Problems and solutions for SK2300 Optical Physics

v. 2014-10-22

This document contains all the problems that are treated during the problem classes. The problems consist of old examination questions that have been selected to match the topic of each problem class. All old exams are available at the course homepage (https://www.kth.se/social/course/SK2300/).

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Contents

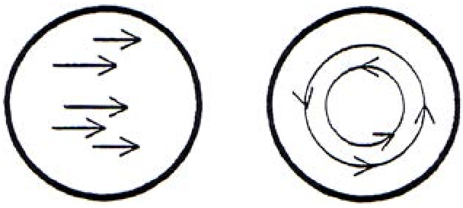
Session 1

Electromagnetic waves

1.1 Problems

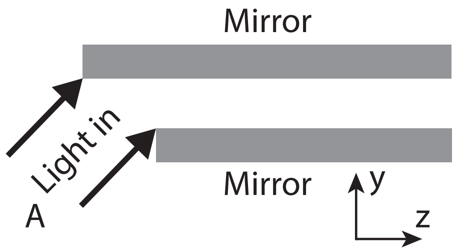
930205:3

The circles in bold in the figure below is meant to illustrate a cross section of a beam of light. The left one illustrates a circularly polarized wave. In this case the electric field vector, E, turns around with constant amplitude. The question is whether the electric field vector can look like the ones in the right circle.



930402:5

Light is reflected back and forth between two lossless mirrors like in the figure below. The incoming light, at A, is a plane wave with the electric field vector in the plane of the paper. The incidence angle of the light is 45°. The main propagation direction of the light wave is in the z-direction. Find the ratio between the longitudinal and the transverse electric field component! What is the phase speed in the z-direction?



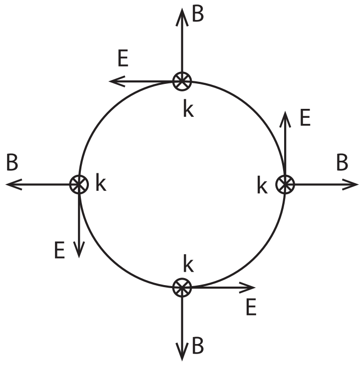
940824:2

A poor student in physics is performing calculations on a problem where the wave equation describing the propagation of light is involved. He/She ends up with a solution where the **D**- and **E**-field is not in phase with each other, but has a small phase difference ($\Delta\delta \sim 1^\circ$). Confusion arises in his/her head which is why we want you to explain what phenomenon that can give rise to this! Some equations would be helpful to make him/her understand.

1.2 Solutions

930205:3

The propagation direction ($\mathbf{k}$) is perpendicular to the board. Also, we know that $\mathbf{E}$, $\mathbf{B}$ and $\mathbf{k}$ are all perpendicular. The only possible configuration for the case in the right circle is then



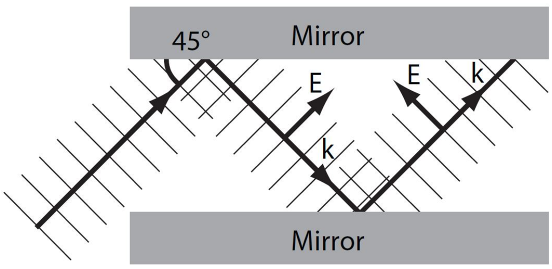
However, from Maxwell's equations we know that

$$\nabla \cdot \mathbf{B} = 0 \qquad \text{(no magnetic monopoles)}$$

$$\oiint \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 0 \qquad \text{(no net magnetic flux through a closed surface)}$$

A cylindrical surface around the beam would have a net flux
⇒ The $\mathbf{E}$-field cannot look like the one in the right circle.

930402:5



Lets start with the expression describing an arbitrary plane wave,

$$\mathbf{E}(\mathbf{r}, t) = \mathrm{Re}\left\{\mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\right\}$$

where $\mathbf{E}_0 = [E_x, E_y, E_z]$, $\mathbf{r} = [x, y, z]$ and $\mathbf{k} = [k_x, k_y, k_z]$. It it often easier to perform the calculations with complex waves and then take the real part of the end result to find the physical real wave. The total field $\mathbf{E}_{\text{tot}}$ between the mirrors is the sum of two fields $\mathbf{E}_{\nearrow}$ and $\mathbf{E}_{\searrow}$, where the second is a reflection of the first. Since the incident light is p-polarized the reflected wave will not undergo a phase shift (according to the Fresnel equations). The problem is in 2D so we will only consider the y and z components of the electric field.

$$\begin{aligned}
\mathbf{E}_{\text{tot}} &= \mathbf{E}_{\nearrow} + \mathbf{E}_{\searrow} \\
&= [E_y, -E_z]\, e^{i(k_y y + k_z z - \omega t} + [E_y, E_z]\, e^{i(-k_y y + k_z z - \omega t} \\
&= \left\{ k_y = k_z = \frac{k}{\sqrt{2}}, E_y = E_z = \frac{E_0}{\sqrt{2}} \right\} \\
&= \frac{E_0}{2} \underbrace{e^{i(kz/\sqrt{2} - \omega y)}}_{e^{i\varphi}} \left[\underbrace{e^{ky/\sqrt{2}} + e^{-ky/\sqrt{2}}}_{=2\cos(\underbrace{ky/\sqrt{2}}_{=\theta})}, \underbrace{-e^{ky/\sqrt{2}} + e^{-ky/\sqrt{2}}}_{=-2i\sin(\underbrace{ky/\sqrt{2}}_{=\theta})} \right] \\
&= \sqrt{2} E_0 (\cos\varphi + i\sin\varphi)[\cos\theta, -i\sin\theta]
\end{aligned}$$

Now we find the physical field by taking the real part of this result,

$$\mathbf{E}_{\text{tot}} = \sqrt{2} E_0 [\cos\varphi\cos\theta, \sin\varphi\sin\theta]$$

The ratio between the z and the y component $\mathbf{E}_{\text{tot}}$ is then,

$$\frac{E_{\text{tot},z}}{E_{\text{tot},y}} = \tan\varphi\tan\theta = \tan(kz/\sqrt{2} - \omega t)\tan(ky/\sqrt{2})$$

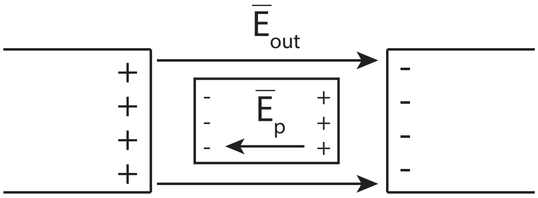
The phase velocity in the z direction is found by calculating how fast a point with constant phase is moving,

$$\varphi = kz/\sqrt{2} - \omega t = C \quad \Rightarrow \quad z = \sqrt{2}\frac{(C + \omega t)}{k}$$

$$\frac{\partial z}{\partial t} = \frac{\omega}{k}\sqrt{2} = c\sqrt{2}$$

This does not mean that information can be transfered faster than the speed of light! The wave fronts are just positions in space where the phase is constant. These positions can move with infinite velocity. Modulations of the initial wave will propagate like a short light pulse would.

940824:2



First we have to know how the outer electric field $\mathbf{E}_{\text{out}}$, the so called displacement field $\mathbf{D} = \varepsilon_0 \mathbf{E}_{\text{out}}$, and the electric field inside the dielectric material $\mathbf{E}$

are related. If an external electric field is applied to a dielectric (not conducting) material the material will be polarized, resulting in a polarization $\mathbf{P}$. The relation is then,

$$\mathbf{E} = \frac{\mathbf{D}}{\varepsilon_0} - \frac{\mathbf{P}}{\varepsilon_0} \qquad \Rightarrow \qquad \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

If $\mathbf{P} = 0$, as in vacuum, $\mathbf{E}$ and $\mathbf{D}$ are in phase. If they are out of phase this must be due to the polarization of the material. This depends on the electrical susceptibility χ_e of the material, which describes how easy the material is polarized $\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}$.

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \varepsilon_0 \chi_e \mathbf{E} = \varepsilon_0 \underbrace{(1 + \chi_e)}_{\varepsilon_r} \mathbf{E} = \varepsilon \mathbf{E}$$

where ε_r is the relative permittivity (dielectric constant) of the material. If ε_r is real there will be no phase difference between $\mathbf{E}$ and $\mathbf{D}$, but if ε_r is complex (or negative) $\varepsilon_r = |A|e^{i\varphi}$ it will introduce a phase difference of φ. Now, what is the physical meaning of a complex ε_r? ε is related to the speed of propagation c, which is related to the index of refraction n.

$$n = \frac{c_0}{c} = \frac{\sqrt{\varepsilon \mu}}{\varepsilon_0 \mu_0} = \sqrt{\varepsilon_r \mu_r} = \{\mu_r \approx 1 \text{ for most optical materials}\} \approx \sqrt{\varepsilon_r}$$

So a complex ε_r results in a complex index of refraction $n = \alpha + i\beta$. The real part α of the complex index of refraction is related to the propagation speed and the imaginary part is related to absorption or amplification.

$$\begin{aligned} \mathbf{E} = \mathbf{E}_0 e^{i(kz - \omega t)} &= \left\{ k = \frac{2\pi}{\lambda} = \frac{2\pi n}{\lambda_0} = k_0 n \right\} \\ &= \mathbf{E}_0 e^{i(k_0 z(\alpha + i\beta) - \omega t)} = \underbrace{\mathbf{E}_0 e^{-k_0 \beta z}}_{\text{amplitude}} \cdot e^{i(k_0 \alpha z - \omega t)} \end{aligned}$$

The intensity $I(z)$ will then decrease, or increase, exponentially in the following way,

$$I(z) \propto |\mathbf{E}|^2 = \mathbf{E} \cdot \mathbf{E}^* = |\mathbf{E}_0|^2 e^{-2k_0 \beta z}$$

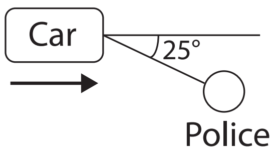
Session 2

Reflection, thin lenses

2.1 Problems

940406:3

A police standing beside the road that wants to study the use of seatbelts in cars has good use for a pair of polarizing sunglasses. How much is the reflection from the windscreen reduced when he wears them, if he is positioned according to the figure below?



960111:1

Assume that we have a 45-45-90-degree prism where a parallel laser beam is incident against one of the cathetus at normal incidence. The refractive index is 1.54 and the prism is surrounded by air. Is it possible to design an AR-coating (anti reflection) such that the hypotenuse has a transmittance higher than 10%?

031024:1

Modern cameras unfortunately often have a fixed objective that can not be changed. However, one can change the focal length of the system by attaching an afocal lens combination in front of the camera objective. The system focal length is thereby changed by a factor equal to the telescope magnification of the lens combination that was attached. Show this by using a ray diagram and conformal triangles! Make a nice figure that is easy to understand!

2.2 Solutions

940406:3

Polarizing sunglasses works by absorbing one polarization and transmitting the other. Sunlight is unpolarized but the reflection will be partly polarized so it can be reduced by the glasses.

The reflectance of the two polarizations are given by

$$R_\perp = \left[-\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}\right]^2, \quad R_\parallel = \left[\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}\right]^2$$

Without the glasses the intensity will be

$$I_{\text{w/o}} = \frac{1}{2} I_0 R_\perp + \frac{1}{2} I_0 R_\parallel .$$

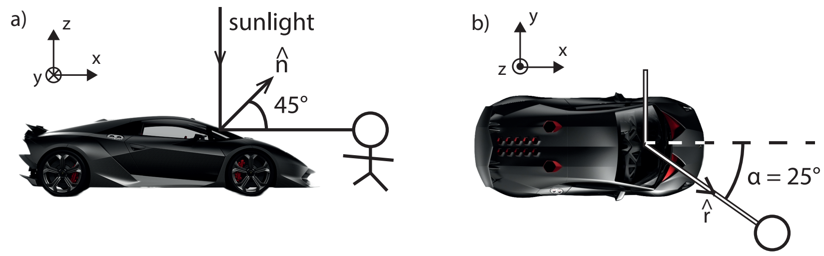
With glasses one polarization is blocked,

$$I_{\text{with}} = \frac{1}{2} I_0 R_*, \quad * = R_\perp \text{ or } R_\parallel$$

so the glasses will reduce the intensity from the reflection by a factor of

$$\frac{I_{\text{w/o}}}{I_{\text{with}}} = \frac{R_\perp + R_\parallel}{R_*}.$$

We now have to find θ_i and θ_t. Assume the angle of the windscreen to be 45°:



The normal to the windscreen is then

$$\hat{n} = \cos 45° \hat{x} + \sin 45° \hat{z} = \frac{1}{\sqrt{2}}(\hat{x} + \hat{z}),$$

and the direction of the reflected light is

$$\hat{r} = \cos\alpha\hat{x} + \sin\alpha\hat{y}$$

The angle between them is the reflection angle

$$\theta_r = \theta_i = \cos^{-1}(\hat{n} \cdot \hat{r}) = \cos^{-1}\left(\frac{\cos\alpha}{\sqrt{2}}\right) = 50°$$

We can now calculate the transmission angle from Snell's law. Assume that the refractive index of the windscreen is $n_1 = 1.5$.

$$n_0 \sin\theta_i = n_1 \sin\theta_t$$

$$\Rightarrow \theta_t = \sin^{-1}\left(\frac{n_0}{n_1}\sin\theta_i\right) = 31°$$

This gives us $R_\perp = 0.11$ and $R_\parallel = 0.003$.

If $R_\perp$ is blocked we get a reduction by a factor of

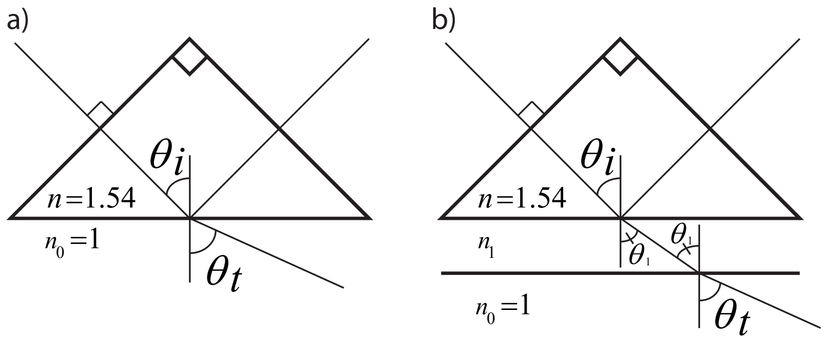
$$\frac{R_\perp + R_\parallel}{R_\parallel} = \frac{0.11 + 0.003}{0.003} \approx 37,$$

and if $R_\parallel$ is blocked

$$\frac{R_\perp + R_\parallel}{R_\perp} = \frac{0.11 + 0.003}{0.003} \approx 1.03.$$

In order to completely block one of the polarizations the police has to tilt his/her head $\tan^{-1}(\sin\alpha) = 23°$ to the right.

960111:1



Start by looking at the case without any coating (figure a). Snell's law gives us

$$n_0 \sin\theta_t = n \sin\theta_i = 1.09 > 1 \quad \Rightarrow \text{ Total internal reflection!}$$

If we add one layer with a refractive index of n_1 (figure b), we get

$$\begin{cases} n \sin\theta_i = n_1 \sin\theta_1 & \text{First surface} \\ n_1 \sin\theta_1 = n_0 \sin\theta_t & \text{Second surface} \end{cases}$$

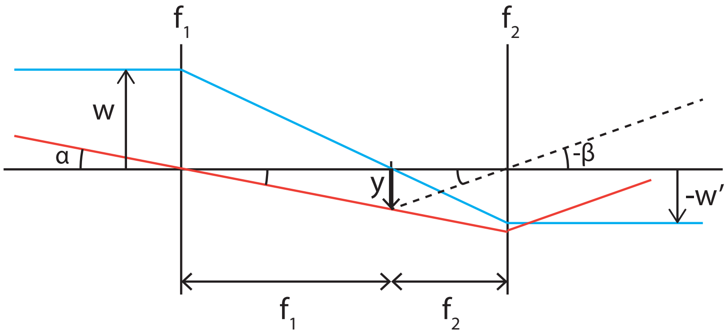
$$\Rightarrow n_0 \sin\theta_t = n \sin\theta_i = 1.09$$

Conclusion: We get total internal reflection independently of coating so no light can be transmitted. It is therefore not possible to design AR-coating so that the hypotenuse would have T>10%.

031024:1

Start by finding an expression for M_t.

Parallel beams entering an afocal system will exit the system in parallel:



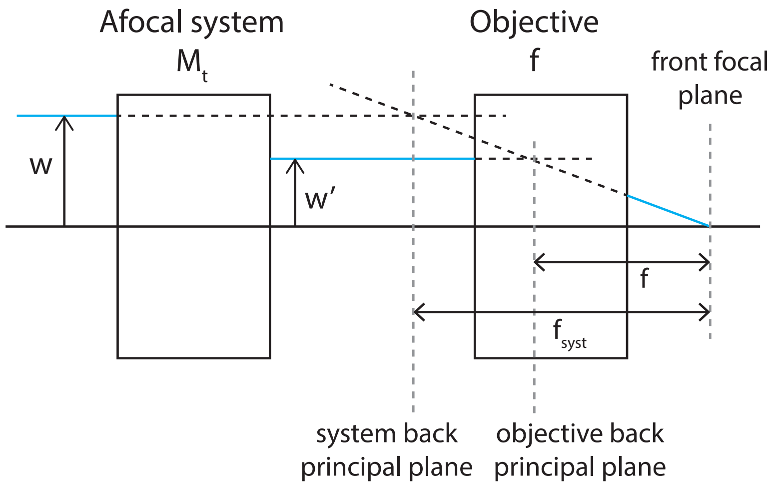
From the figure we can see that the angular magnification is given by

$$M_\mathrm{t} = \frac{\beta}{\alpha} = \frac{-y/f_2}{y/f_1} = -\frac{f_1}{f_2}.$$

Using similar triangles we see that this is equal to the ratio of the beam width before and after the system.

$$\frac{w}{f_1} = \frac{-w'}{f_2} \Rightarrow \frac{w}{w'} = -\frac{f_1}{f_2} = M_\mathrm{t}.$$

The combined system with both the telescope and camera objective will then look like



Again, using similar triangles we see that

$$\frac{f_\mathrm{syst}}{f} = \frac{w}{w'} = M_\mathrm{t}$$

which was to be demonstrated.

Session 3

Thin lenses, beam limiters

3.1 Problems

031020

Task 1-3 treat the same problem which is described below:
You have a laser beam which can be treated as circular, i.e. you don't have consider a Gaussian profile, and you want to build a telescope that can be adjusted in such a way that the magnification can be varied between a value smaller than 1 and one which is bigger than 1. You have the following lens package at your disposal.

a. a f = -20 mm lens with a diameter of 10 mm
b. a f = 50 mm lens with a diameter of 20 mm

The distance between the two lenses, d, can be varied from 35 mm to 50 mm.
Another lens, with f = -100 mm and a diameter of 40 mm, is placed at a distance (for you to calculate) after the lens system.
The laser beam is parallel (not divergent or convergent) both when entering and exiting the telescope.

031020:1-2
Construct the system, i.e. draw a figure with all distances marked for the case where d = 40 mm.
Between which values can the magnification be varied?

031020:3
Assume that the beam has a radius bigger than 4 mm, in which case diffraction will occur. What angular spread does this give rise to? Assume d to be 40 mm and that $\lambda = 514$ nm.

040112:1

An ophthalmologic biomicroscope is used for looking into the human eye. It has a special illumination system, the details of which are not given here. It has an objective with a focal length of 50 mm and an eyepiece with 20 mm focal length. The distance between the back principal plane of the objective and the front principal plane of the eyepiece is 220 mm.

The objective has a front vertex focal length of 90 mm.
How far into the eye is it possible to see if the eye is modeled by a sphere with a radius of curvature of 5 mm at cornea and a refractive index of 1.3306?

The distance from cornea to retina is such that a sharp image of a distant object is obtained on the retina.

040112:2

When the previous task is solved you will have found that it is not possible to see the retina since it is approximately 20 mm from cornea.
One way to observe the retina is to use a strong positive lens (40 D) to image the retina into a real image which is in turned viewed by the doctor who at the same time images the pupil of the patients eye onto his/her own pupil using the same 40 D lens. The patient's eye is relaxed, i.e. focused at distant objects.
How large is the field of view, i.e. how much of the retina can be observed in the following case?
Distances:
Patient's eye with pupil in front of cornea
26.7 mm distance
40 D lens
400 mm distance
The eye of the doctor
The lens has a diameter of 25 mm, i.e. the f# is 1

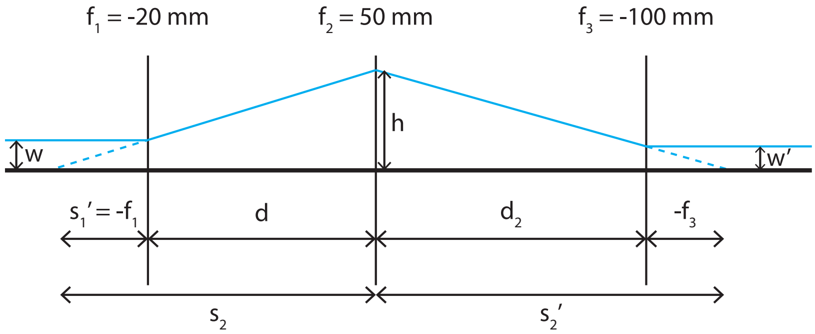
031024:2

A microscope has an objective with f = 3.00 mm and a diameter of 2 mm and use a Huygens eyepiece with f = 20 mm. The tube length (distance from objective to the front focal plane of the eyepiece) is 200 mm. What limitations on the size of the field lens and the eye-lens are there if the field of view must be larger than 0.25 mm?

3.2 Solutions

031020

1-2



1) Calculate d_2 for d = 40 mm

Lens 3 should be placed so that the final image lies in infinity. This is done by placing it so that its front focal plane is at the same position as the intermediate image from lens 2 (see figure). For d = 40 mm,

$$\begin{aligned} & s_2 = d - f_1 = 60 \text{ mm} \\ \Rightarrow \quad & s_2' = \frac{s_2 f_2}{s_2 - f_2} = 300 \text{ mm} \\ \Rightarrow \quad & d_2 = s_2' + f_3 = 200 \text{ mm} \end{aligned}$$

Answer: For $d = 40$ mm, $d_2 = 200$ mm.

2) How can the magnification be varied?
The angular magnification is given by

$$M_t = \frac{\beta}{\alpha} = \frac{w}{w'}$$

From similar triangles in the figure:

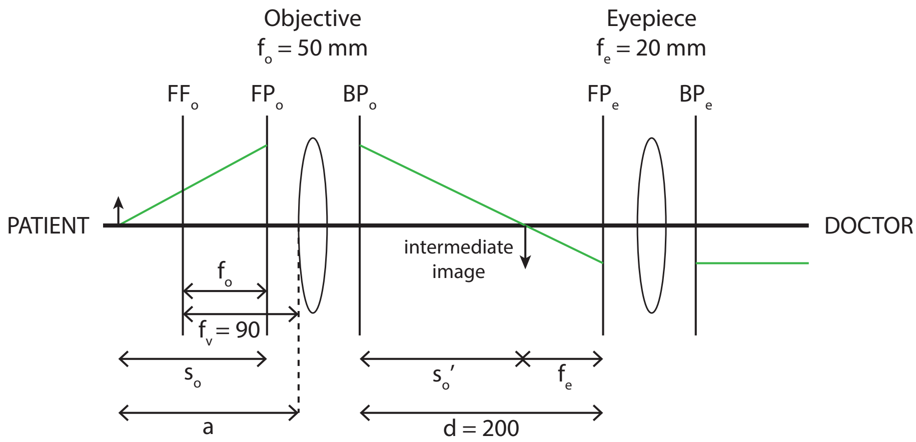
$$\begin{aligned} & \frac{w}{h} = \frac{-f_1}{s_2}, \quad \frac{h}{w'} = \frac{s_2'}{-f_3} \\ \Rightarrow \quad & M_t = \frac{w}{w'} = \frac{s_2'}{s_2}\frac{f_1}{f_3} = \left\{ s_2' = \frac{s_2 f_2}{s_2 - f_2} \right\} = \\ & = \frac{f_2}{s_2 - f_2}\frac{f_1}{f_3} = \{s_2 = d - f_1\} = \frac{f_1 f_2}{f_3(d - f_1 - f_2)} \end{aligned}$$

The distance could be varied between 35 mm and 50 mm which then corresponds to $M_t = 2$ and $M_t = 0.5$, respectively.

Answer: $0.5 \geq M \geq 2$

3
0.094 mrad

040112:1



We have a microscope so we want the final image to be in infinity. Combining that with the information in the text gives a system like the one in the figure above.

We want to find the working distance of the camera, a. From the figure:

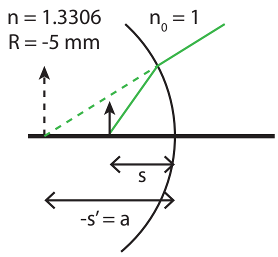
$$a = f_v + s_o - f_o$$

The object distance, s_o, can be calculated from the lens formula

$$s_o = \frac{s_o' f_o}{s_o' - f_o} = \{s_o' = d - f_e\} = 66.7 \text{ mm}$$

so the working distance becomes $a = 106.7$ mm.

An image at a distance $s' = -a$ from the cornea corresponds to an object at a distance s inside the eye.



Refraction in a spherical surface:

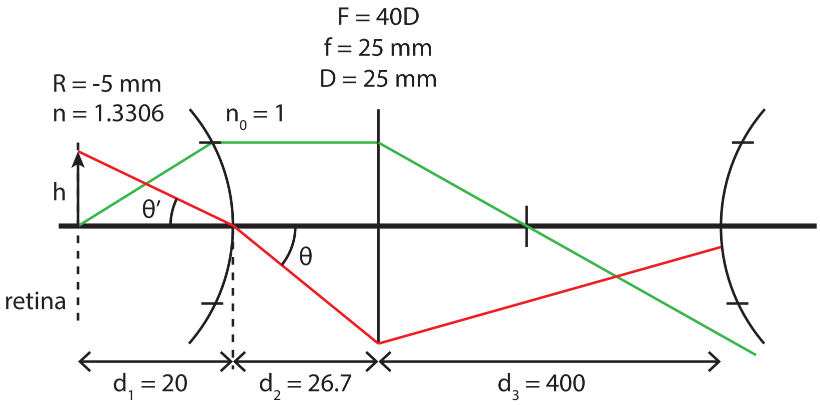
$$\frac{n}{s} + \frac{n_0}{s'} = \frac{n_0 - n}{R} \Rightarrow s = \frac{n}{\frac{n_0 - n}{R} - \frac{n_0}{s}} = 17.6 \text{ mm}$$

It's possible to see 17.6 mm into the eye!

$s' = \infty$ gives the position of the retina

$$s_r = \frac{nR}{n_0 - n} = 20 \text{ mm}$$

040112:2



Recipe for finding the field of view (FOV):

1. Find the aperture stop (AS); the element that limits the rays from the center of the object
2. Find the entrance pupil (ENP); the image of the AS to the left
3. Find the field stop (FS); the element that limits rays from the center of the ENP (or AS)
4. The ray going from the center of the ENP to the edge of the FS is a good measure of the FOV.

Solution:

1. The doctor's eye is the AS
2. The ENP is the image of the AS:

$$s' = \frac{sf}{s-f} = 26.7 \text{ mm}$$

   ENP is located at the patient's eye!
3. The lens is the FS
4. The FOV is given by θ in the figure.

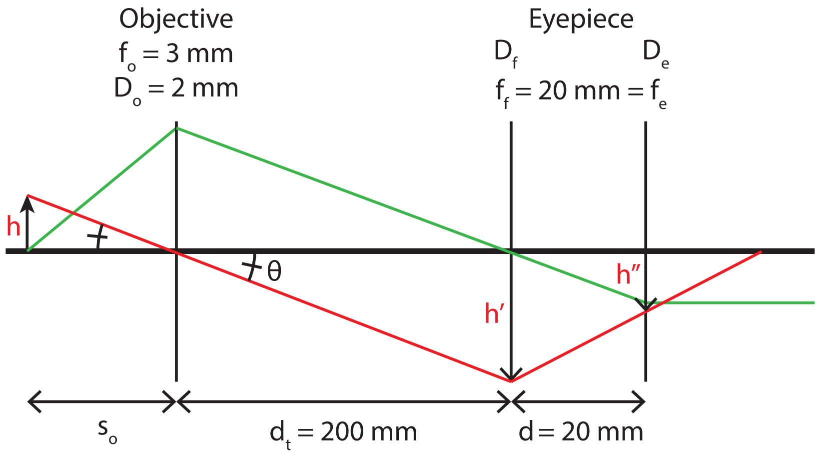
$$\theta = \tan^{-1}\left(\frac{D/2}{d_2}\right) = 25^\circ$$

How much of the retina can be observed? We need to find θ'!

$$\begin{aligned} & n \sin\theta' = n_0 \sin\theta \\ \Rightarrow \quad & \theta' = \sin^{-1}\left(\frac{n_0}{n}\sin\theta\right) = 19^\circ \\ \Rightarrow \quad & h = d_1 \tan(\theta') = 6.7 \text{ mm} \end{aligned}$$

We can observe $2h = 13.4$ mm of the retina.

031024:2



The Huygens eyepiece has a field lens with the same focal length as the eye lens. The distance between the lenses is the same as the focal length so that the lenses are placed in each others focal planes. We have a microscope so the final image should be in infinity and the objective should be the aperture stop (AS). From the green ray figure

$$\frac{D_e}{d} \geq \frac{D_o}{d_f} \quad \Rightarrow \quad D_e \geq \frac{dD_o}{d_t} = 0.2 \text{ mm}$$

The field lens will then become the field stop (FS). The field of view (FOV) is given by the ray going through the center of the AS to the edge of the FS (the red ray in the figure). Since the FOV should be larger than $2h = 0.25$ mm we can calculate the lower limit of the diameter of the field lens, $D_f \geq 2h'$

$$s_o = \frac{s'_o f_o}{s'_o - f_o} = \{s'_o = d_t\} = \frac{d_t f_o}{d_t - f_o} = 3.05 \text{ mm}$$
$$\frac{h'}{d_t} = \frac{h}{s_o} \quad \Rightarrow \quad D_f \geq 2h' = \frac{2hd_t}{s_o} = 16.4 \text{ mm}$$

The eye lens must also be large enough to include the ray from the edge of the FS. This will give us an additional limitation on its diameter, $D_e \geq 2h''$

$$f'_f = \frac{d_t f_f}{d_t - f_f} = 22.2 \text{ mm}$$
$$\frac{h''}{s'_f - d} = \frac{h'}{s'_f} \quad \Rightarrow \quad D_e \geq 2h'' = \frac{2h'(s'_f - d)}{s'_f} = 1.64 \text{ mm}$$

Answer: $D_f \geq 16.4$ mm, $D_e \geq 1.64$ mm.

Session 4

Think lenses, superposition

4.1 Problems

031024:6

An optimistic person tries to manufacture a telescope using a lens with f = 1200 mm, and a diameter of 50 mm, as the objective. The lens is shaped to minimize spherical aberrations when turned correctly, and it's made of crown glass (use the book for data). He uses a commercially available ocular with f = 20 mm. Assume that Jupiter is viewed using this telescope and that it has an angular diameter as seen from Earth ~0.3 mrad. How much bigger (%) is the image in blue and red compared with yellow/green (D-wavelength)? Jupiter is placed symmetrically around the symmetry axis of the telescope.

040112:6

Three different situations occur when imaging a detailed image using a slide projector onto a screen.

a. Only the central part of the image is sharp. Refocusing makes the peripheral part of the image sharp but blurs the central part.

b. Radial structures in the image are sharp. Refocusing leads to structures being orthogonal against lines from the center of the image sharp.

c. Straight lines in the periphery of the image are bent.

Which aberrations occur in the three different cases? Motivate!

930825:5

The atoms in a sodium lamp emit light according to $E = E_0 \sin(\omega_0 t) e^{-\gamma t}$ unless they are not disturbed. However, in a high pressure lamp the many collisions force the emission of light to start over. This means that the expression above is changed by multiplying with a rect function with a width equal to the mean time τ between the collisions. Derive an expression for how the spectrum is broadened by this. (Express the spectra in frequency and not in wavelength)

940406:2

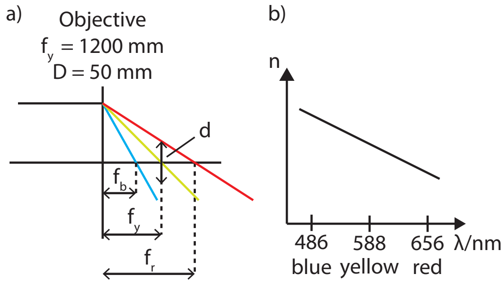
Several technologies can be used when the police measure the speed of an approaching car using a laser. The most common method is to measure the change in pulse-travel-time between two pulses in a pulse train from a laser. Assume that one want to generate pulses by letting the modes in a laser interfere thereby creating a beating.

Assume that these are separated in frequency with 300 MHz and that their initial phases are locked against each other (this can be arranged). How many modes must contribute in order to give 100 ps long pulses (full width at half maximum)? The wavelength is 900 nm.

One might find it useful to look at the derivation of the diffraction pattern from a grating!

4.2 Solutions

031024:6



Aberrations in the optics will degrade the image. A "commercially available ocular" implies that all aberrations have been minimized in the ocular. Therefore, all aberrations in the image will come from the objective.

For the objective we know that:

- No spherical aberrations
- Object on axis → no off-axis aberrations (astigmatism, coma, field curvature and distortion)

⇒ Only chromatic aberrations in the objective will degrade the image.

Chromatic aberrations arise due to the material having a wavelength dependent refractive index (dispersion). Light of different wavelengths will therefore be refracted differently (figure a) Dispersion can be characterized by the Abbe number,

$$V_d = \frac{n_y - 1}{n_b - n_r} = \left\{ \begin{matrix} \text{Figure b:} \\ \Delta n \approx n_b - n_y \approx n_y - n_r \end{matrix} \right\} = \frac{n_y - 1}{2\Delta n}$$

The focal length for red light is then

$$\begin{aligned} \frac{1}{f_r} &= (n_r - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n_y - \Delta n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \\ &= \left(1 - \frac{\Delta n}{n_y - 1}\right)(n_y - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(1 - \frac{1}{2V_d}\right)\frac{1}{f_y} \\ \Rightarrow \quad f_r &= \frac{1}{1 - \frac{1}{2V_d}} f_y = \left\{ \begin{matrix} f_y = 1200 \text{ mm} \\ V_d = 59.6 \text{ (p. 270)} \end{matrix} \right\} = 1210 \text{ mm} \end{aligned}$$

The image is in focus for yellow. At this distance, a red point will become a disc with diameter d (figure a)

$$d = (f_r - f_y)\frac{D}{f_r} = 0.42 \text{ mm}$$

An object with angular diameter α is shown in figure c. The yellow image is sharp, so its size is

$$d_y = f_y \alpha = 0.36 \text{ mm}$$

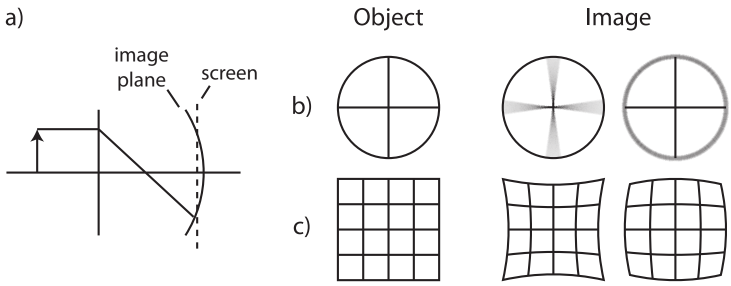
In red we will see the convolution, with a diameter of

$$d_r = d_y + d = 0.78 \text{ mm}$$

The red image is thus $\frac{d_r}{d_y} - 1 \approx 120\%$ larger than the yellow image. Due to symmetry the red and the blue images will have the same sizes.

Answer: The red/blue images are 120% larger than the yellow image.

040112:6



a) The image plane is bent → Field curvature

b) Different focus in two orthogonal planes → Astigmatism

c) Straight lines are bent → Distortion

930825:5

The field is given by

$$E(t) = \begin{cases} E_0 \sin(\omega_0 t) e^{-\gamma t}, & 0 < t < \tau \\ 0, & \text{otherwise} \end{cases}$$

The spectrum is the Fourier transform of the field.

$$\begin{aligned}
\hat{E}(\omega) &= \int_{-\infty}^{\infty} E(t) e^{-i\omega t} dt = \int_0^\tau E_0 \sin(\omega_0 t) e^{-\gamma t - i\omega t} dt \\
&= \int_0^\tau \frac{E_0}{2i} \left(e^{i\omega_0 t} - e^{-i\omega_0 t}\right) e^{-\gamma t - i\omega t} dt \\
&= \frac{E_0}{2i} \int_0^\tau e^{[-\gamma - i(\omega - \omega_o)]t} - e^{[-\gamma - i(\omega + \omega_o)]t} dt \\
&= \frac{E_0}{2i} \left[\frac{e^{[-\gamma - i(\omega - \omega_o)]t}}{-\gamma - i(\omega - \omega_o)} - \underbrace{\frac{e^{[-\gamma - i(\omega + \omega_o)]t}}{-\gamma - i(\omega + \omega_o)}}_{\omega + \omega_0 > \omega - \omega_0 \Rightarrow \approx 0} \right]_0^\tau \\
&= \{\Delta\omega = \omega - \omega_0\} \approx \frac{E_0}{2i} \frac{e^{-(i\Delta\omega + \gamma)\tau} - 1}{(\Delta\omega)^2 + \gamma^2}
\end{aligned}$$

The intensity is then

$$I(\omega) = |E(\omega)|^2 = \frac{E_0^2}{4} \frac{\left|e^{(i\Delta\omega+\gamma)\tau} - 1\right|^2}{(\Delta\omega)^2 + \gamma^2}$$

940406:2

Interference between modes gives beating.

$$E_{tot} = \sum_{n=0}^{N-1} E_n e^{i[kx-(\omega+n\Delta\omega)t]} = \{E_n = E_0\} = \underbrace{E_0 e^{i(kx-\omega t)}}_{A} \sum_{n=0}^{N-1} e^{(-i\Delta\omega t)^n} =$$

$$= \left\{ \sum_{n=0}^{N-1} x^n = \frac{x^N - 1}{x - 1} \right\} = A \, \frac{e^{-iN\Delta\omega t} - 1}{e^{-i\Delta\omega t} - 1} =$$

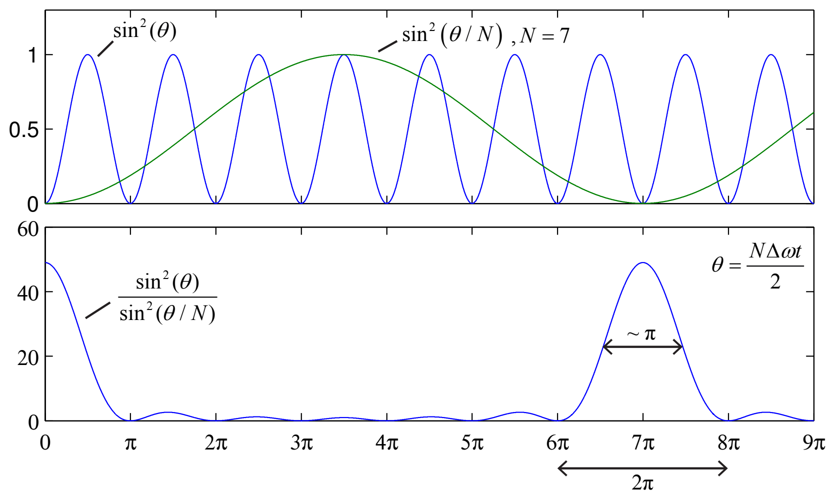
$$= A \, \frac{e^{-iN\Delta\omega t/2}\left(e^{-iN\Delta\omega t/2} - e^{iN\Delta\omega t/2}\right)}{e^{-i\Delta\omega t/2}\left(e^{-i\Delta\omega t/2} - e^{i\Delta\omega t/2}\right)} = \left\{ \sin x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right) \right\} =$$

$$= A e^{-i(N-1)\Delta\omega t/2} \frac{-2i\sin(N\Delta\omega t/2)}{-2i\sin(\Delta\omega t/2)}.$$

The intensity is

$$I = |E_{tot}|^2 = E_0^2 \, \frac{\sin^2(N\Delta\omega t/2)}{\sin^2(\Delta\omega t/2)}.$$

$\sin^2(N\Delta\omega t/2)$ oscillates rapidly and decides the pulse length.
$\sin^2(\Delta\omega t/2)$ oscillates slowly and decides the pulse repetition rate.



The pulse width is $\Delta\theta \approx \pi$

$$\pi = \Delta\theta = \frac{N\Delta\omega\Delta t}{2}$$

$$\Rightarrow \quad N = \frac{2\pi}{\Delta\omega\Delta t} = \frac{1}{\Delta f \Delta t} = \left\{ \begin{matrix} \Delta f = 300 \text{ MHz} \\ \Delta t = 100 \text{ ps} \end{matrix} \right\} = 33$$

Answer: We need about 33 modes.

Session 5

Polarization, birefringence

5.1 Problems

031020:5

Quarter- and half-wavelength plates are described in the book, but there also exist full-wavelength plates in the so called reality. These are commonly of the first order, i.e., the phase difference between *o* and *eo* is a wavelength for the design wavelength, and they are positioned between two crossed polarizers. Assume that we have such a full-wavelength plate, with a design wavelength of 550 nm, and that it is illuminated by white light. The optical axis is oriented 45° against the let-through direction of the polarizer. Additional transparent objects are positioned between the two polarizers, which partially cover the field of view with a weak birefringence, i.e., $\sim$1/6 wavelength for 550 nm. What does the background look like and what does the object look like? Observe that different situations occur depending on the direction of the optical axis of the object.

040112:4

Assume that we have a birefringent crystal that can be controlled optically. It is positioned at a 45° angle between two crossed polarizers. The geometrical path length through the crystal is 5.00 mm. What value on the birefringence (n_{eo}-n_o) makes the transmitted light appear:

a. Yellow

b. Blue

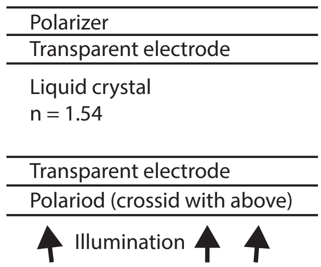
950419:3

Two polarizers can be useful to limit the amount of transmitted light. However, they are seldom ideal but has a limited transmission, T_p, also in the let-through direction, and the transmission T_s is not zero for light polarized orthogonal against it. Assume that we have two polarizers with $T_p = 0.94$ and $T_s = 0.03$.

They are positioned with their let-through directions orthogonal to each other. How much light passes through this combination and how is the transmitted light polarized?

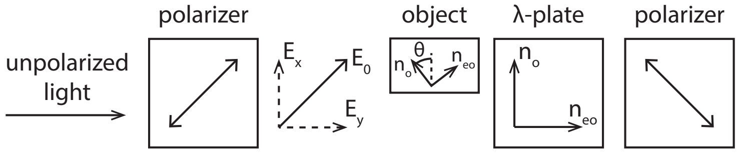
950301:1

Older LCD screens for notebooks use liquid crystals to influence the polarization according to the figure below. A certain color was obtained by adapting the voltage to give transmission maxima for that certain color. However, this produces very pale colors since also surrounding wavelengths are partly transmitted. Assume that one want to produce green (546 nm). Determine analytically and plot the transmitted intensity as a function of wavelength.



5.2 Solutions

031020:5



λ-plate only
Since the E-field is directed 45° relative the optical axis of the λ-plate, the x and y components of the E-field will experience different refractive indices when transversing the crystal. This will induce a relative phase shift of the two components,

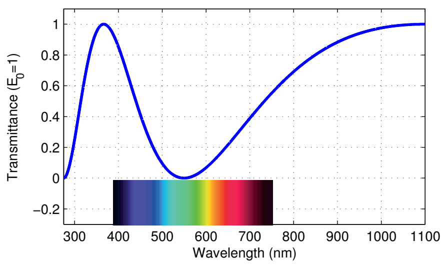
$$\Delta\varphi = \frac{2\pi}{\lambda} d\Delta n, \quad \Delta n = n_o - n_{eo}$$

It's designed for green, so $d\Delta n = \lambda_g = 550$ nm
$\Rightarrow \Delta\varphi_g = 2\pi$
$\Rightarrow$ no transmission for green

Which wavelengths give max/min transmittance?

$$\lambda = \frac{2\pi}{\Delta\varphi} d\Delta n = \{d\Delta n = \lambda_g\} = \frac{2\pi}{\Delta\varphi}\lambda_g$$

$\Delta\varphi$	λ/nm	T
2π	550	0%
π	1100	100%
3π	366	100%
4π	275	0%

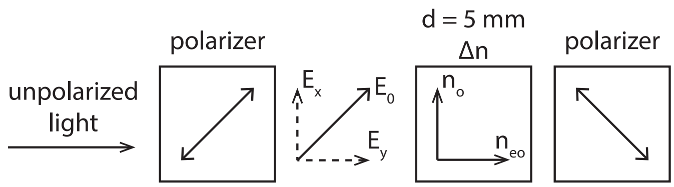


Green will not be transmitted but some blue and red $\Rightarrow$ it will look purple

With object (λ/6-plate) We get three cases:

- $\theta = 45°$: No effect
- $\theta = 0°$: Phase shifts add up.
  Designed for green $\Rightarrow \ \Delta\varphi_g = 2\pi(1 + 1/6)$
  Minimum transmission when $\Delta\varphi = 2\pi$
  $$\frac{\Delta\varphi}{\Delta\varphi_g} = \frac{\lambda_g}{\lambda} \quad \Rightarrow \quad \lambda = \frac{\Delta\varphi_g}{\Delta\varphi}\lambda_g = (1 + 1/6)\lambda_g = 642 \text{ nm}$$
  Red is absorbed and blue transmitted so it will look blue.
- $\theta = 90°$: Phase shifts subtract.
  Minimum transmission at $\lambda = (1 - 1/6)\lambda_g = 458$ nm.
  Blue is absorbed so it will look red.

040112:4



The phase shift introduced by the crystal is

$$\Delta\varphi = \frac{2\pi}{\lambda} d\Delta n, \quad \Delta n = n_o - n_{eo}$$

Maximum transmission: $\Delta\varphi = (2m+1)\pi$
Minimum transmission: $\Delta\varphi = 2\pi m$

a) Yellow light
Yellow contains red and green but not blue so we want a minima at $\lambda = 450$ nm and a maxima around 600 nm. No blue:

$$2\pi m = \frac{2\pi}{\lambda_b} d\Delta n \quad \Rightarrow \quad \Delta n = \frac{m\lambda_b}{d}$$

Next max occurs at

$$2\pi m - \pi = \frac{2\pi}{\lambda_{\max,2}} d\Delta n \quad \Rightarrow \quad \lambda_{\max,2} = \frac{2d\Delta n}{2m-1} = \frac{2m\lambda_b}{2m-1}$$

m	1	2	3
$\lambda_{\max,2}$	900 nm	600 nm	540 nm

m=2 gives yellow and $\Delta n = 2\lambda_b/d = 1.8 \cdot 10^{-4}$

b) Blue light
Blue has no red and green. Max for $\lambda_b = 450$ nm:

$$(2m+1)\pi = \frac{2\pi}{\lambda_b} d\Delta n \quad \Rightarrow \quad \Delta n = \frac{(2m+1)\lambda_b}{2d}$$

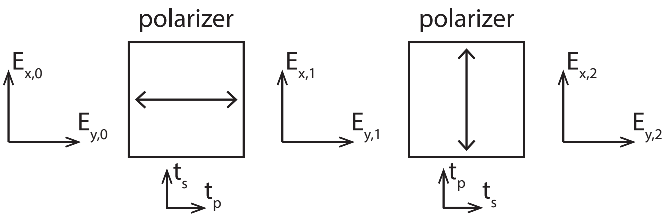
Next min and max at

$$2\pi m = \frac{2\pi}{\lambda_{\min}} d\Delta n \quad \Rightarrow \quad \lambda_{\min} = \frac{2m+1}{2m}\lambda_b$$
$$2\pi m - \pi = \frac{2\pi}{\lambda_{\max,2}} d\Delta n \quad \Rightarrow \quad \lambda_{\max,2} = \frac{2m+1}{2m-1}\lambda_b$$

m	1	2	3
$\lambda_{\min}$	675 nm	563 nm	525 nm
$\lambda_{\max,2}$	1350 nm	750 nm	630 nm

m=1 will look greenish, m=2 blue and m=3 purple.
We wanted blue so m=2 gives $\Delta n = 1.35 \cdot 10^{-4}$.

950419:3



The amplitude transmittances are given by $t_s = \sqrt{T_s}$ and $t_p = \sqrt{T_p}$. The x and y components of the electric field after the last polarizer is then given by

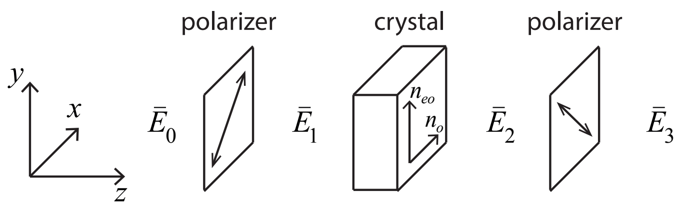
$$\begin{cases} E_{x,2} = t_s E_{x_1} = t_s t_p E_{x,0} \\ E_{y,2} = t_p E_{y_1} = t_p t_s E_{y,0}. \end{cases}$$

The transmitted field will have the same polarization!
The total transmittance is

$$\frac{I_2}{I_0} = \frac{|E_2|^2}{|E_1|^2} = \frac{(E_{x,2})^2 + (E_{y,2})^2}{(E_{x,0})^2 + (E_{y,0})^2} = (t_s t_p)^2 = T_s T_p = 0.0028$$

Answer: 2.8% is transmitted and the field will have the same polarization.

950301:1



We have unpolarized light incident on the first polarizer. The field after the polarizer, $\hat{E}_1$, will be polarized and have its intensity reduced to half.

$$I_1 = \frac{I_0}{2} \quad \Rightarrow \quad \hat{E}_1 = \frac{E_0}{\sqrt{2}} \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \underbrace{e^{i(kx - \omega t)}}_{e^{i\varphi}} = \frac{E_0}{2}(1,1)e^{i\varphi}$$

The crystal will introduce a phase shift, $e^{-i\Delta\varphi}$, to one of the components

$$\hat{E}_2 = \frac{E_0}{2}(1, e^{-i\Delta\varphi})e^{i\varphi}$$

The total field behind the second polarizer is then

$$E_3 = \hat{E}_3 \cdot \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right) = \frac{E_0}{2\sqrt{2}} \left(1 - e^{-i\Delta\varphi}\right) e^{i\varphi},$$

and the intensity

$$I_3 \propto |E_3|^2 = E_3 E_3^* = \frac{E_0^2}{8}\left(1 - e^{-i\Delta\varphi}\right)\left(1 - e^{i\Delta\varphi}\right)$$
$$= \frac{E_0^2}{8}\left(1 - e^{i\Delta\varphi} - e^{-i\Delta\varphi} + 1\right)$$
$$= \frac{E_0^2}{4}\left(1 - \cos(\Delta\varphi)\right)$$

The phase shift is given by

$$\Delta\varphi = \frac{2\pi}{\lambda} d\Delta n.$$

We want maximum transmission for green, $\lambda_g = 546$ nm, so $\Delta\varphi = (2m+1)\pi$

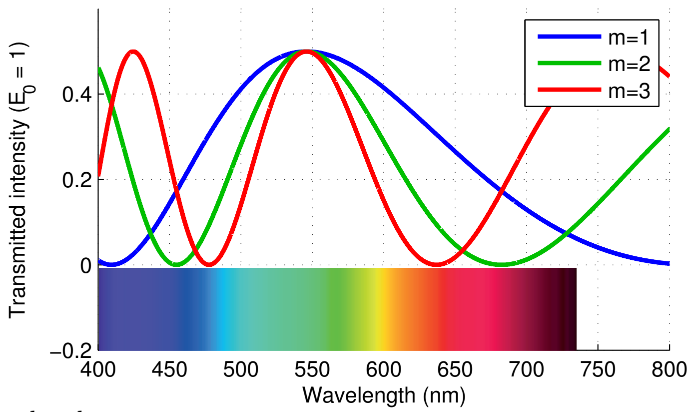
$$(2m+1)\pi = \frac{2\pi}{\lambda_g} d\Delta n \quad \Rightarrow \quad d\Delta n = \frac{(2m+1)\lambda_g}{2}$$

The phase shift for an arbitrary wavelength is thus

$$\Delta\varphi = \frac{2\pi}{\lambda} d\Delta n = \left\{ d\Delta n = \frac{(2m+1)\lambda_g}{2} \right\} = \frac{\lambda_g(2m+1)\pi}{\lambda}$$

so the transmitted intensity as function of the wavelength can be written as

$$I_3 \propto \frac{E_0^2}{4}\left(1 - \cos\left(\frac{\lambda_g(2m+1)\pi}{\lambda}\right)\right)$$



m=2 gives the clearest green.

Session 6

Interference

6.1 Problems

031020:6

Assume you have a laser based Michelson interferometer (the same kind as in the laboration in this course). What happens to the interference pattern if you turn the beam splitter just a bit (a few mrad)? The interferometer was perfectly aligned from the beginning.

031024:4

In a Mach-Zendner interferometer two interference patterns are obtained which are said to be complementary (from the book). Show this for a simple case, for example a beam splitter with a high index of refraction which has one side AR coated (the effect of the AR coating can be neglected).

930402:4

This Michelson interferometer is based on a laser with a wavelength of 513 nm. The laser beam first passes though a negative lens with f = -20 mm and then through a positive lens with f = 200 mm. The distance between the lenses is 160 mm. The optical path length from the last lens to the end screen is for the first arm 1200 mm and for the second 1210 mm. How many interference fringes are visible with a radius of 80 mm?

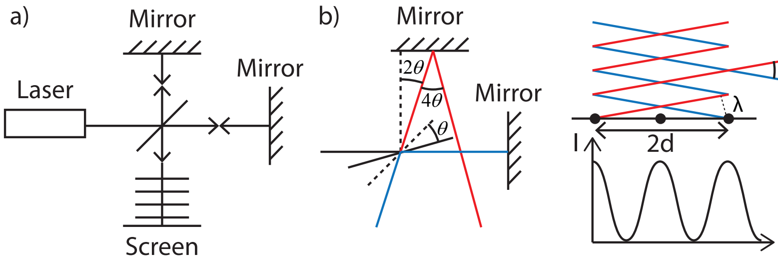
030107:3

The image of a red (620 nm) text display is to be mirrored into the path of light (in the projector mentioned above. This is done with a glass plate in 45° covered with a thin layer with n=1.80. How thick should this be in order to get maximum reflectance for red but minimal for blue through green?

6.2 Solutions

031020:6

In a perfectly aligned interferometer the wavefronts are paralle (figure a).

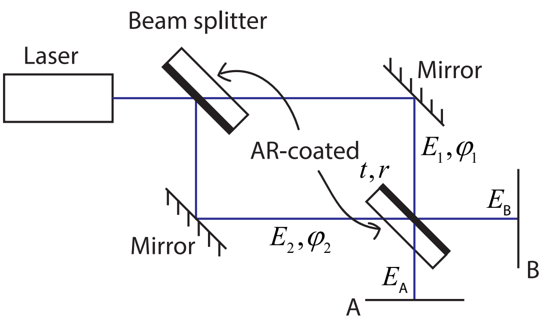


Turning the mirror slightly will change the angle between the wavefronts (figure b). This will give fringes with a period of

$$2\theta = \frac{\lambda}{2d} \quad \Rightarrow \quad d = \frac{\lambda}{4\theta}$$

Ex: If $\lambda = 600$ nm and $\theta = 5$ mrad, the distance between the fringes is 30 µm.

031024:4



Interference:

$$I_{\text{tot}} = \frac{\epsilon_0 c}{2} |E_{\text{tot}}|^2 = |E_1|^2 + |E_2|^2 + 2E_1 E_2 \cos(\Delta\varphi)$$

The intensity at the screens is

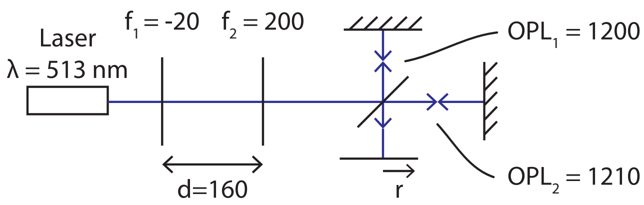
$$I_{\text{A}} = \frac{\epsilon_0 c}{2} |E_{\text{A}}|^2 = \frac{\epsilon_0 c}{2} \left(|tE_1|^2 + |rE_2|^2 + 2tE_1 rE_2 \cos(\varphi_2 - \varphi_1) \right)$$
$$I_{\text{B}} = \frac{\epsilon_0 c}{2} |E_{\text{B}}|^2 = \frac{\epsilon_0 c}{2} \Big(|rE_1|^2 + |tE_2|^2 + 2rE_1 tE_2 \underbrace{\cos(\varphi_2 - (\varphi_1 + \pi))}_{-\cos(\varphi_2 - \varphi_1)} \Big)$$

The sum of the fields at the two screens is then

$$I_{\text{A}} + I_{\text{B}} = \frac{\epsilon_0 c}{2} \Big[\underbrace{(t^2 + r^2)}_{1} E_1^2 + \underbrace{(t^2 + r^2)}_{1} E_2^2 \Big] = \frac{\epsilon_0 c}{2} \left(E_1^2 + E_2^2 \right)$$

The sum is constant (independent of phase), so they are complementary.

930402:4

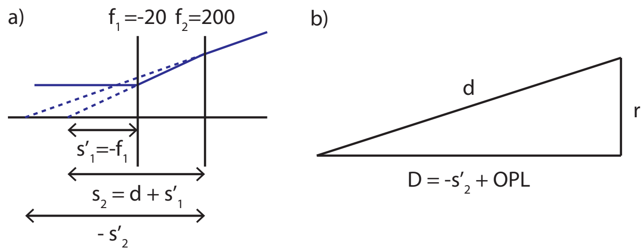


Without the lenses, the beams going through the two arms will be parallel and the difference in optical path length (ΔOPL) at the screen will be independent of the position of the screen. The lenses will refract the light giving a different change in the OPL at different parts of the screen, $\Delta\text{OPL}(r)$.

The number of fringes within two radii r_1 and r_2 will thus be

$$m = \frac{\Delta\text{OPL}(r_2) - \Delta\text{OPL}(r_1)}{\lambda}.$$

We need to find $\Delta\text{OPL}(r)$!



The laser is imaged through the lens system (figure a). The final image of the laser after the second lens is located at

$$s_2' = \frac{s_2 f_2}{s_2 - f_2} = \left\{ \begin{matrix} s_2 = 160 + 20 = 180 \text{ mm} \\ f_2 = 200 \text{ mm} \end{matrix} \right\} = -1800 \text{ mm}.$$

The light therefore seems to come from a point source 1800 mm before the lens. The distance from this point to a radius r on the screen is then (figure b)

Arm 1

$$d_1 = \sqrt{r^2 + D_1^2}, \quad D_1 = 1800 + 1200 = 3000 \text{ mm}$$

Arm 2

$$d_2 = \sqrt{r^2 + D_2^2}, \quad D_2 = 1800 + 1210 = 3010 \text{ mm}$$

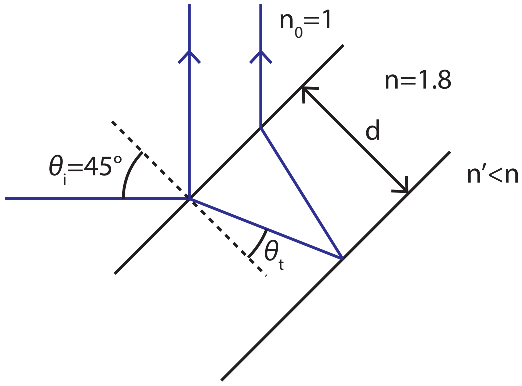
The difference in path length is thus

$$\Delta\text{OPL}(r) = d_2 - d_1 = \sqrt{r^2 + D_2^2} - \sqrt{r^2 + D_1^2}$$

For $r_2 = 80$ mm and $r_1 = 0$ we get

$$m = \frac{\Delta\text{OPL}(80 \text{ mm}) - \Delta\text{OPL}(0)}{\lambda} = 6.9.$$

030107:3



The phase shift between the two reflections is

$$\Delta\varphi = \frac{2\pi}{\lambda} 2nd\cos\theta_t + \pi$$

where the additional π is due to an odd number of reflections against a denser medium. We can calculate the transmission angle from Snell's law:

$$n_0 \sin\theta_i = n\sin\theta_t \quad \Rightarrow \quad \theta_t = \sin^{-1}\left(\frac{n_0}{n}\sin\theta_i\right) = 23°$$

We want constructive interference for $\lambda_r = 620$ nm, so $\Delta\varphi_r$ should be a multiple of 2π

$$2\pi m = \frac{2\pi}{\lambda_r} 2nd\cos\theta_t + \pi \quad \Rightarrow \quad d = \frac{(2m-1)\lambda_r}{4n\cos\theta_t}$$

For this thickness, the wavelengths giving destructive interference is

$$(2m+1)\pi = \frac{2\pi}{\lambda_{\min}} 2nd\cos\theta_t + \pi$$
$$\Rightarrow \quad \lambda_{\min} = \frac{2nd\cos\theta_t}{m} = \frac{(2m-1)\lambda_r}{2m}$$

Different values of m gives different $\lambda_{\min}$:

m	1	2	3
$\lambda_{\min}$/nm	310	465	516

$m = 1$ reflects both blue and green. $m = 3$ doesn't reflect green but has a maxima for blue, so $m = 2$ is best. This gives us

$$d = \frac{3\lambda_r}{4n\cos\theta_t} = 280 \text{ nm}.$$

Session 7

Diffraction

7.1 Problems

031024:3

A ring aperture has some advantages over normal apertures when it comes to resolution. Investigate what these advantages can be by plotting (in the same graph) the intensity distribution of:

a. a circular aperture with diameter D

b. a circular aperture with diameter 0.8D

c. a ring aperture with inner diameter 0.8D and outer diameter D

It is important that the zero intensity positions of the different patterns can be seen. In which way is the ring aperture better?

031024:5

Assume you have a phase grating where the amplitude transmission varies between 1 and -1. The period is 1000 nm and the light is incident perpendicular to the plane of the grating. The wavelength of the light is 500 nm. In which angles do you get intensity maxima?

040112:3

One way to make contact lenses is to add zone plates on top of the normal contact lens. Zone plates have a very strong reversed dispersion compared to normal lenses. The power of a zone plate increases with increasing wavelength. What is the dispersion of zone plates expressed in Abbe number?

040112:5

An object consisting of a row of 10 equidistant small holes is imaged using a diffraction limited lens with NA = 0.20. For which distance, expressed in wavelengths, between the holes will the 1st diffraction order fall outside the numerical aperture of the lens? Compare this result with Rayleigh's resolution criteria and comment on differences if there are any.

940824:4

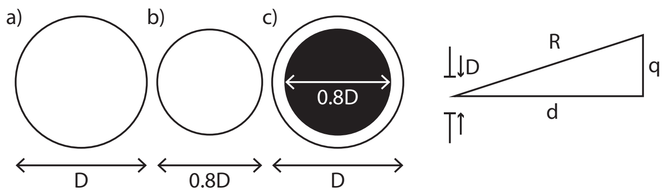
In some electronic components a copper wire grid is used (about 10 wires/mm in a squared grid pattern). It is very important that the wires are intact and that they have the correct position in the grid. It is suggested that the grid will pass by the object plane of a Fourier optical arrangement. The detector placed in the image plane of the arrangement should only detect light if there is an error in the copper grid, not otherwise. Discus different placements and designs of possible filters and select the one you would choose. Make a drawing with distances marked.

930825:4

Three point-like light sources are placed at a distance B from a double slit. The distance between the point sources is A, the central one being centered in front of the double slit. The distance between the two slits is D. A screen is placed on the other side of the double slit at distance C. If the conditions are correct a diffraction pattern will be visible on the screen. Which is the smallest distance A (except A = 0) where the visibility of the pattern is close to 1? The wavelength of the light can be set to λ.

7.2 Solutions

031024:3



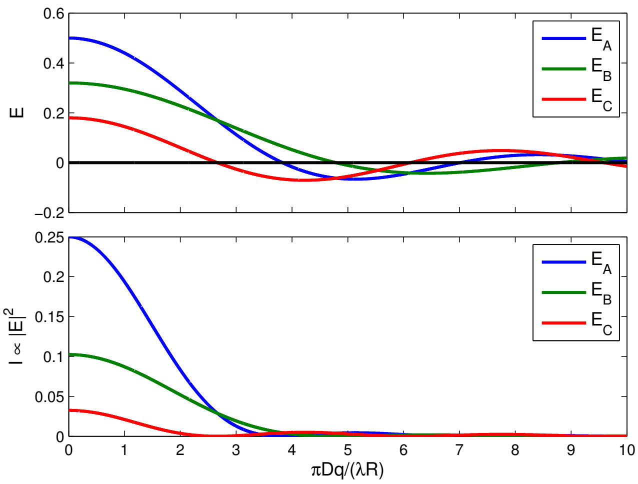
The Fraunhofer diffraction pattern is the Fourier transform of the aperture function.

$$E_{\mathrm{A}} = \mathcal{F}\{A\} \propto D^2 \frac{J_1\left(\frac{\pi D q}{\lambda R}\right)}{\frac{\pi D q}{\lambda R}}$$

$$E_{\mathrm{B}} = \mathcal{F}\{B\} \propto (0.8D)^2 \frac{J_1\left(\frac{\pi 0.8 D q}{\lambda R}\right)}{\frac{\pi 0.8 D q}{\lambda R}}$$

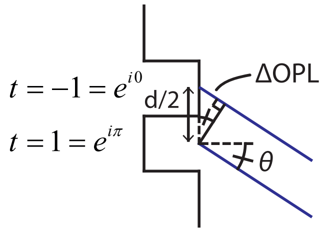
$$E_{\mathrm{C}} = \mathcal{F}\{C\} = \mathcal{F}\{A - B\} \propto E_{\mathrm{A}} - E_{\mathrm{B}}$$

The Bessel function $J_1(u)$ has its first zero at $u = 3.83$.



C has higher Rayleigh resolution!

031024:5



Amplitude transmission between 1 and -1 corresponds to a phase shift of $\Delta\varphi = (2m-1)\pi$. This corresponds to an optical path length difference of

$$\Delta\varphi = \frac{2\pi}{\lambda}\Delta\text{OPL}$$
$$\Rightarrow \quad \Delta\text{OPL} = \frac{\Delta\varphi\lambda}{2\pi} = \frac{(2m-1)\pi\lambda}{2\pi} = \left(m - \frac{1}{2}\right)\lambda$$

From the figure:

$$\sin\theta_m = \frac{\Delta\text{OPL}}{d/2} = \frac{\lambda}{d}(2m-1) = \begin{Bmatrix} \lambda = 500 \text{ nm} \\ d = 1000 \text{ nm} \end{Bmatrix} = m - \frac{1}{2}$$

The only possible solutions are

$$\theta = \sin^{-1}\left(\pm\frac{1}{2}\right) = \pm 30^\circ$$

040112:3

The Abbe number, V, is a measure of a material's dispersion. It can also be seen as a measure of chromatic aberration, i.e. how the power of a lens varies with wavelength. High values of V indicates low dispersion (low chromatic aberration). Typical values for glass are in the range of 20-60.

A zone plate is a lens that uses diffraction instead of refraction to focus light. They are often used for wavelengths that are opaque for glass, e.g. x-rays.

The Abbe number is

$$V = \frac{n_y - 1}{n_b - n_r}.$$

We can rewrite this in terms of the power of the lens,

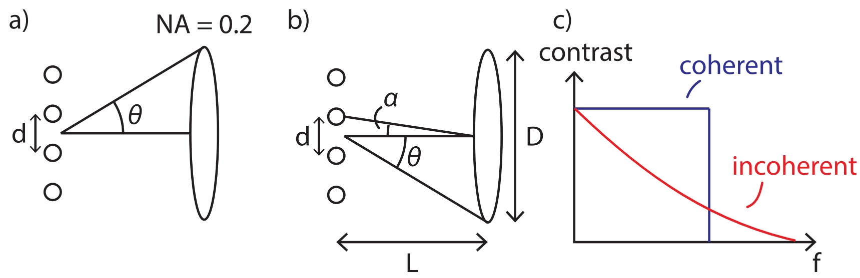
$$F = \frac{1}{f} \propto n - 1$$
$$\Rightarrow \quad V = \frac{F_y}{F_b - F_r}.$$

The power of a zone plate is proportional to the wavelength, so

$$V_{zp} = \frac{\lambda_y}{\lambda_b - \lambda_r} = \begin{Bmatrix} \text{Hecht, Table 6.1} \\ \lambda_y = 587.56 \text{ nm} \\ \lambda_b = 486.13 \text{ nm} \\ \lambda_r = 656.28 \text{ nm} \end{Bmatrix} = -3.45$$

A zone plate has very strong anomalous dispersion (chromatic aberration)!

040112:5



We have two cases: coherent and incoherent illumination

Coherent illumination (figure a)
The first diffraction order is given by

$$\sin\theta = \frac{\lambda}{d}$$

The resolution limit is when the first order is at the edge of the lens

$$d = \frac{\lambda}{\sin\theta} = \{\mathrm{NA} = n\sin\theta = \sin\theta\} = \frac{\lambda}{\mathrm{NA}} = 5\lambda$$

Incoherent illumination (figure b)
We get no interference between the holes so the patterns will just add up. In this case, the resolution limit is given by the Rayleigh criterion,

$$\alpha = 1.22\frac{\lambda}{D}$$

The NA is

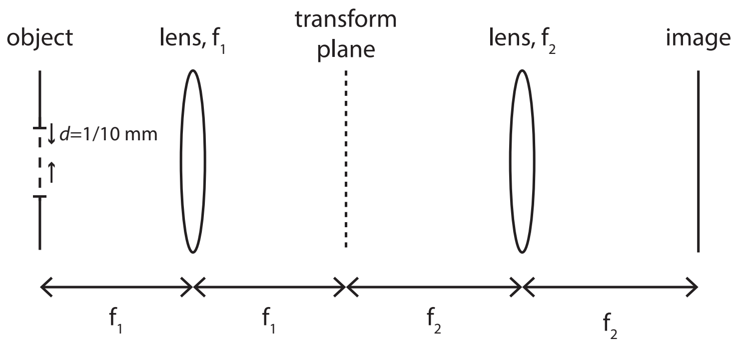
$$\mathrm{NA} = \sin\theta \approx \frac{D/2}{L},$$

so the smallest distance between the holes is

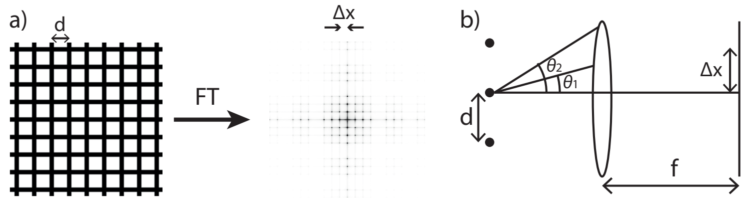
$$d = L\alpha = \frac{D/2}{\mathrm{NA}}1.22\frac{\lambda}{D} = \frac{1.22\lambda}{2\mathrm{NA}} = 3.05\lambda$$

Incoherent illumination gives higher resolution but lower contrast (figure c).

940824:4



A typical Fourier optical arrangement is shown in the figure above. The transform plane is the Fraunhofer diffraction pattern of the object. We want to construct a filter which absorbs the diffraction pattern from a perfect wire grid. Any errors in the grid will change the diffraction pattern so that some light is transmitted through the filter, reaching the image plane. If a detector is placed in the image plane, any signal will then correspond to a faulty grid.



We can try to estimate the distance between the fringes in the transform plane. Maxia occurs for

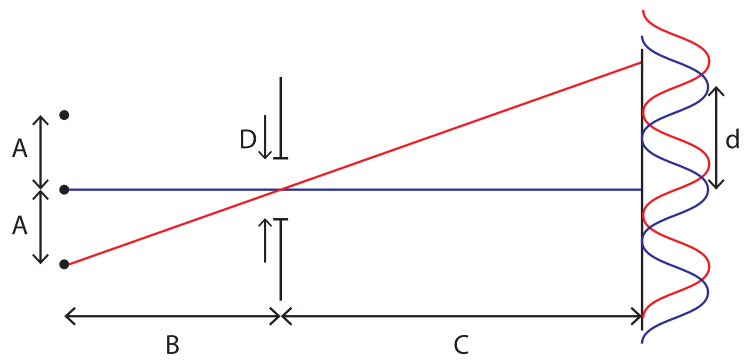
$$\theta_m \approx \sin\theta_m = \frac{m\lambda}{d}$$

The distance between two fringes is then (figure b)

$$\Delta x = f\Delta\theta = f\frac{\lambda}{d}.$$

Ex: If $f = 200$ mm and $\lambda = 500$ nm, the distance between the fringes is 1 mm.

930825:4



For one source, the angle to the first intensity maxima is

$$\theta \approx \sin\theta = \frac{\lambda}{D}.$$

The distance between the fringes is then

$$d = C\theta = C\frac{\lambda}{D}.$$

The sources are mutually incoherent so their diffraction patterns add without interference.

To get a visibility close to 1 the patterns from the different sources should be shifted one period d. From similar triangles in the figure we get

$$\frac{A}{B} = \frac{d}{C} \quad \Rightarrow \quad A = B\frac{d}{C} = B\frac{\lambda}{D}.$$