Problems with the use of Monte Carlo for AMT sharing studies

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Daniel G. Jablonski
Dan.Jablonski@jhuapl.edu

ITEA - Las Vegas

Johns Hopkins
Applied Physics Laboratory

What is Monte Carlo?

- A technique for using coin-tossing techniques to model the numerical behavior of complex systems
  - Used in the Manhattan Project
- It is described in great detail in ITU-R Recommendation SM.2028-1, which is available at ITU.INT, free of charge
- The software often used for running Monte Carlo spectrum simulations is called SEAMCAT, for Spectrum Engineering Advanced Monte Carlo Analysis Tool
- SEAMCAT is available free of charge at www.seamcat.org

SEAMCAT Modules

- The SEAMCAT software modules include
  - Event Generation Engine
  - Distribution Evaluation
  - Interference Calculation
  - Limits Evaluation
- The validity of SEAMCAT analyses depends on the models, distributions, and interference criteria provided to the software
  - The SEAMCAT software itself seems to be a mature, reliable, validated product

How has Monte Carlo been used for Aeronautical Mobile Telemetry (AMT) simulations?

- The Medical Body Area Networks (MBANs) community used SEAMCAT to estimate interference from wireless medical devices to AMT ground stations
  - The simulations combined an experimentally validated probability distribution of AMT signal fades taken from Rec. M.1459 with several hypothetical models of notional MBANs deployments.
- CSMAC used Monte Carlo techniques based on industry-provided probability distribution functions for the power transmitted by an ensemble of LTE handsets
- The Joint Task Group 4-5-6-7 at the ITU has conducted similar studies using slightly different data and assumptions
- Commercial software propagation products often have built-in Monte Carlo capabilities for simulating random features of terrain and clutter.

Are Monte Carlo studies accurate?

- Often, Monte Carlo studies do not properly account for the physics of flight test.
  - In particular, aircraft are allowed to hop randomly from place to place in a manner that is thought, by Monte Carlo advocates, to yield the same results as an analytical model based on validated probability distributions for fades and on accurate aircraft and airspace models
- The studies usually deviate from the technical models and specifications given in ITU-R Recommendation M.1459
  - This is a feature of how the Monte Carlo parameters are defined, not of the Monte Carlo approach itself

Monte Carlo accuracy, cont'd

- Many times, the studies utilize interference protection criteria (IPCs) other than those given in M.1459
  - It is common for models to assume that AMT antennas never point at the horizon
  - AMT signal fades are often ignored in the analyses
- The studies often use over-simplified propagation models
  - Path loss is too high
  - Ground multipath is ignored
    - The existence of multipath effects in data provided by AMT operators has been challenged as being physically impossible.

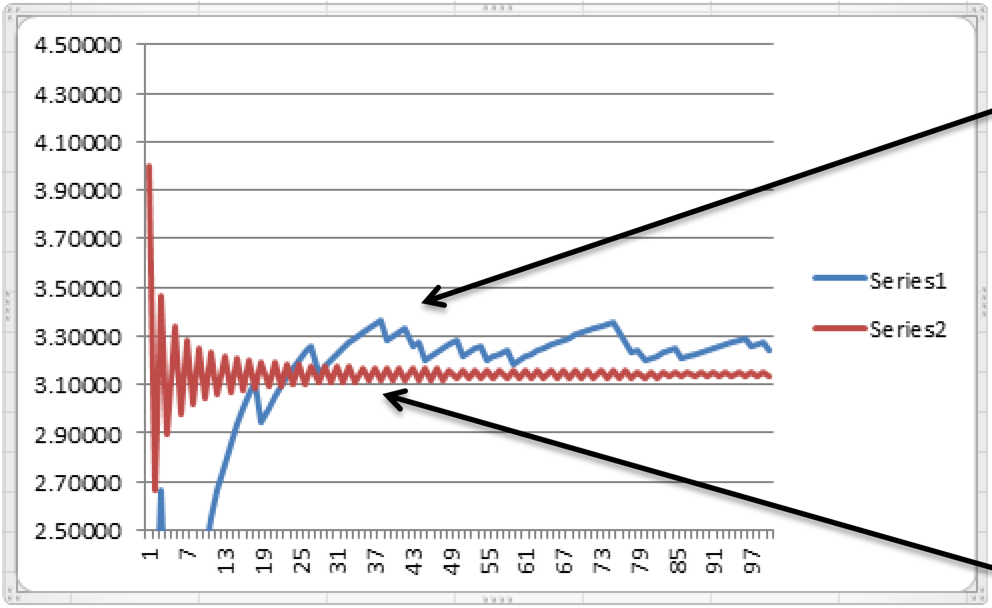
Example of a Monte Carlo simulation

- **As an example, one can calculate a value for π using Monte Carlo:**
  - **Generate two independent random numbers from a distribution that is uniform over the interval [0,1], such as that implemented by the Excel RAND() function**
  - **Use these numbers to define a point x,y**
  - **Test whether ($x^2 + y^2 \leq 1$)**
  - **Repeat , keeping a running tally of the percentage of x,y pairs that lie within this “unit” circle**
  - **This ratio converges, for a sufficiently large number of coin tosses, to the value of $\pi/4$ as $1/\sqrt{N}$**

What is an alternative approach to computing π?

- **Write the Fourier coefficients for a square wave of period T and amplitude 1**
  - $f(t) = \sum_{m\ odd} \frac{4}{m\pi} sin\left(\frac{2\pi mt}{T}\right)$
- **Set $t = T/4$, where f($T/4$) = 1 and sin($2\pi mt/T$) = ±1**
- **Compute $\pi = 4(1 - 1/3 + 1/5 - 1/7 + 1/9 - \ldots)$**
- **Note that no random processes are involved**
- **This is an analytical approach, rather than a Monte Carlo approach, to computing π**
- **This converges as 1/N**

Convergence to a value for π



Monte Carlo algorithm

Fourier Series algorithm

The difference between 1/N versus 1/(√N) convergence
is significant in terms of computational speed. To increase the
precision of "incoherent" Monte Carlo runs by a factor of 2 requires a factor
of 4 increase in the number of computations required. This is in contrast to the
factor of 2 increase in work required for the "coherent" Fourier approach.

When is Monte Carlo better?

- When desperation dictates an alternative to an analytical approach
  - as is the case for highly nonlinear problems that are “computationally large”
- For convenience, when modeling a collection of mathematical distributions is a hassle

How does one know it works?

- **Confidence testing:**
  - **Chi-squared or Kolmogorov-Smirnov tests**
- **Running large numbers of tests until the answer “appears” to have converged**

Just some of the things that can go wrong with any simulation, but especially Monte-Carlo models:

1. **Incorrectly accounting for conditional probabilities and Bayes theorem**
2. **Incorrectly assuming that a process is or is not stationary (i.e., that the probabilities do or do not change over time or space)**
3. **Assuming that data points are random, or assuming that different data points are uncorrelated**
4. **Assuming that there are no Markov models, in which the state of a system at $t + \Delta t$ or $x + \Delta x$ depends on the state of the system at time t or position x.**
5. **Using the wrong metric for answering the question of interest**
6. **Choosing the wrong independent parameter when averaging**
7. **Using the wrong statistical distribution**

Bayes Theorem

- The Pulitzer prize winning book, "The Emperor of All Maladies," notes that cancer statistics have ignored the large number of persons with cancer who would <u>not</u> have died without surgical intervention
  - After these persons undergo surgery that has no impact on their prognosis (e.g., removal of a benign lump after a mammogram), they are often claimed to be "survivors" and included in the statistics used to justify the screening procedure that led to their surgery in the first place.
- Testing of the polio vaccine required a huge number of test subjects since most individuals, even without vaccination, would not have contracted polio.

Stationary processes

- In the film "21", based on the book, "Bringing down the House," a team of student card counters takes on the major casinos.
- By shuffling multiple decks of cards together, the casinos wrongly assume that card counters cannot keep track of the number of remaining face cards.
- The casinos also neglect to note in those "rare" cases in which many face cards remain at the bottom of the "shoe", the deck <u>stays "hot"</u> for a long time and can be exploited for multiple rounds of betting. Dealing cards "without replacement" is not a stationary process (i.e., it is the hypergeometric distribution)
- The students do not bet high under these circumstances. Instead, a roving participant, who always bets high, is signaled to join the table and exploit the "hot" shoe.
- Since no one changes their betting habits, casino staff assume the betting habits remain "stationary". The staff are slow to recognize that, when averaged across an individual table of gamblers, the betting parameters for that <u>table</u> have indeed changed, even though the betting parameters for the <u>individuals</u> at the table have not.

Not all data points are random or uncorrelated

- Social security numbers are supposedly protected by showing only the last four digits: xxx-xx-1234
  - However, the first three digits are, for most baby-boomers, correlated with where the cardholder lived when the card was issued. It is the last four numbers, not the first 5, that are unique and need to be protected.
- Credit cards are similarly insecure
  - The first four numbers identify the issuing bank, and the remaining numbers contain check sums that are not statistically independent of each other.
- The last digit of an international standard book number is a check sum, in Base 11, that is computed from the earlier digits.
  - In base 11, a ten is represented as a single *character* using the Roman numeral X.

Markov Models

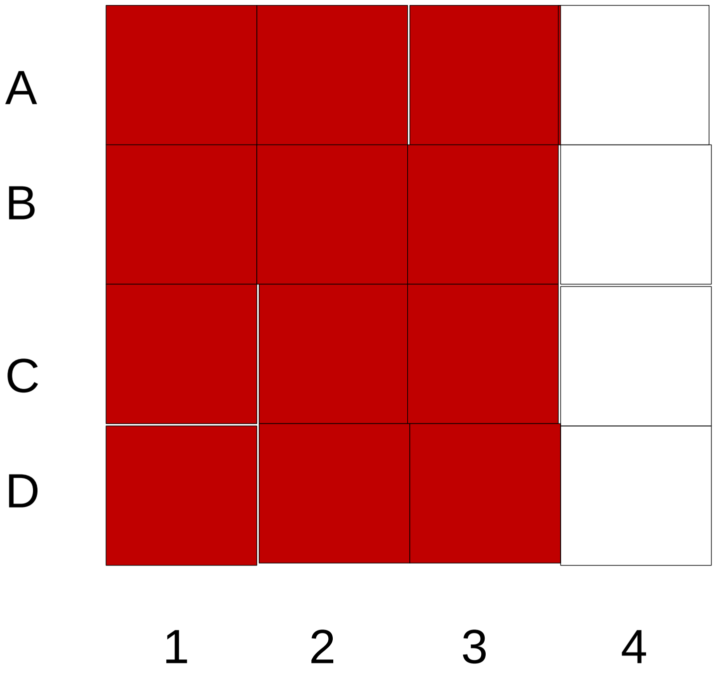
- RSA's secure ID token system was compromised when the details by which a code at time t converted to a new code at time $t + \Delta t$ became known. This is a classic example of a system in which the degree of randomness is much lower than what was presumed to exist
- This can happen in Monte Carlo simulations in which pseudorandom number generators, similar to those used to generate GPS signals, are used for the "coin-tossing" part of the algorithm.
  - High levels of randomness are difficult to achieve

Using the wrong metric

- Such as using as a metric the percentage of the time/airspace for which there is interference, rather than the percentage of time/airspace for which interference causes the AMT link to fail.
- Consider a 10 x 10 2-D mine field containing ten rows and ten columns of 1 x 1 squares. Let the probability that a square contains a mine be 0.1. Thus, only ten of the 100 squares, on average, contains a mine, and interference is "deemed" to be 10%.
- However, the probability of crossing the mine field in a randomly chosen row without hitting a mine is $(1 – 0.1)^{10}$, yielding a probability of a successful mission of 35%, with a corresponding probability of failure of 65%.
- For AMT, short-term interference causes long term dropouts, and the "impact" of interference is thus 65%, not 10%.

Choosing the wrong independent variable when averaging

- **For example, a state is equally divided into four districts**
- **If the majority color of a district is “red”, the state gets a Republican representative**
- **If averaging is done across rows, all four districts become Republican**
- **If averaging is done across columns, there are three Republicans and one Democrat.**
- **Which of these two approaches is “right” is irrelevant here; the issue is whether all parties to the discussion understand what model is being used.**



Using the Wrong Statistical Distribution

- Many different "distributions" are routinely used for Monte Carlo and other probability models:
  - Normal
  - Log-normal
  - Hypergeometric
  - Poisson
  - Weibull
- Great care needs to be taken when choosing and using any of these distributions
- Use of the Poisson distribution, in particular, caused difficulties in ITU work leading up to the 2012 World Radio Conference.

Conclusion

- **Monte Carlo techniques can be extremely useful when properly implemented.**
- **However, their use often implies a fundamental lack of understanding of key features of the problem being solved.**
- **Identifying and understanding anomalies, including coding errors, in Monte Carlo simulations is difficult.**
- **Monte Carlo is an analysis, not a synthesis tool.**
  - **Its usefulness for design is limited.**

Finally…

- **The author has never seen a Monte Carlo analysis related to spectrum that couldn't have been accomplished better using analytical techniques**
- **For example, once the cumulative distribution function of the transmit power of an LTE handset is known, there is no subsequent need for Monte Carlo techniques**
- **Instead, convolution techniques, such as those used in Rec. M.1459, should be used.**
- **This eliminates guess-work and heuristics.**

JOHNS HOPKINS
APPLIED PHYSICS LABORATORY