Problem Solving

As I researched for latest readings on problem solving, I stumbled into a set of rules, the student's misguide to problem solving. One might find these rules absurd, or even funny. But as I went through each rule, I realized these very same rules seem to be the guidelines of the non-performing students in problem solving! And these are the rules that we, teachers, try to delete from our students memory.

Student's Misguide to Problem Solving
A joke by Lynn Nordstrom
Source: Web article, Home School Math

Rule 1: If at all possible, avoid reading the problem. Reading the problem only consumes time and causes confusion.

Rule 2: Extract the numbers from the problem in the order they appear. Be on the watch for numbers written in words.

Rule 3: If rule 2 yields three or more numbers, the best bet is adding them together.

Rule 4: If there are only 2 numbers which are approximately the same size, then subtraction should give the best results.

Rule 5: If there are only two numbers and one is much smaller than the other, then divide if it goes evenly -- otherwise multiply.

Rule 6: If the problem seems like it calls for a formula, pick a formula that has enough letters to use all the numbers given in the problem.

Rule 7: If the rules 1-6 don't seem to work, make one last desperate attempt. Take the set of numbers found by rule 2 and perform about two pages of random operations using these numbers. You should circle about five or six answers on each page just in case one of them happens to be the answer. You might get some partial credit for trying hard.

Going over these rules, I asked myself. What could have caused these students to form these ideas? Could teachers be contributory? Or is this just a manifestation of the kind of values our students have today?

What is Problem Solving?

Problem solving is a process. It is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation. The process begins with the initial confrontation and concludes when an answer has been obtained and considered with regards to the initial conditions. The student must synthesize what he or she has learned and apply it to the new situation.

In the book Problem Solving Strategies by Ted Kerr, problem solving is defined as *what to do when you don't know what to do.*

What is a Problem?

A problem is a situation, quantitative or otherwise, that confronts an individual or group, and for which *no path to the answer is known*.

Question: a situation that can be resolved by recall from memory

Exercise: a situation that involves drill and practice to reinforce previously learned skills or algorithms

Problem: a situation that requires thought and synthesis of previously learned knowledge to resolve

A problem must be perceived as such by the student, regardless of the reason. If the student refuses to accept the challenge, then at that time, it is not a problem for that student. Thus a problem must satisfy the following criteria: acceptance, blockage, goal.

1. Acceptance: The individual accepts the problem. There is a personal involvement, which maybe due to any of a variety of reasons, including internal motivation, external motivation, or simply the desire to experience the enjoyment of solving a problem
2. Blockage: The individual's initial attempts at solutions are fruitless. His or her habitual responses and patterns of attack do not work.
3. Exploration: The personal involvement identified in (1) forces the individual to explore new methods of attack.

The existence of a problem implies that the individual is confronted by something he or she does not recognize, and to which he or she cannot merely apply a model. A situation cannot be considered a problem when it can be solved by merely applying algorithms that have been previously learned when the situation is like one previously remembered.

Textbook Problems

Although most mathematics textbooks contains sections labeled *word problems*, many of these should not really be considered problems. In many cases, a model solution has been presented in class by the teacher. The student merely applies this model to the series of similar exercises in order to solve them. Essentially, the student is practicing an *algorithm,* a technique that applies to a single class of problems that guarantees success if mechanical errors are avoided. Few of these so called problems require reasoning by the students. Yet, the first time a student sees these word problems, they could be problem for him or her, if presented in a non- algorithmic fashion. In many cases, the very placement of these exercises prevents them from being real problems, since they follow an algorithmic development designed specifically for their solution.

We consider these word problems to be exercises or routine problems. Should they be removed from the textbook? The answer is NO. They do have purposes such as: provide exposure to problem situations; practice in the use of algorithm; and drill in associated mathematical processes. A teacher should not think, however, that students who have been solving these exercises through the use of a carefully developed model or algorithm are learning to become problem solvers. Creative teachers, by their approaches, can utilize these to help develop problem solving skills.

Why Teach Problem Solving

Problem solving is fundamental to everyday life. All of our students will face problems, quantitative or otherwise, everyday of their lives. Rarely, if ever, can these problems be resolved by merely referring to an arithmetic fact or a previously learned algorithms without reasoning. The words “add me” or “multiply me” do not appear in a store window. Problem solving provides the link between facts, algorithm and the real life situation we all face.

In spite of the obvious relationship between the mathematics of the classroom and the quantitative situations in life, we know that children of all ages see little connection between what happens in school and what happens in real life. An emphasis on problem solving can lessen the gap between the real world and the classroom world and set a positive mood in the classroom.

In many mathematics classes, students do not see any connection between the various ideas taught during the year. Most regard each topic as a separate entity. Problem solving shows the interconnection between mathematical ideas. Problems are never solved in a vacuum, but are related in someway to something seen before, something learned earlier, or something to be learned at a later time. Thus, good problems can be used to review past mathematical ideas, as well as to sow the seeds for ideas to be presented at a future time.

Problem solving is more exciting, more challenging, and more interesting to children than barren exercises.

When Do We Teach Problem Solving

Problem solving is a lifetime activity. The child encounters problem solving almost from birth. However, the formal teaching and learning of the problem-solving process begin as soon as the child enters school and continues throughout his entire school experience. The elementary school teacher has the responsibility for beginning this instruction and thus laying the foundation for building the child’s capacity to deal successfully with his or her future problem solving encounters.

Experiences in problem solving are always at hand. Thus, the teaching of problem solving should be continuous. Discussion of problems, proposed solutions, methods of attacking problems, etc, should be considered at all times.

Naturally, there will be times when studies of algorithmic skills and drill and practice sessions will be called for. Students must be proficient in the basic computational skills. Problem solving is not a substitute to computational skills.

Heuristics of Problem Solving

Heuristics should not be confused with algorithms. Algorithms, normally presented to children in classrooms, are schemata that are applied to a single class of problems. For each problem or classes of problems, there is a specific algorithm. If on or mechanical errors, the correct answer will be obtained. In contrast, heuristics are general and are applicable to all classes of problems. They provide the direction needed by all people to approach, understand and resolve problems that confront them. The following heuristics were developed by George Polya, a contemporary mathematician.

- **Understanding the Problem**
  What is the unknown? What are the data? What is the condition?
  Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?
  Draw a figure. Introduce suitable notation. Separate the various parts of the condition. Can you write them down?

- **Devising the Plan**
  Find the connection between the data and the unknown. You maybe obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a plan of the solution.
  Have you seen it before? Or have you seen the same problem in a slightly different form?
  Do you know a related problem? Do you know a theorem that could be useful?
  Look at the unknown. Try to think of a familiar problem having the same or similar unknown.
- **Carrying out the Plan**
  Carry out the plan. Check each step.
  Can you see clearly that the step is correct? Can you prove that it is correct?
- **Looking Back**
  Examine the solution obtained.
  Can you check the result? Can you check the argument? Can you use the result or the method for some other problem?

Teaching of Problem Solving

"The first rule of teaching is to know what you are supposed to teach. The second rule of teaching is to know a little more of what you are supposed to teach."

"A teacher of mathematics should know some mathematics, and that a teacher wishing to impart the right attitude of mind toward problems to his students should have acquired that attitude himself."

Suggestions to Teachers

1. Create a non-threatening environment
2. Have your students work together in a variety of groupings
3. Raise creative, constructive, thought- provoking questions
4. Encourage creativity of thought and imagination
5. Create an atmosphere of success
6. Encourage your students to solve problems
7. Help your students to become critical readers
8. Involve your students in both the problem and the process
9. Introduce drawings and manipulatives into the solution process
10. Suggest alternatives when students have been thwarted in their solution efforts
11. Develop pupils skills in estimation
12. Encourage students to make conjectures
13. Have students reflect in their own thought processes
14. Require your students to create their own problem
15. Use strategy games in class

Problem Solving Strategies

Here are some strategies in solving problems:

1. Drawing a Diagram

   A diagram has certain advantages over verbal communications. A diagram can show positional relationships far more easily and clearly than a verbal description.

Example: Betty, Cathy, Isabel, Lani, Alma and Ursula ran an 800-meter race. Alma beat Isabel by 7 meters. Betty finished 12 m behind Ursula. Alma finished 5 meters ahead of Lani but 3 meters behind Ursula. Cathy finished halfway between the first and last person. In what order did they finish? Indicate the order and the distances between each girl.

2. Systematic Lists

   A systematic list is exactly what the name implies: a list generated through some kind of system. The system should be clear enough so that the person making the list can quickly verify its accuracy. It should also be possible for another person to understand the sytem and verify it without too much effort.

   Example: A rectangle has an area of 120 sq m. Its length and width are whole numbers. What are the possibilities for the length and width? Which possibility gives the smallest perimeter?

3. Eliminate possibilities

   "Once you have eliminated the impossible, then whatever is left, no matter how improbabale, must be the solution." Sherlock Holmes

   Example: Jun threw 5 darts. The possible scores on the target were 2, 4, 6, 8 , 10. Each dart hit the target. Which of these total scores are you certain is not possible? 38, 23, 58, 30, 42, 31, 26, 6, 14, 15

4. Matrix logic

   Most problems can be solved using charts or tables called matrix. The matrix serves to organize the information in the problem in a useful way.

   Example 1: Jeff, Amy and Tracy each have a different pet. One child has a bird, one has a cat and the last child has a dog. Match each pet with its owner.

   Amy's pet has 4 legs.
   Jeff's pet doesn't bark.
   Tracy is allergic to cats.
   Jeff's pet doesn't fly.

   Example 2: Tom, John, Fred and Bill are friends whose occupations are (in no particular order) nurse, secretary, teacher and piot. They attended a church picnic recently, and each one brought his favourite meat (hamburger, chicken, steak, and hot dogs.) to barbecue. From the clues shown, determine each man's name, occupation and favourite meat.

   1. *Tom is neither the nurse nr the teacher.*
   2. *Fred and the pilot play golf together. The burger lover and the teacher hate golf.*
   3. *Tom brought hot dogs.*
   4. *Bill sat next to the burger fan and across from the steak lover.*
   5. *The secretary hates golf.*

5. Look for a pattern

   Finding patterns enables one to reduce a complex problem to a pattern and then use the pattern to derive a solution. Often the key to finding a pattern is to organize information.

   Example: Gino liked to jog late at night. One night, he noticed an unusual phenomenon: as he jogged, dogs would hear him and bark. After the first dog had barked for about 15 minutes, *two other dogs would join in and bark. And then in about another 15 seconds, each seemed that each barking dog would inspire two more dogs to start barking. Of course, long after Gino passed the first dog, it continued to bark, as dogs are inclined to do. After about 3 minutes, how many dogs were barking (as a result of Gino passing the first dog?)*

6. Guess and check

   The strength of the guess and check strategy comes from organizing the information in the problem into refined guesses.

   Example: Cloe is two years less than four times as old as Zoe. Cloe is also one year more than three times as old as Zoe. How old is each?

7. Solve an easier related problem

   The major part of the strategy is to change the focus away from the original problem to an easier related problem. Then, after solving the easier problem, decide the plan for the original problem.

   Example: How many squares are there on a checkerboard?

8. Physical representations

   A physical representation differs from a diagram, in the sense that you can touch the problem and not represent it with a drawing.

   a. Acting it out – The solver is involved and walk through the problem

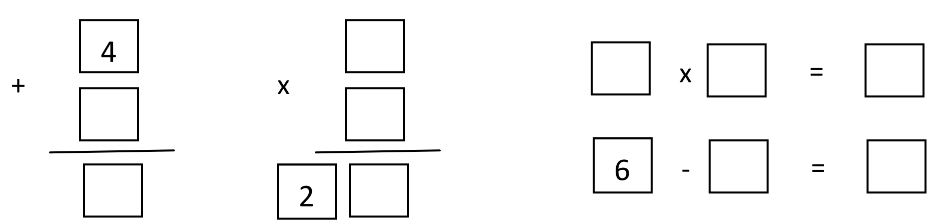
   Example: A group of 10 kids got together at the playground to play basketball. Before the game, every kid shool hands with each of the other kid exactly once. How many handshakes took place?

   b. Making a model – The solver explores using physical objects called models or manipulatives.

   Example: In how many ways can 4 stamps be attached together?

   c. Use manipulatives

   Example: Use the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, once each to fill in the blanks in this puzzle.



9. Work Backwards

   Example: A man competing on a game show ran into a losing streak. First he bet half of his money on one question, and lost it. Then he lost half of his remaining money on another question. Then he lost Php300 on another question. Then he lost half of his remaining money on another question. Finally he got a question right and won Php200. At this point, the show ended and he had Php1 200 left. How much did he have before his losing streak began?

10. Venn Diagram

Example: There are 23 students in a homeroom. Eighteen are taking math and 15 are taking science. Six students are taking math but not science. How many are taking neither subject?

References:

Reasoning and Problem Solving by Stephen Krulik and Jesse Rudnick
Problem Solving Strategies: Crossing the Rivers With Dogs by Tedd Herr and Ken Johnson
How to Solve It by George Polya