

## Problem Set- Chapter 2 Solutions

1. Ch 2, Problem 2.1

The demand for beer in Japan is given by the following equation:  $Q^d = 700 - 2P - P_N + 0.1I$ , where  $P$  is the price of beer,  $P_N$  is the price of nuts, and  $I$  is average consumer income. Assume B is a normal good.

- a) What happens to the demand for beer when the price of nuts goes up? Are beer and nuts demand substitutes or demand complements?

The sign in front of the price of nuts,  $P_N$ , is negative. This means when the price of nuts goes up, the beer quantity demanded falls for all levels of price (demand shifts left). Beer and nuts are demand complements.

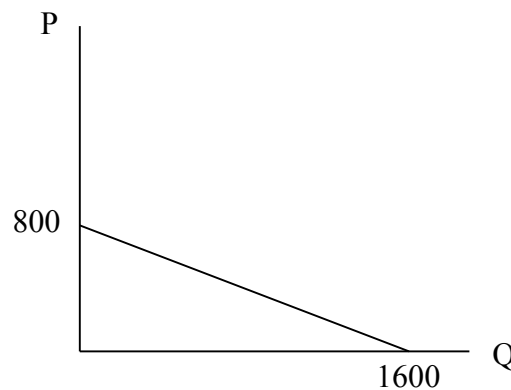
- b) What happens to the demand for beer when average consumer income rises?

The sign in front of income,  $I$ , is positive. This means when income rises, quantity demanded increases for all levels of price (demand shifts rightward).

- c) Graph the demand curve for beer when  $P_N = 100$  and  $I = 10,000$ .

$$\text{Now: } Q^d = 700 - 2P - 100 + 0.1 \cdot 10,000 = 1,600 - 2P \Rightarrow P = 800 - 0.5 Q^d$$

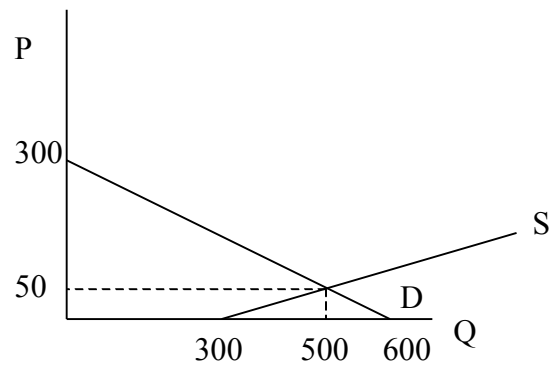
So when  $Q^d$  or  $Q$  is zero  $P=800$ , When  $P=0$ ,  $Q^d$  or  $Q$  is 1600.



2. Ch 2, Problem 2.3

The demand and supply curves for coffee are given by  $Q^d = 600 - 2P$  and  $Q^s = 300 + 4P$ .

- a) Plot the supply and demand curves on a graph and show where the equilibrium occurs.



- b) Using algebra, determine the market equilibrium price and quantity of coffee. Indicate the equilibrium price and quantity on the graph in part a.

$$\begin{aligned}600 - 2P &= 300 + 4P \\300 &= 6P \\50 &= P\end{aligned}$$

Plugging  $P = 50$  back into either the supply or demand equation yields  $Q = 500$ .

3. Ch 2, Problem 2.13

Consider a linear demand curve,  $Q = 350 - 7P$ .

- a) Derive the inverse demand curve corresponding to this demand curve.

$$\begin{aligned}Q &= 350 - 7P \\7P &= 350 - Q \\P &= 50 - \frac{1}{7}Q\end{aligned}$$

- b) What is the choke price?

The choke price occurs at the point where  $Q = 0$ . Setting  $Q = 0$  in the inverse demand equation above yields  $P = 50$ .

c) What is the price elasticity of demand at  $P = 50$ ?

a. At  $P = 50$ , the choke price, the elasticity will approach negative infinity.

4. Ch 2, Problem 2.17

Consider the following demand and supply relationships in the market for golf balls:

$$Q^d = 90 - 2P - 2T \text{ and}$$

$$Q^s = -9 + 5P - 2.5R,$$

where  $T$  is the price of titanium, a metal used to make golf clubs, and  $R$  is the price of rubber.

a) If  $R = 2$  and  $T = 10$ , calculate the equilibrium price and quantity of golf balls.

Substituting the values of  $R$  and  $T$ , we get

$$\text{Demand : } Q^d = 70 - 2P$$

$$\text{Supply : } Q^s = -14 + 5P$$

In equilibrium,  $70 - 2P = -14 + 5P$ , which implies that  $P = 12$ . Substituting this value back,  $Q = 46$ .

b) At the equilibrium values, calculate the price elasticity of demand and the price elasticity of supply.

$$\text{Elasticity of Demand} = \frac{\partial Q^d}{\partial P} * \frac{P}{Q} = -2 * \frac{12}{46} = -0.52$$

$$\text{Elasticity of Supply} = \frac{\partial Q^s}{\partial P} * \frac{P}{Q} = 5 * \frac{12}{46} = 1.30$$

c) At the equilibrium values, calculate the cross-price elasticity of demand for golf balls with respect to the price of titanium. What does the sign of this elasticity tell you about whether golf balls and titanium are substitutes or complements?

$\varepsilon_{golf,titanium} = -2\left(\frac{10}{46}\right) = -0.43$ . The negative sign indicates that titanium and golf balls are complements, i.e., when the price of titanium goes up the demand for golf balls decreases.

d)

5. Suppose there are only two goods ( $X$  and  $Y$ ) and only two individuals (numbered 1 and 2) in an economy. Let  $P_X$  be the price of good  $X$  and  $P_Y$  be the price of good  $Y$ . And finally, let  $I_1$  represent the income of individual 1 and  $I_2$  the income of individual 2.

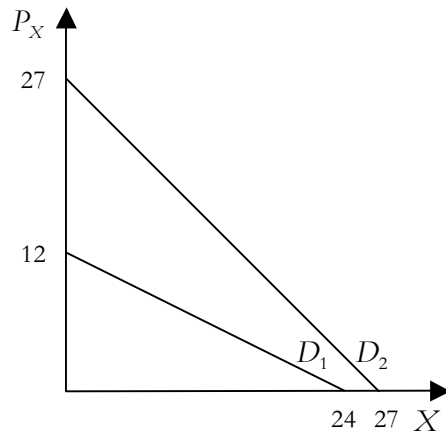
Suppose the quantity of good  $X$  demanded by individual 1 is given by

$$X_1 = 10 - 2 P_X + 0.01 I_1 + 0.4 P_Y,$$

and the quantity of  $X$  demanded by individual 2 is

$$X_2 = 5 - P_X + 0.02 I_2 + 0.2 P_Y.$$

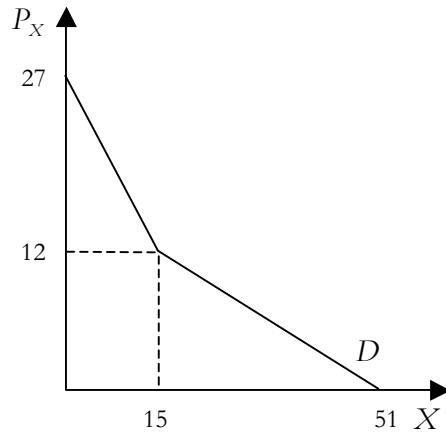
a. Graph the two individual demand curves (with  $X$  on the horizontal axis and  $P_X$  on the vertical axis) for the case  $I_1 = 1000$ ,  $I_2 = 1000$ , and  $P_Y = 10$ .



The algebraic equation for this curve was derived in part a: after plugging in  $I_1 = 1000$ ,  $I_2 = 1000$ , and  $P_Y = 10$ , we obtain

$$X = \begin{cases} 51 - 3P_X & \text{if } 0 \leq P_X \leq 12 \\ 27 - P_X & \text{if } 12 < P_X \leq 27 \\ 0 & \text{if } P_X > 27 \end{cases}$$

- b. Using the individual demand curves obtained in part b, graph the market demand curve for total  $X$ . What is the algebraic equation for this curve?



6. Suppose the demand for lychees is given by the following equation:

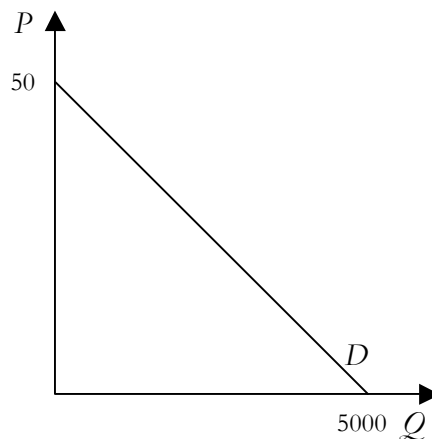
$$Q^d = 4000 - 100P + 500P_M,$$

where  $P$  is the price of lychees and  $P_M$  is the price of mangoes.

- a. What happens to the demand for lychees when the price of mangoes goes up? Are lychees and mangoes substitutes or complements?

The demand for lychees increases when the price of mangoes goes up. Therefore, lychees and mangoes are substitutes.

- b. Graph the demand curve for lychees when  $P_M = 2$ .

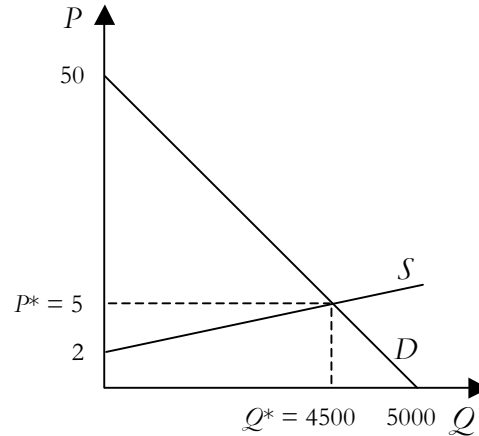


Now suppose that the quantity of lychees supplied is given by the following equation:

$$Q^d = 1500P - 60R,$$

where  $R$  is the amount of rainfall.

- c. On the same graph you drew for part b, graph the supply curve for lychees when  $R = 50$ . Label the equilibrium price and quantity with  $P^*$  and  $Q^*$  respectively.



- d. Calculate the equilibrium price and quantity of lychees.

Setting the quantity supplied equal to the quantity demanded, we obtain

$$\begin{aligned} Q^d &= Q^s \\ 5000 - 100P &= 1500P - 3000 \\ 8000 &= 1600P \\ P^* &= 5 \end{aligned}$$

Plugging the equilibrium price ( $P^* = 5$ ) into the demand curve, we obtain

$$\begin{aligned} Q^d &= 5000 - 100P \\ Q^* &= 4500 \end{aligned}$$

- e. At the equilibrium values, calculate the price elasticity of demand and the price elasticity of supply. Is the demand for lychees elastic, unit elastic, or inelastic? Is the supply of lychees elastic, unit elastic, or inelastic?

The price elasticity of demand is

$$\begin{aligned} \epsilon_{Q,P} &= \frac{\partial Q^d}{\partial P} \frac{P^*}{Q^*} \\ &= -100 \left( \frac{5}{4500} \right) \\ &= -0.\bar{1} \end{aligned}$$

The demand for lychees is inelastic. The price elasticity of supply is

$$\begin{aligned}\epsilon_{Q^s, P} &= \frac{\partial Q^s}{\partial P} \frac{P}{Q^s} \\ &= 1500 \left( \frac{5}{4500} \right) \\ &= 1.\bar{6}\end{aligned}$$

The supply of lychees is elastic.

- f. At the equilibrium values, calculate the cross-price elasticity of demand for lychees with respect to the price of mangoes. What does the sign of this elasticity tell you about whether lychees and mangoes are substitutes or complements? (*Hint*: Check to make sure that your answer is consistent with your answer to part a.)

The cross-price elasticity of demand for lychees with respect to the price of mangoes is

$$\begin{aligned}\epsilon_{Q, P_M} &= \frac{\partial Q^d}{\partial P_M} \frac{P_M}{Q^s} \\ &= 500 \left( \frac{2}{4500} \right) \\ &= 0.\bar{2}\end{aligned}$$

Since the cross-price elasticity of demand is positive, the two goods are substitutes.

7. Consider the demand curve  $Q = aP^{-b}$ , where  $a$  and  $b$  are positive constants. Use the formula for price elasticity of demand given in class,

$$\epsilon_{Q, P} = \frac{\partial Q}{\partial P} \frac{P}{Q},$$

to show that the price elasticity of demand is equal to  $-b$  at every point on the demand curve.

We start by calculating the partial derivative of  $Q$  with respect to  $P$ :

$$\frac{\partial Q}{\partial P} = -abP^{-(b+1)}.$$

Making the appropriate substitutions using  $\partial Q/\partial P = -abP^{-(b+1)}$  and  $Q = aP^{-b}$ , we obtain

$$\begin{aligned}\epsilon_{Q,P} &= -abP^{-(b+1)}\left(\frac{P}{aP^{-b}}\right) \\ &= -abP^{-(b+1)}\left(\frac{1}{aP^{-(b+1)}}\right) \\ &= -b\end{aligned}$$