Probabilistic Investigation of Sensitivities of Advanced Test-Analysis Model Correlation Methods

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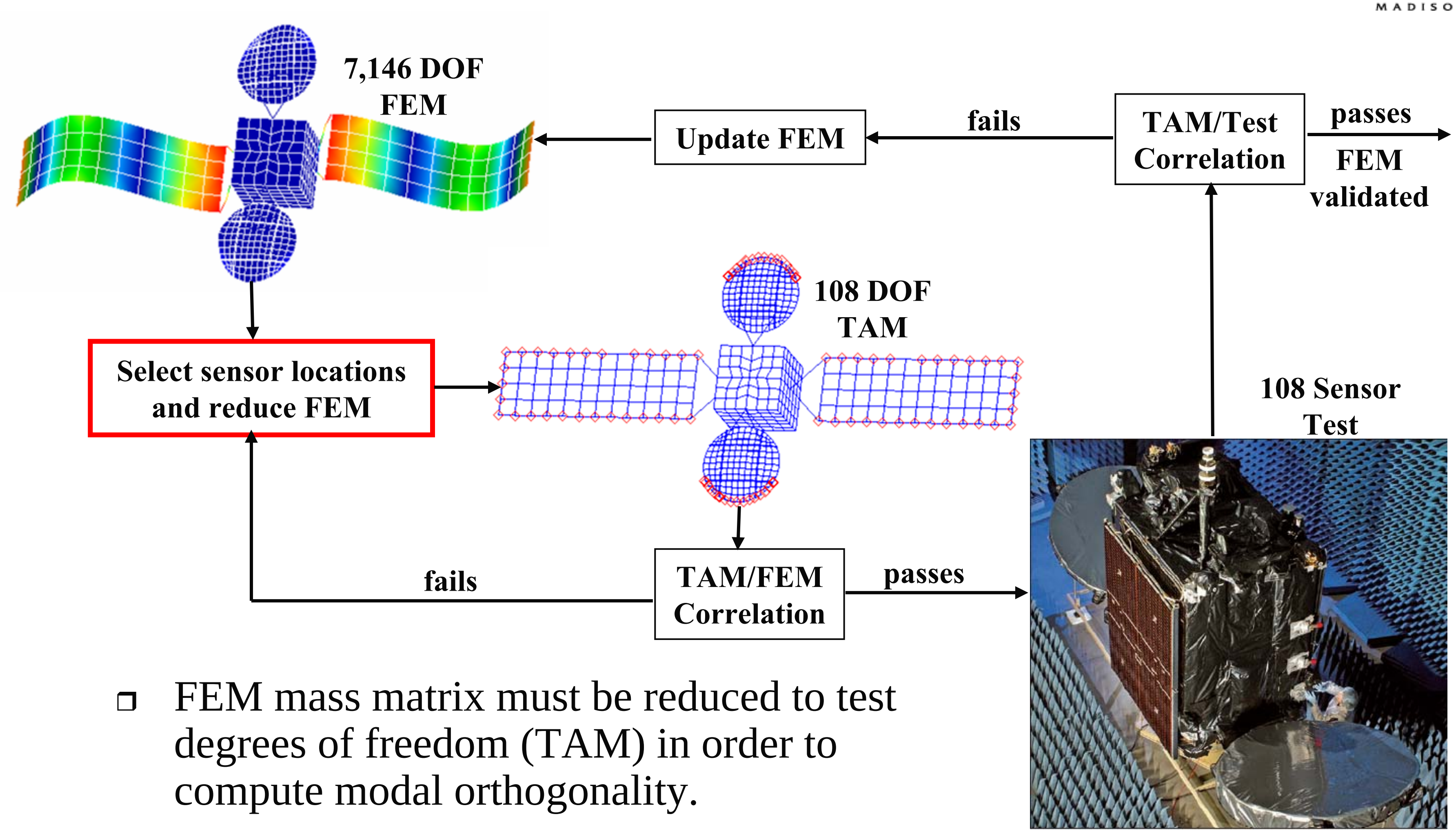
Test Analysis Correlation

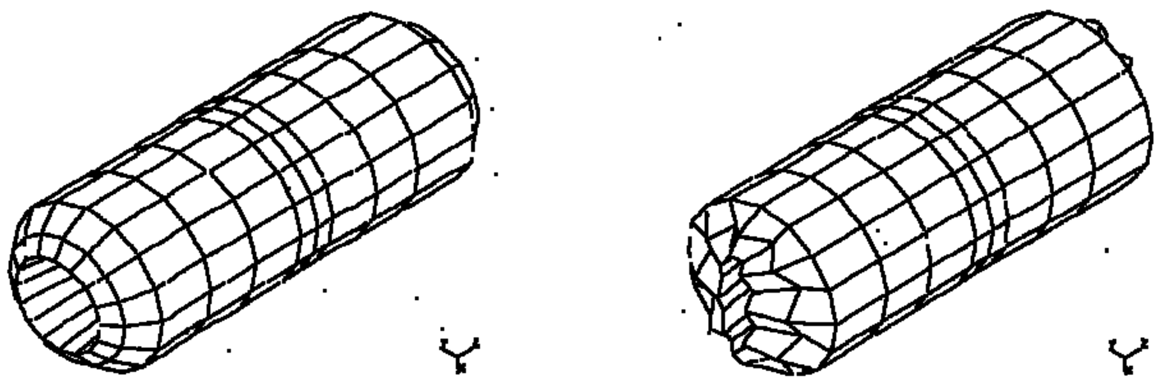


Orbital Sciences

- FEM mass matrix must be reduced to test degrees of freedom (TAM) in order to compute modal orthogonality.

The Controversy

- Current FEM reduction algorithms
  - Static TAM: fails for heavy, soft structures. May be difficult to achieve good TAM/FEM correlation



Fundamental FEM propellant mode (left) and fundamental FEM propellant mode predicted by Static TAM (right)

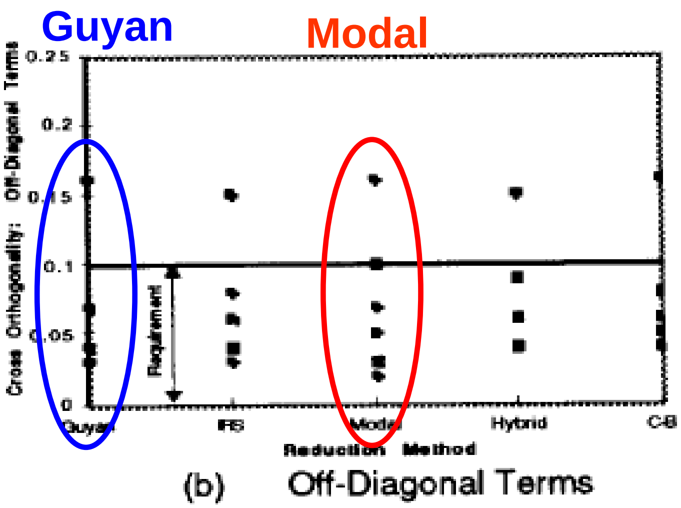
  - Improved Reduced Static (IRS) TAM: ill-conditioned under certain circumstances
  - Modal TAM: Trivial to achieve perfect TAM/FEM correlation, however it has a reputation of being highly sensitive to experimental or modal-mismatch errors

Purpose of Research

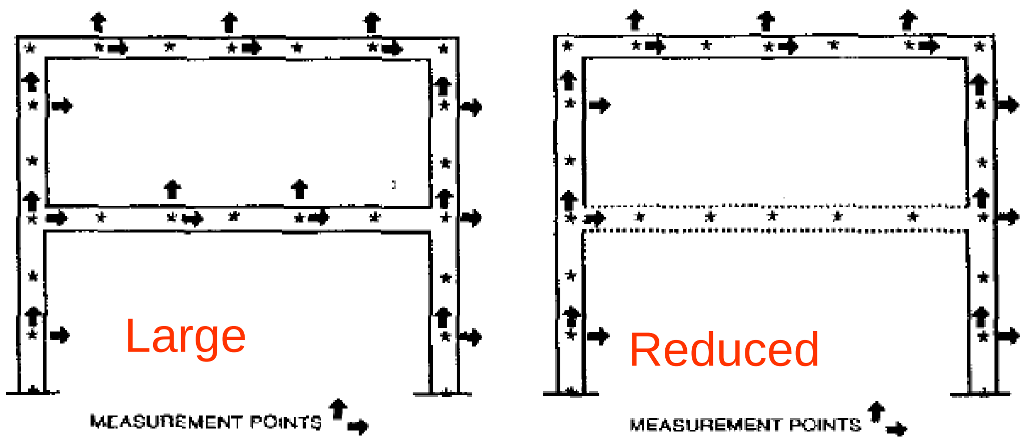
- Study the sensitivity of various TAMs to gain insight into factors that strongly affect sensitivity
- A probabilistic analysis will be used to characterize the effect of measurement errors on TAM sensitivity

Relevant Literature

- Freed, AM and Flanigan, CC (1990): Modal TAM most sensitive, sensors placed using modal kinetic energy
- Avitabile, P, Pechinsky, F, and O'Callahan, J (1992): Sensor placement is vital to TAM performance, SEREP and Hybrid perform better than Static TAM for small sensor sets
- Chung, YT (1998): Sensor placement was not discussed and no significant difference could be seen between the TAMs



Cross Orthogonality of Test Tower (Chung 1998)



Avitabile, P, Pechinsky, F, and O'Callahan, J (1992)

Relevant Literature

- Gordis, JH (1992), Blelloch, P and Vold, H (2005) :
  - Notes ill-conditioning in dynamic reduction equation:

$$\{\phi_{io}\} = -\left[K_{oo} - \omega_i^2 M_{oo}\right]^{-1}\left[K_{oa} - \omega_i^2 M_{oa}\right]\{\phi_{ia}\}$$

  - Proposes that IRS TAM will be ill conditioned if the natural frequencies of the structure with the o-set DOF pinned are similar to the frequencies of the structure of interest.
  - Recently, this theory seems to have been applied to other TAM techniques such as the Modal TAM.

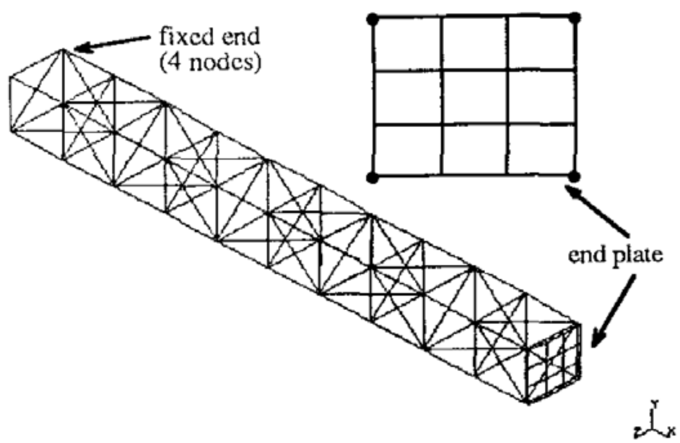


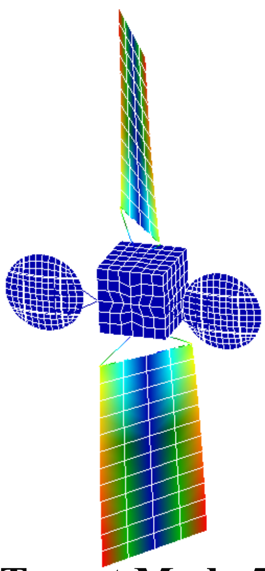
Figure 1. The NASA/Langley 10-Bay Truss.

Table 3b. TBT #1 Orthogonality:
$\Phi^T_{test} \cdot m_{IRS} \cdot \Phi_{test}$.

	1	2	3	4	5	6	7	8	9
1	100	82	22	-17	8	84	17	58	-45
2		100	-7	-24	4	66	3	65	-43
3			100	16	15	29	31	-19	3
4				100	-1	-8	4	-21	8
5					100	4	4	-2	-4
6						100	30	40	-47
7		*symmetric*					100	-13	-3
8								100	-39
9									100

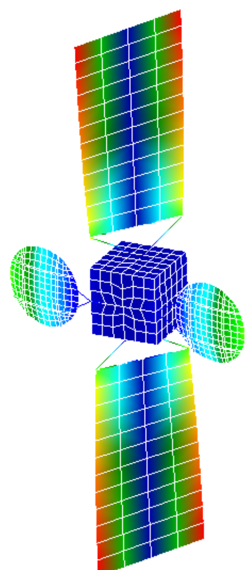
Model

- Generic Satellite
  - 7,146 DOF
  - Target modes: first 18 consecutive flexible modes (0.3-11.8 Hz)
  - 108 sensors



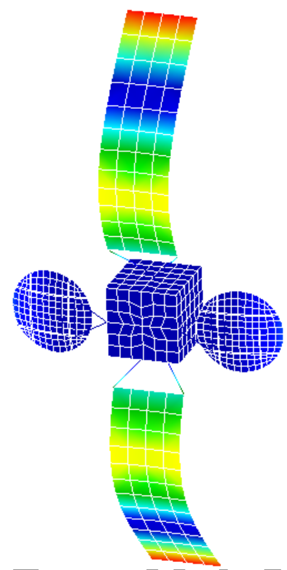
Target Mode 5

2.7 Hz



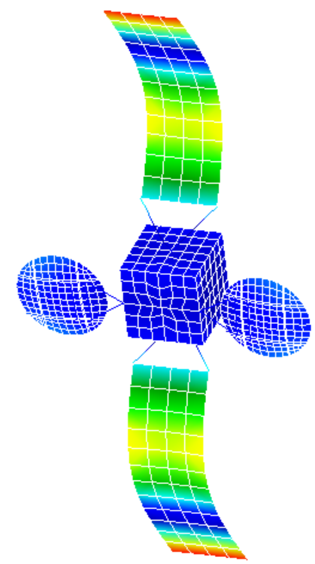
Target Mode 6

2.8 Hz



Target Mode 7

3.5 Hz



Target Mode 8

3.7 Hz

Test Analysis Models – Static TAM

a = sensor location

o = omitted DOF

Eigenvalue problem

$$-\omega_i^2 \begin{bmatrix} M_{aa} & M_{ao} \\ M_{oa} & M_{oo} \end{bmatrix} \begin{Bmatrix} \phi_{ia} \\ \phi_{io} \end{Bmatrix} + \begin{bmatrix} K_{aa} & K_{ao} \\ K_{oa} & K_{oo} \end{bmatrix} \begin{Bmatrix} \phi_{ia} \\ \phi_{io} \end{Bmatrix} = 0$$

Lower partition equation

$$\left[K_{oa} - \omega_i^2 M_{oa}\right]\{\phi_{ia}\} + \left[K_{oo} - \omega_i^2 M_{oo}\right]\{\phi_{io}\} = 0$$

Neglect the mass of the o-set DOF

$$\{\phi_{io}\} = -\left[K_{oo} - \cancelto{0}{\omega_i^2 M_{oo}}\right]^{-1}\left[K_{oa} - \cancelto{0}{\omega_i^2 M_{oa}}\right]\{\phi_{ia}\}$$

Static Transformation Matrix (each column represents a constraint mode)

$$[T_S] = \begin{bmatrix} I \\ -K_{oo}^{-1} K_{oa} \end{bmatrix}$$

Test Analysis Models – IRS TAM

$$\{\phi_{io}\} = -\left[K_{oo} - \omega_i^2 M_{oo}\right]^{-1}\left[K_{oa} - \omega_i^2 M_{oa}\right]\{\phi_{ia}\}$$

Ill-conditioned when ω_i^2 is near any of the eigenvalues of the *Koo*, *Moo* system

Approximate the frequency terms

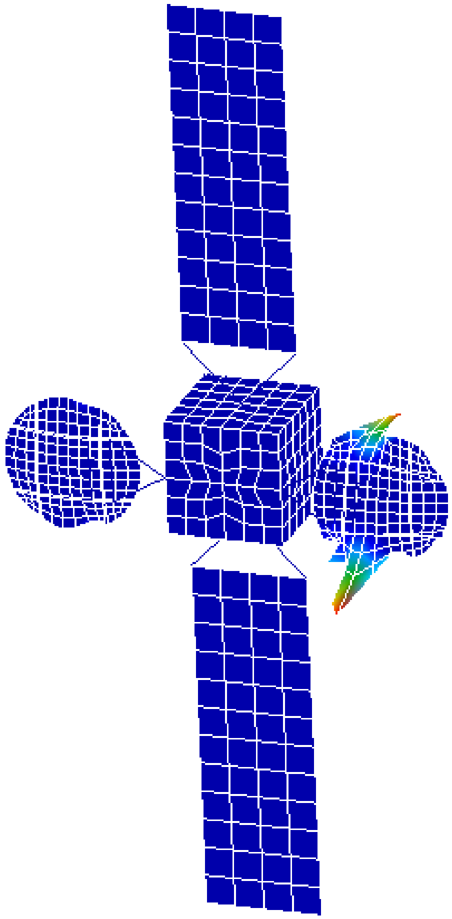
$$\omega_i^2\{\phi_{ia}\} = \tilde{M}_S \tilde{K}_S^{-1}\{\phi_{ia}\}$$

Calculate the IRS transformation matrix

$$[T_{IRS}] = [T_S] + [T_i]$$

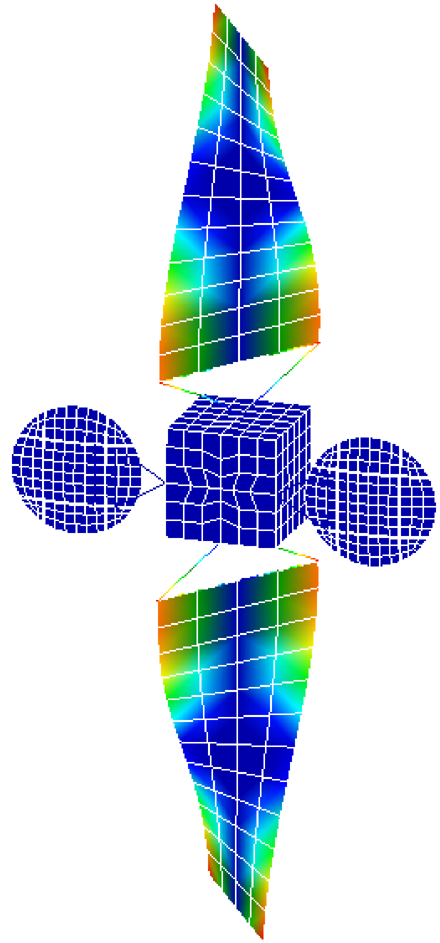
$$[T_i] = -\begin{bmatrix} 0 & 0 \\ 0 & -K_{oo}^{-1} \end{bmatrix}\begin{bmatrix} M_{aa} & M_{ao} \\ M_{oa} & M_{oo} \end{bmatrix}\begin{bmatrix} I \\ -K_{oo}^{-1}K_{oa} \end{bmatrix}\tilde{M}_S^{-1}\tilde{K}_S$$

Test Analysis Models – IRS TAM



O-set system Mode 1

16.8 Hz

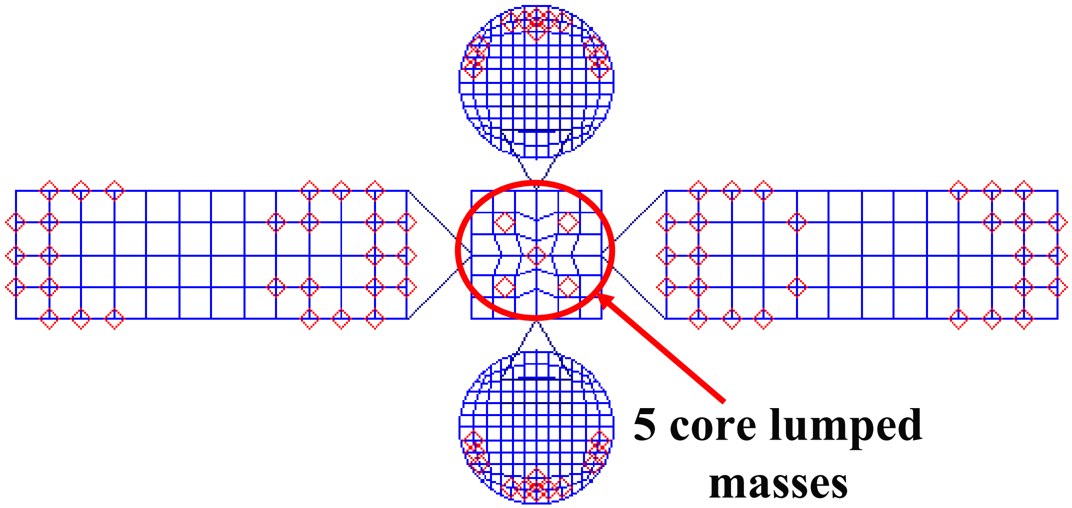


FEM Target Mode 18

11.8 Hz

Test Analysis Models – Static and IRS TAM

- Mass weighted effective independence did not select the lumped masses (the lumped masses were essential to TAM-FEM correlation)
- Modal kinetic energy applied to all 18 target modes was not sufficient
- A significant amount of hand selection and engineering judgment was used (modified modal kinetic energy method)



Test Analysis Models – Modal TAM

Physical coordinates in terms of modal coordinates

$$\begin{Bmatrix} x_a \\ x_o \end{Bmatrix} = \begin{bmatrix} \phi_a \\ \phi_o \end{bmatrix} \{q\}$$

Partitioned Equations

$$\{x_a\} = [\phi_a]\{q\}$$

$$\{x_o\} = [\phi_o]\{q\}$$

Solve for modal coordinates in terms of the sensor DOF

$$\{q\} = \left[\phi_a^T \phi_a\right]^{-1} \left[\phi_a^T\right]\{x_a\}$$

Modal transformation matrix

$$[T_M] = \begin{bmatrix} I \\ \phi_o(\phi_a^T \phi_a)^{-1} \phi_a^T \end{bmatrix}$$

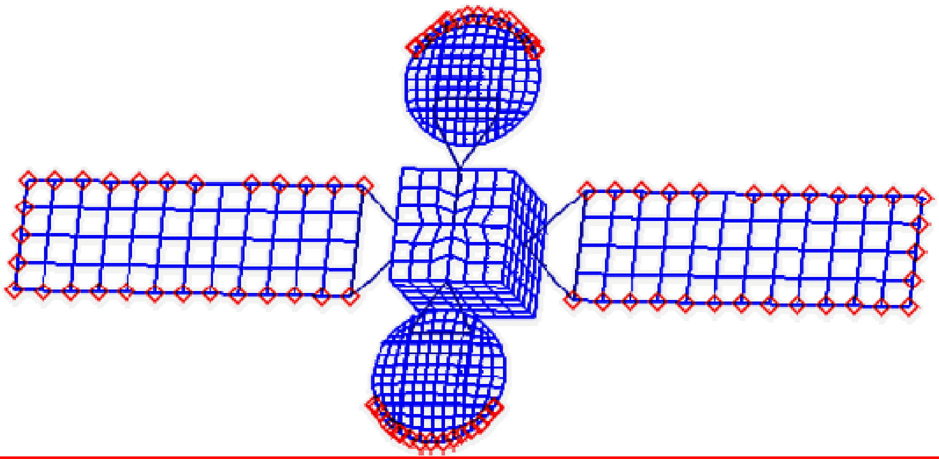
Test Analysis Models – Modal TAM

- Sensor placement achieved with Effective Independence

Maximize the determinant of the Fisher information matrix

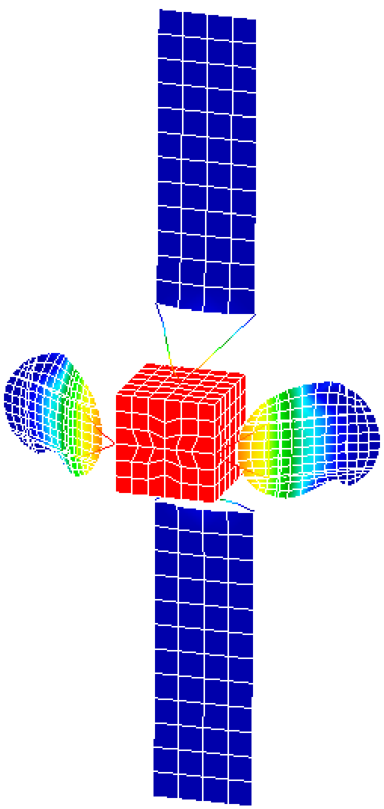
$$\max\|Q\| = \max\left\|\phi_a^T \phi_a\right\|$$

Effective Independence $E_{Di} = \phi_a^i Q^{-1} \phi_a^{iT}$ $\quad 0.0 \le E_{Di} \le 1.0$



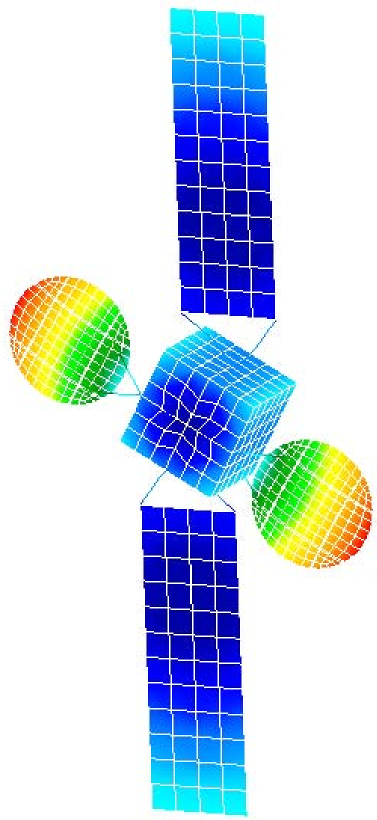
Test Analysis Models – Modal TAM

Modal TAM o-set frequencies are similar to the FEM frequencies, so the theory of Gordis suggests that this TAM will be sensitive.



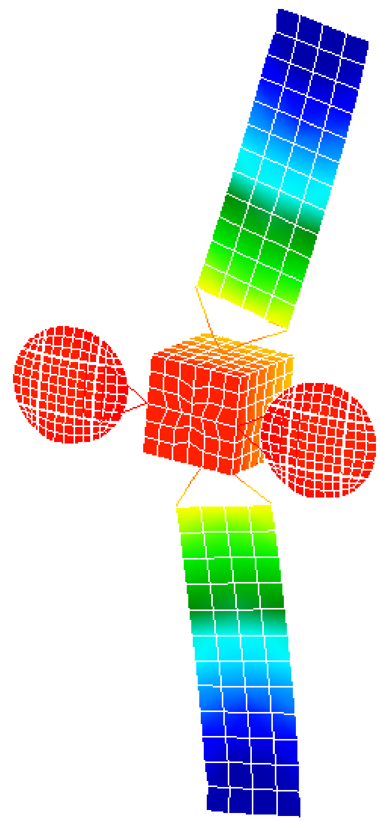
O-set system Mode 2

1.2 Hz



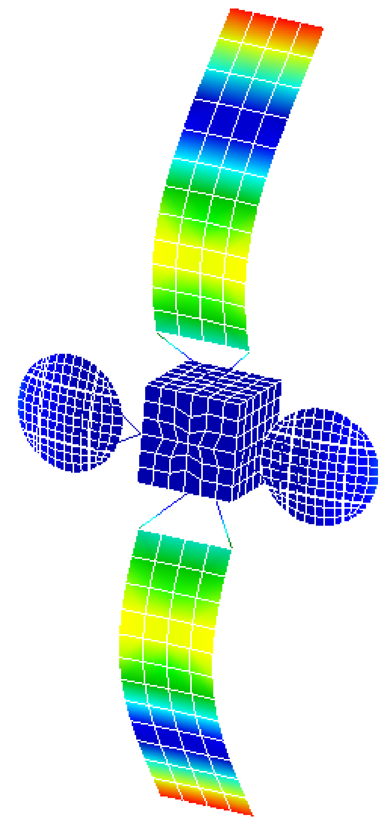
FEM Target Mode 4

1.8 Hz



O-set system Mode 5

3.2 Hz



FEM Target Mode 7

3.5 Hz

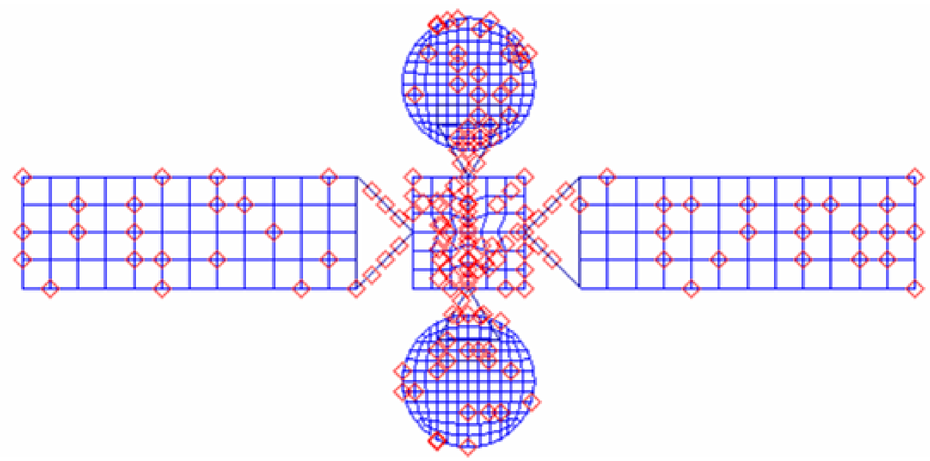
Test Analysis Models – Modal using Condition Number Sensor Placment

Modal coordinates in terms of the sensor DOF

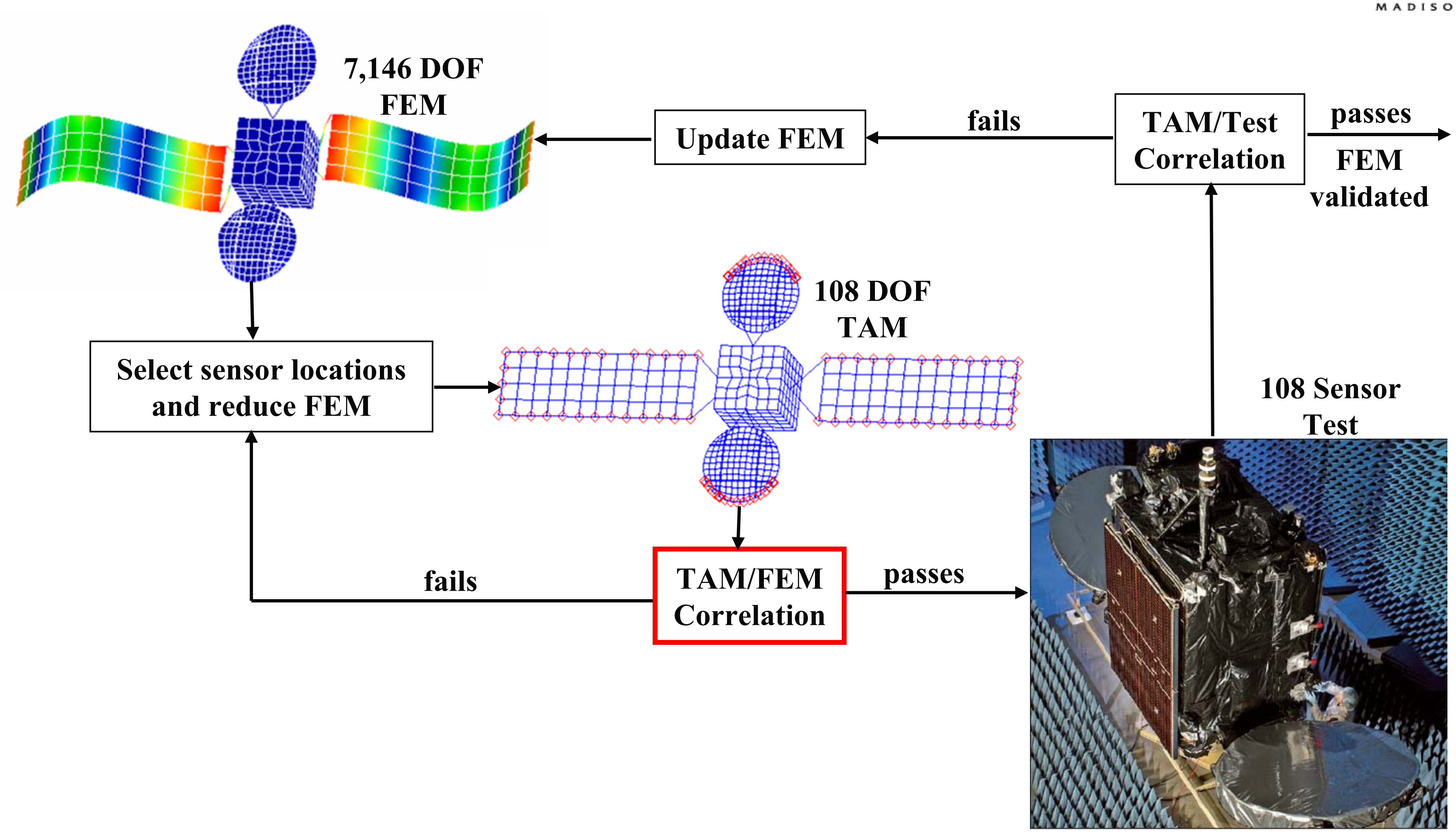
$$\{q\} = \left[\phi_a^T \phi_a\right]^{-1}\left[\phi_a^T\right]\{x_a\}$$

Solution is more sensitive if the condition number of ϕ_a , is large.

Begin with a visualization set, and add sensors that minimize the condition number of ϕ_a



Test Analysis Correlation



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Correlation Metrics

- **Orthogonality**
  - **Criteria:** **0 ≤ off diagonal term ≤ 0.1**

$$O = \left[\phi_{FEM}\right]^T \left[\tilde{M}_{TAM}\right]\left[\phi_{FEM}\right]$$

- Cross Orthogonality
  - Criteria: **0 ≤ off diagonal term ≤ 0.1**

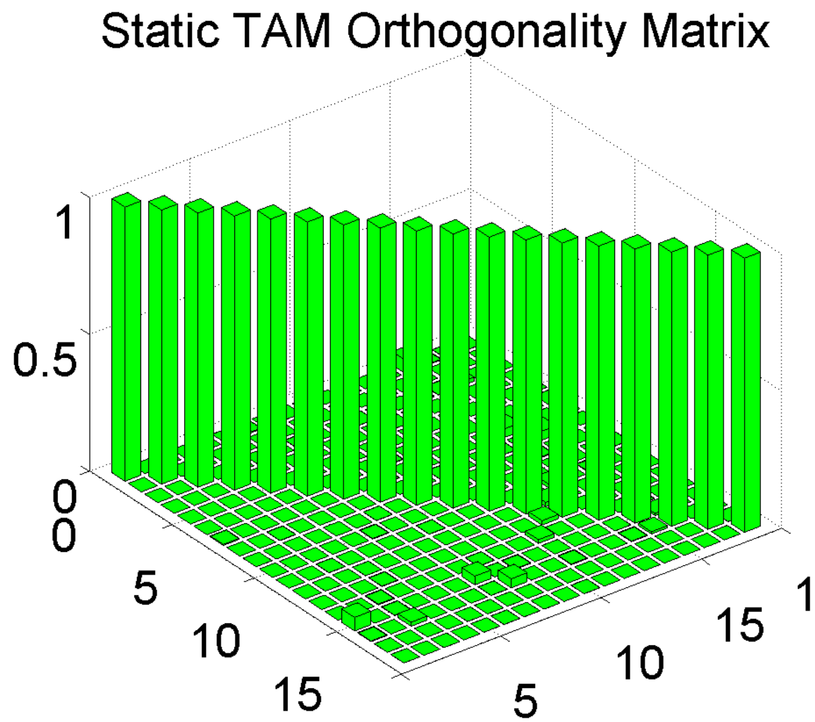
    0.95 ≤ diagonal term ≤ 1.0

$$CO = \left[\phi_{FEM}\right]^T \left[\tilde{M}_{TAM}\right]\left[\phi_{TAM}\right]$$

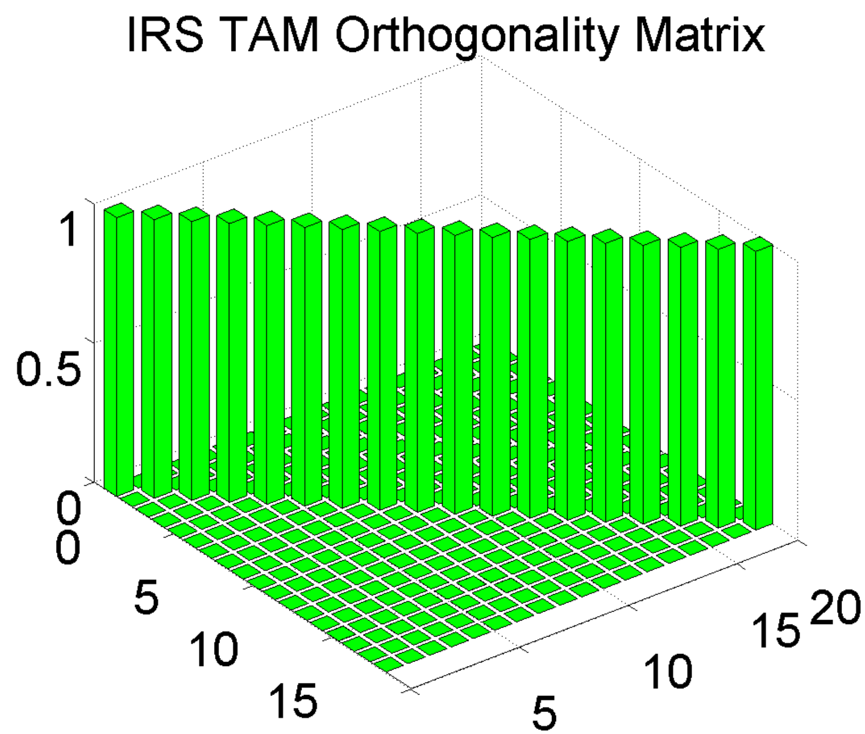
- Frequency Comparison
  - Criteria:

$$f_{error} = \frac{f_{FEM} - f_{TAM}}{f_{FEM}} * 100 \leq 3\%$$

TAM-FEM Correlation



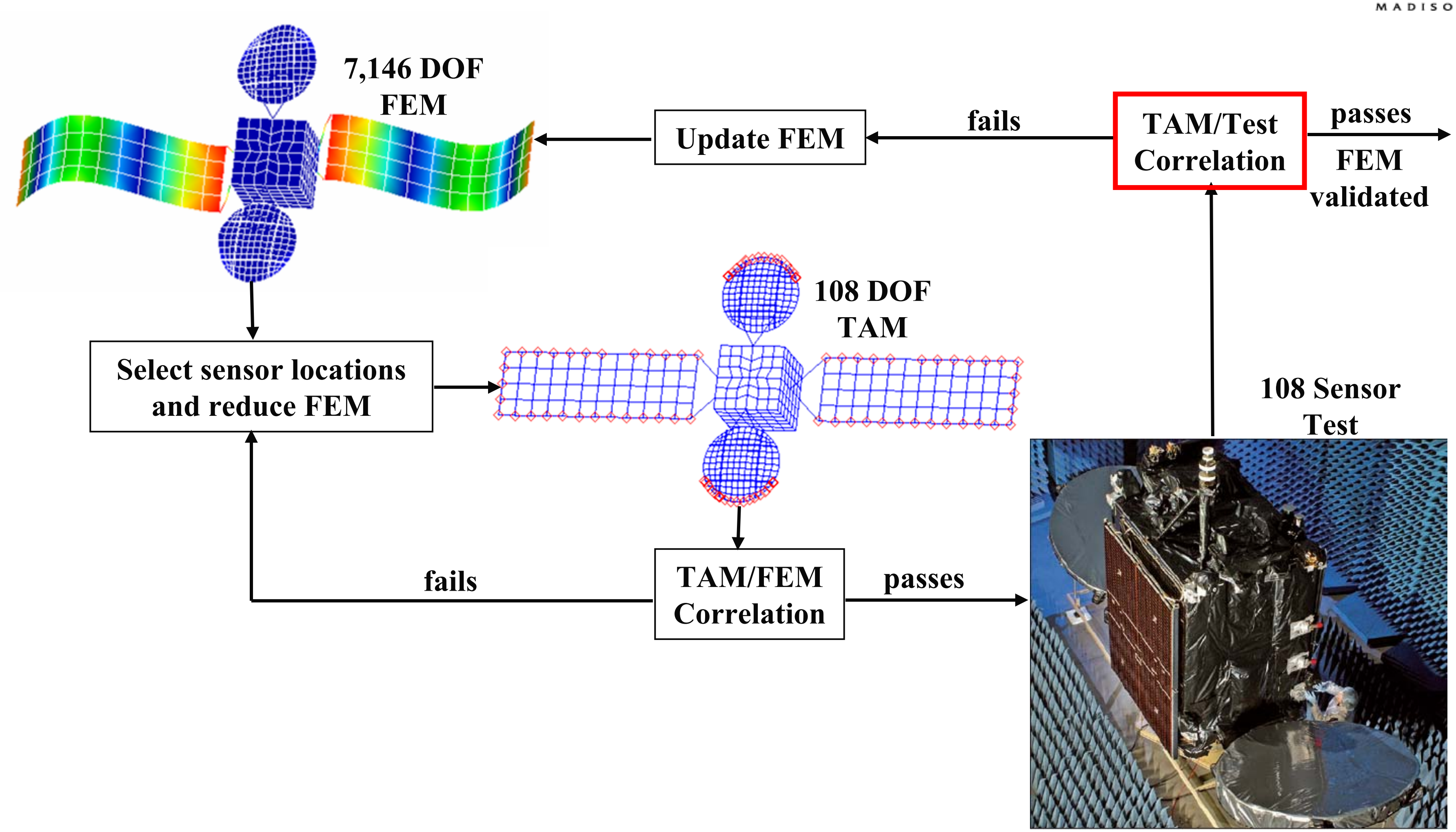
Max off diagonal term: 0.05



Max off diagonal term: 6e-4

***Modal TAM always produces perfect orthogonality for TAM-FEM correlation**

Test Analysis Correlation



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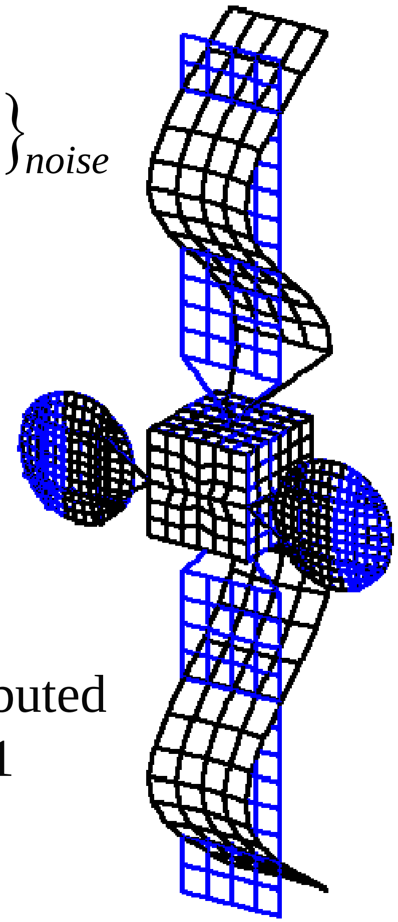
Noise Model and Simulated Test Mode Shapes

$$\{\phi\}^i_{FEM} = \begin{Bmatrix} \phi_1^i \\ \phi_2^i \\ \vdots \\ \phi_{n-1}^i \\ \phi_n^i \end{Bmatrix}$$

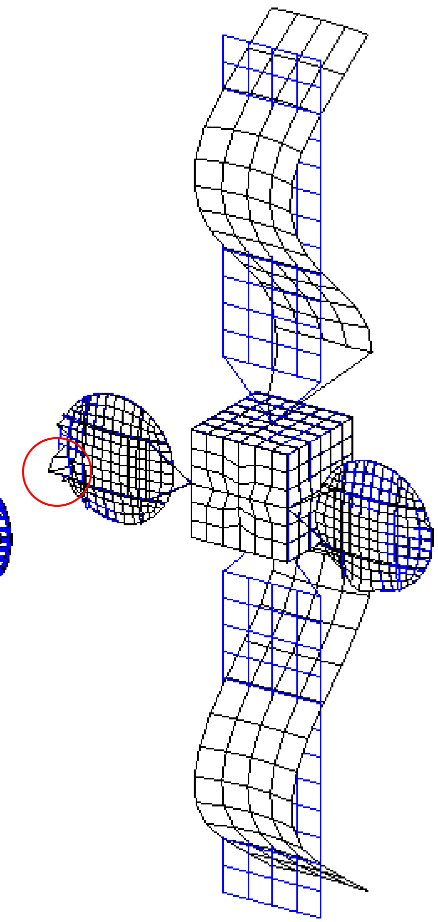
$$\phi_2^i \xrightarrow{\text{Max value}} * 2\% * \{U\} = \{\phi\}_{noise}$$

$$\{\phi\}^i_{Test} = \{\phi\}_{noise} + \{\phi\}^i_{FEM}$$

$\{U\} =$ column vector of uniformly distributed random numbers between -1 and 1

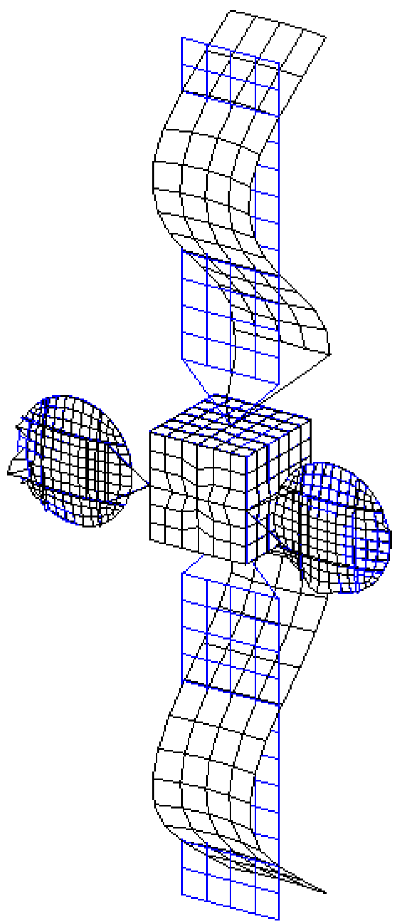


FEM target mode 15



Test target mode 15

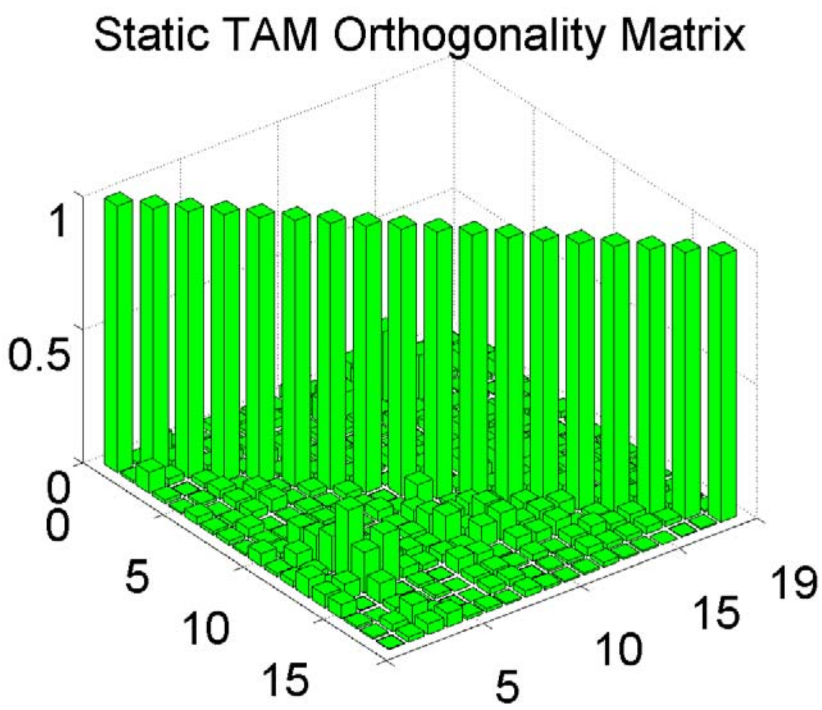
Noise Model



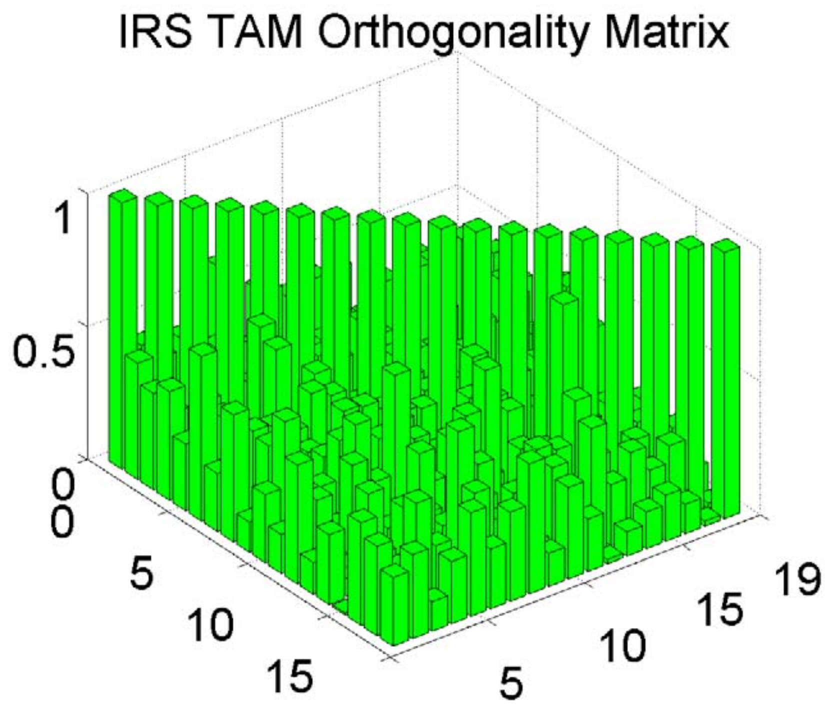
Noise contaminated mode shape

- FEM assumed to be perfect
- Noise vector models the net effect of all errors that cause the FEM mode shapes to disagree with the test mode shapes.
  - Noise Distribution: Uniform – no assumption is made about the distribution of noise
  - Noise Amplitude: Sensors with the smallest motion have the largest noise to signal ratio
  - Noise is small on average: ± 2% at sensor locations with the largest motion.

TAM-Test Correlation Results (1 case of Random Noise)

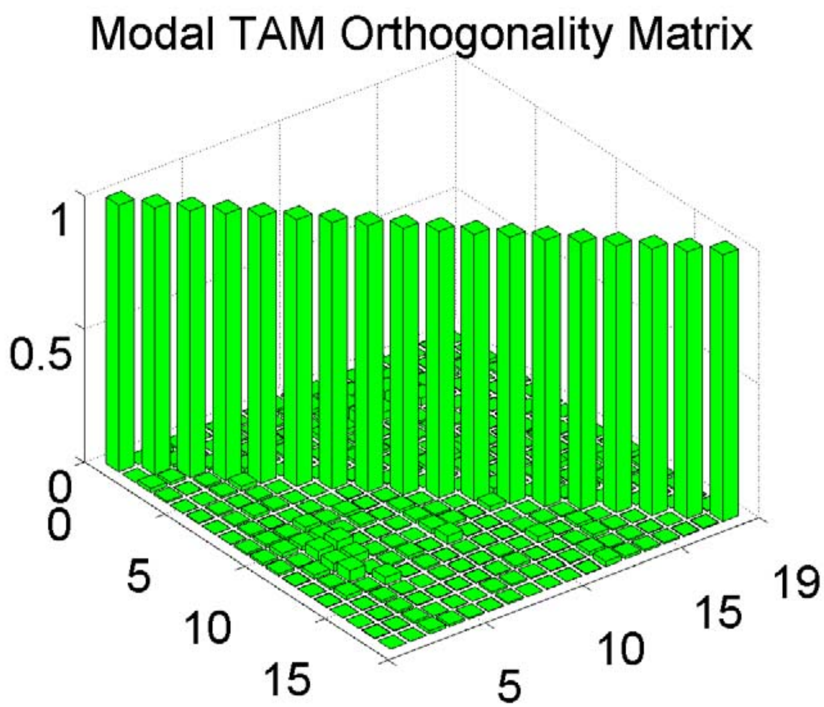


Max off diagonal term: 0.27

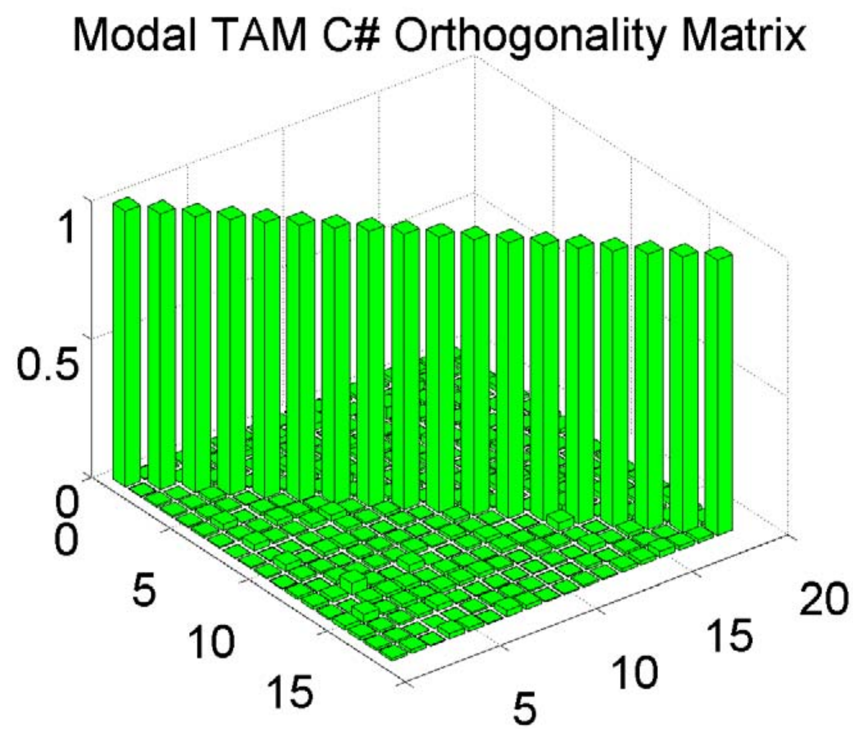


Max off diagonal term: 0.79

TAM-Test Correlation Results (1 case of Random Noise)



Max off diagonal term: 0.05

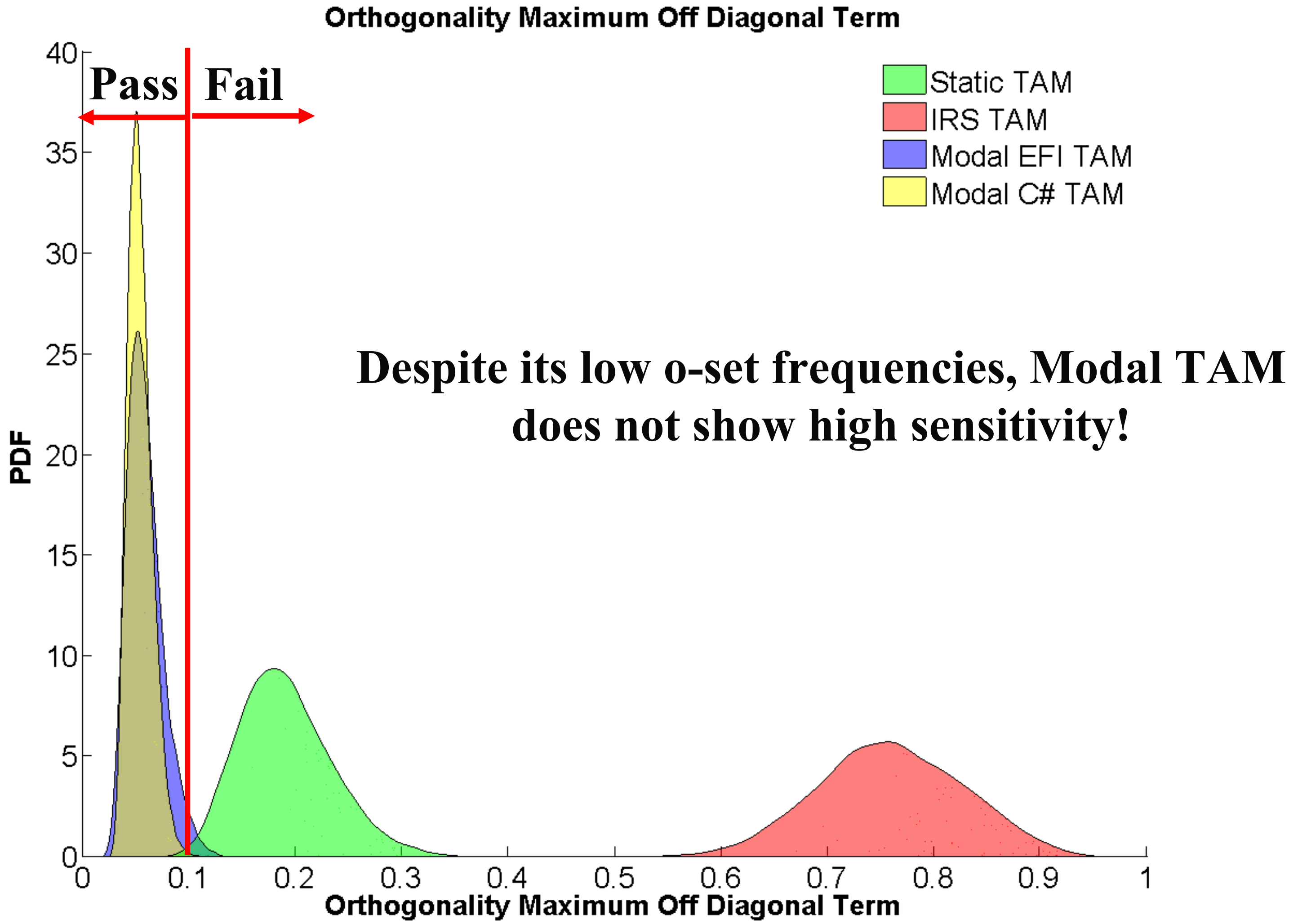


Max off diagonal term: 0.04

Monte Carlo Simulation

- Thus far, TAM-Test correlation has been studied using only one noise profile
- Random noise added in 10,000 iterations
- Orthogonality computed for each iteration
- Maximum off-diagonal term of orthogonality was stored

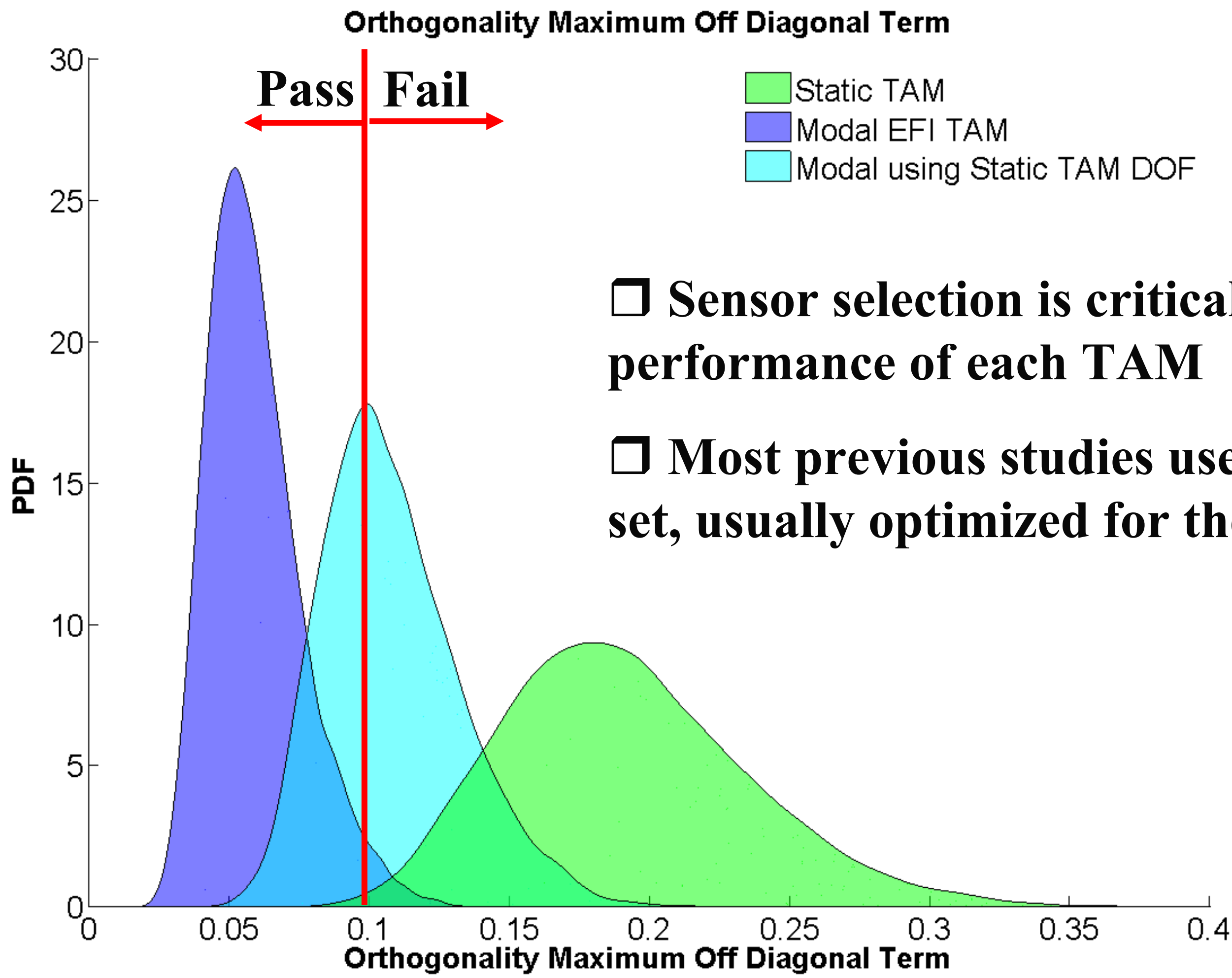
TAM-Test Correlation Results



TAM-Test Correlation Results

- If Orthogonality > 0.1 one might
  - Refine FEM before exiting test
  - Repeat test and/or look for errors
  - Update the FEM
- In this case, the FEM was perfect (errors in test modes were purely random)
- Note: The specific ranking of different TAM methods may depend on:
  - The structure of interest
  - The characteristics of the noise
  - Systematic errors between the test and FEM

TAM-Test Correlation Results



- **Sensor selection is critical to the performance of each TAM**
- **Most previous studies used the same sensor set, usually optimized for the Static TAM**

Predicting Standard Deviation

- Recently, we have developed formulas to analytically predict sensitivity of a TAM based on simple metrics
- For example, for the noise model used in this study:

$$O_{ij} = \left[\phi_i + n_i\right]^T \left[\tilde{M}_{TAM}\right]\left[\phi_i + n_i\right] \qquad n_i = noise$$

$$\sigma(O_{ij}) = \sqrt{\sum_m \left(\tilde{M}_{TAM}\phi_j\right)_m^2 \sigma_i^2 + \sum_m \left(\phi_i^T \tilde{M}_{TAM}\right)_m^2 \sigma_j^2 + \sum_m \sum_n \left(\tilde{M}_{TAM}\right)_{mn}^2 \sigma_i \sigma_j}$$

	Maximum Orthogonality Off-Diagonal	
	Predicted STD	Actual STD
Static	0.03	0.03
Modal EFI	0.009	0.01
Modal C#TAM	0.006	0.006
Modal with Static DOF	0.02	0.02

Conclusions and Future Work

- Conclusions
  - IRS TAM was ill-conditioned, as predicted by Gordis
  - Modal TAM did not show high sensitivity even though its o-set frequencies were near those of the target modes
  - Probabilistic analysis more fully explains TAM sensitivity
    - One can even predict the sensitivity of the TAMs analytically given the TAM Mass matrix, mode shapes and noise model.
- Future Work
  - Develop more accurate noise models
  - Study the effect of systematic mismatch between FEM and test due to modeling errors.
    - May need the Hybrid TAM in these cases
  - Apply these methods to other physical systems, analytically and experimentally.
    - Investigate systems with non-consecutive target modes