

Problem Solving in Mathematics

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Mathematics, Learning &
Technology

Problem Solving

1

Definitions
Just what is problem solving?

2

Ask the students

3

Teaching Ideas

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Making it stick
The importance of recall

6

Further Resources

Problem Solving

What's a problem anyway?

Assessment Objectives

AO1

Higher
40%

Found
50%

Use and apply standard techniques

Students should be able to:

- accurately recall facts, terminology and definitions
- use and interpret notation correctly
- accurately carry out routine procedures or set tasks requiring multi-step solutions

Images & Text Hyperlinks on
this & all remaining slides

[DfE GCSE Maths](#)
[Assessment Objectives](#)

AO2Higher
30%Found
25%**Reason, interpret and communicate mathematically**

Students should be able to:

- make deductions, inferences and draw conclusions from mathematical information
- construct chains of reasoning to achieve a given result
- interpret and communicate information accurately
- present arguments and proofs
- assess the validity of an argument and critically evaluate a given way of presenting information

Where problems require candidates to 'use and apply standard techniques' or to independently 'solve problems' a proportion of those marks should be attributed to the corresponding Assessment Objective

 **AO3**Higher
30%Found
25%**Solve problems within mathematics and in other contexts**

Students should be able to:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
- make and use connections between different parts of mathematics
- interpret results in the context of the given problem
- evaluate methods used and results obtained
- evaluate solutions to identify how they may have been affected by assumptions made

Where problems require candidates to 'use and apply standard techniques' or to 'reason, interpret and communicate mathematically' a proportion of those marks should be attributed to the corresponding Assessment Objective.

What's a problem anyway?

Polya (1945 & 1962) described mathematical problem solving as finding a way around a difficulty and finding a solution to a problem that is unknown.

What's a problem anyway?

Working out what to do, when you don't know what to do...

Hiebert et al:

“A mathematical problem solving task must be problematic for a student to be viewed as legitimate mathematical problem solving.”

What's a problem anyway?

A Level Mathematics Working Group

Tasks have little or no scaffolding: there is little guidance given to the candidate beyond a start point and a finish point. Questions do not explicitly state the mathematical process(es) required for the solution.

Tasks provide for multiple representations, such as the use of a sketch or a diagram as well as calculations.

The information is not given in mathematical form or in mathematical language; or there is a need for the results to be interpreted or methods evaluated, for example, in a real-world context.

[A Level Mathematics Working Group Report](#)

What's a problem anyway?

A Level Mathematics Working Group

Tasks have a variety of techniques that could be used.

The solution requires understanding of the processes involved rather than just application of the techniques.

The task requires two or more mathematical processes or may require different parts of mathematics to be brought together to reach a solution

A Level Mathematics - Overarching themes

OT2 Mathematical problem solving

	Knowledge/Skill
OT2.1	[Recognise the underlying mathematical structure in a situation and simplify and abstract appropriately to enable problems to be solved]
OT2.2	[Construct extended arguments to solve problems presented in an unstructured form, including problems in context]
OT2.3	[Interpret and communicate solutions in the context of the original problem]
OT2.4	Understand that many mathematical problems cannot be solved analytically, but numerical methods permit solution to a required level of accuracy
OT2.5	[Evaluate, including by making reasoned estimates, the accuracy or limitations of solutions] , including those obtained using numerical methods
OT2.6	[Understand the concept of a mathematical problem solving cycle, including specifying the problem, collecting information, processing and representing information and interpreting results, which may identify the need to repeat the cycle]
OT2.7	[Understand, interpret and extract information from diagrams and construct mathematical diagrams to solve problems, including in mechanics]

Ask the students

*Students
on good
teachers*



Good problem solvers need good teachers. Students on good teachers.

Good at explanations and lecturing.

Someone who can explain in different ways.

Someone who won't just tell you how to do something, but will explain how and why it works.

Provokes your mind to think beyond the syllabus



Students on good teachers

A teacher who provides the student with the opportunity to see what they need to revise. Regular tests and quizzes do this.

Doesn't mind repeating things.

Pushes you to work on harder questions to extend your abilities.



Students on good teachers

Patient.

Understanding.

Approachable.

Firm but kind.

Someone you can feel comfortable with.

Recognises achievements.

Genuinely caring about the students.

Someone who knows who you are.

Someone who you know won't judge you.

Good teachers ...



Teaching Ideas

For
problem
solving in
the GCSE
classroom

Developing a problem solving classroom

Teacher / Student relationships?

What sort of questions do you ask? Open, closed?

Are students comfortable to ask & answer questions?

How do you respond to student answers? Acknowledgement?

Are problems a regular part of your lessons? From KS3?

Written into schemes of work?

Do students have thinking time?

Time to play and experiment with problems?

Is it OK to be stuck? Do your students persevere?

Are your students confident? Determined?

Do you model problem solving techniques?

Problem Solving

Understand the problem.

Do your students understand all the words used in stating the problem? Can they restate the problem in their own words?

Do something!

Polya mentions that there are many reasonable ways to solve problems. The skill at choosing an appropriate strategy is best learned by **solving many problems**.

Draw a diagram

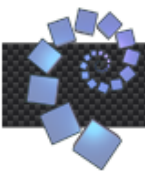
Would a picture or diagram help to understand the problem?

To Try

Guess and check. Look for a pattern. Make an orderly list.

Solve a simpler problem. Consider special cases.

Work backwards. Be ingenious.



Thinking Mathematically

Developing Mathematical Habits of Mind

We learn better when we are curious, thoughtful, determined and collaborative.

Here are some collections of mathematical activities designed to help you develop these characteristics.



Being Curious

Here are some problems that we hope will appeal to curious and inquisitive students. Take a look, we think you'll get hooked on them!



Being Thoughtful

Here are some problems that require careful consideration. Immerse yourself in them - we think they are worth the effort!



Being Collaborative

Here are some problems that are ideal for working on with others. Find a friend, share ideas, and see if two heads really are better than one!



Being Determined

Here are some problems that require you to be determined and persistent. We hope you'll stick with them and feel a sense of achievement at the end!

Elevenses



In this grid, how many pairs of numbers can you find that add up to a multiple of 11?

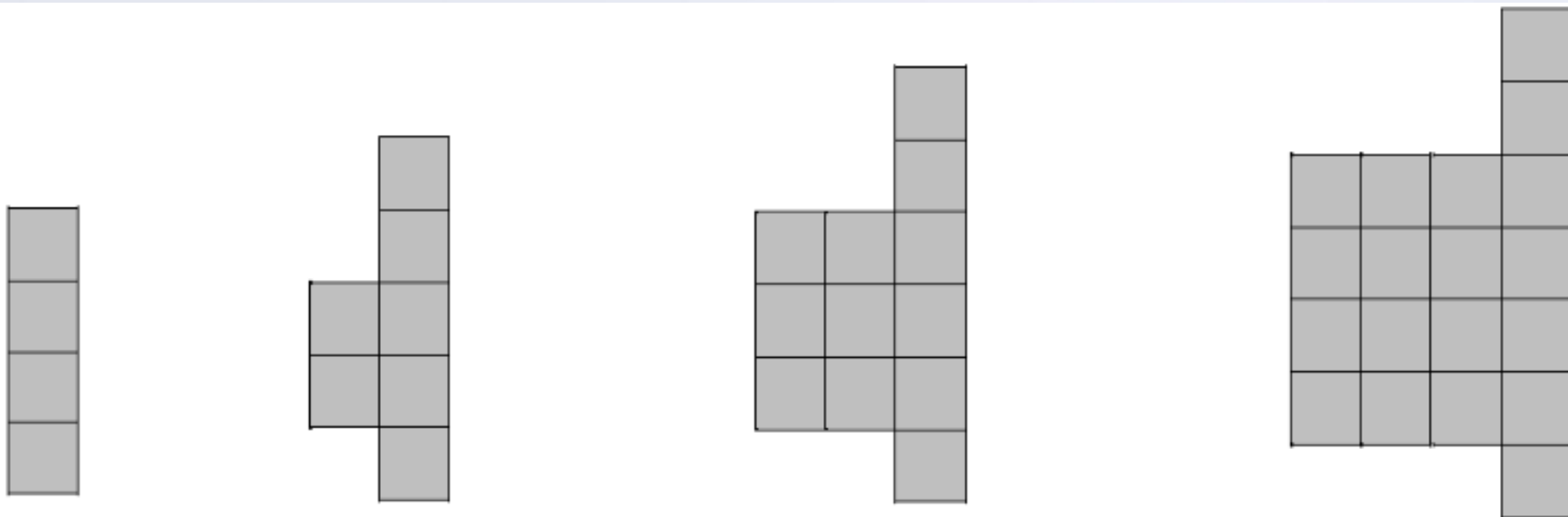
Could you convince someone that you've found them all?

9	46	79	13
64	90	2	97
25	31	20	22
4	52	55	7



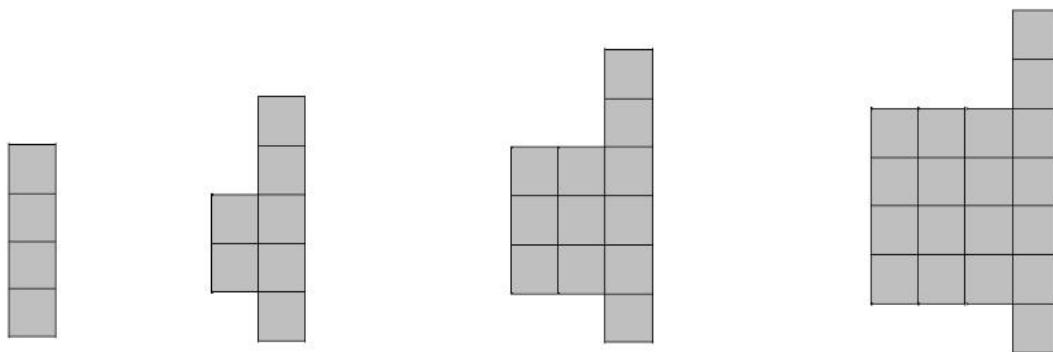
Being Determined

nrich.maths.org



What do you think the question might be?

The diagram shows the first four patterns of a sequence.



1

4

2

7

3

12

4

19

Find an expression for the number of squares in the n th pattern of the sequence. [2]

n	1	2	3	4
	4	7	12	19

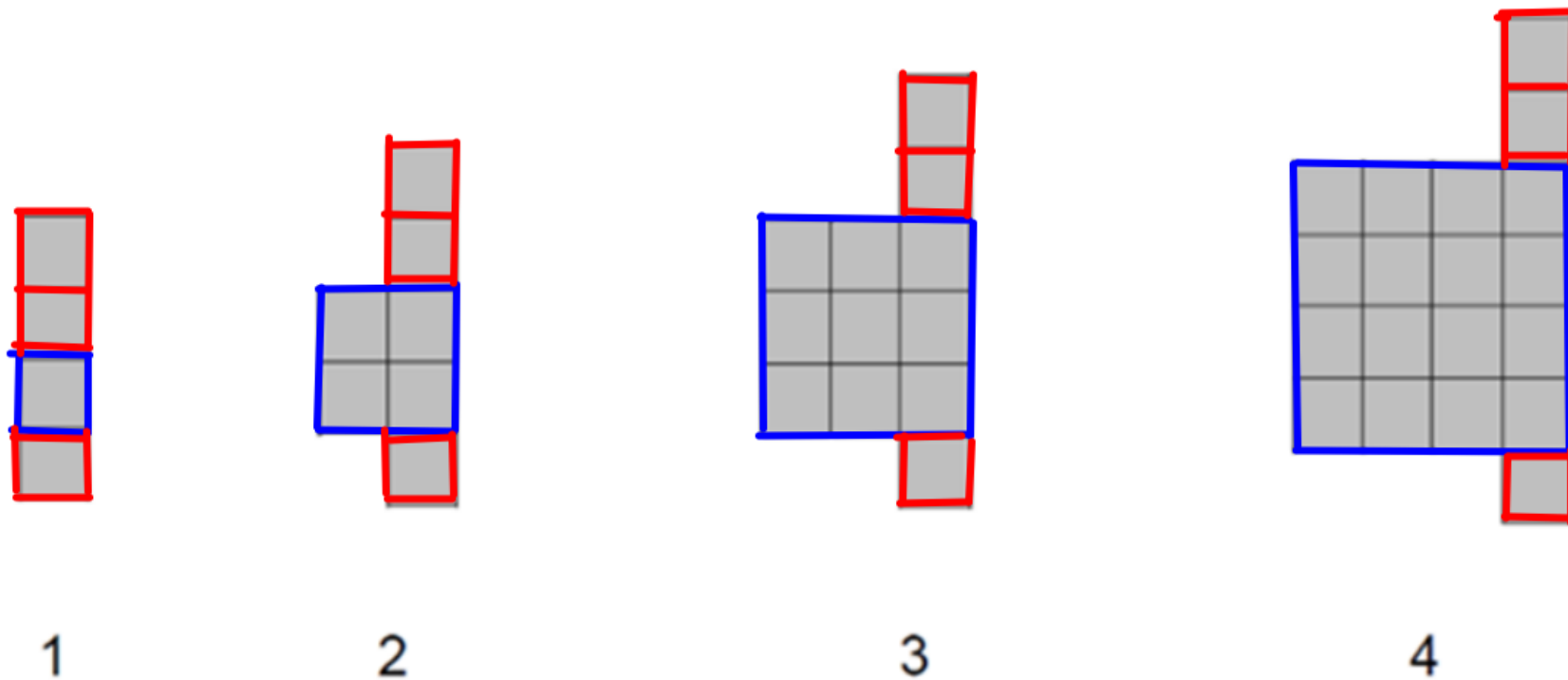
$4 \rightarrow 7$ difference 3
 $7 \rightarrow 12$ difference 5
 $12 \rightarrow 19$ difference 7
 $3 \rightarrow 5$ difference 2
 $5 \rightarrow 7$ difference 2

1st. difference is not
the same so its
quadratic.

n^2	1	4	9	16
	+3	+3	+3	+3

$n^2 + 3$

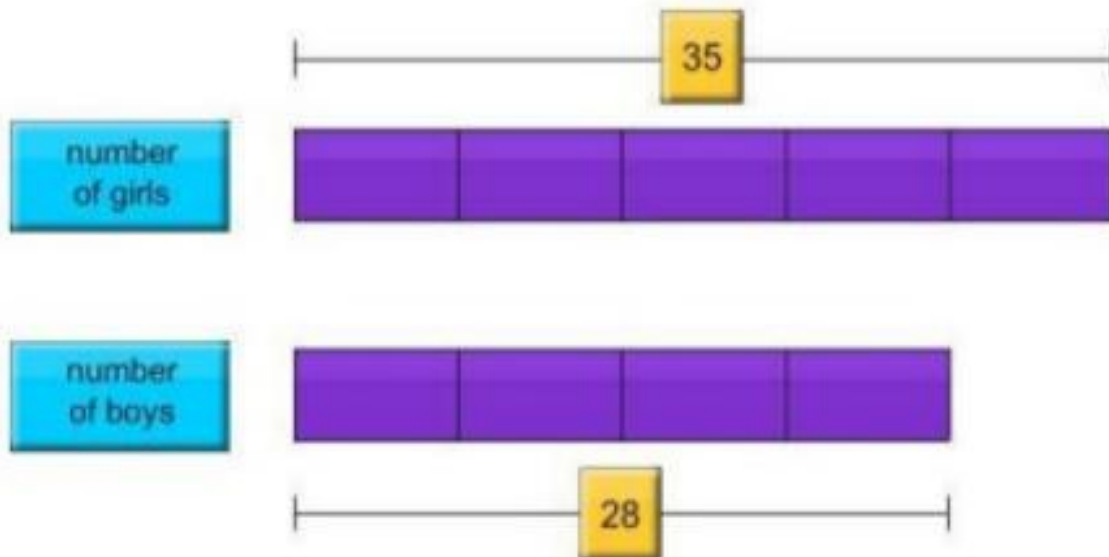
Or less formally



This is from a Year 7 student.

Do we sometimes get too formal too quickly?

Word Problem: The ratio of girls to boys who participated in the essay writing contest was 5:4. There were 35 girls. How many boys participated?

Thinking
Blocks

ANSWER: boys

Way
to Go!

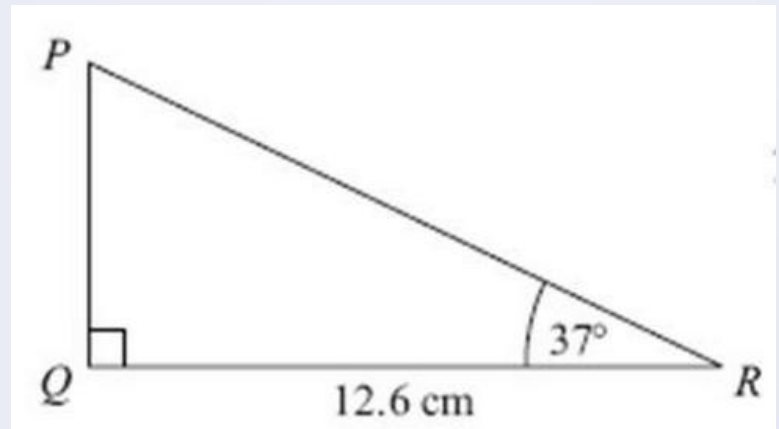
Next

Try diagrams to model problems

Here's the diagram..
... and ask "what's the question"

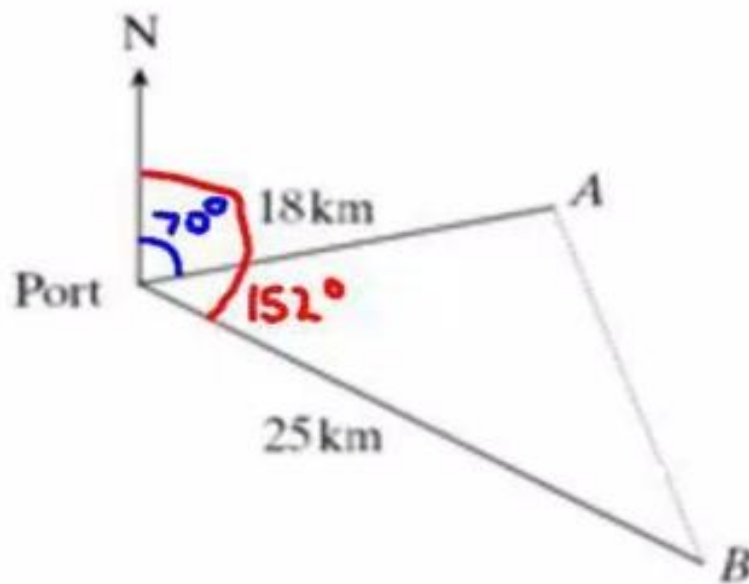
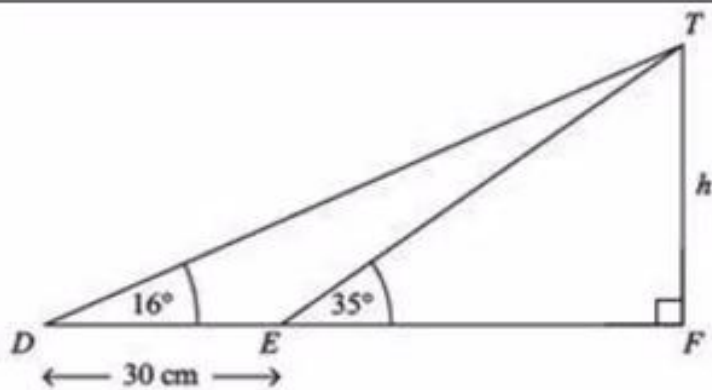
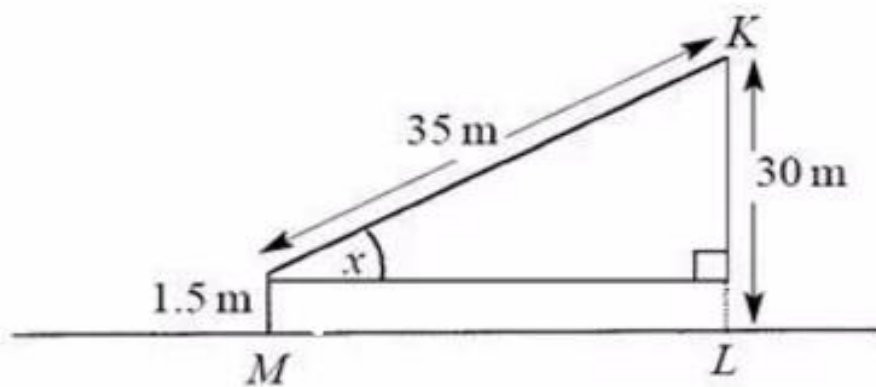
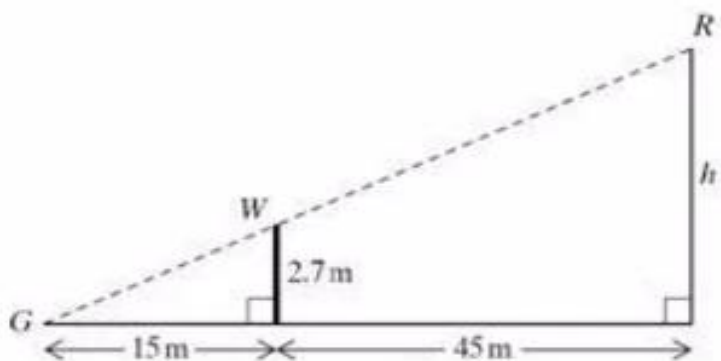
Try, here's the diagram what's the question?

Or try Algebra Snippets

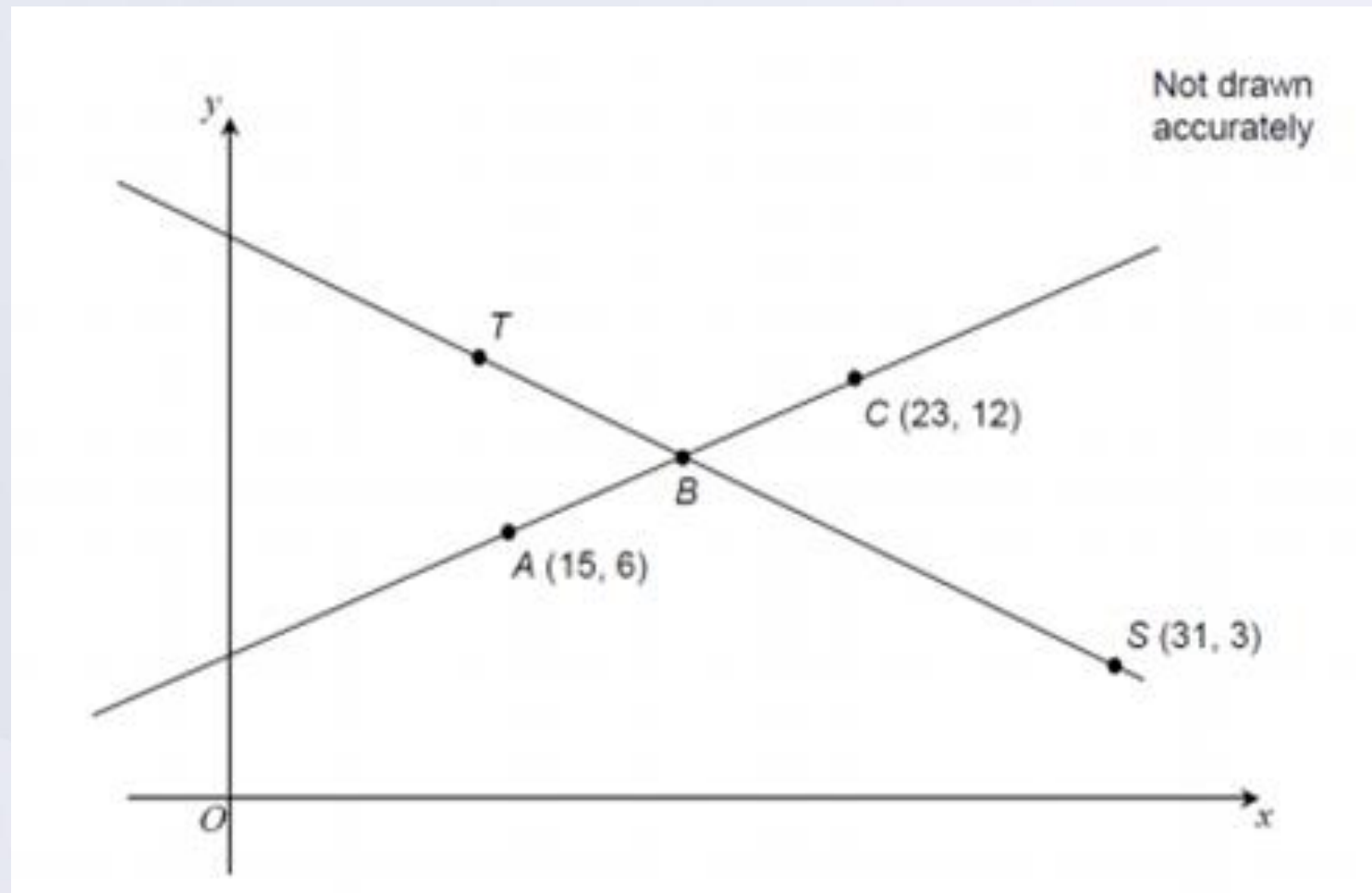


$$c = \frac{10(t - d)}{d}$$

What might the question be?



Here's the diagram....



What could you work out?

What else could we work out if we had more information?

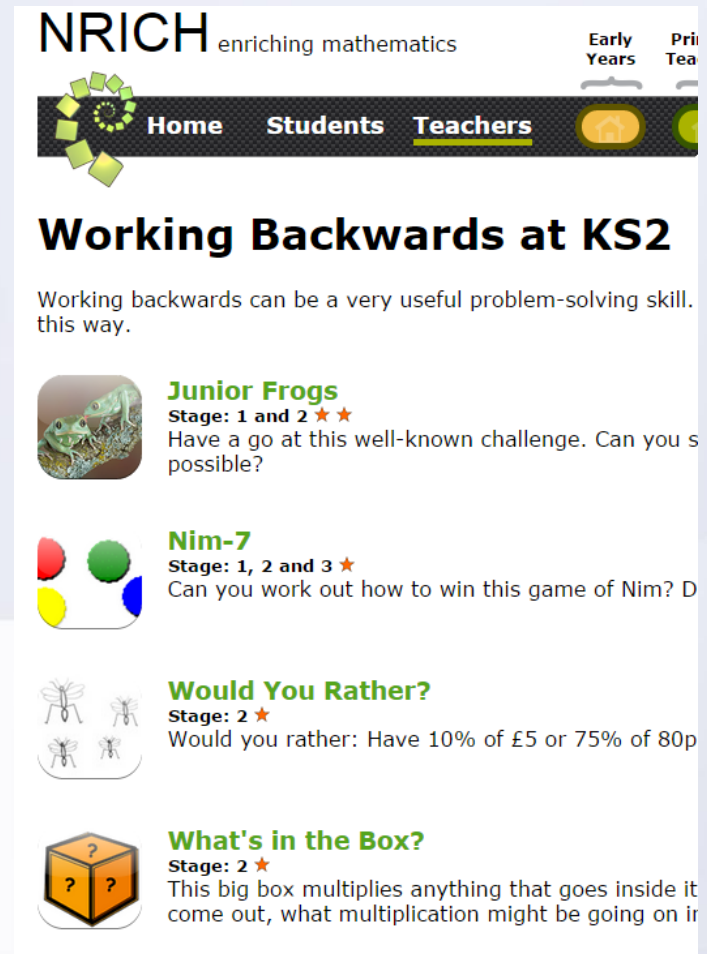
Help students with vocabulary

What we say...	What we mean...
Estimate	Round numbers to 1 significant figure and use these to obtain an answer.
Explain	Use words to explain an answer.
You must show your working	You will be penalised if you do not show your working.
Simplify	Collect terms together.
Simplify fully	Collect terms together and factorise the answer.
Show that	Use words, numbers or algebra to show an answer.

Good mathematicians can go backwards!
From Nrich...

Working backwards
at KS2; the ideas
here could also be
used at KS3.

See this Nrich article
from Liz Woodham
on Developing
Problem-solving skills
which includes the
link above



SMART Board



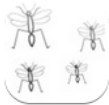

NRICH enriching mathematics

Early Years Pri Tea

Home Students Teachers

Working Backwards at KS2

Working backwards can be a very useful problem-solving skill. this way.

-  **Junior Frogs**
Stage: 1 and 2 ★★
Have a go at this well-known challenge. Can you solve it?
possible?
-  **Nim-7**
Stage: 1, 2 and 3 ★
Can you work out how to win this game of Nim? D
-  **Would You Rather?**
Stage: 2 ★
Would you rather: Have 10% of £5 or 75% of 80p
-  **What's in the Box?**
Stage: 2 ★
This big box multiplies anything that goes inside it
come out, what multiplication might be going on in

Arithmagons

Perfect for any topic
for thinking
backwards.

[See the ideas and resources here](#) -
everything from
simple arithmetic to
Calculus!

Click the tiles to hide or show the missing values.

The value in each square is equal to the **sum** of the circles on either side of it.

Decimals?
Negatives?

New numbers?

© flashmaths.co.uk

[Arithmagons from Flash Maths](#)

Problems

See for example the [AQA problems](#) included here.

This type of [backwards problem](#) really helps students think deeply.

Meanset

A set of five numbers has:

a mode of 12

a median of 11

a mean of 10

What could the numbers be?

Strategy Identification	Item	Tier
Set out cases	Cube	Foundation
	Crate	Either
	Dice	Higher
	Repeater	Higher
Work back - familiar	Apples	Foundation (easiest)
	Stationery	Foundation
	Boat Hire	Foundation
	Half Take	Either
	Hotel	Either
	Flight cost	Higher

GCSE Mathematics: 90 maths problem solving questions

The new Maths GCSE has an increased focus on problem solving. So that you can help your students practice this type of question, we've refreshed our *90 maths problems* resource so that it's relevant to the new GCSE.

Visit All About Maths aqa.maths.aqa.org.uk our free maths resource website to access other resources and guidance.

Great support
+ Great resources
AQA Maths GCSE

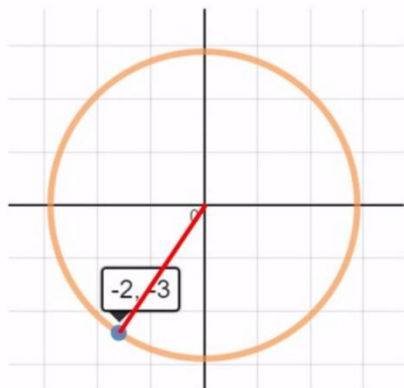
Content area	Item	Tier	Strategy Identification
N Quadratic equations	Expand	Higher	Find mathematical features
	Quadratic Function Graph	Higher	Find mathematical features
	Form	Higher (hardest)	Work back – unfamiliar
G Area	T Grid	Either	Find key relationships
	Bubble	Either	Find mathematical features
	Inside circle	Higher	Find key relationships
	Stretcher	Higher (hardest)	Find key relationships
G Co-ordinates	Equal Points	Either	Find key relationships
	Isosceles Grid	Either	Find key relationships
	Identical Rectangles	Higher	Find key relationships

Questions

For
problem
solving in
the GCSE
classroom



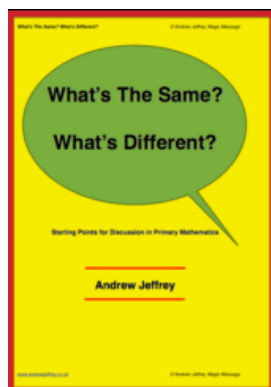
What is the gradient of the tangent to the circle at the point $(-2, -3)$?



- A $\frac{2}{3}$
 B $\frac{3}{2}$
 C $-\frac{2}{3}$
 D $-\frac{3}{2}$

Which numbers less than 100 have exactly three factors?

When you add two numbers, you get the same result as when you multiply them.



The answer is 7, what
is the question?

Give me a pair of
equations whose
solutions differ by three.

Can you construct a
triangle with sides 3, 4
and 9?

Is a square a rectangle?



Can you give me a
quadratic equation
whose roots differ by 3?
And another ...

Making it Stick

The importance of recall.

Apply all the basics.

Mini-Tests

Use the very important idea of *making things stick* and give students a mini-test. Reading Make it Stick (The Science of Successful Learning) which discusses the use of testing as a learning tool convinces me even more that mini-tests are a good idea! Students need to recall information and the evidence suggests that testing is a better way of doing this than simply rereading material, a method often favoured by students. Students like the alternative name 'Self-checks' which I hope helps them realise they are as the authors suggest a *learning tool, not something to be stressed by.*

“exercise in repeatedly recalling a thing strengthens the memory.”

Aristotle

Making it stick

Mini-Tests – the students


“I got it right in the test because I got it wrong in a mini-test”

On good teachers: “A teacher who provides the student with the opportunity to see what they need to revise. Regular tests and quizzes do this.”

What did you learn today?
What did you find tricky?
What can we do next time?

Today's Lesson Tweet.

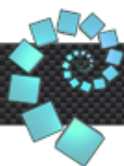
Don't forget the hashtag if describes the lesson or something memorable from the lesson.



@Maths Department
Made summary notes on quadratics
using mini test # enlightened

Resources

Further
resources
for problem
solving in
the GCSE
classroom

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Short Problems for Practice and Revision

The links below take you to a selection of short problems based on UKMT junior and intermediate mathematical challenge questions. You may wish to use them for practice, revision or a mathematical workout!

Longer NRICH problems can be found on the [Topics in Secondary Mathematics](#) page.



Number - Short Problems

A collection of short Stage 3 and 4 problems on number.



Algebra - Short Problems

A collection of short Stage 3 and 4 problems on algebra.



Geometry and Measure - Short Problems

A collection of short Stage 3 and 4 problems on geometry and measure.



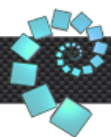
Handling Data - Short Problems

A collection of short Stage 3 and 4 problems on handling data.



Working Mathematically - Short Problems

A collection of short Stage 3 and 4 problems on Working Mathematically.

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Working Mathematically - Short Problems

The collections below contain short problems about Working Mathematically.

For other curriculum areas, visit our collection of [Short Problems](#).

You may also be interested in our longer problems on [Working Mathematically](#).



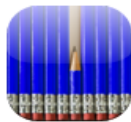
Exploring and Noticing Structure - Short Problems

A collection of short Stage 3 and 4 problems on Exploring and Noticing Structure.



Thinking Strategically - Short Problems

A collection of short Stage 3 and 4 problems on Thinking Strategically.



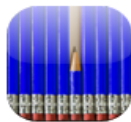
Visualising - Short Problems

A collection of short Stage 3 and 4 problems on Visualising.



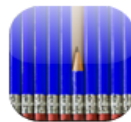
Representing - Short Problems

A collection of short Stage 3 and 4 problems on Representing.



Working Systematically - Short Problems

A collection of short Stage 3 and 4 problems on Working Systematically.



Posing Questions and Making Conjectures - Short Problems

A collection of short Stage 3 and 4 problems on Posing Questions and Making Conjectures.



Mathematical Modelling - Short Problems

A collection of short Stage 3 and 4 problems on Mathematical Modelling.



Reasoning, Justifying, Convincing and Proof - Short Problems

A collection of short Stage 3 and 4 problems on Reasoning, Justifying, Convincing and Proof.



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Problem
Solution
Printable page

You may also like

Consecutive Numbers

An investigation involving adding and subtracting sets of consecutive numbers. Lots to find out, lots to explore.

14 Divisors

What is the smallest number with exactly 14 divisors?

Summing Consecutive Numbers

Many numbers can be expressed as the sum of two or more consecutive integers. For example, $15=7+8$ and $10=1+2+3+4$. Can you say which numbers can be expressed in this way?

Almost Constant Digits

Stage: 3 Short ★

How many ten-digit numbers are there which contain only the digits 1, 2 or 3, and in which any pair of adjacent digits differs by 1?

If you liked this problem, [here is an NRICH task](#) which challenges you to use similar mathematical ideas.

*This problem is taken from the [UKMT Mathematical Challenges](#).
[View the archive of all weekly problems grouped by curriculum topic](#)*

[View the previous week's solution](#)

[View the current weekly problem](#)

Interactivities.

Working systematically.

Addition & subtraction.

Selecting and using information.

Factors and multiples. Games.

Mathematical reasoning & proof. Visualising.

smartphone. Combinations.



Exploring and Noticing Structure - Lower Secondary

What mathematical structures do you notice as you explore these problems?



Working Systematically - Lower Secondary

Work on these problems to improve your ability to work systematically.



Thinking Strategically - Lower Secondary

Work on these problems to improve your strategic thinking.



Posing Questions and Making Conjectures - Lower Secondary

Work on these problems to improve your questioning skills.



Visualising - Lower Secondary

Work on these problems to improve your visualisation skills.



Mathematical Modelling - Lower Secondary

Work on these problems to improve your mathematical modelling skills.



Representing - Lower Secondary

Consider how alternative representations can help us to understand the underlying mathematical concepts.



Reasoning, Justifying, Convincing and Proof - Lower Secondary

Work on these problems to improve your reasoning skills.

Standards Unit

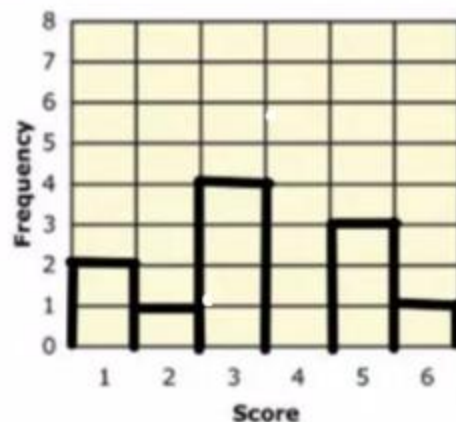
Raw scores

1	3
2	5
3	1
4	6
5	5
6	5
7	3
8	3
9	3
10	2
11	1

Frequency table

Score	1	2	3	4	5	6
Frequency	2	1	4	0	3	1

Bar chart



Statistics

Mean	3
Median	3
Mode	3
Range	5

The task (S4) is to match the bar charts with the statistics (mean, median, mode and range are all given). This has worked really well with my students and I feel leads to a deeper understanding than just simply calculating these statistics.

[Standards Unit](#)

GCSE (9-1) **MATHEMATICS**

Topic Check Ins

Questions 1-5 cover procedural calculations (AO1).

Questions 6-8 assess the learner's ability to reason and communicate mathematically (AO2).

Questions 9-10 relate to problem solving tasks (AO3).



GCSE (9–1) MATHEMATICS

Topic Check Ins

Questions 1-5 cover procedural calculations (AO1).

Questions 6-8 assess the learner's ability to reason and communicate mathematically (AO2).

Questions 9-10 relate to problem solving tasks (AO3).

Each *Check In* also contains an open-ended extension task at the end which will allow more able students to stretch themselves once they have completed the *Check In* questions. Extension questions may include links to related areas of mathematics or extend to higher level techniques. These tasks could serve as a possible start for a whole class discussion or for more able students to develop individually.



GCSE (9-1) MATHEMATICS

Topic Check In - 1.04 Inverse operations

6. Ruth is thinking of a number. If she multiplies it by 5 and then adds 3 the answer will be 58. Describe how you could work out what number she is thinking of.
7. What are the missing digits in the following calculation? $3 \square \times \square = \square 70$
8. Given that $32^2 = 1024$, find $7 + \sqrt{1024}$.
9. Amy and Brian play football. If Amy scored 4 more goals than Brian and they scored 22 goals in total, find the number of goals scored by Amy.
10. It costs £10 to hire a bicycle plus £2 for every hour. If it cost Darren £18 to hire a bike, how many hours did he hire it for?

Extension

How many different ways can you find to use four different single digit numbers, and any mathematical operations [$+$, $-$, \times , \div and brackets ($)$] to make the answer 14?



GCSE MATHEMATICS

Key Stage 3 - 4 Bridging the Gap
Teacher Guide

Problem solving 1: Function Notation

Print and cut out the cards



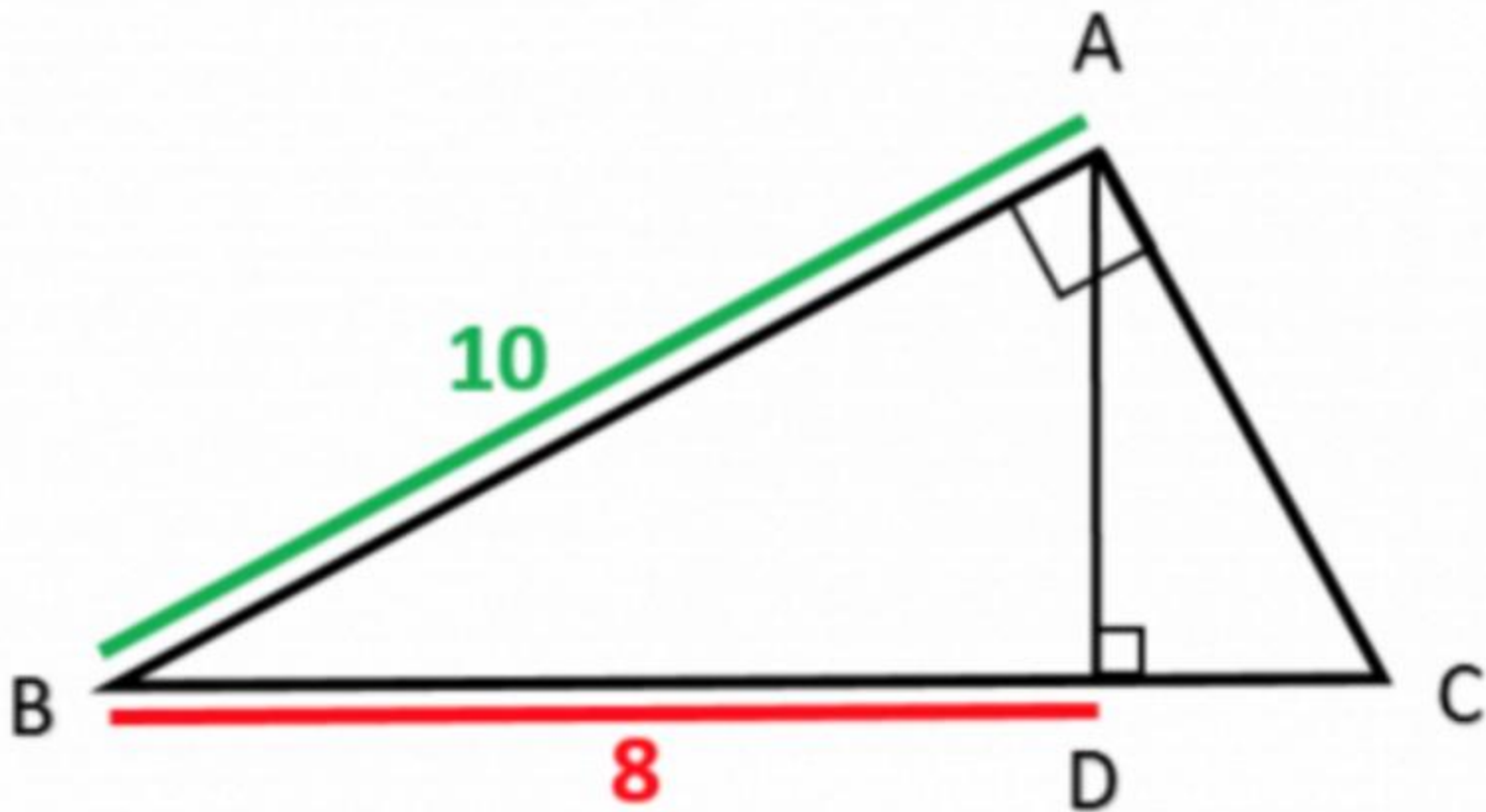
Give one set to each pair or group of students.

Students should match up the cards to make a chain, beginning with the 'Start' card, and working towards the 'Finish' card. All cards are used.

$f(17) = 8$	$f(x) = \frac{5x}{2x-3}$	$f(x) = 3x^2 - 5$	$f(31) = 2$
-------------	--------------------------	-------------------	-------------

Three functions are made into the composite $\sqrt{2x+1}$

Write down three functions and state the order in which they are put together to make the composite



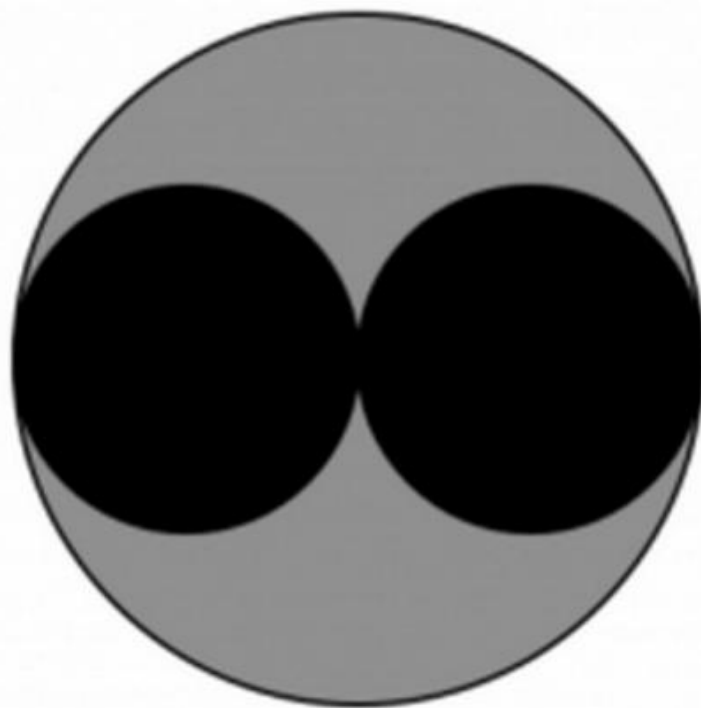
What Is The Area Of Triangle ABC?



How Many Circles Will It Take To Balance The Top Scale?

To draw an alien's head, Mary drew a large circle, and then added 2 equal black circles for the eyes.

Which region is larger, the grey region or the black region?



Can You Answer This Without Guessing?

	High School	Middle School	Total
Boys	a	$2b$	X
Girls	$2a$	b	Y
Total	M	N	T

In the table above, each letter represents the number of students in that category. Which of the following equals $M - N$?

X

Y

$3(Y - X)$

$\frac{T}{3}$

$X + Y$

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The FMSP produces resources for teaching, enrichment and the promotion of mathematics.



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GCSE Problem Solving Materials

20 Problems for GCSE Mathematics

20 Problems for GCSE Mathematics

These materials have been produced by the Further Mathematics Support Programme and are designed to help teachers develop their students' problem solving abilities. Each problem comes with a set of suitable prompts for use with students and a full worked solution.

	<p>Problem 1</p>	<p>27×147</p>	<p>Problem 2</p>	<p>If A is divided by B the result is $\frac{2}{3}$. If B is divided by C the result is $\frac{4}{7}$. What is the result if A is divided by C?</p>	<p>Problem 3</p>
	<p>Problem 4</p>		<p>Problem 5</p>		<p>Problem 6</p>
<p>What is the smallest number divisible by 1, 2, 3, 4, 5, 6, 7, 8 and 9?</p>	<p>Problem 7</p>	<p>Two numbers add together to 6. A number of the size 23 is divided by the other to leave the remainder 3. What number is 2 in the hundreds place if you're considering 2? When is today's 4th birthday in your life? Can you write an equation representing all these?</p>	<p>Problem 8</p>		<p>Problem 9</p>
<p>x, x^3, x^4, x^2, x^0</p>	<p>Problem 10</p>		<p>Problem 11</p>		<p>Problem 12</p>
	<p>Problem 13</p>	<p>$\sqrt{p^2 + 5q} = 8$ $\sqrt{p^2 - 3q} = 6$</p>	<p>Problem 14</p>	<p>a mode of 24 a median of 21 a mean of 20</p>	<p>Problem 15</p>
	<p>Problem 16</p>		<p>Problem 17</p>		<p>Problem 18</p>
<p>Find the sum of any three consecutive numbers. What do you notice about the total? Is this true for any three consecutive numbers? Can you prove why this is true?</p>	<p>Problem 19</p>	<p>Choose any three consecutive numbers. Multiply the first number by the first number. Square the second number. What do you notice? Is this true for any three consecutive numbers?</p>	<p>Problem 20</p>		

You can find the complete set of problems, prompts and solutions within the **GCSE problem solving booklet**.



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Problem 20

Choose any three consecutive numbers.

Multiply the first number by the third number.

Square the second number.

What do you notice?

Is this true for any three consecutive numbers?

Can you **prove** why this is true?

For each problem:

Suggested questions to ask students

How to 'get into the problem'

Complete solution

KS4 Extension and Enrichment

Quadratic functions and expressions

Short activity

All, some or none?

For each question there are 5 related statements. In each case decide which of them are true.

1. The quadratic $y = x^2 - 2x - 3$:

a. rearranges to $y = (x-1)^2 - 2$	d. has an axis of symmetry at $x = 1$
b. Has a y intercept at -3	e. has a minimum value of -3
c. factorises to $y = (x-3)(x+1)$	

2. The quadratic $y = (x+1)^2 + 2$:

a. rearranges to $y = (x+1)(x+2)$	d. has an axis of symmetry
b. has a minimum value of 2	e. doesn't cross the x axis
c. always has positive values for y	

3. All quadratics:

a. have an axis of symmetry	d. cross the y axis once
b. cross the x axis	e. have a minimum value
c. can be arranged to a completed square format	

Challenge: For any statements that are false in question 3, give counter examples and explain when and why they are false.

S6

Use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing

Teaching Guidance

Students should be able to:

- recognise and name positive, negative or no correlation as types of correlation
- recognise and name strong, moderate or weak correlation as strengths of correlation
- understand that just because a correlation exists, it does not necessarily mean that causality is present
- draw a line of best fit by eye for data with strong enough correlation, or know that a line of best fit is not justified due to the lack of correlation
- understand outliers and make decisions whether or not to include them when drawing a line of best fit
- use a line of best fit to estimate unknown values when appropriate.

S6

Use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing

Examples

- 1 From a scatter diagram:
 - (a) Write the down the strength **and** type of correlation shown by the diagram.
 - (b) Interpret your answer to part (a) in the context of the question.
- 2 Draw a line of best fit on a scatter diagram, make predictions, interpolate and comment on the dangers of extrapolation.



Higher tier

non-calculator



Question 6

- 6** The sum of the whole numbers from 1 to 50 inclusive is 1275
Work out the sum of the whole numbers from 2 to 51 inclusive.

[2 marks]



Answer _____

AQA Guidance

GCSE MATHEMATICS

Exemplar Questions for A01, A02 and A03
Assessment Objectives

FOUNDATION and HIGHER TIERS



Higher tier

non-calculator



Question 6

- 6** The sum of the whole numbers from 1 to 50 inclusive is 1275
Work out the sum of the whole numbers from 2 to 51 inclusive.

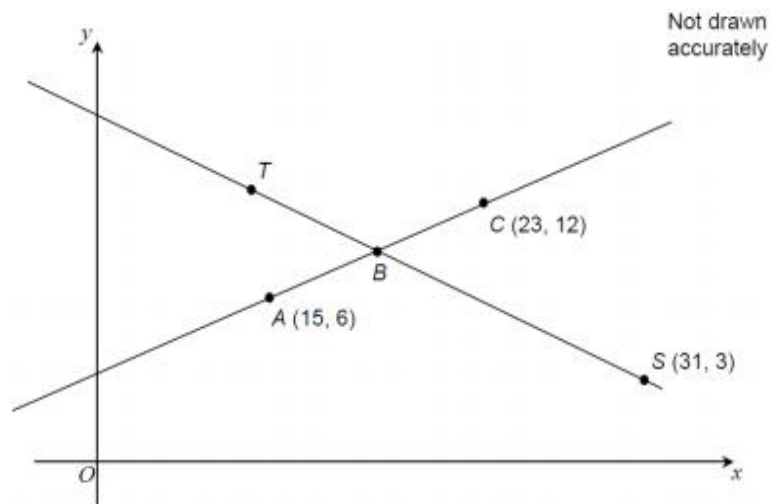
[2 marks]

Mark Scheme

6	1275 – 1 or 1274 or 1275 + 51 or 1326	M1	
	1325	A1	An answer of 1275 scores 0

Example of testing results Paper 1H Question 21

- 21 Two straight lines are shown.
B is the midpoint of AC.
 $TB : BS = 2 : 3$



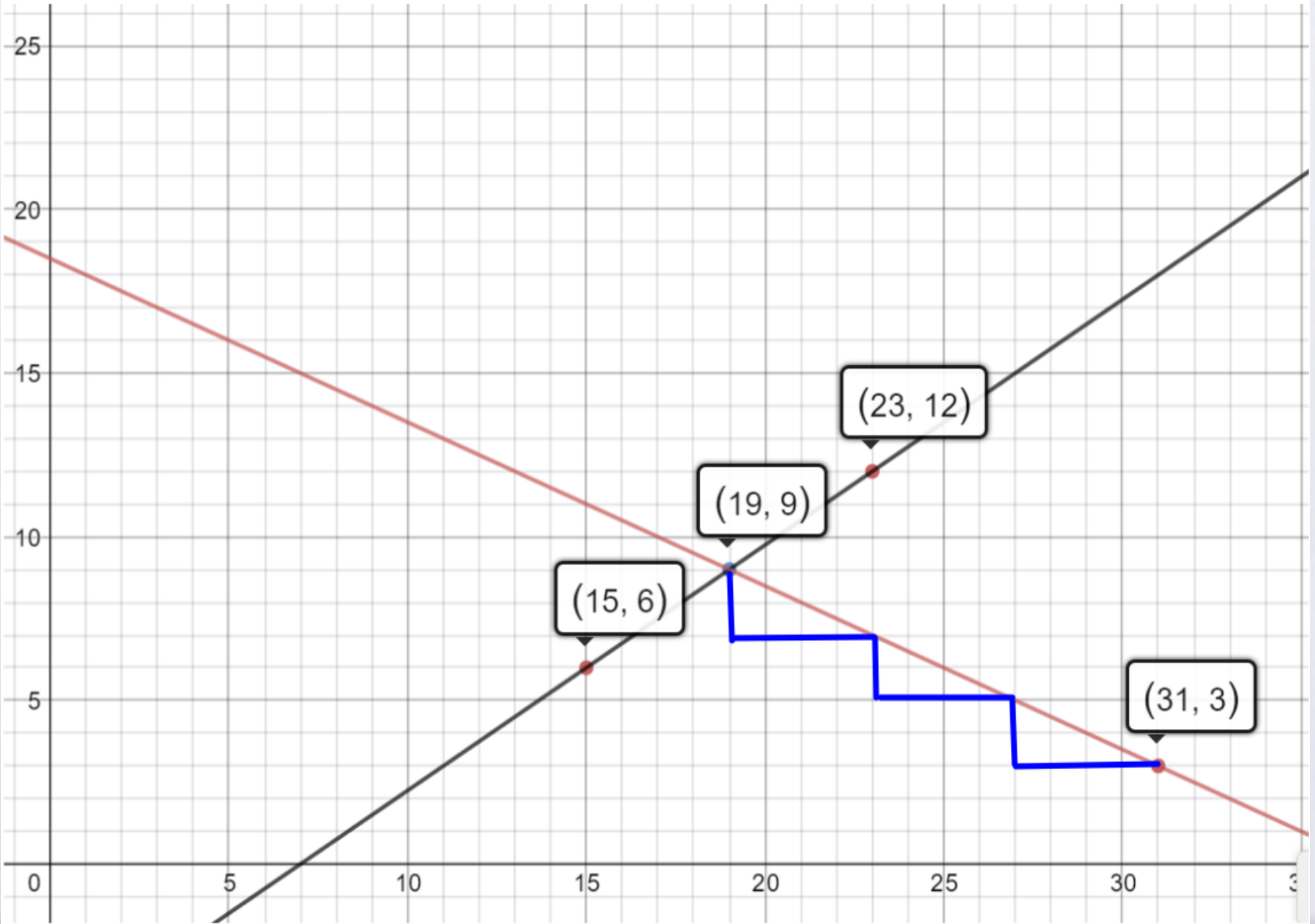
Work out the coordinates of T.

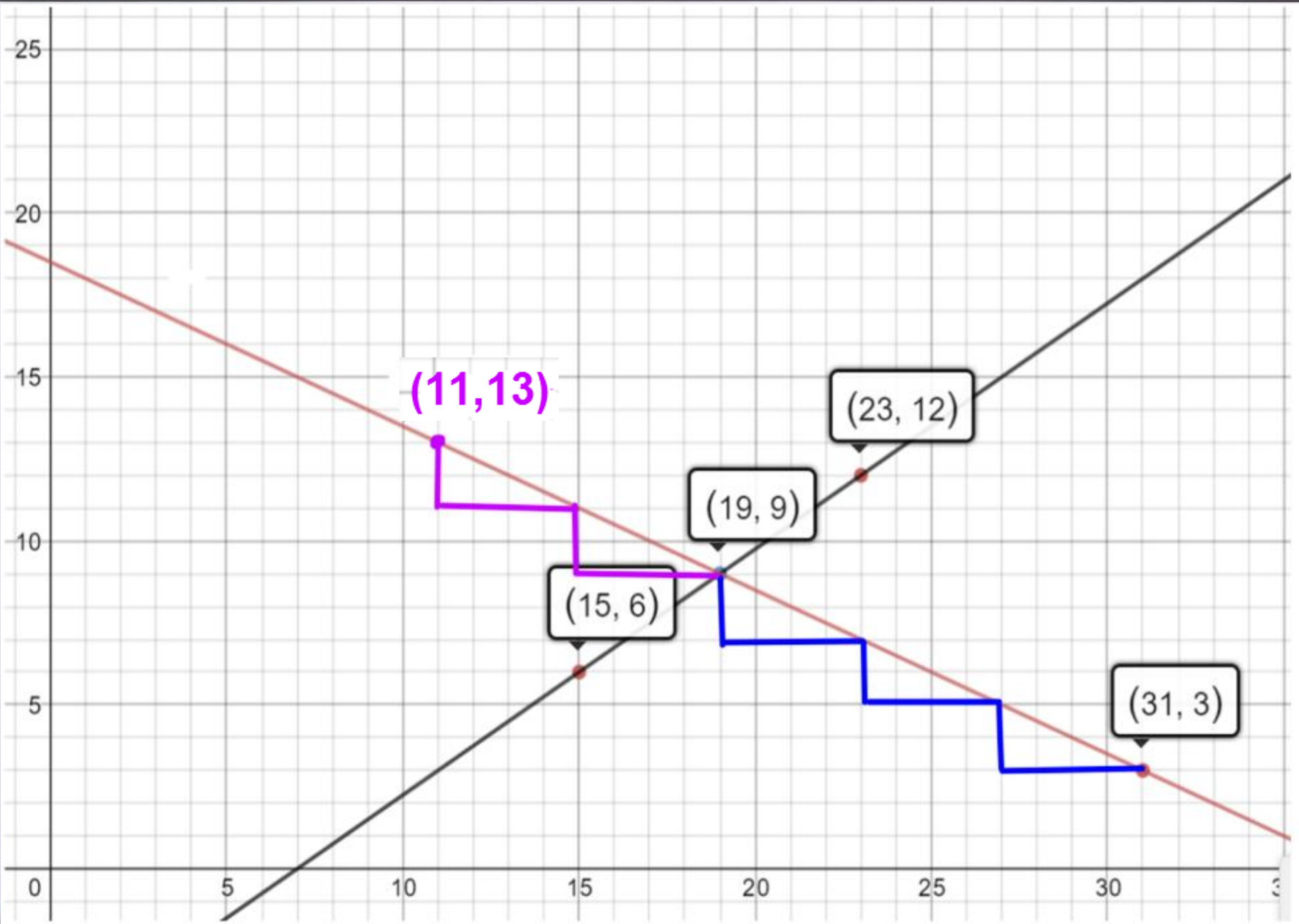
[4 marks]

Problem solving
questions don't need to
be in context or word-
heavy

Still a challenge!

10% scored 4 marks
9% scored 3 marks
10% scored 2 marks
22% scored 1 mark
49% did not score





AQA Mock Exam Analyser

available through
All About Maths

1a	Equation & identity	AO1	1	0.63
1b	Equation & identity	AO1	1	0.88
1c	Equation & identity	AO1	1	0.35
2	Money problem	AO2	5	4.13
3a	Mean and median	AO3	2	1.26
3b	Mean and median	AO1	3	1.44
4a	Quadrilaterals	AO1	3	1.62
4b	Quadrilaterals	AO3	1	0.78
5a	Prime factors, HCF	AO1	2	1.48
5b	Prime factors, HCF	AO1	2	1.28
6	Correlation	AO1	2	1.88
7	Semicircle	AO1	3	1.42
8a	Equations	AO1	2	0.94
8b	Equations	AO1	4	1.45
9a	Formula from line	AO2	2	0.79
9b	Formula from line	AO1	2	0.99
10	Comparison of distributions	AO1	2	0.73
11	Speed and algebra	AO3	4	1.06
12	Tree Diagram	AO3	4	2.08
13	Simultaneous equations	AO2	4	1.72
14a	Quadratic in context	AO2	2	0.66
14b	Quadratic in context	AO2	3	0.5
15	Congruent proof	AO2	4	0.9
16a	Surds area	AO1	1	0.5
16b	Surds area	AO2	3	0.24
17	Volume frustum	AO1	3	0.7
18a	Quad graph	AO3	2	0.26
18b	Quad graph	AO3	2	0.11

Reviewing question 3a

[Return to main screen](#)



The question

- 3 (a) Write down four **different** numbers that have
- a **median** of 5
 - and a **range** of 7.

Put the numbers in order.

[2 marks]

.....

.....

Answer , , ,

Comments

Question 3a

There are many answers to this question. First work out a pair of numbers (that do not have a range of 7) with a middle value of 5 and then 'bracket' them with a pair of numbers that have a range of 7.

The Mark Scheme

<p>3(a)</p> <p>Four different numbers in any order with median 5 and range 7</p> <p>eg 1, 4, 6, 8</p> <p>9, 6, 4, 2</p> <p>3, 10, 6, 4</p> <p>1, 3, 7, 8</p> <p>2, 3, 7, 9</p> <p>0, 4, 6, 7</p> <p>1.5, 4, 6, 8.5</p> <p>-1, 4.5, 5.5, 6</p>	<p>B2</p>	<p>B1 Four numbers in any order with median 5 and range 7 with repeats</p> <p>eg 4, 4, 6, 11</p> <p>3, 3, 7, 10</p> <p>1, 5, 5, 8</p> <p>5, 5, 4, 11</p> <p>B1 Four different numbers in any order with median 5 or range 7</p>
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An answer worth full marks

3 (a) Write down four **different** numbers that have

a median of 5
and a range of 7.

Put the numbers in order.

[2 marks]

Median of 5 \Rightarrow middle 2 could be 4, 6 for example

Range of 7 \Rightarrow outer two could be 1, 8 for example

Answer 1 , 4 , 6 , 8

$A(-2, 1)$, $B(6, 5)$, and $C(4, k)$ are the vertices of a right-angled triangle ABC .

Angle ABC is the right angle.

Find an equation of the line that passes through A and C .

Give your answer in the form $ay + bx = c$ where a , b and c are integers.

plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form $y = mx + c$ to identify parallel **and perpendicular lines**; find the equation of the line through two given points or through one point with a given gradient (A9)

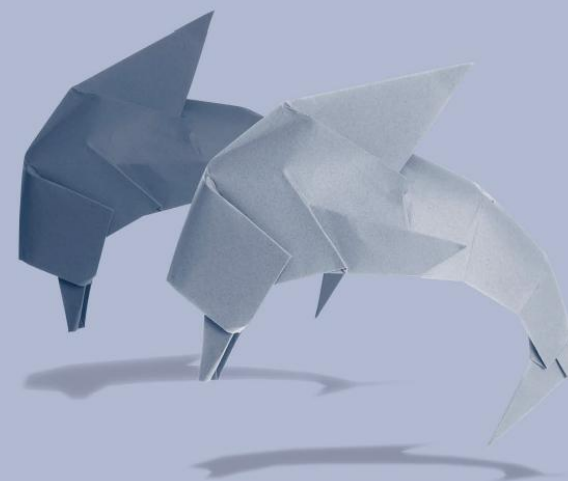
AO1

1.3b – accurately carry out set tasks requiring multi-step solutions (1 mark)

AO3

3.1b – translate problems in mathematical contexts into a series of processes (4 marks)

**Pearson Edexcel Level 1/
Level 2 GCSE (9-1) in
Mathematics (1MA1)**



**EXEMPLIFICATION OF THE
NEW SAMPLE ASSESSMENT MATERIALS**

First certification 2017

In triangle RPQ ,

$$RP = 8.7 \text{ cm}$$

$$PQ = 5.2 \text{ cm}$$

$$\text{Angle } PRQ = 32^\circ$$

know and apply $\text{Area} = \frac{1}{2} ab \sin C$ to calculate the area, sides or angles of any triangle (G23)

AO1

1.3b – accurately carry out set tasks requiring multi-step solutions (1 mark)

AO3

3.1b – translate problems in mathematical contexts into a series of processes (3 marks)

(a) Assuming that angle PQR is an acute angle, calculate the area of triangle RPQ .
Give your answer correct to 3 significant figures.

(b) If you did not know that angle PQR is an acute angle, what effect would this have on your calculation of the area of triangle RPQ ?

New to 1MA1

recognise, sketch and interpret graphs of **the trigonometric functions $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size (A12)**

AO3

3.4a – evaluate methods used (1 mark)

If Lyn did **not** drive along the same roads as Gary, explain how this could affect your answer to part (a).

Common question across both tiers

compare lengths, areas and volumes using ratio notation; make links to similarity and scale factors (R12)

AO3

3.5 – Evaluate solutions to identify how they may have been affected by assumptions made (1 mark)

Axel and Lethna are driving along a motorway.
They see a road sign.

To Junction 8
30 miles
26 minutes

AO1

1.1 – Accurately recall facts, terminology and definitions (1 mark)

The road sign shows the distance to Junction 8
It also shows the average time drivers take to get to Junction 8.
The speed limit on the motorway is 70 mph.

Lethna says

“We will have to drive faster than the speed limit to drive 30 miles in 26 minutes.”

Is Lethna right? **You must show how you get your answer.**

AO3

3.1d – translate problems in non-mathematical contexts into a series of mathematical processes (1 mark)

3.3 – Interpret results in the context of the given problem (1 mark)

The CEO of GE explains why his math degree is more useful than his MBA

December 2015

Immelt says his most invaluable qualification is not the MBA. It is his undergraduate degree in math.

"I use my math major every day — I don't use the MBA quite as much. My intellectual curiosity goes more toward **problem solving** than spreadsheets."

Running a company, to him, is really about **problem solving**. And that's something he learned about in his undergraduate studies, due to "the inherent intellectual curiosity around math and physics." That has stayed with Immelt throughout his career.

"I'm just curious about everything," he said. "I can view every situation as a **problem to be solved**."

Mathematics, Learning and Technology



I'm Looking For...

Lesson Planning

Starters

Problems & Activities

Rich Tasks

Revision Activities



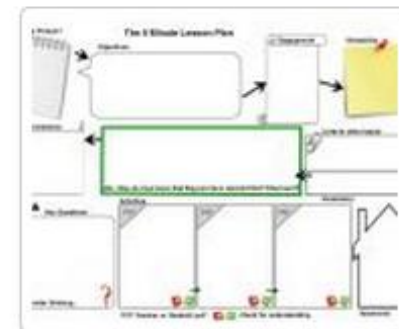
Diagnostic Questions – by
Year 7



Top 100 Tools for
Learning 2013



National STEM Centre



Lesson Planning

Mathematics for Students

Learning Mathematics – Resources for Students



Challenges

Explorations

Graphs

WolframAlpha

Study Tips

Reference

Notes

