Probability, Random Events, and the Mathematics of Gambling

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> Probability theory originated in a supremely practical topic—gambling. Every gambler has an instinctive feeling for "the odds." Gamblers know that there are regular patterns to chance—although not all of their cherished beliefs survive mathematical analysis. (Stewart, 1989, p. 44)

Introduction

Anyone who has worked with people who gamble come to realize that they often have a number of erroneous beliefs and attitudes about control, luck, prediction and chance. The main purpose of this chapter is to draw a connection between the folk beliefs of the individual who gambles and the reality of the physical world, to illustrate where people make errors and to explore the origin of these errors.

The basic problem is that people who gamble often believe they can beat the odds and win. Even those who know the odds still believe they can win. Turner (2000) has argued that much of this is the result of experience with random events: random events fool people into believing they can predict their random outcomes.

Another problem is that the human mind is predisposed to find patterns and does so very efficiently. For example, natural formations like the "face" in the Cydonia Mensae region of Mars or the Sleeping Giant peninsula on the shore of Lake Superior in Northern Ontario, which have human-like features, are interpreted as images of people. In addition, deviations from expected results, such as winning or losing streaks, are often perceived as too unlikely to be a coincidence. As an example of our willingness of find patterns, a few years ago Eric Von Daniken (1969) wrote a book in which he claimed to have found evidence for the influence of extraterrestrial beings on human history. Much of his "evidence" was based on such things as the coincidental similarity between a rock drawing in the Sahara desert and the appearance of a modern astronaut's space suit. The book has sold 7 million copies, testifying to the ease with which people can be swayed by the argument that patterns cannot be random coincidence. Some people believe that "random" events have no cause and are thus mysterious. As a result, they may believe there is a greater opportunity to influence the random outcome through prayer or similar means. In the past, some religions have used dice

games to divine the will of the gods (Gabriel, 2003). Related to this is the notion that everything happens for a reason and thus random outcomes must contain a message.

Some people who gamble believe that there is no such thing as a random event and that they can therefore figure out how to win. In a sense, they are correct in that all random events are the result of physical forces or mathematical algorithms. In practice, however, they are completely wrong. A random event occurs when a difficult problem (e.g., controlling the exact speed, movement and height of a dice throw) is combined with a complex process (e.g., the dice rolling across a table and bouncing against a bumper on its far side). This combination leads to complete uncertainty as to what will actually occur.

Randomness is a mathematical concept used to model the real world. The fact that random events can be described mathematically does not mean they are deterministic, nor does it mean they are non-deterministic—their predictability is irrelevant. To call something random simply means that the observer does not know what the outcome will be (De Finetti, 1990). Most events that we think random are, in fact, deterministic in nature, but so complex that they are impossible to predict. For example, where a ball lands on a roulette wheel is directly related to the amount of force used to throw the ball and the speed of the spinning wheel. In practice, however, it is impossible to predict where the ball will land. The probability of different random events is not equal; some events are more likely to occur than others. This is especially true when we consider the chances of joint events (e.g., three win symbols showing on a slot machine) or the chance of one event compared to all other events (e.g., holding a winning vs. a losing lottery ticket). Taken together, however, the probabilities of all possible events must add up to 100% and each of those percentage points is equally likely.

Unfortunately, many people hold erroneous ideas about the nature of random chance. The best way to get a feeling for what lies at the root of these misconceptions is to explore the basic, interrelated concepts upon which most gambling activity depends: probability and randomness. The goal of this chapter is to help the reader understand probability well enough to identify the errors in thinking of people with a gambling problem and to help the therapist communicate with them. Misunderstanding probability may not be the main cause of an individual's gambling problem. Turner, Littman-Sharp and Zangeneh, (2006) found that problematic gambling was more strongly related to depression, stressful life experiences and a reliance on escape to cope with stress than it was to erroneous beliefs. Correcting misconceptions, however, may be an important part of relapse prevention. If a client really believes that it is possible to beat the odds, the odds are that he or she will try. In addition, the use of escape methods to deal with stress is significantly correlated with erroneous beliefs, suggesting that using gambling to escape negative moods may be directly tied to the belief

that one can beat the odds (Turner et al., 2002). Furthermore, it is argued that prevention requires disseminating accurate information about the reality of gambling and the way these games can fool individuals into believing they can win. A second goal of this chapter is to de-mystify random events.

The following parts of this chapter:

- provide a list of some of the more common misconceptions that people who gamble have about the nature of random chance;
- give definitions and examples of probability, odds and other key concepts related to random chance;
- examine, from a theoretical point of view, how a mechanistic universe filled with cause-and-effect relationships can produce random events;
- describe how specific types of games produce random events, including how slot machines work; and
- discuss the origins of some erroneous beliefs.

Erroneous Beliefs

People hold a number of misconceptions about the nature of random events. Many of these misconceptions are due to the nature of random events and to misunderstandings about the words used to describe the phenomenon. Table 1 summarized these misunderstandings. The first column lists a number of the common misconceptions, or "naive concepts," that individuals with a gambling problem may express concerning random events. The second column provides a series of statements that describe the true nature of these events. The subsequent few pages provide resource information on probability, odds and randomness to help the therapist understand the difference between the naive concept of randomness and the actual nature of random events.

Table 1. Random events: Naive concepts vs. actual nature

Naive Concept of Random Events	Actual Nature of Random Events
Events are consistently erratic.	Events are just plain erratic (fundamental uncertainty). Random events are often described as "clumpy" because clumps of wins or losses sometimes occur.
Things even out.	Things do not *have to* even out, but sometimes *seem to*, as more observations are added (law of large numbers).
If a number hasn't come up, it's due. If heads has occurred too often, tails is due.	Numbers that haven't come up are never due to come up. Coins and dice have no memories (independence of events).

After a few losses a person is due to win.	A player is never due for a win (or a loss). In most games the past tells us nothing about what will occur next (independence of events).
Randomness contains no patterns.	Sometimes random events appear to form patterns. Coincidences *do* happen (fundamental uncertainty).
If there appear to be patterns, then events are not random and are therefore predictable.	Apparent patterns will occur, but these patterns will not predict future events. Patterns that occur in past lottery or roulette numbers are not likely to be repeated (fundamental uncertainty).
If a betting system, lucky charm or superstition appears to work, it actually does work.	Through random chance, betting systems, charms and superstitions may sometimes appear to work. That success is not likely to be repeated (fundamental uncertainty).
Random events are self-correcting.	Random events are not self-correcting. A long winning or losing streak might be followed by ordinary outcomes so that the impact of the streak will appear to diminish as more events are added (law of large numbers; regression to the mean), but there is no force that causes the numbers to balance out.
If a number comes up too often, there must be a bias.	True biases do sometimes occur (e.g., faulty equipment, loaded dice), but more often an apparent bias will just be a random fluke that will not allow one to predict future events (fundamental uncertainty; independence of events).
A player can get an edge by looking for what is due to happen.	Nothing is certain; nothing is ever due to happen (independence of events).

Probability, Odds and Random Chance

Probability: A Definition

Probability is the likelihood or chance that something will happen. Probability is an estimate of the relative average frequency with which an event occurs in repeated independent trials. The relative frequency is always between 0% (the event never occurs) and 100% (the event always occurs). Probability gives us a tool to predict how often an event will occur, but does not allow us to predict when exactly an event will occur. Probability can also be used to determine the conditions for obtaining certain results or the long-term financial prospects of a particular game; it may also help determine if a particular game is worth playing. It is often expressed as odds, a fraction or a decimal fraction (also known as a proportion). Probability and odds are slightly different ways of describing a player's chances of winning a bet.

Probability

Probability is an estimate of the chance of winning divided by the total number of chances available. Probability is an ordinary fraction (e.g., 1/4) that can also be expressed as a percentage (e.g., 25%) or as a proportion between 0 and 1 (e.g., p = 0.25). If there are four tickets in a draw and a player owns one of them, his or her probability of winning is 1 in 4 or 1/4 or 25% or p = 0.25.

Odds

Odds are ratios of a player's chances of losing to his or her chances of winning, or the average frequency of a loss to the average frequency of a win. If a player owns 1 of 4 tickets, his/her probability is 1 *in* 4 but his/her odds are 3 *to* 1. That means that there are 3 chances of losing and only 1 chance of winning. To convert odds to probability, take the player's chance of winning, use it as the numerator and divide by the total number of chances, both winning and losing. For example, if the odds are 4 to 1, the probability equals 1 / (1 + 4) = 1/5 or 20%. Odds of 1 to 1 (50%) are called "evens," and a payout of 1 to 1 is called "even money." Epidemiologists use odds ratios to describe the risk for contracting a disease (e.g., a particular group of people might be 2.5 times more likely to have cancer than the rest of the population).

In gambling, "odds" rarely mean the actual chance of a win. Most of the time, when the word "odds" is used, it refers to a subjective estimate of the odds rather than a precise mathematical computation. Furthermore, the odds posted by a racetrack or bookie will not be the "true odds," but the payout odds. The true odds are the actual chances of winning, whereas the payout odds are the ratio of payout for each unit bet. A favourite horse might be quoted at odds of 2 to 1, which mathematically would represent a probability of 33.3%, but in this case the actual meaning is that the track estimates that it will pay $2 profit for every $1 bet. A long shot (a horse with a low probability of winning) might be quoted at 18 to 1 (a mathematical probability of 5.3%), but these odds do not reflect the probability that the horse will win, they mean only that the payout for a win will be $18 profit for every $1 bet. When a punter says "those are good odds," he or she is essentially saying that the payout odds compensate for the true odds against a horse winning. The true odds of a horse are actually unknown, but most often the true odds against a horse winning are longer (a lower chance of a win) than the payout odds (e.g., payout odds = 3 to 1; true odds = 5 to 1). The posted odds of a horse actually overestimate the horse's chance of winning to ensure that the punter is underpaid for a win.

Equally Likely Outcomes

Central to probability is the idea of equally likely outcomes (Stewart, 1989). Each side of a die or coin is equally likely to come up. Probability, however, does not always seem to be

about events that are equally likely. For example, the bar symbol on a slot machine might have a probability of 25%, while a double diamond might have a probability of 2%. This does not actually contradict the idea of equally likely outcomes. Instead, think of the 25% as 25 chances and the 2% as two chances, for a total of 27 chances out of 100. Each of those 27 chances is equally likely. As another example, in rolling two dice there are 36 possible outcomes: (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1) . . . (6, 6); and each of these combinations is equally likely to happen. A player rolling 2 dice, however, is most likely to get a total of 7 because there are six ways to make a 7 from the two dice: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1). A player is least likely to get a total of either 2 or 12 because there is only one way to make a 2 (1, 1) and one way to make a 12 (6, 6).

Independence of Events

A basic assumption in probability theory is that each event is independent of all other events. That is, previous draws have no influence on the next draw. A popular catch phrase is "the dice have no memory." A die or roulette ball cannot look back and determine that it is due for a 6 or some other number. How could a coin decide to turn up a head after 20 tails? Each event is independent and therefore the player can never predict what will come up next. If a fair coin was flipped 5 times and came up heads 5 times in a row, the next flip could be either heads or tails. The fact that heads have come up 5 times in a row has no influence on the next flip. It is wise not to treat something that is very very unlikely as if it were impossible (see Turner, 1998, and "Incremental Betting Strategies" in Part 1.5, "Games and Systems"). In fact, if a coin is truly random, it must be possible for heads to come up 1 million times in a row. Such an event is extraordinarily unlikely, $p = 1/2^{1,000,000}$, but possible. Even then, the next flip is just as likely to be heads as it is tails. Nonetheless, many people believe that a coin corrects itself; if heads comes up too often, they think tails is due.

To complicate matters, however, there are cases where random events are not completely independent. With cards, the makeup of the deck is altered as cards are drawn from the deck. As a result, the value of subsequent cards is constrained by what has already been drawn. Nonetheless, each of the cards that remains in the deck is still equally probable. If, for example, there are only six cards left in a deck, four 7s and two 8s, a 7 is twice as likely to be drawn as an 8, but the specific card, the 7 of spades, has the same probability of being drawn as the 8 of diamonds.

Opportunities Abound

Another key aspect to computing probability is factoring in the number of opportunities for something to occur. The more opportunities there are, the more likely it is that an event will occur. The more tickets a player buys or the more often a player buys them, the greater the

player's chances of winning. At the same time, the more tickets purchased, the greater the average expected loss. One thousand tickets means 1,000 opportunities to win, so that the chance of winning *Lotto 6/49* goes from 1 in 14 million to 1 in 14,000. However, because the expected return is nearly always negative, the player will still lose money, on average, no matter how many tickets the player purchases (see "Playing Multiple Hands, Tickets or Bets" in Part 1.5, "Games and Systems"). This is true whether the player buys several tickets for the same draw or one ticket for every draw. Adding more opportunities (e.g., more tickets, bingo cards or slot machines) increases a player's chance of a win, but does not allow him/her to beat the odds.

Combinations

One final aspect of probability is the fact that the likelihood of two events occurring in combination is always less than the probability of either event occurring by itself. Friday occurs, once every 7 days (1/7) and the 13th day of the month comes once per month (about 1/30 on average). Friday the 13th, however, only occurs roughly once in 210 days (7 x 30) or once or twice per year.

To compute the joint probability of an event, multiply the probability of each of the two events. For example, the chances of rolling a 4 with a single dice are 1/6, or 16.7%. The chances of rolling a 4 two times in a row are: 1/6 x 1/6 = 1/36 (2.78%). The chances of rolling a 4 three times in a row is 1/6 x 1/6 x 1/6 = 1/216 (0.46%). It is important to note, however, that the joint probability of two events occurring refers only to events that have not happened yet. If something has already happened, then its chance of occurring is 100% because it has already happened. If the number 4 came up on the last two rolls, the chances of rolling another 4 are 1/6 not 1/216 because the new formula is 1 x 1 x 1/6, not 1/6 x 1/6 x 1/6. Each event is an independent event. In addition, the chances of any number coming up twice in a row are 1/6, not 1/36. This is because there are six possible ways (opportunities) of getting the same number twice in a row: (1/6 x 1/6) x 6 = 6/36 = 1/6.

It is the cumulative and multiplicative aspects of probability that lead people to overestimate their chances of winning. People tend to underestimate the chance of getting one or two of the same symbols on a slot machine because they do not take into account the number of opportunities. A number of studies have shown that people can unconsciously learn probability through experience (Reber, 1993). Suppose the chances of getting a diamond on a slot machine are 1 in 32 on each of three reels. The chance of getting at least one diamond is 3 (the number of reels) x 1/32 = 9.4%. That is, the player will see a diamond on the payline roughly one time every 10.6 spins. But their chances of getting three diamonds would be 1/32 x 1/32 x 1/32 = 1/ 32,768 = 0.003%. Because we occasionally see one (9.4%) or two (0.3%)

winning symbols on the payline, we may overestimate the chances of getting three of the big win symbols. This overestimation of the odds is also likely enhanced by seeing the big win symbols spin by on each spin, the occurrence of big win symbols above or below the payline, the distortion of the apparent odds caused by virtual reel mapping, and the larger number of big win symbols on the first two reels (see Turner & Horbay, 2004).

Law of Averages and the Law of Large Numbers

Part of the explanation for the persistent belief among those who gamble that there are patterns in chance, may stem from a misunderstanding of two related "laws" of statistics: the law of averages and the law of large numbers. The first is an informal folk theory of statistics; the second is a statistical law. These laws can be summarized as follows:

> *Law of Averages:* Things average out over time.

> *Law of Large Numbers:* As the sample size increases the average of the actual outcomes will more closely approximate the mathematical probability.

The law of large numbers is a useful way to understand betting outcomes. A coin on average will come up heads 50% of the time. It could nonetheless come up heads 100% of the time or 0% of the time. In a short trial, heads may easily come up on every flip. The larger the number of flips, however, the closer the percentage will be to 50%.

The law of averages is an informal approximation of the law of large numbers. The problem with the law of averages, as it is often understood, is that people assume that if something has not happened it is due to happen. For example, a person who gambles might expect that if heads have come up 10 times in a row, the next flip is more likely to be tails because the flips have to average out to 50%. Many people believe that deviations from chance are corrected by subsequent events and refer to the law of averages in support of their belief. Turner, Wiebe, Falkowski-Ham, Kelly and Skinner (2005) found that 36% of the general population believes that after 5 heads in a row the next flip is more likely to be tails. The law of large numbers, on the other hand, asserts only that the average converges towards the true mean as more observations are added. The average is not somehow corrected to ensure it reflects the expected average. The key difference is in the expectation. After a streak of 10 heads in a row, the law of averages would predict that more tails should come up so that the average is balanced out. The law of large numbers only predicts that after a sufficiently large number of trials, the streak of 10 heads in a row will be statistically irrelevant and the average will be close to the mathematical probability.

Some people accept the idea that the measured average will reflect the probability percentage in the long run, but still expect that if a trial of coin tosses began with a streak of heads, after

a million flips extra tails would have to have occurred for the measured average to be close to 50%. One individual argued that there had to be a "bias" in favor of tails to get the average back to 50%. This is still incorrect. According to the law of large numbers, it is not the *actual* number of flips that converges to the probability percentage, but the *average* number of flips. Suppose we start by getting 10 heads in a row and keep flipping the coin 1 million times. Does the difference of 10 go away? No. In fact, after 1 million flips the number of heads and tails could differ by as much as 1 or 2 thousand. Even a difference of 9,000 more tails than heads would still round off to 50% after one million flips. Consequently, the individual cannot use deviations from the expected average to get an edge.

It is important to realize that this "law" is really only a statement that summarizes what has been observed, most of the time, over a large (in theory, infinite) number of events. It says absolutely nothing about what will happen next or is likely to happen. Suppose a coin was tossed and the first 10 coin tosses resulted in the following sequence of heads and tails: T, H, H, H, H, H, T, H, H, H (20% tails, 80% heads). If the next 40 trials resulted in 19 tails and 21 heads (47.5% tails and 52.5% heads), the cumulative percentage of tails after all 50 trials would have moved from 20% to 42%—even though more heads came up during the subsequent flips. Incidentally, a player who bet $1 on tails on each of the 40 trials, assuming that tails was "due," would have ended up losing $2. The average converges toward the expected mean, but it does not correct itself.

This can be illustrated by comparing Figures 1 and 2. Figure 1 shows the percentage of heads and tails in numerous coin tossing trials, while Figure 2 shows the actual number of heads and tails. In Figure 1, it is clear that the ratio of heads to tails is converging to the average of 50% as the number of tosses increases. Figure 2, however, shows that the actual number of heads and tails is not converging. In fact, as the number of tosses increases, the line depicting the balance of heads vs. tails drifts away from 0. In some cases, the line drifts up (more heads) and in some case it drifts down (more tails). Many people who gamble understand the idea that the average converges towards the mean (Figure 1), but mistakenly believe that the actual number of heads and tails also converges towards the mean. The thick line in both graphs represents an individual coin that started out with more heads than tails. Notice how even though its average converges towards 50% (Figure 1), the line depicting the balance of heads and tails continues to drift upwards away from the mean (Figure 2).

Figure 1. Percentage of heads and tails over an increasing number of coin tosses.
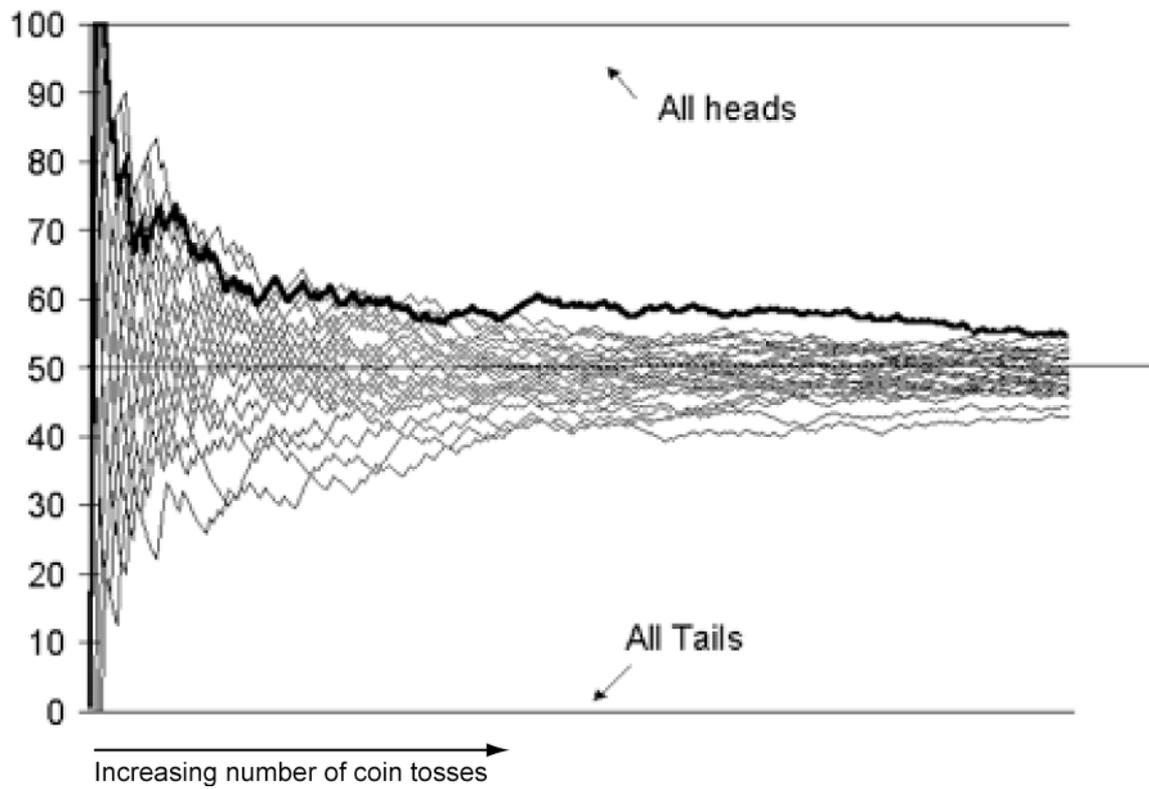
The percentage of heads and tails converges towards the mean.

Figure 2. The balance of heads and tails over an increasing number of coin tosses.
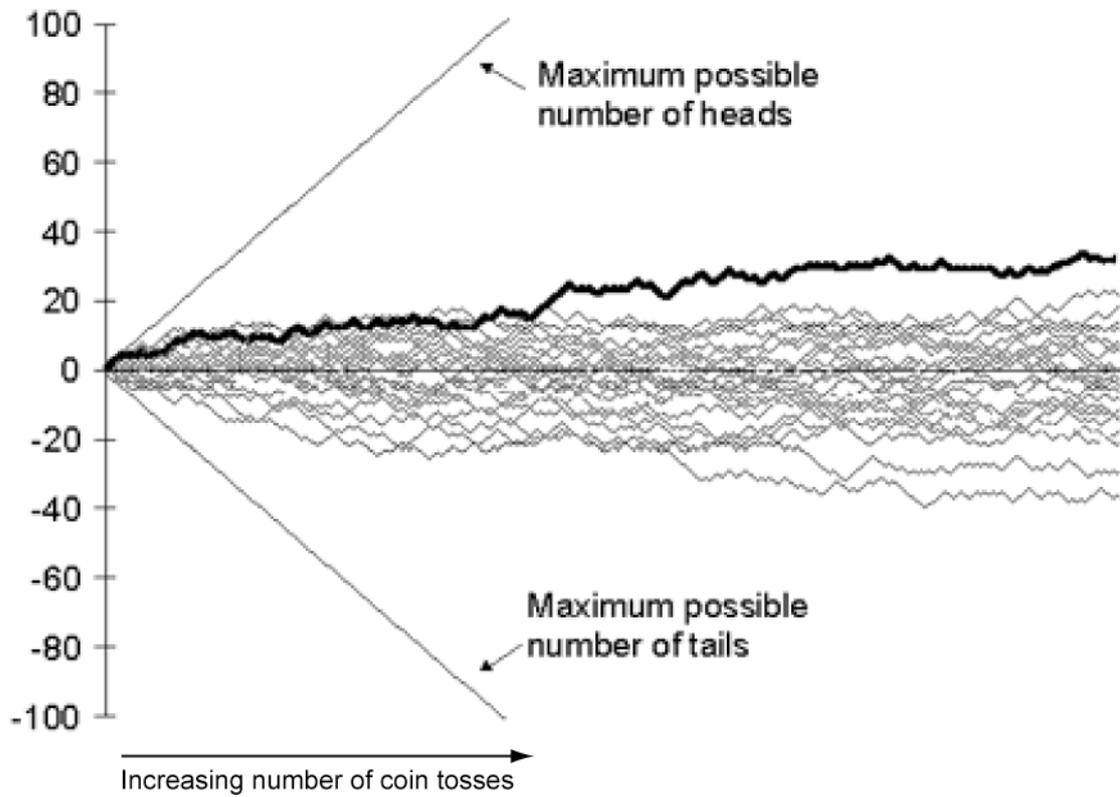
The actual number of heads and tails does not converge towards the mean; rather, it diverges away from the mean.

Random Events

Games of chance are made up of a series of events that are not predictable. If we toss an unbiased coin, the best we can say is that it will land either tails-side-up or heads-side-up and that the two outcomes have the same chance of occurring. Many of the beliefs and "systems" people who gamble develop are based on misconceptions about the nature of random events. So it is worthwhile to examine in more detail the essence of what it means to say that something is random.

Randomness is difficult to define. Random events are unpredictable, erratic, unplanned and independent of each other. However, random events sometimes appear to form a pattern or serve a purpose. For example, there are areas in the night sky, such as Orion's "belt," where stars appear to form a straight line. Given enough opportunity, any pattern could form by chance alone. An infinite number of monkeys on typewriters could eventually type out the complete works of Shakespeare.

Although random events appear to happen without a rule or cause, they are in fact the result of material cause (e.g., gravity and friction), but an exact list of forces acting on a random number generator (e.g., dice) may be unknown or impossible to specify precisely.

Sometimes clients believe that there is no such thing as randomness and that it is therefore possible to predict the outcome of games. Other people believe that random events have no cause, they just happen. This can make random events seem rather mysterious. Interestingly many religions used random events as part of their religious ritual to divine the will of the gods (Gabriel, 2003). There is nothing mysterious about random events. All physical events are determined or caused by something. Mechanical randomizers such as bingo balls, roulette wheels and dice use the laws of physics to maximize uncertainty. The basis of all random-like events is a combination of (1) initial uncertainty and (2) complex or non-linear relationships.

Uncertainty simply means that we do not know the exact values of all the variables with absolute precision. Uncertainty is an inherent part of measurement; nothing is ever 100% certain. A car driving at 70 kilometres per hour in cruise control will vary in speed by 1 or 2 kilometres per hour (more on a hilly highway). Thus there is some uncertainty as to the exact speed at any given moment. Orkin (2000) illustrates this problem with the question "How many fish are exactly 12 inches long?" Suppose a type of fish is usually 12 inches long. In all cases 12 inches is only an approximation. If a fish is 12.000001 inches long, it is not exactly 12 inches long. It is not possible to measure something so precisely as to completely eliminate uncertainty.

A complex or non-linear relationship is one in which a small change in the input causes an unpredictable change in the outcome: sometimes a large change, at other times a small one. For example, there is a non-linear relationship between caffeine and performance. Too little caffeine and a person might have trouble staying awake; too much and the person might become agitated and unable to concentrate on what he or she is doing. Suppose a researcher wanted to know how caffeine affected performance on a task. Initial uncertainties in this example would be factors such as how much sleep the research participants had the night before, how many cups of coffee they had that morning and how much coffee they usually drink per day (i.e., their level of tolerance). If the researcher did not make an effort to control for these factors, the uncertainties combined with the non-linear effect of caffeine could produce chaotic test results.

Over the past 30 years, physicists and mathematicians have come to realize that "tiny differences in input can quickly become overwhelming differences in output" (Gleick, 1987, p. 8). *Chaos* describes the unpredictable effects associated with small changes in a complex system (Gleick, 1987). For example, given the exact same weather conditions, the flapping of a butterfly's wings might make the difference a week later between a thunderstorm and a sunny day. This is a somewhat romantic exaggeration of chaos, and it is unlikely that a butterfly could actually have such a profound effect. However, when uncertainty is combined with complexity, the results can be completely unpredictable. Although this sounds improbable, physicists have found that small changes to the initial conditions within climate models grow in unpredictable ways because of the complexity of the system. All true random events are the result of chaos, but many chaotic patterns would not be sufficiently random to be used in a casino game.

The problem, from the gambler's point of view, is that the precision with which the initial conditions would have to be specified in order to predict the outcome is beyond the gambler's capacity. That is, unless players can control or measure the speed of a roulette ball and the wheel it is rolling around in, they cannot make a precise prediction about where the ball will land (Stewart, 1989). In the late 1970s, a group of engineering and computer science students at Stanford University tried to beat the roulette wheel using a concealed computer (Bass, 1985). Although theoretically possible, their plan ultimately failed because of the practical, legal and safety related difficulties of having to conceal their computer in a shoe. The use of a concealed computer to predict a casino game's outcome is illegal.

In summary, random events are the result of the chaotic interaction of uncertainty and complexity. This begs the question, "Is anything truly random?" If not, then surely one can predict the outcome of "random" events! On the contrary, the fact is that deterministic chaos

does an excellent job of creating random events. The amount of information needed to gain an edge in a game of chance is often extremely large. For example, no dice are perfectly cubed, and this will produce a slight bias. But the bias on a pair of casino dice might not show up until after several thousand bets, and even then would most likely be too small to allow the player to make money. So, while in theory nothing is completely random, in practice many games produce events that are indistinguishable from being purely random.

Generating Random Events

The question "What is random?" descends from the realm of theory into that of practice when we look at how the events which underlie most gambling games are produced. Randomness should be thought of as an ideal that is never really obtained in practice. The gambling devices used by casinos, however, efficiently maximizing uncertainty, and the results produced by these devices are close enough to truly random to be treated as such.

Roulette

The roulette wheel is a very efficient randomizer. Non-linearity is ensured by the combination of friction, gravity, centrifugal motion, and bumps and obstacles. This complexity is magnified many times by the fact that the inner wheel spins in the direction opposite that in which the ball is thrown. Initial uncertainty is introduced into the game by the speed of wheel at the outset, the speed of the ball, the exact position of the ball and wheel, the weight and bounciness of the ball and the air pressure and humidity. The outcome of a roulette wheel would be completely predictable if the ball were always thrown with exactly the same force from the exact same position and the speed of the wheel and all other environmental conditions were held exactly constant. In practice this is impossible; some croupiers, however, can apparently throw the ball with enough accuracy to hit a particular section of the wheel (Bass, 1985). As a result some casinos require that the croupier not look at the wheel when throwing the ball.

Dice

The key to ensuring randomness in dice is the combination of flat surfaces and sharp edges, coupled with the rolling of the dice, which makes it difficult to predict the outcome of a throw. In addition, the house rules for dice games specify that for a throw to be valid, the dice have to hit a bumper on the other side of the table, making it impossible to manipulate the throw's outcome. Dice used in home board games often have small holes drilled into the dice to mark the numbers. As a result the side with 6 dots is lighter than the opposite side, which has only 1 dot. This produces a slight bias, of 1% to 2%, that 6 and 5 will come up somewhat more often than their opposites, 1 and 2 respectively. In addition, the dice of some home

board games we have looked at are not perfectly square and therefore have other biases. However, casino dice have flat sides with no holes, are manufactured to ensure that they are square and are tested for balance regularly by the casino. The bias in home dice may be the source of the belief that energetic rolling leads to large numbers, because a bias will show up more often when a lot of energy is put into the roll.

Bingo and Lotteries

To ensure randomness in bingo and lottery balls, the balls are kept in an enclosed space and moved around by air or the rolling of the cages. Additional bounce may be achieved using a spring at the bottom of the bingo cage. The cage may be made of plastic or wire; the nature of the cage ensures additional variation, adding to the randomness. The numbers on bingo balls are embedded into the ball so that there is no differential weight or drag that would make one ball more likely to be selected than another. While the balls might be entered in the same order, randomness is ensured by tiny differences in the initial position of the balls, the air pressure, dust or smoke in the air, humidity, and the timing of the removal of the balls. Furthermore, the introduction of turbulence through the air jet or rolling adds a great deal of complexity.

Cards

Cards are perhaps the least efficient randomizer currently available. Randomizing cards is a two-step process including shuffling, which mixes up the cards (complexity), and cutting the deck, which ensures uncertainty about how the decks are mixed together. Washing the deck (spreading them out face down and mixing them around the table) is also used to increase the randomness of the cards. With most types of randomizers, past results cannot affect the outcome of the next draw. However, because cards are drawn from a limited pack, each card draw influences the probability of the next card. If, for example, 3 aces out of 4 have already been drawn, the chance that the next card will be an ace is very small. Consequently a skilled blackjack player can make money by card counting (Thorpe, 1966). Most people have some experience of playing cards at home, and this card-playing experience is perhaps a source of the very common belief that random events correct themselves because, with a deck of cards, to some extent they do.

It takes about seven complete shuffles to ensure randomness (Patterson, 1990), but given that games such as blackjack and baccarat often use six or more decks at a time, most casinos do not have enough time to completely randomize their decks. This has given rise to a system called "shuffle tracking," which is a variant on card counting (Patterson, 1990). Recent advances in computer technology have led to the creation of automatic shufflers, which use a random number generator to determine how to cut and sort the deck. In one variation, after

each hand of blackjack, a computer-controlled device "randomly" reinserts each of the discarded cards back into the stack of unused cards so that the dealer never has to shuffle the cards.

Computer Generated Randomness

Computerised games such as video games, electronic slot machines and video lottery terminals (VLTs) use a complex mathematical formula called a congruential iteration to produce "random" events. The formula uses three very large numbers, called A, B and M, which are used over and over again, and a seed value that changes each time. The formula provides complexity, while the seed value provides uncertainty. This system works as follows:

1. The seed number is usually obtained from the computer's clock.
2. This seed number is multiplied by a very large number (A).
3. The result of Step 2 is then added to another large number (B).
4. The result of Step 3 is then divided by a third very large prime number (M).
5. The remainder, or "what is left over" after Step 4 is the first "random" number. This "random" number is usually converted into a range that is convenient for the program, such as a number ranging from 1 to 32 (which would correspond to symbols on a slot reel). This remainder is also used as the seed for the next cycle.
6. The cycle is repeated as many times as needed.

Because the numbers are produced by a formula, they cannot be considered random and are called "pseudo-random." However, a sequence of pseudo-random numbers is difficult to distinguish from one produced by pure chance. Like mechanical randomizers, most computerized random number generators are good enough for practical purposes. The sequence produced by this algorithm is limited to the size of the value of M. If M is 4.1 billion, then the sequence of numbers would repeat in the exact same order after 4.1 billion numbers are output. At 25 cents per spin and a 90% payback, it would cost as much $36 million in bets to track the entire sequence.

As stated above, to achieve randomness a system needs both non-linearity and uncertainty. Slot machines add uncertainty in two ways. First, they seed the sequence with a time function so that the sequence will differ depending on the time of day that the computer was turned on. Second, the random number generator in a slot machine runs all the time, but the numbers are withdrawn from this formula only when the player presses the spin button or lever. As such, the numbers drawn depend on the exact millisecond when the spin button is pressed. A

millisecond later and the outcome might be different. As a result, the outcomes of slot machines are, in effect, random, and waiting for the cycle to repeat itself is not possible.

Games as Complex Systems

The reader might have trouble thinking of a sports game as a randomization process, but just as with dice, coins or computer programs, much of what happens in a sports game follows the rules of chance. How can people be elements in a random number generation system? First, a sports game is a complex system. Successfully playing a game involves a large number of physical actions that when added together result in a great deal of complexity. Second, many elements of a game involve a chance outcome. A highly skilled baseball player may hit a ball only 30% of the time or catch a fly ball only 80% of the time. Third, injuries, health, player composition, weather, time of day, player stress, fans yelling in the stands or even birds landing on the field all add uncertainty to the system. Fourth, the difference between most professional teams is actually very small: even the worst major league baseball team will beat the best on occasion. But this only makes sports games partly random.

If all that people who gamble needed to do to win money was to pick the better team, they could win most of the time by simply betting on the favourites. Unfortunately that prospect is eliminated by the way that racetracks and sports bookies operate. In horse races, the track takes a cut (17%) off the top and distributes the rest of the prize pool to the people who bet on the winners. The chaotic process of the mass betting pool essentially removes the differential ability of the horses. A horse that has a better chance of winning gets more action (bets) and thus less money goes to each individual who bet on that horse. In sports games, the bookies estimate how many points a team will win by. This is called the point spread. A bettor wins only if his/her chosen team beats the point spread. These subjectively estimated lines and odds virtually eliminate the role of a team or a horse's ability in the outcome of the bet. (See "Part 7: Subjective Probability" in the chapter "Games and Systems" for more information on skill and sports betting.)

Why We See Patterns in Sequences of Random Events

This chapter began with a table showing the contrast between what people think random events are like and what they are really like. Here we will explore some of the reasons that people have these erroneous beliefs. It has been well documented that most people—even those who understand that any result of a series of tosses of a fair coin is a random sequence—make errors in their judgements about random sequences. The following is a list of some possible explanations for this tendency. The focus here is not on superstitious beliefs, but on cognitive processes and experiences that might lead a person to hold faulty beliefs. For

a more complete examination of erroneous beliefs in gambling, see Wagenaar (1988); Ladouceur and Walker (1996); Kahneman and Tversky (1982); Toneatto (1999); and Toneatto, Blitz-Miller, Calderwood, Dragonetti, and Tsanos (1997).

People will often judge the coin-tossing sequence of H, H, H, H, H, H as being less random than H, T, H, H, T, H, even though the probability of obtaining each of these given sequences is identical: 1/2 x 1/2 x 1/2 x 1/2 x 1/2 x 1/2 = 0.015625. Note that this is the probability of getting a specific sequence compared to the probability of getting a second specific sequence. Kahneman and Tversky (1982) call this tendency the representativeness heuristic. People who make this error are often computing the probability of getting 6 consecutive heads compared to *every* other possible sequence. Most random sequences of heads and tails do not have an easily recognizable pattern. This tends to reinforce the belief that a sequence of all heads is less likely. However, any one specific arbitrary combination of heads and tails has exactly the same chance of occurring as any other specific combination. Another factor that contributes to this error is that there is only one possible way of getting 6 heads and only one possible way of getting 6 tails, while there are a total of 64 possible ways of tossing six coins, 20 of which produce exactly 50% heads (e.g., H, T, T, T, H, H or H, T, H, H, T, T). This gives the illusion that combinations that "look random," are more likely, but in fact each one of those specific combinations (e.g., H, T, T, T, H, H) has the same chance of occurring as H, H, H, H, H, H.

Another reason for errors in our understanding of randomness may be confusion between the way the word random is used in everyday speech and the way it is used in statistics and mathematics. According to the Merriam-Webster Online Dictionary, the most common meaning of the adjective random is "lacking a definite plan, purpose, or pattern." It also lists "haphazard" as a synonym (http://www.m-w.com/cgi-bin/dictionary). Judging solely by its appearance, a sequence of 6 heads in a row might appear to have a pattern. Probability theory, however, is concerned with how the events in a sequence are produced, not in how they appear after the fact.

A third reason is the tendency of the human brain toward "selective reporting"—the habit of seizing on certain events as significant, while ignoring the other neighbouring events that would give the chosen events context and help to evaluate how likely or unlikely the perceived pattern really is. Big or salient events will be recalled better. We recall plane crashes because they are highly publicized. Uneventful flights are ignored. Because of the occasional well-publicized plane crash many people are afraid to fly, even though plane crashes are much rarer than car crashes. Kahneman and Tversky (1982) call this tendency the availability heuristic.

A closely related tendency is for people to underestimate the likelihood of repeated numbers, sequences, or rare events occurring by pure chance. The basic problem is that we do not take into account the number of opportunities for something to occur, so we are often surprised when random chance produces coincidences. As an example, in a class of 35 students, we assume that the chance of 2 people sharing a birthday is very small, say 1 in 365, or maybe 35 in 365 (Arnold, 1978). The actual probability that at least two people will share the same birthday is close to 100% because there are actually (35 x 34) / 2 = 595 possible combinations of people in the class. Because the possible combinations of people (595) exceed the number of days in the year (365), the chance that at least 1 pair of people will share a birthday is surprisingly high.

Our minds are predisposed to find patterns, not to discount them. It is argued that we have evolved the ability to detect patterns because to do so was often essential for survival. For example, if a person was walking in the jungle and saw a pattern of light and dark stripes in the shadows, it would be prudent to assume that the pattern was a tiger and act accordingly. The consequences of incorrectly assuming that the pattern is *not* a tiger far outweigh those of incorrectly assuming that it is. But when applied to random events, this survival "skill" leads to errors.

Some errors might be the result of the way in which statistics are disseminated. Academics, journalists, advertisers and others often report statistics using terms such as "1 out of every 10," or "1 death every 25 seconds." These statements might lead to the impression that the events reported occur in a regular manner.

We learn through experience and logically induce general rules on that basis. If our experience is limited, we may induce the wrong rule. A chance occurrence may lead to false expectations. As a result, a win the first time one plays a game, or a win after some extraneous event, may lead to the formulation of an erroneous general rule. For example, a bingo player reported that she was once about to buy her bingo booklet, but was called away for some reason. Later, she bought her booklet and then won. Now she has a ritual of going back to the end of the line if she does not feel that the serial numbers are lucky, and she reports that this system has worked for her on at least one other occasion.

Natural human reasoning tends to assume that a premise is reversible. That is, given the premises that all As are Bs and all Bs are Cs, the correct conclusion is that all As are Cs. However, people tend to assume that all Cs must also be As. In fact, this is incorrect. All that we can be certain of is that *some* Cs are As, but there may be many Cs that are not As. This "conversion error" is common and it creates all manner of problems (Johnson-Laird, 1983). Even highly educated individuals frequently make conversion errors. The basic flaw in the

law of averages is the error of converting the correct premise "The number of heads and tails even out in the long run" to the incorrect conclusion "Since the number of heads and tails even out in the long run, I should win if I bet on tails."

Individuals who gamble often think that random events are self-correcting. One possible reason for this is that their experience seems to be consistent with this belief. Closely related to the law of large numbers is the phenomenon of "regression to the mean," which predicts that exceptional outcomes (e.g., very high or very low scores) will most likely be followed by scores that are closer to the mean. For example, a father who is very tall is more likely to have a son who is shorter than he is, not taller. It is true that a tall man is more likely to have a tall son than a short one, because height is partly under genetic control. However, the random factors that influence height (the recombination of the parents' genes, nutrition, accidents, diseases, etc.) will tend to pull the son's height down closer to the average for the general population. The fact is that, by pure chance, there is more room to move down, closer to the mean, than up, away from the mean.

To turn to a gambling example, suppose a coin is tossed 100 times, and 80% are heads. If the coin is tossed another 100 times, the net outcome is more likely to move closer 50% heads than to stay at 80% heads or to increase to 90%. But it is important to understand that regression to the mean does not *have* to occur: the son could be taller or the next 100 flips could all be heads. But it is *more likely* that the son's height or the number of heads and tails will be closer to the mean because the mean is the single most likely outcome. In the context of gambling, regression to the mean might produce the illusion that the random events are "evening out." Unusual events (long losing streaks or winning streaks) seem to be corrected over time, but in fact they are not corrected, only diluted. The average converges towards the mean; it is not pushed there. But the experience from event to event gives the illusion that it is pushed there by some sort of force.

Increased bets may also play a role in convincing those who gamble that random events are self-correcting because "chasing" works. Doubling a bet after a loss has the interesting effect of increasing the player's chance of walking away a winner. The rationale behind this practice is again the law of averages. Since people expect random events to correct themselves, doubling after a loss may seem like a good investment strategy. Incremental betting strategies appear to push around random events so they do not look "random" (Turner & Horbay, 2003). Turner (1998) has shown that a doubling strategy would be successful if random events were self-correcting. The chapter "Games and Systems" discusses this betting system and its flaws in more detail. It is enough to note here that most of the time doubling appears to work, thus reinforcing the idea that random events correct themselves. This system usually

produces a very slow accumulation of money. Eventually, however, the player experiences a disastrous losing streak

One final reason for errors in judging random events is that our minds tend to segment events in ways that are consistent with what we expect. Given a heads and tails sequence of H, H, H, H, T, H, H, T, T, T, T, H, we are likely to divide this string into a segment in which H was more likely to appear (H, H, H, H, T, H, H, H) and one in which T was more likely to appear (T, T, T, T, H). This segmentation process is very often used by sports commentators (e.g., "The Blue Jays have now won 5 of their last 6 games," or "A player has struck out 11 times in his last 15 at bat"). In segmenting the sequence this way, it is very easy to convince oneself that tails did in fact come up more often, to correct for the excess of heads. As noted above, our minds are predisposed to find patterns, not to discount them.

Summary

There are important clues regarding erroneous beliefs in the experiences of the individual who gambles. While it is unlikely that addressing the erroneous beliefs is sufficient for treatment, exploring these beliefs can be an important aid in helping the client understand their gambling experiences—both their wins and their losses. Correcting these beliefs may also help in relapse prevention. If individuals with a gambling problem still believe that they can beat the odds, the odds are they will try again.

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