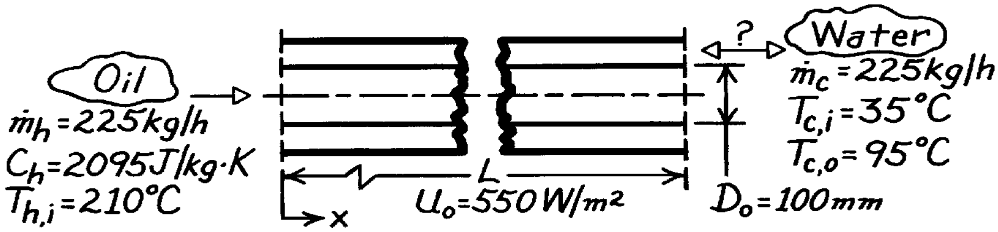
PROBLEM 11.41

KNOWN: Concentric tube heat exchanger.

FIND: Length of the exchanger.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6*, Water ($\overline{T}_c$ = (35 + 95)°C/2 = 338 K): $c_{p,c}$ = 4188 J/kg·K

ANALYSIS: From the rate equation, Eq. 11.14, with $A_o = \pi D_o L$,

$$L = q / U_o \pi D_o \Delta T_{\ell m}.$$

The heat rate, q, can be evaluated from an energy balance on the cold fluid,

$$q = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right) = \frac{225 \text{ kg/h}}{3600 \text{ s/h}} \times 4188 \text{J/kg} \cdot \text{K}\left(95 - 35\right)\text{K} = 15,705 \text{ W}.$$

In order to evaluate $\Delta T_{\ell m}$, we need to know whether the exchanger is operating in CF or PF. From an energy balance on the hot fluid, find

$$T_{h,o} = T_{h,i} - q / \dot{m}_h c_h = 210°\text{C} - 15,705 \text{ W} / \frac{225 \text{ kg/h}}{3600 \text{s/h}} \times 2095 \frac{\text{J}}{\text{kg} \cdot \text{K}} = 90.1°\text{C}.$$

Since $T_{h,o} < T_{c,o}$ it follows that Hxer operation must be CF. From Eq. 11.15,

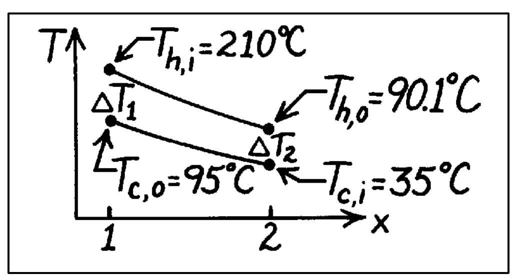
$$\Delta T_{\ell m,CF} = \frac{\Delta T_1 - \Delta T_2}{\ell n\left(\Delta T_1 / \Delta T_2\right)} = \frac{\left(210 - 95\right) - \left(90.1 - 35\right)}{\ell n\left(115/55.1\right)} °\text{C} = 81.4°\text{C}.$$

Substituting numerical values, the HXer length is

$$L = 15,705 \text{ W} / 550 \text{ W/m}^2 \cdot \text{K} \pi \left(0.10\text{m}\right) \times 81.4\text{K} = 1.12\text{m}.$$ <

COMMENTS: The ε-NTU method could also be used. It would be necessary to perform the hot fluid energy balance to determine if CF operation existed. The capacity rate ratio is C_{min}/C_{max} = 0.50. From Eqs. 11.19 and 11.20 with q evaluated from an energy balance on the hot fluid,

$$\varepsilon = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}} = \frac{210 - 90.1}{210 - 35} = 0.69.$$



From Fig. 11.15, find NTU ≈ 1.5 giving

$$L = \text{NTU} \cdot C_{min} / U_o \pi D_o \approx 1.5 \times 130.94 \frac{\text{W}}{\text{K}} / 550 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \cdot \pi \left(0.10\text{m}\right) \approx 1.14\text{m}.$$

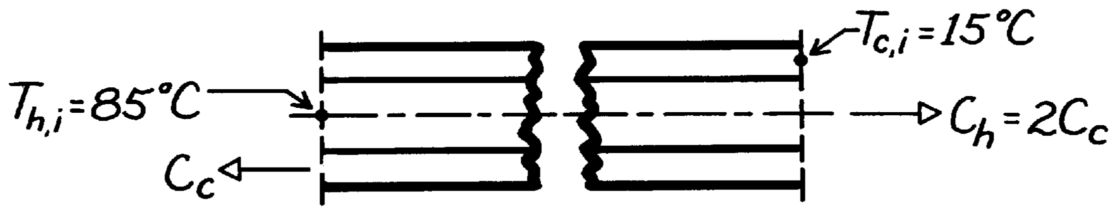
Note the good agreement in both methods.

PROBLEM 11.42

KNOWN: A *very long*, concentric tube heat exchanger having hot and cold water inlet temperatures, 85°C and 15°C, respectively; flow rate of hot water is twice that of the cold water.

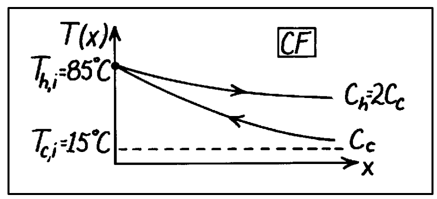
FIND: Outlet temperatures for counterflow and parallel flow operation.

SCHEMATIC:



ASSUMPTIONS: (1) Equivalent hot and cold water specific heats, (2) Negligible kinetic and potential energy changes, (3) No heat loss to surroundings.

ANALYSIS: The heat rate for a concentric tube heat exchanger with very large surface area operating in the *counterflow* mode is



$$q = q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right)$$

where $C_{min} = C_c$. From an energy balance on the hot fluid,

$$q = C_h\left(T_{h,i} - T_{h,o}\right).$$

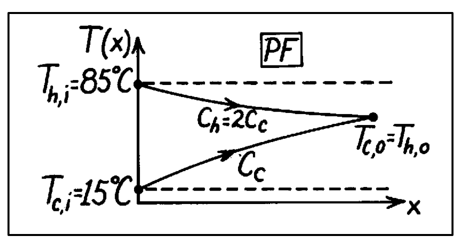
Combining the above relations and rearranging, find

$$T_{h,o} = -\frac{C_{min}}{C_h}\left(T_{h,i} - T_{c,i}\right) + T_{h,i} = -\frac{C_c}{C_h}\left(T_{h,i} - T_{c,i}\right) + T_{h,i}.$$

Substituting numerical values,

$$T_{h,o} = -\frac{1}{2}(85-15)°C + 85°C = 50°C.$$ <

For *parallel flow* operation, the hot and cold outlet temperatures will be equal; that is, $T_{c,o} = T_{h,o}$. Hence,



$$C_c\left(T_{c,o} - T_{c,i}\right) = C_h\left(T_{h,i} - T_{h,o}\right).$$

Setting $T_{c,o} = T_{h,o}$ and rearranging,

$$T_{h,o} = \left(T_{h,i} + \frac{C_c}{C_h}T_{c,i}\right) \Big/ \left(1 + \frac{C_c}{C_h}\right)$$

$$T_{h,o} = \left(85 + \frac{1}{2}\times 15\right)°C \Big/ \left(1 + \frac{1}{2}\right) = 61.7°C.$$ <

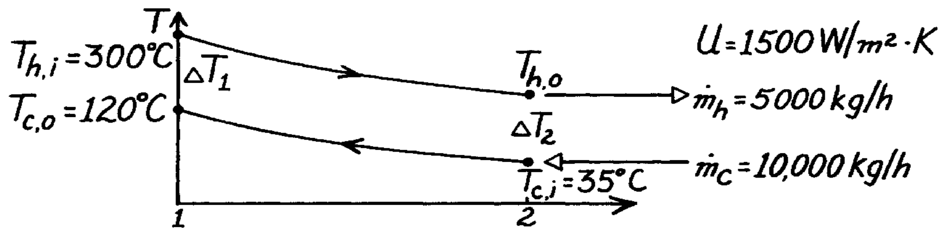
COMMENTS: Note that while $\varepsilon = 1$ for CF operation, for PF operation find $\varepsilon = q/q_{max} = 0.67$.

PROBLEM 11.14

KNOWN: A shell and tube Hxer (two shells, four tube passes) heats 10,000 kg/h of pressurized water from 35°C to 120°C with 5,000 kg/h water entering at 300°C.

FIND: Required heat transfer area, A_s.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6*, Water $(\overline{T}_c = 350K)$: c_p = 4195 J/kg·K; *Table A-6*, Water (Assume $T_{h,o}$ ≈ 150°C, $\overline{T}_h$ ≈ 500 K): c_p = 4660 J/kg·K.

ANALYSIS: The rate equation, Eq. 11.14, can be written in the form

$$A_s = q / U\Delta T_{\ell m} \quad (1)$$

and from Eq. 11.18,

$$\Delta T_{\ell m} = F\Delta T_{\ell m,CF} \qquad \text{where} \qquad \Delta T_{\ell m,CF} = \frac{\Delta T_1 - \Delta T_2}{\ell n\left(\Delta T_1 / \Delta T_2\right)}. \quad (2,3)$$

From an energy balance on the cold fluid, the heat rate is

$$q = \dot{m}_c\, c_{p,c}\left(T_{c,o} - T_{c,i}\right) = \frac{10{,}000\ kg/h}{3600 s/h} \times 4195 \frac{J}{kg \cdot K}(120 - 35)K = 9.905 \times 10^5\ W.$$

From an energy balance on the hot fluid, the outlet temperature is

$$T_{h,o} = T_{h,i} - q / \dot{m}_h\, c_{p,h} = 300°C - 9.905 \times 10^5\ W / \frac{5000}{3600}\frac{kg}{s} \times 4660 \frac{J}{kg \cdot K} = 147°C.$$

From Fig. 11.11, determine F from values of P and R, where P = (120 – 35)°C/(300 – 35)°C = 0.32, R = (300 – 147)°C/(120-35)°C = 1.8, and F ≈ 0.97. The log-mean temperature difference based upon a CF arrangement follows from Eq. (3); find

$$\Delta T_{\ell m} = \left[(300 - 120) - (147 - 35)\right] K / \ell n \frac{(300 - 120)}{(147 - 35)} = 143.3 K. \quad <$$

$$A_s = 9.905 \times 10^5 W / 1500 W/m^2 \cdot K \times 0.97 \times 143.3 K = 4.75 m^2 \quad <$$

COMMENTS: (1) Check $\overline{T}_h$ ≈ 500 K used in property determination; $\overline{T}_h$ = (300 + 147)°C/2 = 497 K.

(2) Using the NTU-*e* method, determine first the capacity rate ratio, C_{min}/C_{max} = 0.56. Then

$$e \equiv \frac{q}{q_{max}} = \frac{C_{max}\left(T_{c,o} - T_{c,i}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{1}{0.56} \times \frac{(120 - 35)°C}{(300 - 35)°C} = 0.57.$$

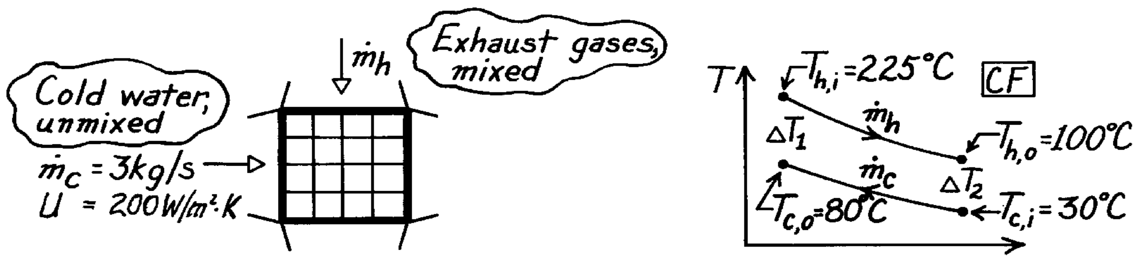
From Fig. 11.17, find that NTU = AU/C_{min} ≈ 1.1 giving A_s = 4.7 m^2.

PROBLEM 11.32

KNOWN: Single pass, cross-flow heat exchanger with hot exhaust gases (mixed) to heat water (unmixed)

FIND: Required surface area.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Exhaust gas properties assumed to be those of air.

PROPERTIES: *Table A-6*, Water ($\overline{T}_c$ = (80 + 30)°C/2 = 328 K): c_p = 4184 J/kg·K; *Table A-4*, Air (1 atm, $\overline{T}_h$ = (100 + 225)°C/2 = 436 K): c_p = 1019 J/kg·K.

ANALYSIS: The rate equation for the heat exchanger follows from Eqs. 11.14 and 11.18. The area is given as

$$A = q / U\Delta T_{\ell m} = q / U F \Delta T_{\ell m,CF} \tag{1}$$

where F is determined from Fig. 11.13 using

$$P = \frac{80-30}{225-30} = 0.26 \text{ and } R = \frac{225-100}{80-30} = 2.50 \text{ giving } F \approx 0.92. \tag{2}$$

From an energy balance on the cold fluid, find

$$q = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right) = 3\frac{kg}{s} \times 4184 \frac{J}{kg \cdot K}(80-30)K = 627{,}600 \text{ W}. \tag{3}$$

From Eq. 11.15, the LMTD for counter-flow conditions is

$$\Delta T_{\ell m,CF} = \frac{\Delta T_1 - \Delta T_2}{\ell n\left(\Delta T_1 / \Delta T_2\right)} = \frac{(225-80)-(100-30)}{\ell n(145/70)} °C = 103.0°C. \tag{4}$$

Substituting numerical values resulting from Eqs. (2-4) into Eq. (1), find the required surface area to be

$$A = 627{,}600\,W / 200\,W/m^2 \cdot K \times 0.92 \times 103.0K = 33.1m^2.$$ <

COMMENTS: Note that the properties of the exhaust gases were not needed in this method of analysis. If the ε-NTU method were used, find first C_h/C_c = 0.40 with $C_{min} = C_h$ = 5021 W/K. From Eqs. 11.19 and 11.20, with $C_h = C_{min}$, $\varepsilon = q/q_{max} = (T_{h,i} - T_{h,o})/(T_{h,i} - T_{c,i})$ = (225 – 100)/(225 – 30) = 0.64. Using Fig. 11.19 with C_{min}/C_{max} = 0.4 and ε = 0.64, find NTU = $UA/C_{min} \approx 1.4$. Hence,

$$A = NTU \cdot C_{min} / U \approx 1.4 \times 5021\,W/K / 200\,W/m^2 \cdot K = 35.2m^2.$$

Note agreement with above result.

Part D. (solution 1)

There are several points worth noting in this problem

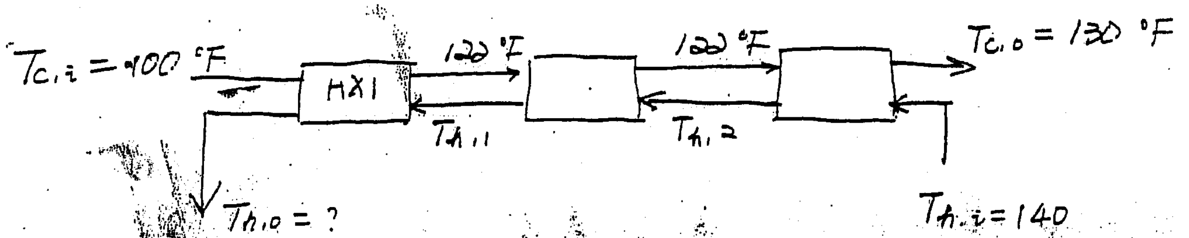
i) $T_{c,i} < T_{melt} < T_{c,o}$ for cempene, which means we need to divide the HX into three sections. (See figure below)

ii) Cold stream consists of compene and water. But the solubility in each other is negligible. So the condition leads to:

$$C_{p,c} = x_w C_{p,w} + (1 - x_w) C_{p,s}$$

($C_{p,c}$ → cold stream; $C_{p,w}$ → water; $C_{p,s}$ → solid (compene))

iii) Outlet temperature of hot stream can tell you if co-current flow is possible.



(Assume it's counter-current flow for the moment).

Overall energy balance:

① $q_t = m_h C_{p,h} (T_{h,i} - T_{h,o}) = \dot{m}_c C_{p,c_1} (122 - T_{c,i})$ } HX1

(C_{p,c_1}: specific heat of cold stream in HX1)

$+ \dot{m}_c x_s \Delta H_f$ } HX2

(ΔH_f → Heat of fusion; x_s → weight percent of compene)

$+ \dot{m}_c C_{p,c3} (T_{c,o} - 122)$ } HX3

$C_{p\,c1} = C_{ps}\, x_s + C_{p,w}\, x_w = 0.38 \times 0.2 + 1.0 \times 0.8 = 0.876$ Btu/lb. °F

$C_{p\,c3} = 0.4 \times 0.2 + 1.0 \times 0.8 = 0.88$ Btu/lb °F.

(2pts)

From eq ①, we solve for $T_{h,o}$

② $$T_{h,o} = T_{h,i} - \frac{q_t}{\dot{m}_h} = 140 - \frac{2.35 \times 10^5}{\dot{m}_h}$$

(a) $\dot{m}_h = 9000$ lb/hr

$$T_{h,o} = 140 - \frac{2.35\times10^5}{9000} = 114\ °F.$$

Since $T_{h,o} < T_{c,i}$ it is impossible to have co-current flow as heat can't be transferred from a colder stream to a hotter stream. Only count-current flow is OK.

From $q = UA\,(LMTD)$

$$\Rightarrow \quad A = A_1 + A_2 + A_3 = \frac{1}{U}\left(\frac{q_1}{LMTD_1} + \frac{q_2}{LMTD_2} + \frac{q_3}{LMTD_3}\right)$$

Energy Balance over each HX:

HX1: $q_1 = \dot{m}_h C_{p,h}(T_{h,1} - T_{h,o}) = \dot{m}_c C_{p,c,l}(122 - T_{c,i}) = 96360$ Btu/hr

$$\Rightarrow \quad T_{h,1} = \frac{q_1}{\dot{m}_h C_{p,h}} + T_{h,o}$$

$$= \frac{96360}{9000\times1} + 114 = 124.7\ °F$$

$(LMTD)_1 = 6.7\ °F$

HX2: $q_2 = \dot{m}_h C_{p,h}(T_{h,2} - T_{h,1}) = \dot{m}_c \chi_3 \Delta H_f = 103000$ Btu/hr

$$T_{h,2} = \frac{103000}{9000\times1} + 124.7 = 136.1\ °F$$

$(LMTD)_2 = 6.9\ °F$

HX3. $q_3 = \dot{m}_c C_{p,cs}(130 - 122) = 35040$ Btu/hr.

$(LMTD)_3 = 11.9\ °F$

Summarize the above results in a table:

HX	1	2	3
q_i	96360	103000	35040
$(LMTD)_i$	6.7	6.9	11.9
A_i (ft²)	71.9	74.6	14.7
A_{total}		161.2	

(2pts)

$$L = \frac{A}{\pi D} = \frac{161.2}{\pi \times 2/12} = 307.9\ ft \qquad (2pts)$$

(b). $\dot{m}_h = 7000$ lb/hr.

From eq ②, we can find (2pts)

$$T_{h,c} = 107\ °F.$$

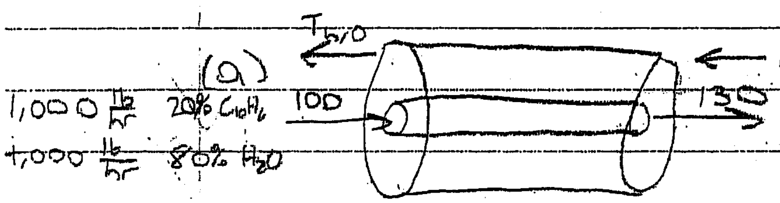
Follow same procedure as in (a), we find (2pts)

$$T_{h,1} = 121\ °F < 122\ °F$$

There is a pinch point in HX2; Cannot work.

Part D (solution 2)

Assumptions: ① Negligible heat loss to the surroundings
② Negligible potential + kinetic energy changes
③ constant properties
④ Fully developed conditions for the H_2O and $C_{10}H_{16}$ (U independent of x)

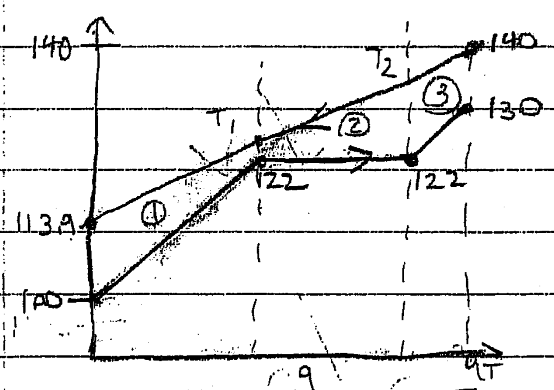


$$q_T = \left[(\dot{m} c_p \Delta T)_{C_6H_{10},S} + (\dot{m} \Delta H_{fusion})_{C_6H_{10}} + (\dot{m} c_p \Delta T)_{C_6H_{10},\ell} + (\dot{m} c_p \Delta T)_{H_2O}\right]_c$$
$$= (\dot{m} c_p \Delta T)_H$$

$\Rightarrow 1000(0.38 \times 22 + 103 + 8 \times 0.4) + 4,000(30) = 9,000(140 - T_{h,o})$

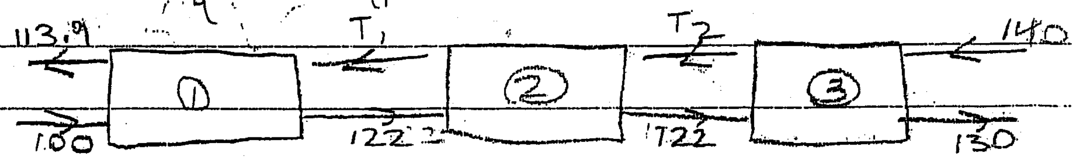
$\Rightarrow \boxed{T_{h,o} = 113.9°F}$

∴ heat exchanger must be counter-current since $T_{c,o} > T_{h,o}$.



To find L, model heat exchanger as 3 seperate heat exchangers

$L = L_1 + L_2 + L_3$



$T_1: \; q_1 = \left[(\dot{m}C_p\Delta T)_{C_6H_{10}, S} + (\dot{m}C_p\Delta T)_{H_2O}\right]_c = (\dot{m}C_p\Delta T)_{H, H_2O}$

$= 1000(22 \times .38) + 4000(22) = 9000(T_1 - 113.9)$

$\Rightarrow \boxed{T_1 = 124.6°F}$

$T_2: \; q_2 = (\dot{m}\Delta H_{fus}) = (\dot{m}C_p\Delta T)_{H, H_2O}$

$= (1000)(103) = 9000(T_2 - 124.6) \Rightarrow \boxed{T_2 = 136.1°F}$

$q_1 = UA\Delta T_{lm} \Rightarrow A_1 = \frac{q_1}{U\Delta T_{lm,1}} \quad A_2 = \frac{q_2}{U\Delta T_{lm,2}} \quad A_3 = \frac{q_3}{U\Delta T_{lm,3}}$

$A_1 = \frac{9000(124.6 - 113.9)}{200\left[\frac{(124.6-122)-(113.9-100)}{\ln\frac{124.6-122}{113.9-100}}\right]} = 70.9 \, ft^2$

$A_2 = \frac{103{,}000}{200\left[\frac{(136.1-122)-(124.6-122)}{\ln\left(\frac{136.1-122}{124.6-122}\right)}\right]} = 75.3 \, ft^2$

$A_3 = \frac{9{,}000(140 - 136.1)}{200\left[\frac{(140-130)-(136.1-122)}{\ln\left(\frac{140-130}{136.1-122}\right)}\right]} = 14.75 \, ft^2$

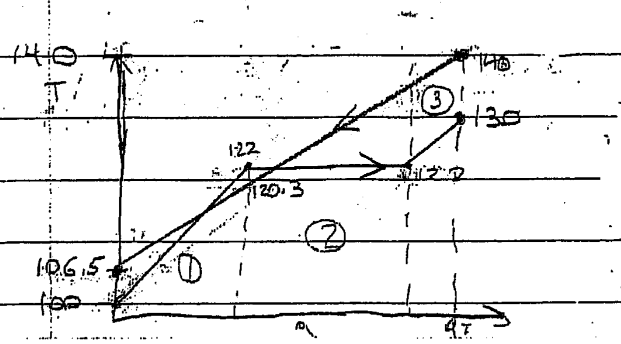
$A = A_1 + A_2 + A_3 \Rightarrow \boxed{L} = \frac{A}{\pi D} = \frac{160.95}{\pi(\frac{1}{6})} = \boxed{307.4 \, ft}$

(b) Redoing calculations for temperatures:

$1000(.38 \times 22 + 103 + 8 \times 0.4) + 4000(30) = 7000(140 - T_{h,o})$

$\Rightarrow \boxed{T_{h,o} = 106.5°F}$

$1000(22 \times .38) + 4000(22) = 7{,}000(T_1 - 106.5) \Rightarrow \boxed{T_1 = 120.3°F}$



We find there is a pinch point ∴ can't work