skiladæmi 9

Due: 11:59pm on Wednesday, November 4, 2015

You will receive no credit for items you complete after the assignment is due. Grading Policy

Problem 10.67

A yo-yo is made from two uniform disks, each with mass m and radius R, connected by a light axle of radius b. A light, thin string is wound several times around the axle and then held stationary while the yo-yo is released from rest, dropping as the string unwinds.

Part A

Find the linear acceleration of the yo-yo.

Express your answer in terms of g, b, R, m.

ANSWER:

$a = \frac{2b^2g}{R^2+2b^2}$

Correct

Part B

Find the angular acceleration of the yo-yo.

Express your answer in terms of g, b, R, m.

ANSWER:

$\alpha = \frac{2bg}{R^2+2b^2}$

Correct

Part C

Find the tension in the string.

Express your answer in terms of g, b, R, m.

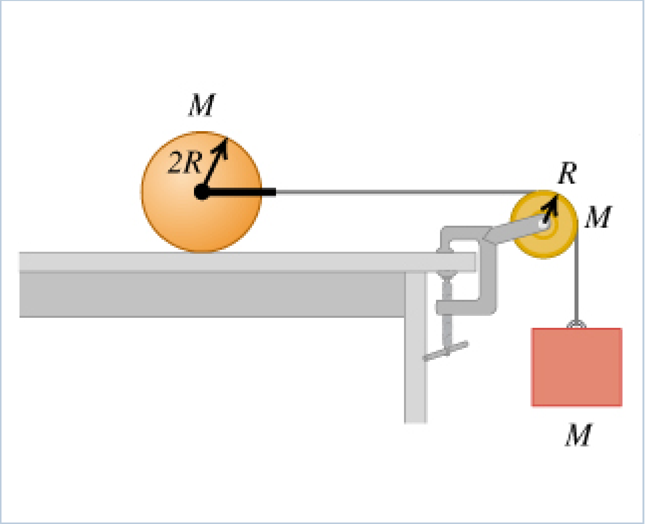
ANSWER:

$T = \frac{m2gR^2}{R^2+2b^2}$

Correct

Problem 10.75

A uniform, solid cylinder with mass M and radius $2R$ rests on a horizontal tabletop. A string is attached by a yoke to a fri Typesetting math: 100% the center of the cylinder so that the cylinder can rotate about the axle. The string runs over a disk-shaped pulley with mass M and radius R that is mounted on a frictionless axle through its center. A block of mass M is suspended from the free end of the string (the figure). The string doesn't slip over the pulley surface, and the cylinder rolls without slipping on the tabletop.



Part A

Find the magnitude of the acceleration of the block after the system is released from rest.

Express your answer in terms of the variables M, R, and appropriate constants.

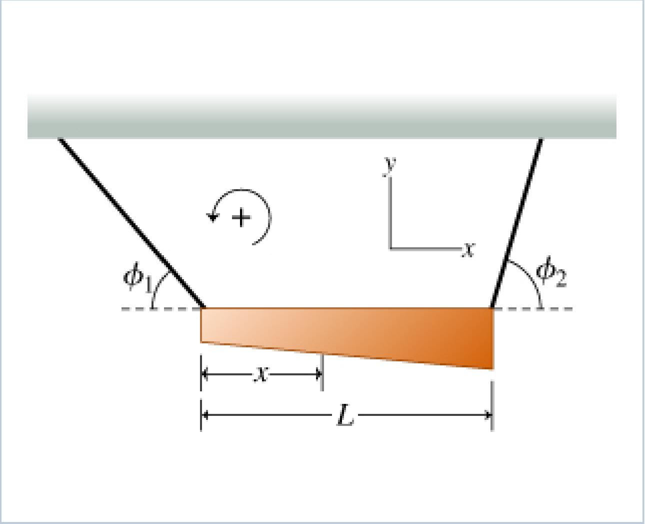
ANSWER:

a_{block} = 3.27

Correct

A Bar Suspended by Two Wires

A nonuniform, horizontal bar of mass m is supported by two massless wires against gravity. The left wire makes an angle ϕ_1 with the horizontal, and the right wire makes an angle ϕ_2. The bar has length L.



Part A

Find the position of the center of mass of the bar, x, measured from the bar's left end.

Express the center of mass in terms of L, ϕ_1, and ϕ_2.

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Hint 1. Nature of the problem

This is a statics problem. There is no net force or torque acting on the bar.

Hint 2. Torques about left end of bar

The net torque is zero about any point you select. Here we ask you to find the net torque of the system about the left end of the bar. Label the tension in the left wire T_1, and label the other wire's tension T_2. The weight of the bar is $W = mg$. Note that the vector sum of T_1, T_2, and W is zero. Using the sign convention shown in the picture, express the sum of the torques about the left end of the bar.

Answer in terms of L, x, W, T_2, T_1, ϕ_2, and/or ϕ_1. Note that not all of these quantities will appear in your answer.

ANSWER:

$\sum \tau_{\rm left} = 0 =$ $LT_2 \sin(\phi_2) - Wx$

Hint 3. Forces: *x* components

Assume that the tensions in the left and right wires are T_1 and T_2, respectively. What is the sum of the *x* components of the forces ΣF_x? Because this is a statics problem, these forces will sum to zero.

Use the sign convention indicated in the figure, and express your answer in terms of L, x, W, T_2, T_1, ϕ_2, and/or ϕ_1. Note that not all of these quantities will appear in your answer.

ANSWER:

$\sum F_x = 0 =$ $-T_1 \cos(\phi_1) + T_2 \cos(\phi_2)$

Hint 4. Forces: *y* components

Assuming that the tensions in the left and right wires are T_1 and T_2, respectively, what is the sum of the *y* components of the forces ΣF_y? Because this is a statics problem, these forces will sum to zero.

Use the sign convention indicated in the figure, and express your answer in terms of L, x, W, T_2, T_1, ϕ_2, and/or ϕ_1. Note that not all of these quantities will appear in your answer.

ANSWER:

$\sum F_y = 0 =$ $T_1 \sin(\phi_1) + T_2 \sin(\phi_2) - W$

Hint 5. Eliminate weight from your equations

You should have found three equations by now. It is possible to eliminate two variables and solve for x in terms of the others. As an intermediate step, solve your torque equation for x in terms of W, T_2, L, etc. and then solve your *y*-component force equation for W and substitute back into your expression for x. In other words, find an expression for x.

Answer in terms of T_1, T_2, ϕ_1, ϕ_2, and L.

ANSWER:

$x = \dfrac{T_2 \sin(\phi_2) L}{T_1 \sin(\phi_1) + T_2 \sin(\phi_2)}$

Typesetting math: 100% l trig identity

The dimensions for the expression you just found for x are correct, since the units of the tensions cancel out, leaving the units of length in the numerator. If you now solve the x-component force equation for T_1 in terms of T_2, and substitute into your equation for x, you should find the following trig identity useful:

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b).$$

Alternatively, you could express your answer in terms of $\tan(\phi_1)$ and $\tan(\phi_2)$.

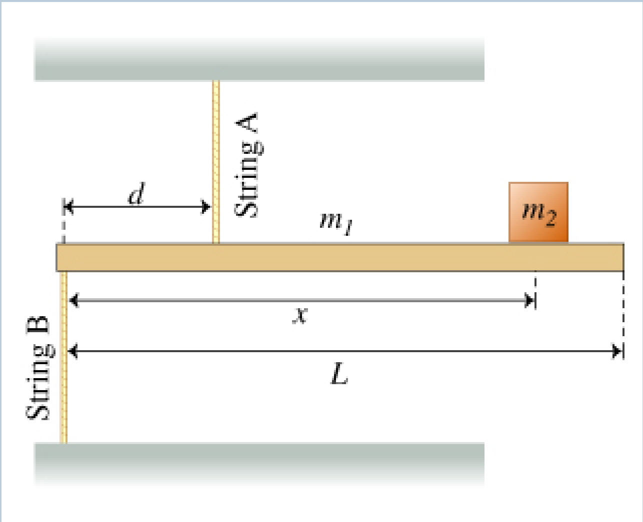
ANSWER:

$x = \frac{L}{\tan(\phi_1)\cot(\phi_2)+1}$

Correct

A Bar Suspended by Two Vertical Strings

A rigid, uniform, horizontal bar of mass m_1 and length L is supported by two identical massless strings. Both strings are vertical. String A is attached at a distance $d < L/2$ from the left end of the bar and is connected to the ceiling; string B is attached to the left end of the bar and is connected to the floor. A small block of mass m_2 is supported against gravity by the bar at a distance x from the left end of the bar, as shown in the figure.



Throughout this problem positive torque is that which spins an object counterclockwise. Use g for the magnitude of the acceleration due to gravity.

Part A

Find T_A, the tension in string A.

Express the tension in string A in terms of g, m_1, L, d, m_2, and x.

Hint 1. Choosing an axis

Choose a rotation axis *p*, about which to apply the requirement $\sum \tau_p = 0$. Since the system is in static equilibrium, the choice of rotation axis is arbitrary; however, there is a convenient choice of *p* to find T_A by eliminating the torque from an unknown force.

Hint 2. Find the torque around the best axis

It is convenient to choose the rotation axis to be through the point where string B is attached to the bar. This eliminates any torque from the tension in string B. Find the total torque about this point.

Answer in terms of T_A, m_1, m_2, L, x, d, and g.

ANSWER:

Typesetting math: 100%

$\sum \tau_B$ = $T_A d - \frac{m_1 g L}{2} - m_2 g x$

Hint 3. Summing the torques

$\sum \tau_p = 0$ for a static system. Solve for T_A.

ANSWER:

T_A = $\frac{\frac{L}{2} m_1 g + x m_2 g}{d}$

Correct

Part B

Find T_B, the magnitude of the tension in string B.

Express the magnitude of the tension in string B in terms of T_A, m_1, m_2, and g.

Hint 1. Two different methods to find T_B

There are two equivalent ways to find T_B. One way is to balance the torques as was done in the calculation of T_A, except using a different rotation axis. In this case, a convenient axis is through the point where string A is attached to the bar. The second, and easier, method is to use the second equation for static equilibrium, $\sum \vec{F} = 0$.

Hint 2. Direction of forces

Since both strings are vertical, all forces on the bar--the tension forces and the weights of the bar and block--act vertically. Thus, only vertical components of forces need be considered.

ANSWER:

T_B = $T_A - m_1 g - m_2 g$

Correct

Part C

If the bar and block are too heavy the strings may break. Which of the two identical strings will break first?

ANSWER:

- (selected) string A
- string B

Correct

Typesetting math: 100%

Part D

If the mass of the block is too large *and* the block is too close to the left end of the bar (near string B) then the horizontal bar may become unstable (i.e., the bar may no longer remain horizontal).

What is the smallest possible value of x such that the bar remains stable (call it $x_{\rm critical}$)?

Express your answer for $x_{\rm critical}$ in terms of m_1, m_2, d, and L.

Hint 1. Nature of the unstable motion

When the bar becomes unstable there are only two points about which the bar can rotate: the points where the strings attach to the bar. About which point will the bar rotate when $x < x_{\rm critical}$?

ANSWER:

- (selected) The point where string A is attached to the bar
- The point where string B is attached to the bar

Hint 2. Tension in string B at the critical point

The tension in string B counteracts the clockwise rotation of the bar about the point where string A is attached to the bar. As x is decreased, T_B is likewise decreased because the clockwise torque about this point decreases. The critical value $x_{\rm critical}$ corresponds to when $T_B = 0$. If x is decreased further, T_B will continue to be zero and the counterclockwise torque due to the weight of the block will be greater than the clockwise torque due to the weight of the bar, causing the system to rotate.

Hint 3. Calculate the torques

Add up the total torque about the point in which string A attaches to the bar when the mass m_2 is at $x_{\rm critical}$. Remember that T_B has a special value at this point and that, owing to the choice of origin, the torque due to string A is 0. Remember to pay attention to the direction of the torques.

Answer in terms of m_2, m_1, d, L, g, and $x_{\rm critical}$.

Hint 1. Find the distance of the center of mass of the bar from string A

What is the distance d_1 of the center of mass of the bar from string A?

Answer in terms of the given variables.

ANSWER:

d_1 = $\frac{L}{2} - d$

Hint 2. Find the distance of m_2 from the string A

What is the distance d_2 of m_2 from the string A?

Answer in terms of the given variables.

ANSWER:

d_2 = $d - x_{\rm critical}$

ANSWER:

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$$\sum \tau_A = 0 = m_2 g(d - x_{\text{critical}}) - m_1 g\left(\frac{L}{2} - d\right)$$

ANSWER:

$$x_{\text{critical}} = d - \frac{m_1}{m_2}\left(\frac{L}{2} - d\right)$$

Correct

Part E

Note that x_{critical}, as computed in the previous part, is not necessarily positive. If $x_{\text{critical}} < 0$, the bar will be stable no matter where the block of mass m_2 is placed on it.

Assuming that m_1, d, and L are held fixed, what is the maximum block mass m_{max} for which the bar will *always* be stable? In other words, what is the maximum block mass such that $x_{\text{critical}} \leq 0$?

Answer in terms of m_1, d, and L.

Hint 1. Requirement of stability

If x is calculated to be less than zero, the solution is unphysical. (The bar does not extend there to support it!) The minimum value that x can have is obviously zero. If m is less than the mass that would give $x_{\text{critical}} = 0$ then the bar will be stable for any physical value of x.

ANSWER:

$$m_{\text{max}} = m_1\left(\frac{L}{2d} - 1\right)$$

Correct

Alternative Exercise 11.106

A nonuniform fire escape ladder is 6.0 m long when extended to the icy alley below. It is held at the top by a frictionless pivot, and there is negligible frictional force from the icy surface at the bottom. The ladder weighs 250 N, and its center of gravity is 2.0 m along the ladder from its bottom. A mother and child of total weight 750 N are on the ladder 1.5 m from the pivot. The ladder makes an angle θ with the horizontal.

Part A

Find the magnitude of the force exerted by the icy alley on the ladder?

Express your answer using two significant figures.

ANSWER:

F = 350 N

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Correct

Part B

What is the direction of the force exerted by the icy alley on the ladder?

ANSWER:

- (selected) upward
- downward

Correct

Part C

Find the magnitude of the force exerted by the ladder on the pivot.

Express your answer using two significant figures.

ANSWER:

F = 650 N

Correct

Part D

What is the direction of the force exerted by the ladder on the pivot.

ANSWER:

- upward
- (selected) downward

Correct

Part E

Do your answers in parts A and C depend on the angle θ?

ANSWER:

- yes
- (selected) no

Correct

Score Summary:

Y[Typesetting math: 100%]nment is 103%.
You received 5.13 out of a possible total of 5 points.