PROBLEMS AND SOLUTIONS IN ENGINEERING MATHEMATICS

PROBLEMS AND SOLUTIONS
IN
ENGINEERING MATHEMATICS

For

B.E./B.Tech. 1st Year (I & II Semesters)

(Volume-I)

By

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PREFACE

I have no words to express my gratitude towards my worthy students on account of whose keen interest and continuous suggestions this book is appearing in its present form as the new Revised Second Edition (Part-I) Keeping in view the changes done by Some Universities in the Syllabus of First and Second Semesters, I have revised it thoroughly to make a Comprehensive Book by rearranging some topics/chapters, adding new and important problems and all the questions asked/set in the previous university examinations.

The response to the First Edition of this book (All the three Volumes for respective Semesters), has been overwhelming and very encouraging which amply indicates that this book has proved extremely useful and helpful to all the B.E./B.Tech. students of Engineering colleges and Institutes throughout the country. Obviously it has helped them to be better equipped and more confident in solving the problems asked in several university examinations.

All the problems have been solved systematically and logically so that even an average student can become familiar with the techniques to solve the mathematical problems independently. Mathematics has always been a problematic subject for the students, hence they have been depending and relying upon private tuitions and coaching academies. It is hoped that this book in its new form will provide utmost utility to its readers.

—Author

SYMBOLS

Greek Alphabets

A	α	Alpha	I	ι	Iota	P	ρ	Rho
B	β	Beta	K	κ	Kappa	Σ	σ	Sigma
Γ	γ	Gamma	Λ	λ	Lambda	T	τ	Tau
D	δ	Delta	M	μ	Mu	Y	υ	Upsilon
E	ε	Epsilon	N	ν	Nu	Φ	φ	Phi
Z	ζ	Zeta	Ξ	ξ	Xi	X	χ	Chi
H	η	Eta	O	ο	Omicron	Ψ	ψ	Psi
Θ	θ	Theta	Π	π	Pi	Ω	ω	Omega
	∃	There exists		∀	For all			

Metric Weights and Measures

LENGTH

10 millimetres	=	1 centimetre
10 centimetres	=	1 decimetre
10 decimetres	=	1 metre
10 metres	=	1 decametre
10 decametres	=	1 hectometre
10 hectometres	=	1 kilometre

CAPACITY

10 millilitres	= 1 centilitre
10 centilitres	= 1 decilitre
10 decilitres	= 1 litre
10 litres	= 1 dekalitre
10 decalitres	= 1 hectolitre
10 hectolitres	= 1 kilolitre

VOLUME

1000 cubic centimetres	=	1 centigram
1000 cubic decimetres	=	1 cubic metre

AREA

100 square metres	= 1 are
100 ares	= 1 hectare
100 hectares	= 1 square Kilometre

WEIGHT

10 milligrams	=	1 centigram
10 centigrams	=	1 decigram
10 decigrams	=	1 gram
10 grams	=	1 decagram
10 dekagrams	=	1 hectogram
10 hectograms	=	1 kilogram
100 kilograms	=	1 quintal
10 quintals	=	1 metric ton (tonne)

ABBREVIATIONS

kilometre	km	tonne	t
metre	m	quintal	q
centimetre	cm	kilogram	kg
millimetre	mm	gram	g
kilolitre	kl	are	a
litre	l	hectare	ha
millilitre	ml	centiare	ca

1

Infinite Series

IMPORTANT DEFINITIONS AND FORMULAE

1. Convergent, Divergent and Oscillating Sequences:

A Sequence $\{a_n\}$ is said to be ***convergent or divergent*** if $\underset{n\to\infty}{\text{Lt}}\ a_n$ is ***finite or not finite*** respectively.

For example, consider the sequence $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \ldots$

Here $$a_n = \frac{1}{2^n},\ \underset{n\to\infty}{\text{Lt}}\ a_n = \underset{n\to\infty}{\text{Lt}}\ \frac{1}{2^n} = 0 \text{ which is finite.}$$

$\Rightarrow$ The sequence $\{a_n\}$ is **convergent.**

Consider the sequences $\{n^2\}$ or $\{-2^n\}$.

Here $$a_n = n^2 \quad \text{or} \quad -2^n$$

$$\underset{n\to\infty}{\text{Lt}}\ a_n = \infty \quad \text{or} \quad -\infty.$$

$\Rightarrow$ Both these sequences are **divergent.**

If a sequence $\{a_n\}$ neither converges to a finite number nor diverges to $+\infty$ or $-\infty$, it is called an ***Oscillatory sequence.***

Oscillatory sequences are of 2 types:

(*i*) A bounded sequence which does not converge, is said to **oscillate finitely.**

For example, consider the sequence $\{(-1)^n\}$.

Here $a_n = (-1)^n$. It is a bounded sequence because there exist two real numbers k and K ($k \le$ K) such that $k \le a_n \le$ K $\forall\ n \in$ N.

$$\{a_n\} = \{-1, 1, -1, 1, -1, \ldots\ldots\} \quad \Rightarrow \quad -1 \le a_n \le 1$$

Now $$\underset{n\to\infty}{\text{Lt}}\ a_{2n} = \underset{n\to\infty}{\text{Lt}}\ (-1)^{2n} = 1$$

$$\underset{n\to\infty}{\text{Lt}}\ a_{2n+1} = \underset{n\to\infty}{\text{Lt}}\ (-1)^{2n+1} = -1$$

Thus $\underset{n\to\infty}{\text{Lt}}\ a_n$ does not exist $\Rightarrow$ The sequence does not converge. Hence this sequence **oscillates finitely.**

(*ii*) An unbounded sequence which does not diverge, is said to **oscillate infinitely.**

Note: When we say $\underset{n\to\infty}{\text{Lt}}\ a_n = l$, it means

$$\underset{n\to\infty}{\text{Lt}}\ a_{2n} = \underset{n\to\infty}{\text{Lt}}\ a_{2n+1} = l.$$

2. **Infinite Series:** If $\{u_n\}$ is a sequence of real numbers, then the **expression $\mathbf{u_1 + u_2 + ... + u_n + ...}$** [*i.e.,* the sum of the terms of the sequence, which are infinite in number] is called **an infinite series**, usually denoted by $\sum_{n=1}^{\infty} u_n$ or more briefly, by Σu_n.
3. **Partial Sums:** If Σu_n is an infinite series where the terms may be +ve or –ve, then $S_n = u_1 + u_2 + ... + u_n$ is called the n^{th} partial sum of Σu_n. Thus, the n^{th} partial sum of an infinite series is the sum of its first n terms. $S_1, S_2, S_3, ...$ are the first, second, third, ... partial sums of the series. Since $n \in$ N (set of natural numbers), $\{S_n\}$ is a sequence called the sequence of partial sums of the infinite series Σu_n. Therefore, *to every infinite series Σu_n, there corresponds a sequence $\{S_n\}$ of its partial sums.*
4. **Behaviour of an Infinite Series:** An infinite series Σu_n converges, diverges or oscillates (finitely or infinitely) according as the sequence $\{S_n\}$ of its partial sums converges, diverges or oscillates (finitely or infinitely).
5. **Geometric Series:** The geometric series $1 + x + x^2 + x^3 + ...$ to ∞
   (*i*) converges if $-1 < x < 1$ *i.e.,* $|x| < 1$
   (*ii*) diverges if $x \geq 1$
   (*iii*) oscillates finitely if $x = -1$
   (*iv*) oscillates infinitely if $x < -1$
6. **Theorem:** If a series Σu_n is convergent, then

$$\underset{n\to\infty}{\text{Lt}}\ u_n = 0.$$

However, converse of the above theorem is not always true *i.e.,* the n^{th} term may tend to zero as $n \to \infty$ even if the series is not convergent.

Thus $\underset{n\to\infty}{\text{Lt}}\ u_n = 0 \quad \Rightarrow \quad \Sigma u_n$ may or may not be convergent.

Also $\underset{n\to\infty}{\text{Lt}}\ u_n \neq 0 \quad \Rightarrow \quad \Sigma u_n$ is not convergent.

7. A positive term series either converges or diverges to $+\infty$.
8. There are six different comparison tests which can be used to examine the nature of infinite series. These are described in detail in question number 18 of this chapter.
9. **General procedure** for testing a series for convergence is given under question 127, depending upon the type of series whether it is alternating, positive term series or a power series.

1. *Give an example of a monotonic increasing sequence which is (i) convergent (ii) divergent.*

SOLVED PROBLEMS

Sol. (*i*) Consider the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, ..., \frac{n}{n+1}, ...$

Since $\frac{1}{2} < \frac{2}{3} < \frac{3}{4} < ...$, the sequence is monotonic increasing.

$a_n = \frac{n}{n+1}, \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{n+1} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = 1$, which is finite.

$\therefore$ The sequence is convergent.

(*ii*) Consider the sequence 1, 2, 3, ..., n, ...

Since $1 < 2 < 3 < ... < n < ...$, the sequence is monotonic increasing,

$a_n = n,\ \lim_{n\to\infty} a_n = \lim_{n\to\infty} n = \infty$

$\therefore$ The sequence diverges to $+\infty$.

2. *Give an example of a monotonic decreasing sequence which is (i) convergent (ii) divergent.*

Sol. (*i*) Consider the sequence $1, \frac{1}{2}, \frac{1}{3}, \ldots \frac{1}{n}, \ldots$

Since $1 > \frac{1}{2} > \frac{1}{3} > \ldots$, the sequence is monotonic decreasing.

$a_n = \frac{1}{n},\ \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$

$\therefore$ The sequence converges to 0.

(*ii*) Consider the sequence $-1, -2, -3, \ldots, -n, \ldots$

Since $-1 > -2 > -3 > \ldots$, the sequence is monotonic decreasing.

$a_n = -n,\ \lim_{n\to\infty} a_n = \lim_{n\to\infty} (-n) = -\infty$

$\therefore$ The sequence diverges to $-\infty$.

3. *Discuss the convergence of the sequence $\{a_n\}$, where*

(*i*) $a_n = \frac{n}{n^2+1}$ (*ii*) $a_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \ldots + \frac{1}{3^n}$ (*iii*) $a_n = \frac{n+1}{n}$.

Sol. (*i*) Here, $a_n = \frac{n}{n^2+1}$

$$\therefore \quad a_{n+1} - a_n = \frac{n+1}{(n+1)^2+1} - \frac{n}{n^2+1} = \frac{(n+1)(n^2+1) - n(n^2+2n+2)}{(n^2+2n+2)(n^2+1)}$$

$$= \frac{-n^2-n+1}{(n^2+2n+2)(n^2+1)} < 0\ \forall\ n \quad \Rightarrow \quad a_{n+1} < a_n$$

$\Rightarrow \{a_n\}$ is a decreasing sequence

Also, $a_n = \frac{n}{n^2+1} > 0\ \forall\ n \quad \Rightarrow \quad \{a_n\}$ is bounded below by 0.

$\because\ \{a_n\}$ is decreasing and bounded below, it is convergent.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{n^2+1} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1+\frac{1}{n^2}} = 0$$

$\therefore$ The sequence $\{a_n\}$ converges to zero.

(*ii*) Here, $a_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \ldots + \frac{1}{3^n}$

$=$ sum of $(n + 1)$ terms of a G.P. whose first term is 1 and common ratio is $\frac{1}{3}$

$$= \frac{1\left(1 - \frac{1}{3^{n+1}}\right)}{\left(1 - \frac{1}{3}\right)} \qquad \left|\ \because\ S_n = \frac{a(1-r^n)}{1-r}\right.$$

$$= \frac{3}{2}\left(1 - \frac{1}{3^{n+1}}\right)$$

Now, $$a_{n+1} = 1 + \frac{1}{3} + \frac{1}{3^2} + \ldots + \frac{1}{3^n} + \frac{1}{3^{n+1}}$$

$\therefore$ $$a_{n+1} - a_n = \frac{1}{3^{n+1}} > 0 \ \forall\, n \quad \Rightarrow \quad a_{n+1} > a_n \ \forall\, n$$

$\Rightarrow$ $\{a_n\}$ is an increasing sequence.

Also, $$a_n = \frac{3}{2}\left(1 - \frac{1}{3^{n+1}}\right) < \frac{3}{2} \ \forall\, n \quad \Rightarrow \quad \{a_n\} \text{ is bounded above by } \frac{3}{2}.$$

$\because$ $\{a_n\}$ is increasing and bounded above, it is convergent.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3}{2}\left(1 - \frac{1}{3^{n+1}}\right) = \frac{3}{2}(1-0) = \frac{3}{2}$$

$\therefore$ The sequence $\{a_n\}$ converges to $\frac{3}{2}$.

(*iii*) $$a_n = \frac{n+1}{n}$$

$\therefore$ $$a_{n+1} - a_n = \frac{n+2}{n+1} - \frac{n+1}{n} = \frac{-1}{n(n+1)} < 0 \ \forall\, n$$

$\Rightarrow$ $$a_{n+1} < a_n \ \forall\, n$$

$\Rightarrow$ $\{a_n\}$ is a decreasing sequence.

Also, $$a_n = \frac{n+1}{n} = 1 + \frac{1}{n} > 1 \ \forall\, n$$

$\Rightarrow$ $\{a_n\}$ is bounded below by 1.

$\because$ $\{a_n\}$ is decreasing and bounded below, it is convergent.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right) = 1$$

$\therefore$ The sequence $\{a_n\}$ converges to 1.

4. *What is an infinite series ? When does it converge, diverge or oscillates (finitely or infinitely) ?*

Sol. If $\{u_n\}$ is a sequence of real numbers, then the expression $u_1 + u_2 + u_3 + \ldots + u_n + \ldots$ (*i.e.*, the sum of the terms of the sequence, which are infinite in number) is called an infinite series. The infinite series $u_1 + u_2 + \ldots + u_n + \ldots.$ is denoted by $\sum_{n=1}^{\infty} u_n$ or more briefly, by Σu_n.

To every infinite series Σu_n, there corresponds a sequence $\{S_n\}$, where $S_n = u_1 + u_2 + u_3 + \ldots + u_n$ is called the partial sum of its first n terms.

The infinite series Σu_n converges, diverges or oscillates (finitely or infinitely) according as the sequence $\{S_n\}$ of its partial sums converges, diverges or oscillates (finitely or infinitely)

(*i*) Series $\sum u_n$ is convergent if $\lim_{n\to\infty} S_n$ = finite.

(*ii*) Series $\sum u_n$ is divergent if $\lim_{n\to\infty} S_n = +\infty$ or $-\infty$

(*iii*) Series $\sum u_n$ oscillates finitely if $\{S_n\}$ is bounded and neither converges nor diverges.

(*iv*) Series $\sum u_n$ oscillates infinitely if $\{S_n\}$ is unbounded and neither converges nor diverges.

5. *Discuss whether the following series converges or otherwise,* $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{n(n+1)}+\ldots\infty$

Sol. Here, $$u_n = \frac{1}{n(n+1)} = \frac{1}{n}-\frac{1}{n+1}$$

Putting $n = 1, 2, 3, \ldots.n$, we have

$$u_1 = 1-\frac{1}{2},\ u_2 = \frac{1}{2}-\frac{1}{3},\ u_3 = \frac{1}{3}-\frac{1}{4}$$

$$u_4 = \frac{1}{4}-\frac{1}{5},\ \ldots.\ u_n = \frac{1}{n}-\frac{1}{n+1}$$

Adding, $$S_n = 1-\frac{1}{n+1}$$

$$\lim_{n\to\infty} S_n = 1-0 = 1$$

$\Rightarrow$ $\{S_n\}$ converges to 1 $\Rightarrow$ $\Sigma\, u_n$ converges to 1.

6. *Examine the convergence of the series* $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$.

Sol. Let, $u_n = \frac{1}{n(n+2)} = \frac{1}{2n}-\frac{1}{2(n+2)} = \frac{1}{2}\left(\frac{1}{n}-\frac{1}{n+2}\right) = \frac{1}{2}\left[\left(\frac{1}{n}-\frac{1}{n+1}\right)+\left(\frac{1}{n+1}-\frac{1}{n+2}\right)\right]$

Putting $n = 1, 2, 3, \ldots.\ n$, we obtain

$$u_1 = \frac{1}{2}\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)\right],\quad u_2 = \frac{1}{2}\left[\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)\right]$$

$$u_3 = \frac{1}{2}\left[\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)\right],\quad \ldots\ldots\ldots\ldots\ldots\ldots$$

$$u_n = \frac{1}{2}\left[\left(\frac{1}{n}-\frac{1}{n+1}\right)+\left(\frac{1}{n+1}-\frac{1}{n+2}\right)\right]$$

Adding, $$S_n = \frac{1}{2}\left[\left(1-\frac{1}{n+1}\right)+\left(\frac{1}{2}-\frac{1}{n+2}\right)\right]$$

$$\lim_{n\to\infty} S_n = \frac{1}{2}\left(1-0+\frac{1}{2}-0\right)$$

$$= \frac{3}{4}, \text{ a finite quantity.}$$

$\Rightarrow$ the given sequence $<S_n>$ converges to $\frac{3}{4}$. Hence the given infinite series $\sum_{n=1}^{\infty} u_n$ converges to $\frac{3}{4}$.

7. *Show that the series* $\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^{n-1}$ *converges to 4.*

Sol. Let $u_n = \left(\frac{3}{4}\right)^{n-1}$

then $S_n = u_1 + u_2 + u_3 + \ldots.. + u_n$

$$= 1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \ldots + \left(\frac{3}{4}\right)^{n-1} = \frac{1\left[1-\left(\frac{3}{4}\right)^n\right]}{1-\frac{3}{4}} = 4\left(1-\left(\frac{3}{4}\right)^n\right)$$

$$\lim_{n\to\infty} S_n = 4[1-0] \qquad [\because \text{ if } |x| < 1, \text{ then } x^n \to 0 \text{ as } n \to \infty]$$

$$= 4, \text{ a finite quantity.}$$

$\Rightarrow$ the sequence $<S_n>$ converges to 4. Hence the given series $\sum_{n=1}^{\infty} u_n$ converges to 4.

8. *Examine convergence or otherwise of the series,* $1^2 + 2^2 + 3^2 + \ldots + n^2 + \ldots$

Sol. $S_n = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

$$\lim_{n\to\infty} S_n = +\infty$$

$\Rightarrow$ $<S_n>$ diverges to $+\infty$

$\Rightarrow$ the given series diverges to $+\infty$.

9. *Show that the series* $-1-2-3-\ldots-n-\ldots$ *diverges to* $-\infty$.

Sol. $S_n = -1-2-3 \ldots. - n = -(1+2+3+\ldots+n)$

$$= -\frac{n(n+1)}{2}$$

$$\lim_{n\to\infty} S_n = -\infty \Rightarrow <S_n> \text{ diverges to } -\infty.$$

$\Rightarrow$ the given series diverges to $-\infty$.

10. *Examine the convergence or otherwise of the series* $\sum_{n=1}^{\infty}(-1)^{n-1}$

Sol. $S_n = 1-1+1-1+1-1+\ldots.$ to n terms

$= 1$ or 0 according as n is odd or even.

The subsequence $<S_{2n-1}>$ converges to 1 while the subsequence $<S_{2n}>$ converges to 0.

$\Rightarrow$ $<S_n>$ is not convergent.

Since $<S_n>$ is bounded, $\therefore$ $<S_n>$ oscillates finitely

$\Rightarrow$ $\sum_{n=1}^{\infty}(-1)^{n-1}$ oscillates finitely.

11. *Test the convergence of the series* $5-4-1+5-4-1+5-4-1+\ldots$ *to* ∞.

Sol. Here $S_n = 5-4-1+5-4-1+5-4-1+\ldots$ to n terms

$= 0, 5$ or 1 according as the number of terms is $3m$, $3m+1$, $3m+2$.

Clearly, S_n does not tend to a unique limit. Since $<S_n>$ is bounded, it oscillates finitely.

$\therefore$ the given series oscillates finitely.

12. *Show that the series* $\sum_{n=1}^{\infty} n\,(-1)^n$ *oscillates infinitely.*

Sol. Here,
$$S_n = -1 + 2 - 3 + 4 - 5 + 6 + \ldots + \text{to } n \text{ terms}$$
$$= \begin{cases} -\left(\dfrac{n+1}{2}\right), & \text{if } n \text{ is odd} \\ \dfrac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

The subsequence $<S_{2n-1}>$ diverges to $-\infty$, while the subsequence $<S_{2n}>$ diverges to $+\infty$

$\therefore$ $<S_n>$ oscillates infinitely.

$\Rightarrow$ $\sum_{n=1}^{\infty} n\,(-1)^n$ oscillates infinitely.

13. *Test the nature of the series* $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots. + \infty$.

Sol. Here,
$$u_n = \frac{1}{2^{n-1}}$$
$$S_n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots \frac{1}{2^{n-1}} = \frac{1\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}} = 2\left[1-\frac{1}{2^n}\right]$$
$$\lim_{n\to\infty} S_n = 2(1-0) = 2, \text{ a finite quantity.}$$

$\Rightarrow$ The sequence $<S_n>$ converges to 2.

$\Rightarrow$ The infinite series $\sum_{n=1}^{\infty} u_n$ converges to 2.

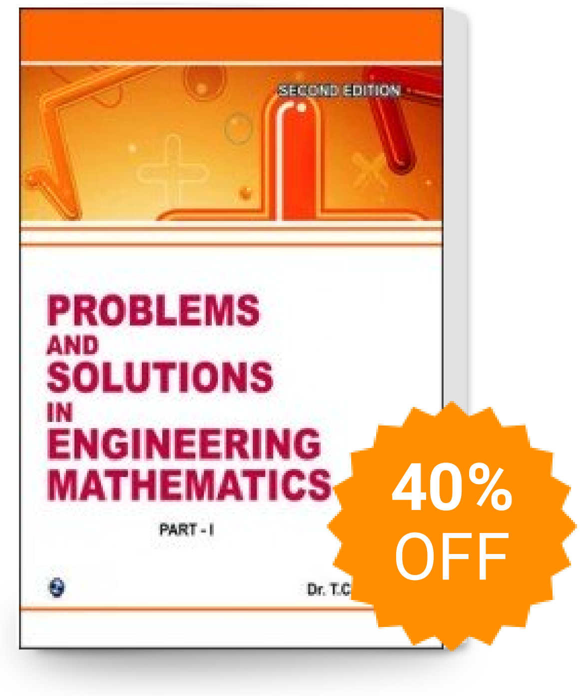
14. *Test the nature of the series* $-1-8-27-64-\ldots.\ \infty$

Sol.
$$S_n = -1-8-27-64-\ldots-n^3 = -(1^3+2^3+3^3+4^3+\ldots+n^3)$$
$$= -\left[\frac{n(n+1)}{2}\right]^2 \qquad \text{(Sum of cubes of } n \text{ natural numbers)}$$
$$= -\frac{n^2}{4}(n^2+2n+1) = -\frac{n^4}{4}\left[1+\frac{2}{n}+\frac{1}{n^2}\right]$$
$$\lim_{n\to\infty} S_n = -\infty \;\Rightarrow\; <S_n> \text{ diverges to } -\infty$$

$\therefore$ The given series diverges to $-\infty$.

15. *Examine the series* $1 - \frac{1}{5} + \frac{1}{5^2} - \frac{1}{5^3} + \ldots\infty$ *for its nature.*

Problems And Solutions In Engineering Mathematics Volume-I By Dr. T.C. Gupta



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