay, time to learn more mathematics." And I laughed and said that I really liked his definition. I asked him if I could quote him on that? And he said "yes".

I **Calabi himself is a pure mathematician.**

D Of course, so I liked the fact that he said it that way. But I think it is true. As an applied mathematician, you want to solve problems and so the problems guide you. As a pure mathematician, very often you want to build a theory and see where the theory can take you. And it's immensely useful because if it weren't for all the work that the pure mathematicians had done, the applied mathematicians wouldn't have many of the tools that they can use for solving problems.

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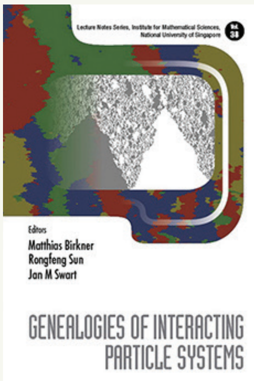

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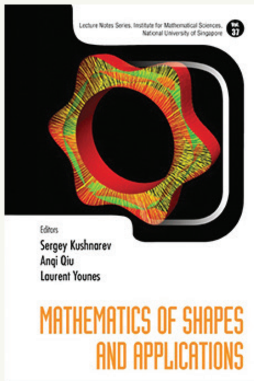

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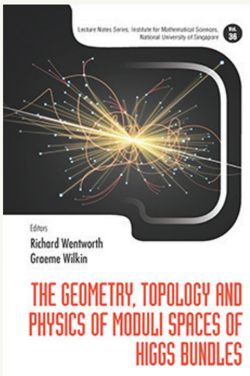

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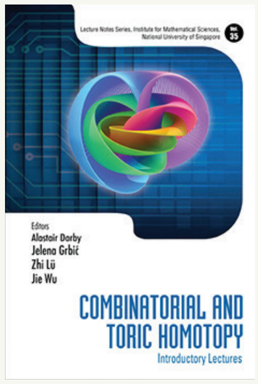

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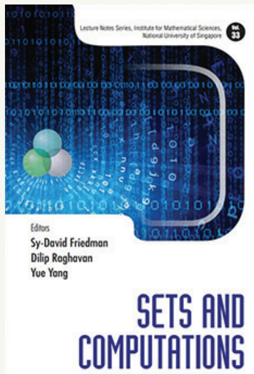

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