Decision Analysis

Sesi 05-07

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Problem Solving and Decision Making

- 7 Steps of Problem Solving
  (First 5 steps are the process of decision making)
  – Define the problem.
  – Identify the set of alternative solutions.
  – Determine the criteria for evaluating alternatives.
  – Evaluate the alternatives.
  – Choose an alternative (make a decision).

  – Implement the chosen alternative.
  – Evaluate the results.

Introduction to Decision Analysis

- The field of decision analysis **provides framework** for making **important decisions**.
- Decision analysis allows us to **select a decision** from **a set of possible decision alternatives** when **uncertainties** regarding the future exist.
- The goal is to **optimized the resulting payoff** in terms of a decision criterion.
- Maximizing expected profit is a common criterion when probabilities can be assessed.
- When risk should be factored into the decision making process, Utility Theory provides a mechanism for analyzing decisions in light of risks.

- Decision theory and decision analysis help people (including business people) make better decisions.
  – They identify the best decision to take.
  – They assume an ideal decision maker:
    - Fully informed about possible decisions and their consequences.
    - Able to compute with perfect accuracy.
    - Fully rational.
- Decisions can be difficult in two different ways:
  – The need to use game theory to predict how other people will respond to your decisions.
  – The consequence of decisions, good and bad, are stochastic.
    - That is, consequences depend on decisions of nature.

Decision Analysis Definitions

- **Actions** – alternative choices for a course of action
- **Events** –possible outcomes of chance happenings
- **Payoffs** – a value associated with the result of each event
- **Decision criteria** – rule for selecting an action

Decision Analysis Definitions

- Decision analysis = explicit, quantitative method to make (or think about) decisions in the face of uncertainty.
  – Portray *options* and their *consequences*
  – Quantify *uncertainty* using *probabilities*
  – Quantify the *desirability of outcomes* using *utilities*
  – Calculate the *expected utility* of each option (alternative course of action)
  – Choose the option that *on average* leads to most desirable outcomes

Decision Analysis Definitions

- A set of alternative actions
  - We may chose whichever we please
- A set of possible states of nature
  - Only one will be correct, but we don't know in advance
- A set of outcomes and a value for each
  - Each is a combination of an alternative action and a state of nature
  - Value can be monetary or otherwise

- A decision problem is characterized by decision alternatives, states of nature (decisions of nature), and resulting payoffs.
- The decision alternatives are the different possible actions or strategies the decision maker can employ.
- The states of nature refer to possible future events (rain or sun) not under the control of the decision maker.
  - States of nature should be defined so that they are mutually exclusive (one or the other) and collectively exhaustive (one will happen).
    - There will be either rain or sun, but not both.

Indications for Decision Analysis

- Uncertainty about outcomes of alternative courses of action.
  1. Developing policies, treatment guidelines, etc.
  2. At the bedside (i.e. helping patients make decisions)
  3. Focus discussion and identify important research needs
  4. In your life outside of medicine
  5. As teaching tool to discourage dogmatism and to demonstrate rigorously the need to involve patients in decisions

Decision Making Criteria

- Certainty
  - Decision Maker knows with certainty what the state of nature will be - only one possible state of nature
- Ignorance
  - Decision Maker knows all possible states of nature, but does not know probability of occurrence
- Risk
  - Decision Maker knows all possible states of nature, and can assign probability of occurrence for each state

Criteria for decision making

- Maximize expected monetary value
- Minimize expected monetary opportunity loss
- Maximize return to risk ratio
  - E monetary V/σ
- Maximize maximum monetary value (maximax) – best best case monetary value
- Maximize minimum monetary value (maximin) – best worst case monetary value
- Minimize maximum opportunity loss (minimax) – best worst case for opportunity loss

Problem Formulation

- A decision problem is characterized by decision alternatives, states of nature, and resulting payoffs.
- The decision alternatives are the different possible strategies the decision maker can employ.
- The states of nature refer to future events, not under the control of the decision maker, which will ultimately affect decision results. States of nature should be defined so that they are mutually exclusive and contain all possible future events that could affect the results of all potential decisions.

Decision Theory Models

- Decision theory problems are generally represented as one of the following:
  - – Influence Diagram
  - – Payoff Table/Decision Table
  - – Decision Tree
  - – Game Theory

Influence Diagram

Influence Diagrams

- An influence diagram is a graphical device showing the relationships among the decisions, the chance events, and the consequences.
- Squares or rectangles depict decision nodes.
- Circles or ovals depict chance nodes.
- Diamonds depict consequence nodes.
- Lines or arcs connecting the nodes show the direction of influence.

Pay-Off Table

Payoff Tables

- The consequence resulting from a specific combination of a decision alternative and a state of nature is a payoff.
- A table showing payoffs for all combinations of decision alternatives and states of nature is a payoff table.
- Payoffs can be expressed in terms of profit, cost, time, distance or any other appropriate measure.

Payoff Table Analysis

- Payoff Tables
  - – Payoff Table analysis can be applied when -
    - There is a finite set of discrete decision alternatives.
    - The outcome of a decision is a function of a single future event.
  - – In a Payoff Table -
    - The rows correspond to the possible decision alternatives.
    - The columns correspond to the possible future events.
    - Events (States of Nature) are mutually exclusive and collectively exhaustive.
    - The body of the table contains the payoffs.

Payoff Table

Alternatives	States of Nature: State 1	State 2
Alternative 1	Outcome 1	Outcome 2
Alternative 2	Outcome 3	Outcome 4

Event i	Market A1	Do not market A2
Success	$45.00	-$3
Failure	-$36	-$3

Decision Making Model

The types of decision models:
- Decision making under certainty
  - The future state of nature is assumed known.
- Decision making under uncertainty (no probabilities)
  - There is no knowledge about the probability of the states of nature occurring.
- Decision making under risk (with probabilities)
  - There is (some) knowledge of the probability of the states of nature occurring.
- Decision making with perfect information
  - The future state of nature is assumed known with certain probability.
- Decision making with imperfect information (Bayesian Theory)
- Decision making in light of competitive actions (Game theory)
  - All the actors (players) are seeking to maximize their return.

Decision Making Under Uncertainty

- *Maximax* - Choose the alternative that maximizes the maximum outcome for every alternative (Optimistic criterion)
- *Maximin* - Choose the alternative that maximizes the minimum outcome for every alternative (Pessimistic criterion)
- *Equally likely* - chose the alternative with the highest average outcome.

Ex: Decision Making Under Uncertainty

Alternatives	States of Nature: Favorable Market	Unfavorable Market	Maximum in Row	Minimum in Row	Row Average
Construct large plant	200,000	-180,000	200,000	-180,000	10,000
Construct small plant	100,000	-20,000	100,000	-20,000	40,000
Do nothing	0	0	0	0	0

Maximax Maximin Equally likely

Ex: SI KASEP INVESTMENT DECISION

- Si Kasep has inherited $1000.
- He has decided to invest the money for one year.
- A broker has suggested five potential investments.
  - Gold.
  - Company A
  - Company B
  - Company C
  - Company D
- Si Kasep has to decide how much to invest in each investment.

SOLUTION

- Construct a Payoff Table.
- Select a Decision Making Criterion.
- Apply the Criterion to the Payoff table.
- Identify the Optimal Decision

Construct a Payoff Table

The States of Nature are Mutually Exclusive and Collective Exhaustive

…t a Payoff Table
…e set of possible decision alternatives.
…he set of five investment opportunities.
– Defined … ature.
• Kasep cons… ock market states (expressed by changes in the DJA)

State of Nature	DJA Correspondence
S.1: A large rise in the stock market	Increase over 1000 points
S.2: A small rise in the stock market	Increase between 300 and 1000
S.3: No change in the stock market	Change between -300 and +300
S.4: A small fall in stock market	Decrease between 300 and 800
S5: A large fall in the stock market	Decrease of more than 800

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The Payoff Table

	States of Nature				
Decision Altern	Large Rise	Small Rise	No Change	Small Fall	Large Fall
Gold	-100	100	200	300	0
Bond	250	200	150	-100	-150
Stock	500	250	100	-200	-600
C/D Account	60	60	60	60	60
Stock Option H	200	150	150	-200	-150

The Stock Option Alternative is dominated by the Bond Alternative

Sistem Pendukung Keputusan

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The Payoff Table

Decision	States of Nature				
Alternatives	Large Rise	Small Rise	No Change	Small Fall	Large Fall
Gold	-100	100	200	300	0
Company A	250	200	150	-100	-150
Company B	500	250	100	-200	-600
Company C	60	60	60	60	60
Company D	200	150	150	-200	-150

Decision	States of Nature				
Alternatives	Large Rise	Small Rise	No Change	Small Fall	Large Fall
Gold	-100	100	200	300	0
Company A	250	200	150	-100	-150
Company B	500	250	100	-200	-600
Company C	60	60	60	60	60
Company D	200	150	150	-200	-150

• The Company D alternative is dominated by the Company A alternative
• The Company D alternative can be dropped

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Decision Making Under Uncertainty

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Decision Making Under Uncertainty

- The decision criteria are based on the decision maker's attitude toward life.
- These include an individual being pessimistic or optimistic, conservative or aggressive.
- Criteria
  - Maximin Criterion - pessimistic or conservative approach.
  - Minimax Regret Criterion - pessimistic or conservative approach.
  - Maximax criterion - optimistic or aggressive approach.
  - Principle of Insufficient Reasoning.

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The Maximin Criterion

- This criterion is based on the worst-case scenario.
- It fits both a pessimistic and a conservative decision maker.
  - A pessimistic decision maker believes that the worst possible result will always occur.
  - A conservative decision maker wishes to ensure a guaranteed minimum possible payoff.

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The Maximin Criterion

- To find an optimal decision
  - Record the minimum payoff across all states of nature for each decision.
  - Identify the decision with the maximum "minimum payoff".

Decisions	The Maximin Criterion: Large Rise	Small rise	No Change	Small Fall	Large Fall	Minimum Payoff
Gold	-100	100	200	300	0	-100
Company A	250	200	150	-100	-150	-150
Company B	500	250	100	-200	-600	-600
Company C	60	60	60	60	60	60

Optimal Decision

The Minimax Regret Criterion

- This criterion fits both a pessimistic and a conservative decision maker.
- The payoff table is based on "lost opportunity," or "regret".
- The decision maker incurs regret by failing to choose the "best" decision.
- To find an optimal decision
  - For each state of nature.
    - Determine the best payoff over all decisions.
    - Calculate the regret for each decision alternative as the difference between its payoff value and this best payoff value.
  - For each decision find the maximum regret over all states of nature.
  - Select the decision alternative that has the minimum of these "maximum regrets".

The Minimax Regret Criterion

The Payoff Table

Decision	Large rise	Small rise	No change	Small fall	Large fall
Gold	-100	100	200	300	0
Company A	250	200	150	-100	-150
Company B	500	250	100	-200	-600
Company C	60	60	60	60	60

The Regret Table

Decision	Large rise	Small rise	No change	Small fall	Large fall
Gold	600	150	0	0	60
Company A	250	50	50	400	210
Company B	0	0	100	500	660
Company C	440	190	140	240	0

The Minimax Regret Criterion

The Payoff Table

Decision	Large rise	Small rise	No change	Small fall	Large fall
Gold	-100	[illegible]	[illegible]	[illegible]	[illegible]
Company A	250	[illegible]	[illegible]	[illegible]	[illegible]
Company B	500	[illegible]	[illegible]	[illegible]	[illegible]
Company C	60	60	60	[illegible]	60

Investing in gold generates a regret of 600 when the market exhibits a l... rise

500 – (-100) = 600

The optimal decision

The Regret Table

Decision	Large rise	Small rise	No change	Small fall	La...	Maximum Regret
Gold	600	150	0	0	60	600
Company A	250	50	50	400	210	400
Company B	0	0	100	500	660	660
Company C	440	190	140	240	0	440

The Maximax Criterion

- This criterion is based on the best possible scenario.
- It fits both an optimistic and an aggressive decision maker.
  - An optimistic decision maker believes that the best possible outcome will always take place regardless of the decision made.
  - An aggressive decision maker looks for the decision with the highest payoff (when payoff is profit)
- To find an optimal decision.
  - Find the maximum payoff for each decision alternative.
  - Select the decision alternative that has the maximum of the "maximum" payoff.

The Maximax Criterion

Decision	The Maximax Criterion: Large rise	Small rise	No change	Small fall	Large fall	Maximum Payoff
Gold	-100	100	200	300	0	300
Company A	250	200	150	-100	-150	250
Company B	500	250	100	-200	-600	500
Company C	60	60	60	60	60	60

Optimal Decision

The Principle of Insufficient Reason

- This criterion might appeal to a decision maker who is neither pessimistic nor optimistic.
- It assumes all the states of nature are equally likely to occur.
- The procedure to find an optimal decision.
  - For each decision add all the payoffs.
  - Select the decision with the largest sum (for profits).

The Principle of Insufficient Reason

- Sum of Payoffs
  - Gold 500
  - Company A 350
  - Company B 50
  - Company C 300
- Based on this criterion the optimal decision alternative is to invest in gold.

The Principle of Insufficient Reason

Payoff Table

	Large Rise	Small Rise	No Change	Small Fall	Large Fall
Gold	-100	100	200	300	0
Company A	250	200	150	-100	-150
Company B	500	250	100	-200	-600
Company C	60	60	60	60	60

RESULTS

Criteria	Decision	Payoff
Maximin	Company C	60
Minimax Regret	Company A	400
Maximax	Company B	500
Insufficient Reason	Gold	100

- Decision under uncertainty can present a problem if the attitude toward life (optimistic, pessimistic, or somewhere in between) change rapidly
- Solution: obtain probability estimates for the states of nature and implement decision making under risk.

Decision Making Under Risk

Decision Making Under Risk

- Probabilistic decision situation
- States of nature have probabilities of occurrence
- The probability estimate for the occurrence of each state of nature (if available) can be incorporated in the search for the optimal decision.
- For each decision calculate its expected payoff.

Expected Payoff = Σ(Probability)(Payoff)

- Select the decision with the best expected payoff

The Expected Value Criterion

- The expected value criterion is useful generally in the case where the decision maker is risk neutral.
- This criterion does not take into account the decision maker's attitude toward possible losses. We will see that utility theory offers an alternative to the expected value approach.
- When to Use the Expected Value Approach
  - The Expected Value Criterion is useful in cases where long run planning is appropriate, and decision situations repeat themselves.
  - One problem with this criterion is that it does not consider attitude toward possible losses.

The Expected Value Criterion

Optimal Decision

Decision	The Expected Value Criterion Large rise	Small rise	No change	Small fall	Large fall	Expected Value
Gold	-100	100	200	300	0	100
Company A	250	200	150	-100	-150	130
Company B	500	250	100	-200	-600	125
Company C	60	60	60	60	60	60
Prior Prob.	0.2	0.3	0.3	0.1	0.1	

EV(Company A) = (0.2)(250) + (0.3)(200) + (0.3)(150) + (0.1)(-100) + (0.1)(-150) = 130

Expected Return of The EV (EREV) = max (100, 130, 125, 60) = 130

Decision Making With Perfect Information

Expected Value of Perfect Information

- The Gain in Expected Return obtained from knowing with certainty the future state of nature is called:

Expected Value of Perfect Information

Therefore, the EVPI is the expected regret corresponding to the decision selected using the expected value criterion

- It is also the Smallest Expect Regret of any decision alternative.

Expected Value of Perfect Information

Decision	The Expected Value of Perfect Information Large rise	Small rise	No change	Small fall	Large fall
Gold	-100	100	200	300	0
Company A	250	200	150	-100	-150
Company B	500	250	100	-200	-600
Company C	60	60	60	60	60
Prob.	0.2	0.3	0.3	0.1	0.1

- If we know with certainty that the market were going to "Large Rise" (resp. small fall) the optimal decision would be to invest in Company B (Gold).
- Expected Return with Perfect information ERPI
  = (Probability of 1st state of nature)*(best outcome of 1st state of nature)
  +…+ (Probability of 5th state of nature)*(best outcome of 5th state of nature)
  = 0.2(500)+0.3(250)+0.3(200)+0.1(300)+0.1(60) = 271

EREV Expected Return of the EV criterion = 130

EVPI = ERPI - EREV = 271 - 130 = 141

Expected Value of Perfect Information

If Kasep knew in advance the Market would undergo	His optimal decision	With a gain of Payoff (with respect to risk case) of
a large rise	Company B	500-250=250
a small rise	Company B	250-200=50
no c hange	Gold	200-150=50
a small fall	Gold	300-(-100)=400
a large fall	Company C	60-(-150)=210

EVPI =.2(250)+.3(50)+.3(50)+.1(400)+.1(210) = 141

Decision Making With Perfect Imperfect Information

Decision Making with Imperfect Information (Bayesian Analysis)

- Some statisticians argue that is unnecessary to practice decision making under uncertainty coz one always has at least some probabilistic info related to the states of nature.
- Bayesian Statistics play a role in assessing additional information obtained from various sources.
- This additional information may assist in refining original probability estimates, and help improve decision making.

Ex: SI KASEP INVESTMENT DECISION (continued)

Should Kasep purchase the Forecast ?

- Kasep can purchase econometric forecast results for $50.
- The forecast predicts "negative" or "positive" econometric growth.
- Statistics regarding the forecast.

The Forecast predicted	When the Company showed a...				
	Large Rise	Small Rise	No Change	Small Fall	Large Fall
Positive econ. growth	80%	70%	50%	40%	0%
Negative econ. growth	20%	30%	50%	60%	100%

When the Company B showed a large rise the Forecast predicted a "positive growth" 80% of the time.

P(forecast predicts "positive " | small rise in market) = .7
P(forecast predicts "negative" | small rise in market) = .3

Solution

- Kasep should determine his optimal decisions when the forecast is "positive" and "negative".
- If his decisions change because of the forecast, he should compare the expected payoff with and without the forecast.
- If the expected gain resulting from the decisions made <u>*with*</u> the forecast *exceeds $50*, he should purchase the forecast.

- Kasep needs to know the following probabilities
  - P(Large rise | The forecast predicted "Positive")
  - P(Small rise | The forecast predicted "Positive")
  - P(No change | The forecast predicted "Positive ")
  - P(Small fall | The forecast predicted "Positive")
  - P(Large Fall | The forecast predicted "Positive")
  - P(Large rise | The forecast predicted "Negative ")
  - P(Small rise | The forecast predicted "Negative")
  - P(No change | The forecast predicted "Negative")
  - P(Small fall | The forecast predicted "Negative")
  - P(Large Fall) | The forecast predicted "Negative")
- Bayes' Theorem provides a procedure to calculate these probabilities

Bayes Theorem

$$P(A|B)=\frac{P(B|A)P(A)}{P(B)}$$

<u>Proof :</u> P(A | B)=P(A and B)/P(B) (1)

P(B | A)=P(A and B)/P(A)
➔ P(A and B) =P(B | A)*P(A)

(1) ➔ P(A | B)=P(B | A)*P(A) / P(B)

Bayes, Thomas (1763) An essay towards solving a problem in the doctrine of chances. *Philosophical Transactions of the Royal Society of London,* **53:370-418**

Bayes Theorem

- Often we begin probability analysis with initial or prior probabilities.
- Then, from a sample, special report, or a product test we obtain some additional information.
- Given this information, we calculate revised or posterior probabilities.
- Bayes' theorem provides the means for revising the prior probabilities.

Prior Probabilities → New Information → Application of Bayes' Theorem → Posterior Probabilities

Bayes Theorem

- Knowledge of sample (survey) information can be used to revise the probability estimates for the states of nature.
- Prior to obtaining this information, the probability estimates for the states of nature are called prior probabilities.
- With knowledge of conditional probabilities for the indicators of the sample or survey information, these prior probabilities can be revised by employing Bayes' Theorem.
- The outcomes of this analysis are called posterior probabilities or branch probabilities for decision trees.

Bayes Theorem

$A_1, A_2, \ldots, A_n$ are mutually exclusive and collectively exhaustive (e.g., events)

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \ldots + P(B|A_n)P(A_n)}$$

Posterior Probabilities
Probabilities determined after the additional info becomes available.

Prior probabilities
Probability estimates determined based on current info, before the new info becomes available.

Ex:

- The tabular approach to calculating posterior probabilities for "positive" economical forecast

Case 1: Forecast predicts "Positive"						
States of Nature	Prior Prob.		Prob. (Positive|State)		Joint Prob.	Posterior Prob.
Large Rise	0.2		0.8		0.16	0.286
Small Rise	0.3	X	0.7	=	0.21	0.375
No Change	0.3		0.5		0.15	0.268
Small Fall	0.1		0.4		0.04	0.071
Large Fall	0.1		0		0	0.000
			Sum	=	0.56	

$\frac{0.16}{0.56}$

Revision of state of nature probability

P(large rise | Forecast = positive)

Probability(Forecast = positive) = .16 + .21 + .15 + .04 +.0 = .56

Ex:

- The tabular approach to calculating posterior probabilities for "positive" economical forecast

Case 1: Forecast predicts "Positive"						
States of Nature	Prior Prob.		Prob. (Positive|State)		Joint Prob.	Posterior Prob.
Large Rise	0.2		0.8		0.16	0.286
Small Rise	0.3	X	0.7	=	0.21	0.375
No Change	0.3		0.5		0.15	0.268
Small Fall	0.1		0.4		0.04	0.071
Large Fall	0.1		0		0	0.000
			Sum	=	0.56	

$\frac{0.16}{0.56}$

Revision of state of nature probability

P(large rise | Forecast = positive)

Probability(Forecast = positive) = .16 + .21 + .15 + .04 +.0 = .56

Ex:

Bayesian Analysis

Indicator 1 Positive Eonomical Forecast

States of Nature	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Posterior Probabilites
Large Rise	0.2	0.8	0.16	0.286
Small Rise	0.3	0.7	0.21	0.375
No Change	0.3	0.5	0.15	0.268
Small Fall	0.1	0.4	0.04	0.071
Large Fall	0.1	0	0	0.000
s6	0		0	0.000
s7	0		0	0.000
s8	0		0	0.000
		P(Indicator 1)	0.56	

Indicator 2 Negative Economical Forecast

States of Nature	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Posterior Probabilites
Large Rise	0.2	0.2	0.04	0.091
Small Rise	0.3	0.3	0.09	0.205
No Change	0.3	0.5	0.15	0.341
Small Fall	0.1	0.6	0.06	0.136
Large Fall	0.1	1	0.1	0.227
s6	0		0	0.000
s7	0		0	0.000
s8	0		0	0.000
		P(Indicator 2)	0.44	

Decision	The revised probabilities payoff table				
	Large rise	Small rise	No change	Small fall	Large fall
Gold	-100	100	200	300	0
Company A	250	200	150	-100	-150
Company B	500	250	100	-200	-600
Company C	60	60	60	60	60
P(State|Positive)	0.286	0.375	0.268	0.071	0
P(State|Negative)	0.091	0.205	0.341	0.136	0.227

Revision of state of nature probability

Expected Value of Sample Information

- The expected gain from making decisions based on Sample Information.
- With the forecast available, the Expected Value of Return is revised.
- Calculate Revised Expected Values for a given forecast as follows.

EV(Invest in Comp A |"Positive" forecast) =

=.286(250)+.375(200)+.268(150)+.071(-100)+0(-150)

= $180

EV(Invest in Comp A | "Negative" forecast) =

=.091(250)+.205(200)+.341(150)+.136(-100)+.227(-150)

= $ 65

The Reversed Expected Value

Positive forecast	Negative forecast
EV(Gold | Positive) = 84	EV(Gold | Negative) = 120
EV(C A | Positive) = 180	EV(C A | Negative) = 65
EV(C B | Positive) = 250	EV(C B | Negative) = -37
EV(C C | Positive) = 60	EV(C C | Negative) = 60
If the forecast is "Positive" Invest in Company B.	If the forecast is "Negative" Invest in Gold.

EREV = Expected Value Without Sampling Information = 130 a

similar manner.

Expected Value of Sample Information

Decisio	large	small	no ch	small	large	Prior EV	Revised EV Pos	Neg
Gold	-100	100	200	300	0	100	84	120
Company A	250	200	150	-100	-150	130	180	65
Company B	500	250	100	-200	-600	125	250	-37
Company C	60	60	60	60	60	60	60	60
Pr								
Po								
Ne								0.44

So,

Should Kasep purchase the Forecast ?

ERSI = Expected Return with sample Information =

(0.56)(250) + (0.44)(120) = $193

e"

- EVSI = Expected Value of Sampling Information

  = ERSI - EREV = 193 - 130 = $63.

Yes, Kasep should purchase the Forecast.

His expected return is greater than the Forecast cost.($63>$50)

- Efficiency = EVSI / EVPI = 63 / 141 = 0.45

EVPI = ERPI – EREV

EVSI = ERSI – EREV

Efficiency = EVSI / EVPI

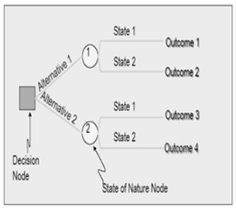
Decision Tree

Decision Trees

- The Payoff Table (Decision Table) approach is useful for a non-sequential or single stage.
- Decision Tree is useful in analyzing multi-stage decision problems consisting of a sequence of dependent decisions.
- A Decision Tree is a chronological representation of the decision process.

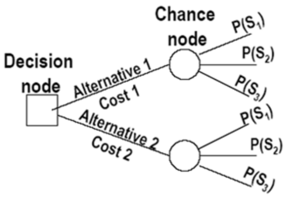
Decision Trees

- A decision tree is a chronological representation of the decision problem.
- Each decision tree has two types of nodes; round nodes correspond to the states of nature while square nodes correspond to the decision alternatives.
- The branches leaving each round node represent the different states of nature while the branches leaving each square node represent the different decision alternatives.
- At the end of each limb of a tree are the payoffs attained from the series of branches making up that limb.



Decision Tree

- The tree is composed of nodes and branches.



A branch emanating from a **decision node** corresponds to a decision alternative. It includes a cost or benefit value.

A branch emanating from a **state of nature (chance)** node corresponds to a particular state of nature, and includes the probability of this state of nature.

Motivating Case:

Ms. Brooks is a 50 year old woman with an incidental cerebral aneurysm. She presented with new vertigo 3 weeks ago and her primary MD ordered a head MRI. Her vertigo has subsequently resolved and has been attributed to labyrinthitis.

Her MRI suggested a left posterior communicating artery aneurysm, and a catheter angiogram confirmed a 6 mm berry aneurysm.

Case Presentation (cont'd)

Past medical history is remarkable only for 35 pack-years of cigarette smoking.
Exam is normal.
Ms. Brooks: "I don't want to die before my time."

Question is: Do we recommend surgical clipping of the aneurysm or no treatment?

Alternative ways of dealing with uncertainty

- Dogmatism. *All aneurysms should be surgically clipped.*
- Policy. *At UCSF we clip all aneurysms.*
- Experience. *I've referred a number of aneurysm patients for surgery and they have done well.*
- Whim. *Let's clip this one.*
- Nihilism. *It really doesn't matter.*
- Defer to experts. *Vascular neurosurgeons say clip.*
- Defer to patients. *Would you rather have surgery or live with your aneurysm untreated?*

OR

Decision Analysis

Overview of DA Steps

1. Formulate an explicit question
2. Make a decision tree.
   (squares = decision nodes, circles = chance nodes)
   a) Alternative actions = branches of the decision node.
   b) Possible outcomes of each = branches of chance nodes.
3. Estimate probabilities of outcomes at each chance node.
4. Estimate utilities = numerical preference for outcomes.
5. Compute the expected utility of each possible action
6. Perform sensitivity analysis

1. FORMULATE AN EXPLICIT QUESTION

- Formulate explicit, answerable question.
- May require modification as analysis progresses.
- The simpler the question, without losing important detail, the easier and better the decision analysis.

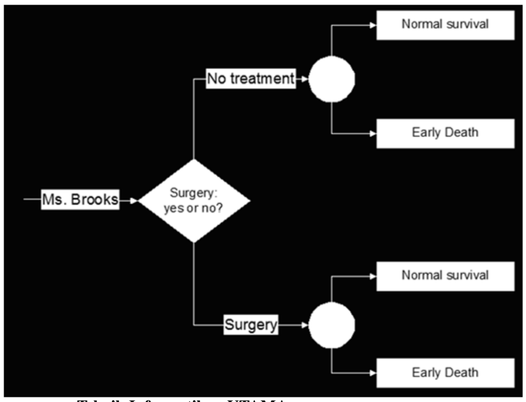
In the aneurysm example, our interest is in determining what's best for Ms. Brooks so we'll take her perspective. We will begin with the following question:

Which treatment strategy, surgical clipping or no treatment, is better for Ms. Brooks considering her primary concern about living a normal life span?

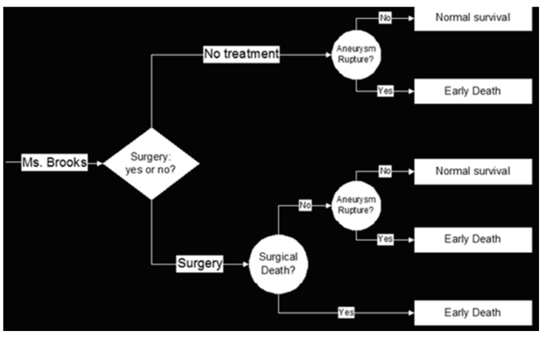
2. MAKE A DECISION TREE

- Creating a decision tree = *structuring the problem*
- Provide a reasonably complete depiction of the problem.
- Best is one decision node (on the left, at the beginning of the tree).
- Branches of each chance node -- *exhaustive* and *mutually exclusive*.
- Proceed incrementally. Begin simple.

Decision Trees: Simple to …



… to Less Simple…



…to Complex

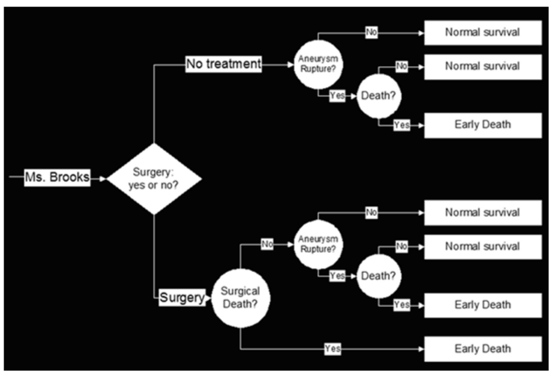
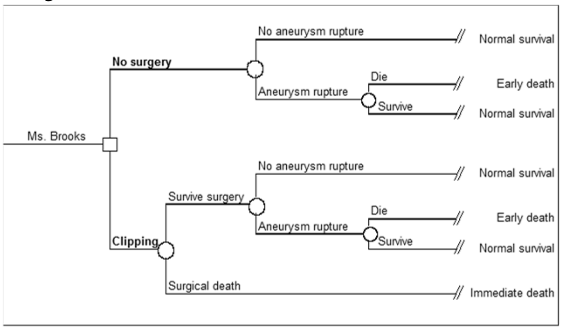


Figure 1



3. ESTIMATE PROBABILITIES

- From the most reliable results applicable to the patient or scenario of interest.
- Standard hierarchies of data quality

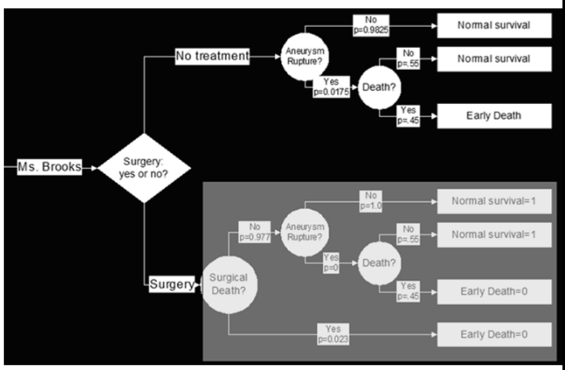
  Definitive trials → Meta-analysis of trials → Systematic review → Smaller trials → Large cohort studies → Small cohort studies → Case-control studies → Case series → Expert opinion

3. Fill in the probabilities: No treatment node

- Prob rupture =exp life span x rupture/yr
  - Expected life span:
- From US mortality figures: 35 years
  - Probability of untreated aneurysm rupture.
  - Cohort study
    - 0.05%/yr for <10 mm
  - Lifetime prob rupture = 0.05%/y x 35 y = 1.75%

- Case fatality of rupture
  - Meta-analysis: 45%

3. Fill in the probabilities

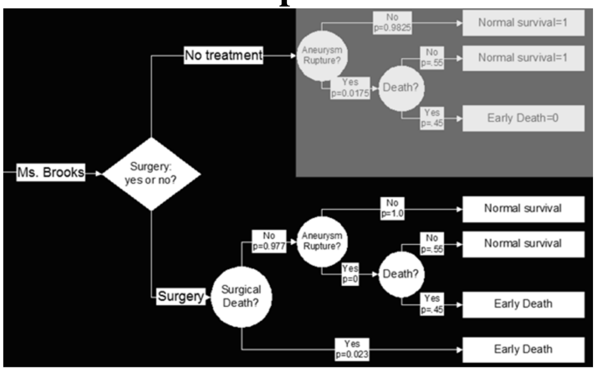


3. Fill in the probabilities: Surgery node

- Probability of treated aneurysm
  – rupture.
  – No data: probably very small ~ 0 (Opinion)

- Surgical mortality. Options:
- Meta-analysis of case series: 2.6%
  - Clinical databases: 2.3%
  - The numbers at UCSF: 2.3%

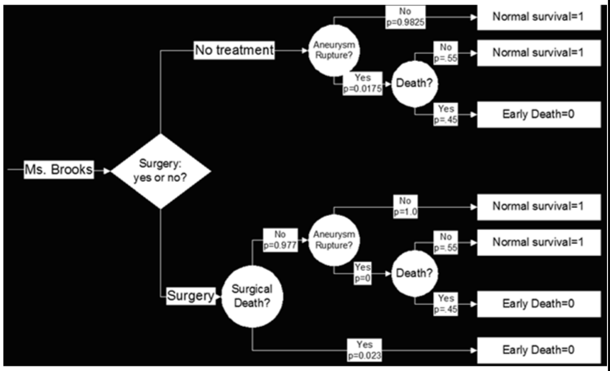
3. Fill in the probabilities



4. Estimate utilities

- Valuation of an outcome (more restrictive use in the next lecture).
- Best = 1
- Worst = 0
- In this case, she wants to avoid early death:
  - Normal survival = 1
  - Early death = 0

4. Fill in the utilities



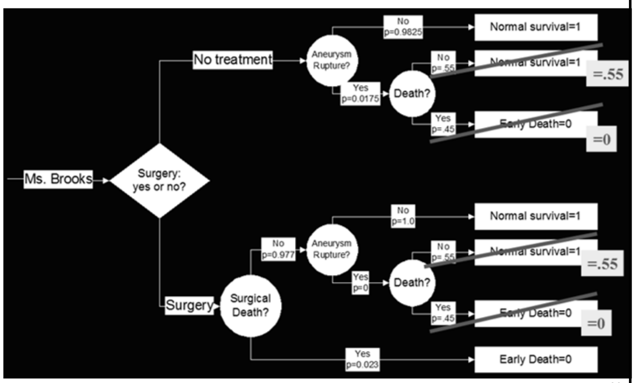
5. COMPUTE THE EXPECTED UTILITY OF EACH BRANCH

Called "folding back" the tree.

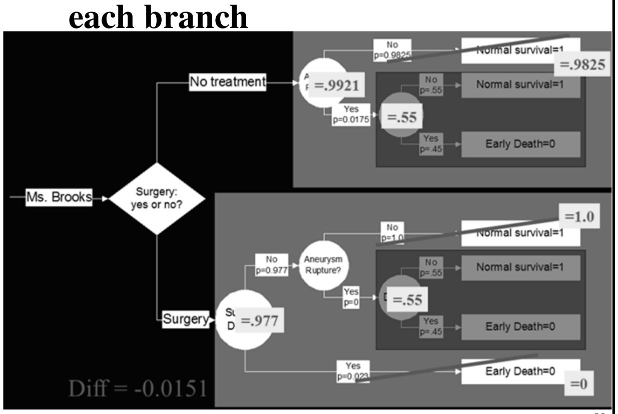
Expected utility of action = each possible outcome weighted by its probability.

Simple arithmetic calculations

5. Compute expected utility of each branch



5. Compute expected utility of each branch

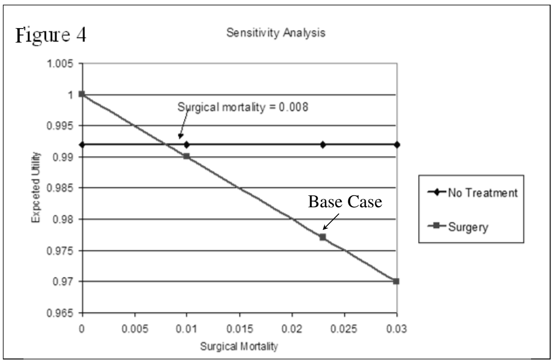


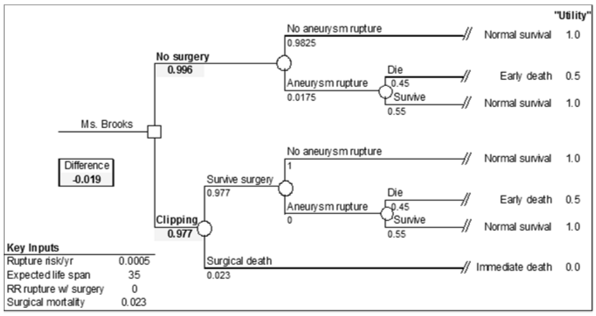
.865 vs .977

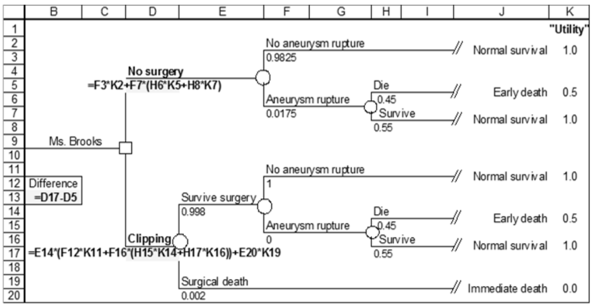
6. Perform sensitivity analysis

- How certain are we of our recommendation?
- Change the input parameters to see how they affect the final result.
  - What if her life expectancy were shorter?
  - What if the rupture rate of untreated aneurysms were higher?
  - How good a neurosurgeon is required for a toss up?

Point at which the two lines cross =
***treatment threshold*.**







STEP BACK AND REVIEW THE ANALYSIS

As each iteration is completed, step back …
Have we answered the question?
Did we ask the right question?
Are there other details that might be important?
Consider adding complexity to improve accuracy.

Ms. Brooks

We recommend NO surgery.

- "Thanks… But I meant I wanted to live the most years possible. Dying at age 80 isn't as bad as dying tomorrow…"

Improve the Analysis

Add layers of complexity to produce a more realistic analysis.

Game Theory

97

Game Theory

- Game theory can be used to determine optimal decision in face of other decision making players.
- All the players are seeking to maximize their return.
- The payoff is based on the actions taken by all the decision making players.

Classification of Games

- Number of Players
  - Two players - Chess
  - Multiplayer - More than two competitors (Poker)
- Total return
  - Zero Sum - The amount won and amount lost by all competitors are equal (Poker among friends)
  - Nonzero Sum -The amount won and the amount lost by all competitors are not equal (Poker In A Casino)
- Sequence of Moves
  - Sequential - Each player gets a play in a given sequence.
  - Simultaneous - All players play simultaneously.

IGA SUPERMARKET

- The town of Gold Beach is served by two supermarkets: IGA and Sentry.
- Market share can be influenced by their advertising policies.
- The manager of each supermarket must decide weekly which area of operations to discount and emphasize in the store's newspaper flyer.

Data

- The weekly percentage gain in market share for IGA, as a function of advertising emphasis.

		Sentry's Emphasis			
		Meat	Produce	Grocery	Bakery
IGA's Emphasis	Meat	2	2	-8	6
	Produce	-2	0	6	-4
	Grocery	2	-7	1	-3

- A gain in market share to IGA results in equivalent loss for Sentry, and vice versa (i.e. a zero sum game)

IGA needs to determine an advertising emphasis that will maximize its expected change in market share *regardless* of Sentry's action.

SOLUTION

- IGA's Perspective - A Linear Programming model
  - Decision variables
    - X1 = the probability IGA's advertising focus is on meat.
    - X 2 = the probability IGA's advertising focus is on produce.
    - X 3 = the probability IGA's advertising focus is on groceries.
  - Objective Function For IGA
    - Maximize expected market change (in its own favor) regardless of Sentry's advertising policy.
    - Define the actual change in market share as V.

- Constraints
  - IGA's market share increase for any given advertising focus selected by Sentry, must be at least V.
- The Model

Maximize V

ST

		IGA expected change in market share				
Sentry's	**Meat**	2X1	- 2X2	+ 2X3	≥	V
Advertising	**Produce**	2X1		- 7X3	≥	V
Emphasis	**Groceries**	(-8X1)	+6X2	+ X3	≥	V
	Bakery	6X1	- 4X2	- 3X3	≥	V
The variables are probabilities		X1	+ X2	+ X3	=	1

X1, X2, X3, are non negative: V is unrestricted

Sentry's Perspective - A Linear Programming model

- Decision variables
  - Y1 = the probability that Sentry's advertising focus is on meat.
  - Y2 = the probability that Sentry's advertising focus is on produce.
  - Y3 = the probability that Sentry's advertising focus is on groceries.
  - Y4 = the probability that Sentry's advertising focus is on bakery.
- Objective function
  - Minimize changes in market share in favor of IGA
- Constraints
  - Sentry's market share decrease for any given advertising focus selected by IGA, must not exceed V.

- The Model

Minimize V

ST

		Sentry's expected change in market share					
IGA's	**Meat**	2Y1	+ 2Y2	- 8Y3	+ 6Y4	≤	V
Advertising	**Produce**	-2Y1		+ 6Y3	- 4Y4	≤	V
Emphasis	**Groceries**	2Y1	- 7Y2	+ Y3	- 3Y4	≤	V
		Y1	+ Y2	+ Y3	+ Y4	=	1

Y1, Y2, Y3, Y4 are non negative; V is unrestricted

Optimal Solution

- For IGA
  - X1 = 0.3889; X2 = 0.5; X3 = 0.111
- For Sentry
  - Y1 = 0.6; Y2 = 0.2; Y3 = 0.2; Y4 = 0
- For both players V =0 (a fair game).

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