# Problems and Solutions to
# **Physics of Semiconductor Devices**

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## Contents



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# 1 Problems

## 1.1 Properties of Semiconductors

**1.** Which of the following semiconductors are transparent, partially transparent, non-transparent for visible light ($\lambda = 0.4$–$0.7\ \mu$m): Si, GaAs, GaP, and GaN?

**2.** Band gap of Si depends on the temperature as

$$E_g = 1.17\,\text{eV} - 4.73 \times 10^{-4}\frac{T^2}{T + 636}\,.$$

Find a concentration of electrons in the conduction band of intrinsic (undoped) Si at $T = 77$ K if at 300 K $n_i = 1.05 \times 10^{10}$ cm$^{-3}$.

**3.** Electron mobility in Si is 1400 cm$^2$ V$^{-1}$s$^{-1}$. Calculate the mean free time in scattering (Relaxationszeit) of electrons. Effective mass is $m_e^*/m_0 = 0.33$.

**4.** Calculate thermal velocity of electrons and holes in GaAs at room temperature. Effective masses are $m_e^*/m_0 = 0.063$ and $m_h^*/m_0 = 0.53$.

**5.** Hole mobility in Ge at room temperature is 1900 cm$^2$ V$^{-1}$s$^{-1}$. Find the diffusion coefficient.

**6.** Calculate dielectric relaxation time in $p$-type Ge at room temperature. Assume that all acceptors are ionized. $N_a = 10^{15}$ cm$^{-3}$, $\epsilon = 16$, $\mu_p = 1900$ cm$^2$ V$^{-1}$s$^{-1}$.

**7.** Calculate dielectric relaxation time in intrinsic Si at 300 K. $\epsilon = 12$, $\mu_n = 1400$ cm$^2$ V$^{-1}$s$^{-1}$, $\mu_n = 3.1\,\mu_p$.

**8.** Find Debye length in $p$-type Ge at 300 K if $N_a = 10^{14}$ cm$^{-3}$. Assume that all acceptors are ionized, $\epsilon = 16$.

**9.** Calculate the ambipolar diffusion coefficient of intrinsic (undoped) Ge at 300 K. $\mu_n/\mu_p = 2.1$, $\mu_n = 3900$ cm$^2$ V$^{-1}$s$^{-1}$.

**10.** Holes are injected into $n$-type Ge so that at the sample surface $\Delta p_0 = 10^{14}$ cm$^{-3}$. Calculate $\Delta p$ at the distance of 4 mm from the surface if $\tau_p = 10^{-3}$ s and $D_p = 49$ cm$^2$/s.

## 1.2 Schottky Diode

**1.** Find a hight of the potential barrier for a Au-$n$-Ge Schottky contact at room temperature ($T = 293$ K) if $\rho = 1\,\Omega\,$cm, $\psi_{\text{Au}} = 5.1$ eV, and $\chi_{\text{Ge}} = 4.0$ eV. Electron mobility in Ge is 3900 cm$^2$ V$^{-1}$ s$^{-1}$, density of the states in the conduction band is $N_c = 1.98 \times 10^{15} \times T^{3/2}$ cm$^{-3}$.

**2.** Calculate the depletion width for a Pt-$n$-Si Schottky diode ($T = 300$ K) at $V = 0$, $+0.4$, and $-2$ V. Concentration of doping impurity in Si equals $4 \times 10^{16}$ cm$^{-3}$. Work function of Pt is 5.65 eV, electron affinity of Si is 4.05 eV, $\epsilon_{\mathrm{Si}} = 11.9$, density of the states in the conduction band is $N_c = 6.2 \times 10^{15} \times T^{3/2}$ cm$^{-3}$.

**3.** For a Schottky contact Au-GaAs calculate the maximum electric field within the space charge region at $V = 0$, $+0.3$, and $-100$ V. $N_d = 10^{16}$ cm$^{-3}$, $\chi_{\mathrm{GaAs}} = 4.07$ eV, $\epsilon_{\mathrm{GaAs}} = 12.9$. Work function of Au is 5.1 eV, $T = 300$ $K$, density of the states in the conduction band is $N_c = 8.63 \times 10^{13} \times T^{3/2}$ cm$^{-3}$.

**4.** What is the electric field $E$ for a Schottky diode Au-$n$-Si at $V = -5$ V at the distance of 1.2 $\mu$m from the interface at room temperature if $\rho = 10\,\Omega\,$cm, $\mu_n = 1400$ cm$^2$ V$^{-1}$ s$^{-1}$, $N_c = 6.2 \times 10^{15} \times T^{3/2}$ cm$^{-3}$.

**5.** Find current densities $j$ at room temperature for a Schottky diode Pt-$n$-GaAs at $V = +0.5$ and $-5$ V if $\rho = 50\,\Omega\,$cm. $\mu_n = 8800$ cm$^2$ V$^{-1}$ s$^{-1}$, $m_n/m_0 = 0.063$, work function of Pt is 5.65 eV, $\chi_{\mathrm{GaAs}} = 4.07$ eV, $N_c = 8.63 \times 10^{13} \times T^{3/2}$ cm$^{-3}$. Apply thermionic-emission theory.

**6.** The capacitance of a Au-$n$-GaAs Schottky diode is given by the relation $1/C^2 = 1.57 \times 10^{15} - 2.12 \times 10^{15}$ V, where $C$ is expressed in F and $V$ is in Volts. Taking the diode area to be 0.1 cm$^2$, calculate the barrier height and the dopant concentration.

**7.** From comparison of the de Broglie wavelength of electron with the depletion width of a contact metal-$n$-Si, estimate the electron concentration at which Schottky diode loses its rectifying characteristics. For the estimate, assume that the height of the potential barrier a the contact is half the value of the band gap at room temperature ($E_g = 1.12$ eV), $m_e^* = m_0$, $T = 300$ K, and $\epsilon_{\mathrm{Si}} = 11.9$.

### 1.3 Ideal *p-n* Junction

**1.** Find the built-in potential for a $p$-$n$ Si junction at room temperature if the bulk resistivity of Si is 1 $\Omega\,$cm. Electron mobility in Si at RT is 1400 cm$^2$ V$^{-1}$ s$^{-1}$; $\mu_n/\mu_p = 3.1$; $n_i = 1.05 \times 10^{10}$ cm$^{-3}$.

**2.** For the $p$-$n$ Si junction from the previous problem calculate the width of the space charge region for the applied voltages $V = -10$, 0, and $+0.3$ V. $\epsilon_{\mathrm{Si}} = 11.9$

**3.** For the parameters given in the previous problem find the maximum electric field within the space charge region. Compare these values with the electric field within a shallow donor: $E \approx e/\epsilon_{\mathrm{Si}} a_{\mathrm{B}}^2$, where $a_{\mathrm{B}}$ is the Bohr radius of a shallow donor, $a_{\mathrm{B}} = \epsilon_{\mathrm{Si}} \hbar^2/m_e^* e^2$ and $m_e^*/m_0 = 0.33$.

**4.** Calculate the capacity of the $p$-$n$ junction from the problem 2 if the area of the junction is 0.1 cm$^2$.

**5.** $n$-Si of a $p$-$n$ Si junction has a resistivity of 1 $\Omega\,$cm. What should be the resistivity of $p$-Si so that 99 % of the total width of the space charge region would be located in $n$-Si ($p^+$-$n$ junction)? For the parameters needed see problem 1.

**6.** At room temperature under the forward bias of 0.15 V the current through a $p$-$n$ junction is 1.66 mA. What will be the current through the junction under reverse bias?

**7.** For a $p^+$-$n$ Si junction the reverse current at room temperature is 0.9 nA/cm$^2$. Calculate the minority-carrier lifetime if $N_d = 10^{15}$ cm$^{-3}$, $n_i = 1.05 \times 10^{10}$ cm$^{-3}$, and $\mu_p = 450\ \ \mathrm{cm^2\,V^{-1}\,s^{-1}}$.

**8.** How does the reverse current of a Si $p$-$n$ junction change if the temperature raises from 20 to 50 °C? The same for a Ge $p$-$n$ junction. Band gaps of Si and Ge are 1.12 and 0.66 eV, respectively.

**9.** Estimate temperatures at which $p$-$n$ junctions made of Ge, Si, and GaN lose their rectifying characteristics. In all cases $N_a = N_d = 10^{15}$ cm$^{-3}$. Assume that $E_g$ are independent of the temperature and are 0.66, 1.12, and 3.44 eV for Ge, Si, and GaN, respectively. Intrinsic carrier concentrations at room temperature are $n_i^{\mathrm{Ge}} = 2 \times 10^{13}$, $n_i^{\mathrm{Si}} = 10^{10}$, and $n_i^{\mathrm{GaN}} = 10^{-9}$ cm$^{-3}$.

### 1.4 Nonideal *p-n* Junction

**1.** $n$-Si with $N_d = 7 \times 10^{15}$ cm$^{-3}$ additionally contains $N_t = 10^{15}$ cm$^{-3}$ generation-recombination centers located at the intrinsic Fermi level with $\sigma_n = \sigma_p = 10^{-15}$ cm$^2$ and $v_t = 10^7$ cm/s. Calculate generation rate, if

1. $n$ and $p$ are low as compared to the equilibrium value
2. only $p$ is below the equilibrium value.

For Si, $n_i = 1.05 \times 10^{10}$ cm$^{-3}$.

**2.** Illumination of $n$-type Si ($N_d = 10^{16}$ cm$^{-3}$) generates $10^{21}$ cm$^{-3}$/s electron-hole pairs. Si has $N_t = 10^{15}$ cm$^{-3}$ generation-recombination centers with $\sigma_n = \sigma_p = 10^{-16}$ cm$^2$. Calculate equilibrium concentration of electrons and holes if $E_t = E_i$, where $E_i$ is the Fermi level of intrinsic Si, and $v_t = 10^7$ cm/s.

**3.** A $p^+$-$n$ Si junction ($n_i = 1.05 \times 10^{10}$ cm$^{-3}$, $\epsilon = 11.9$) is formed in an $n$-type substrate with $N_d = 10^{15}$ cm$^{-3}$. If the junction contains $10^{15}$ cm$^{-3}$ generation-recombination centers located at the intrinsic Fermi level with $\sigma_n = \sigma_p = 10^{-15}$ cm$^2$ ($v_t = 10^7$ cm/s), calculate generation current density at a reverse bias of 10 V.

**4.** For a $p$-$n$ Si junction with the $p$-side doped to $10^{17}$ cm$^{-3}$, the $n$-side doped to $10^{19}$ cm$^{-3}$ ($n^+$-$p$ junction), and a reverse bias of $-2$ V, calculate the generation current density at room temperature, assuming that the effective lifetime is $10^{-5}$ s.

**5.** For a $p$-$n$ GaAs junction at room temperature find the donor/acceptor concentration at which de Broglie wavelength ($\lambda = 2\pi\hbar/\sqrt{2m^*E}$) of electrons/holes is equal to the width of the space charge region. Assume $\langle E\rangle = 3kT/2$, $m_e^*/m_0 = 0.063$, $m_h^*/m_0 = 0.53$, and $\epsilon_{\mathrm{GaAs}} = 12.9$, $n_i^{\mathrm{GaAs}} = 2.1 \times 10^6$ cm$^{-3}$, and $N_a = N_d$.

**6.** When a silicon $p^+$-$n$ junction is reverse-biased to 30 V, the depletion-layer capacitance is 1.75 nF/cm$^2$. If the maximum electric field at avalanche breakdown is $3 \times 10^5$ V/cm, find the breakdown voltage. $\epsilon_{\mathrm{Si}} = 11.9$.

**7.** For a $p^+$-$n$ Si junction with $N_d = 10^{16}$ cm$^{-3}$, the breakdown voltage is 32 V. Calculate the maximum electric field at the breakdown. $\epsilon_{\mathrm{Si}} = 11.9$.

### 1.5 Solar Cells

**1.** The spectrum of Sun could be reasonably well modelled by that of the black body with $T \approx 5800$ K. In this case, the number of photons and power per unit energy could be approximated as

$$dN_\omega = g(\omega)d\omega \sim \frac{\omega^2 d\omega}{e^{\hbar\omega/kT} - 1}\,, dE_\omega = \hbar\omega\, g(\omega)d\omega \sim \frac{\omega^3 d\omega}{e^{\hbar\omega/kT} - 1}\,.$$

Find the maximum flux density and power per photon *energy* coming to Earth from Sun (find maxima of $g(\omega)$ and $\omega\, g(\omega)$). What are the corresponding maxima in *wavelength*? Hint: use the relation $\omega = 2\pi c/\lambda$ in $\omega\, g(\omega)$.

**2.** Consider a Si $p$-$n$ junction solar sell of area 2 cm$^2$. If the dopings of the solar cell are $N_a = 1.7 \times 10^{16}$ cm$^{-3}$ and $N_d = 5 \times 10^{19}$ cm$^{-3}$, and given $\tau_n = 10$ $\mu$s, $\tau_p = 0.5$ $\mu$s, $D_n = 9.3$ cm$^2$/s, $D_p = 2.5$ cm$^2$/s, and $I_L = 95$ mA, (i) calculate the open-circuit voltage, and (ii) determine the maximum output power of the solar cell at room temperature.

**3.** At room temperature, an ideal solar cell has a short-circuit current of 3 A and an open-circuit voltage of 0.6 V. Calculate and sketch its power output as a function of operation voltage and find its fill factor from this power output.

**4.** What happens to the short-circuit current, the open-circuit voltage, and the maximum output power of the solar cell from the previous problem if it is employed as a power supply for the Mars Pathfinder mission? Mean distance from the Mars to the Sun is approximately a factor of 1.5 longer than that of between the Earth and the Sun. Assume that in both cases the solar cell operates at room temperature.

**5.** At room temperature, an ideal solar cell has a short-circuit current of 2 A and an open-circuit voltage of 0.5 V. How does the open-circuit voltage change if the short-circuit current drops by a factor of 2, 5, or 10?

**6.** At 300 K, an ideal Si $p$-$n$ junction solar cell has a short-circuit current of 2 A and an open-circuit voltage of 0.5 V. How does the maximum output power of the solar cell change if the temperature raises to 400 K?

### 1.6 Bipolar Transistor

**1.** A silicon $p^+$-$n$-$p$ transistor has impurity concentrations of $5\times10^{18}$, $10^{16}$, and $10^{15}$ cm$^{-3}$ in the emitter, base, and collector, respectively. If the metallurgical base width is 1.0 $\mu$m, $V_{EB} = 0.5$ V, and $V_{CB} = 5$ V (reverse), calculate (i) the neutral base width, and (ii) the minority carrier concentration at the emitter-base junction. Transistor operates at room temperature.

**2.** For the transistor from the previous problem calculate the emitter injection efficiency, $\gamma$, assuming that $D_E = D_B$ and the neutral base and emitter widths are equal ($x_E = x_B$).

**3.** For the same transistor calculate the base transport factor ($\alpha_T$) assuming the diffusion length of the minority carriers in the base of 3.5 $\mu$m.

**4.** Diffusion length of the minority carriers in the base region is 4 $\mu$m. Calculate he base width at which the base transport factor is 0.99, 0.9, and 0.5.

**5.** A Si $n^+$-$p$-$n$ transistor has dopings of $10^{19}$, $3\times10^{16}$, and $5\times10^{15}$ cm$^{-3}$ in the emitter, base, and collector, respectively. Find the upper limit of the base-collector voltage at which the neutral base width becomes zero (punch-through). Assume the base width (between metallurgical junctions) is 0.5 $\mu$m.

**6.** Empirically the band gap reduction $\Delta E_g$ in Si can be expressed as

$$\Delta E_g = 18.7 \ln\left(\frac{N}{7\times10^{17}}\right) \text{ meV}\,.$$

Compare the emitter injection efficiency at room temperature for emitter dopings of $10^{19}$ and $10^{20}$ cm$^{-3}$. The base doping in both cases is $10^{18}$ cm$^{-3}$. Assume that $x_E = x_B$ and $D_E = D_B$.

**7.** What profile of the base doping results in a uniform electric field in the base?

**8.** For a nonuniform doping profile of the base resulting in a mean electric field of $10^4$ V/cm compare the drift and diffusion transport time at room temperature of the minority carriers through the base ($x_B = 0.5$ $\mu$m).

**9.** For a Si transistor with $D_B = 50$ cm$^2$/s and $L_B = 3.5$ $\mu$m in the base and $x_B = 0.5$ $\mu$m estimate the cut-off frequencies in common-emitter and common-base configurations.

### 1.7 MIS/MOS Capacitor and MOSFET

**1.** For an ideal Si-$SiO_2$ MOS capacitor with $d = 10$ nm, $N_a = 5 \times 10^{17}$ cm$^{-3}$, find the applied voltage at the $SiO_2$-Si interface required (a) to make the silicon surface intrinsic, and (b) to bring about a strong inversion. Dielectric permittivities of Si and $SiO_2$ are 11.9 and 3.9, respectively. $T = 296$ K.

**2.** A voltage of 1 V is applied to the MOS capacitor from the previous problem. How this voltage is distributed between insulator and semiconductor?

**3.** An ideal Si-$SiO_2$ MOSFET has $d = 15$ nm and $N_a = 10^{16}$ cm$^{-3}$. What is the flat-band capacitance of this system? $S = 1$ mm$^2$, and $T = 296$ K.

**4.** For the MOSFET from the previous problem find the turn-on voltage ($V_T$) and the minimum capacitance under high-frequency regime.

**5.** For a metal-$SiO_2$-Si capacitor with $N_a = 10^{16}$ cm$^{-3}$ and $d = 8$ nm, calculate the minimum capacitance on the $C$-$V$ curve under high-frequency condition. $S = 1$ mm$^2$, and $T = 296$ K.

**6.** Find a number of electrons per unit area in the inversion region for an ideal Si-$SiO_2$ MOS capacitor with $N_a = 10^{16}$ cm$^{-3}$, $d = 10$ nm, $V = 1.5$ V, $T = 296$ K.

**7.** Turn-on voltage of the MOS from the previous problem was found to be shifted by 0.5 V from the ideal value. Assuming that the shift is due entirely to the fixed oxide charges at the $SiO_2$-Si interface, find the number of fixed oxide charges.

### 1.8 Low-dimensional Structures

**1.** Electric field at the surface of a semiconductor in the inversion layer is $\mathcal{E} = 5 \times 10^4$ V/cm. Using the variational principle with the probe function $\psi \sim z \exp(-z/a)$ estimate the lowest energy of an electron in the triangle potential well ($V = 0$ for $z \leq 0$ and $V = e\mathcal{E}z$ for $z > 0$) formed by the electric field.[1] Effective mass of the electron is $m^* = 0.063\, m$.

**2.** A potential well has a hight of 0.05 eV. What should be the width of the well so that the binding energy of the electron ($m^* = 0.063\, m_e$) would be equal to 0.025 eV.

**3.** A potential well of width 10 nm is formed by GaAs and $Al_xGa_{1-x}As$. Band gap of GaAs is 1.42 eV, the band gap of $Al_xGa_{1-x}As$ is $1.42 + 1.247\, x$ ($x \leq 0.45$). The band gap discontinuity is $\Delta E_c = 0.78\, x$. What should be $x$ so that the binding energy of an electron ($m^* = 0.063\, m_e$) in the well is $5\, kT$ at room temperature?

[1] The ground state energy is $E = \min \frac{\langle\psi|\widehat{H}|\psi\rangle}{\langle\psi|\psi\rangle}$

**4.** Two barriers with the hight of 0.1 eV and width of 20 nm are separated by the distance of 5 nm. Calculate at which bias voltage a resonance tunneling diode made of this structure has the first local maximum on the I/V curve. Effective mass of the electron is $m^* = 0.063\, m$.

**5.** Estimate the ground state lifetime of an electron trapped between the two barriers from the previous problem.

### 1.9 LEDs and Lasers

**1.** The spectrum for spontaneous emission is proportional to

$$(E - E_g)^{1/2} \exp(-E/kT) \, .$$

Find (a) the photon energy at the maximum of the spectrum and (b) the full width at half maximum (FWHM) of the emission spectrum.

**2.** Find the FWHM of the spontaneous emission in wavelength. If the maximum intensity occurs at 0.555 $\mu$m, what is the FWHM at room temperature?

**3.** Assume that the radiative lifetime $\tau_r$ is given by $\tau_r = 10^9/N$ s, where $N$ is the semiconductor doping in cm$^{-3}$ and the nonradiative lifetime $\tau_{nr}$ is equal to $10^{-7}$ s. Find the cutoff frequency of an LED having a doping of $10^{19}$ cm$^{-3}$.

**4.** For an InGaAsP laser operating at a wavelength of 1.3 $\mu$m, calculate the mode spacing in nanometer for a cavity of 300 $\mu$m, assuming that the group refractive index is 3.4.

**5.** Assuming that the refractive index depends on the wavelength as $n = n_0 + dn/d\lambda(\lambda - \lambda_0)$, find the separation $\Delta\lambda$ between the allowed modes for a GaAs laser at $\lambda_0 = 0.89$ $\mu$m, $L = 300$ $\mu$m, $n_0 = 3.58$, $dn/d\lambda = 2.5$ $\mu$m$^{-1}$.

**6.** An InGaAsP Fabry-Perot laser operating at a wavelength of 1.3 $\mu$m has a cavity length of 300 $\mu$m. The refractive index of InGaAsP is 3.9. If one of the laser facets is coated to produce 90 % reflectivity, what should be the minimum gain for lasing, assuming the absorption coefficient of the material $\alpha$ to be 10 cm$^{-1}$?

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# 3 Tables

Table 1: SI vs. CGS units.

| Quantity | SI | CGS |
|---|---|---|
| Force | 1 Newton (N) | 1 dyne (dyn) = $10^{-5}$ N |
| Work, energy | 1 Joule (J) | 1 erg = $10^{-7}$ J |
| Dynamic viscosity | 1 Pa·s | 1 Poise (P) = 0.1 Pa·s |
| Kinematic viscosity | 1 $m^2/s$ | 1 Stokes (St) = $10^{-4}$ $m^2/s$ |
| Pressure | 1 Pascal (Pa) | 1 barye (ba) = 0.1 Pa |
| Charge | 1 Coulomb (C) | 1 esu = $10/c \approx 3.3356 \cdot 10^{-10}$ C |
| Current | 1 Amperes (A) | 1 esu/s = $10/c \approx 3.3356 \cdot 10^{-10}$ A |
| Voltage | 1 Volt (V) | 1 Statvolt = $10^{-8}c \approx 300$ V |
| Resistance | 1 Ohm ($\Omega$) | 1 s/cm = $10^{-9}c^2 \approx 9 \cdot 10^{11}\,\Omega$ |
| Capacitance | 1 Farad (F) | 1 cm = $10^9/c^2 \approx 10^{-11}/9$ F |
| Magnetic field strength | 1 A/m | 1 Oersted (Oe) = $10^3/(4\pi) \approx 79.6$ A/m |
| Magnetic flux density | 1 Tesla (T) | 1 Gauss (G) = $10^{-4}$ T |
| Magnetic flux | 1 Weber (Wb) | 1 Maxwell (Mx) = $10^{-8}$ Wb |

Table 2: Basic parameters of some semiconductors at room temperature.

| | | | Effective mass,[a] $m_0$ | | Mobility, $cm^2$/V sec | | |
|---|---|---|---|---|---|---|---|
| Semiconductor | $E_g$, eV | Band | $m_e^*$ | $m_h^*$ | $\mu_e$ | $\mu_h$ | $\epsilon$ |
| Ge | 0.66 | I | 0.57 | 0.37 | 3900 | 1900 | 16.0 |
| Si | 1.12 | I | 1.08 | 0.59 | 1400 | 450 | 11.9 |
| GaAs | 1.42 | D | 0.063 | 0.53 | 8800 | 400 | 12.9 |
| GaP | 2.26 | I | 0.8 | 0,83 | 250 | 150 | 11.4 |
| GaN | 3.44 | D | 0.22 | 0,61 | 8500 | 400 | 10.4 |

[a]Effective mass in the expression for the density of the states of the conduction/valence band: $N_{c(v)} = 2(m^*_{e(h)}kT/2\pi\hbar^2)^{3/2}$.

Table 3: Work function of some metals.

| | Au | Ag | Al | Cu | Pt |
|---|---|---|---|---|---|
| $\psi_m$, eV | 5.1 | 4.3 | 4.25 | 4.7 | 5.65 |

Table 4: Electron affinity of some semiconductors.

| | Si | Ge | GaAs | GaP | GaN |
|---|---|---|---|---|---|
| $\chi$, eV | 4.05 | 4.0 | 4.07 | 3.8 | 4.2 |

Table 5: Properties of $SiO_2$ and $Si_3N_4$ at room temperature.

| Property | $SiO_2$ | $Si_3N_4$ |
|---|---|---|
| Energy gap, eV | 9 | 5 |
| Electron affinity, eV | 0.9 | – |
| Dielectric constant | 3.9 | 7.5 |
| Refractive index | 1.46 | 2.05 |
| Resistivity, $\Omega$·cm | $10^{14}$–$10^{16}$ | $10^{14}$ |

# 4 Answers and Solutions

## 4.1 Properties of Semiconductors

1. It follows from Table 2 that Si and GaAs are not transparent, GaP is partially transparent, and GaN is transparent for the visible light.
2. $n_i^2 = N_c N_v \exp\left(-E_g/kT\right) \sim T^3 \exp\left(-E_g/kT\right)$. Therefore
$$n_i(T_2) = n_i(T_1)\left(\frac{T_2}{T_1}\right)^{3/2} \exp\left(-\frac{E_g(T_2)}{2kT_2} + \frac{E_g(T_1)}{2kT_1}\right).$$
Putting the proper values in the formula we obtain that $n_i(77\,\mathrm{K}) \approx 10^{-20}\mathrm{cm}^{-3}$.
3. From $\mu = e\tau/m^*$ we get that $\tau = 2.6 \times 10^{-13}$ s.
4. Since
$$v_t = \frac{\int_0^\infty v \exp\left(-m^* v^2/2kT\right) d^3v}{\int_0^\infty \exp\left(-m^* v^2/2kT\right) d^3v} = \sqrt{\frac{8kT}{\pi m^*}},$$
thermal velocities of electrons and holes are $4.3 \times 10^7$ and $1.5 \times 10^7$ cm/s, respectively.
5. From $eD = \mu kT$, it follows that $D = 49$ cm$^2$/s.
6. $\tau_\mathrm{r} = \epsilon/4\pi e N_a \mu_p = 4.7 \times 10^{-12}$ s.
7. In this case,
$$\tau_\mathrm{r} = \frac{\epsilon}{4\pi e n_i(\mu_n + \mu_p)} = 3.4 \times 10^{-7}\,\mathrm{s}.$$
8. $L_D = 0.48$ $\mu$m.
9. $D = 65$ cm$^2$/s.
10. $\Delta p = \Delta p_0 \times \exp\left(-\frac{L}{\sqrt{D_p \tau_p}}\right) = 1.6 \times 10^{13}$ cm$^{-3}$.

## 4.2 Schottky Diode

1. $eV_d = 0.88$ eV.
2. $w = 0.22$, 0.19, and 0.34 $\mu$m for $V = 0$, $+0.4$, and $-2$ V, respectively.
3. $E = 5.1 \times 10^4$, $4.2 \times 10^4$, and $5.1 \times 10^5$ V/cm for $V = 0$, $+0.3$, and $-100$ V, respectively.
4. $E = 2 \times 10^4$ V/cm.
5. From $n = 1/e\rho\mu_n$ we obtain that $n = 1.4 \times 10^{13}$ cm$^{-3}$. Thus,
$$e\varphi_d = \psi_\mathrm{Pt} - \chi_\mathrm{GaAs} - kT \ln N_c/n = 1.32\,\mathrm{eV}.$$
The average thermal velocity is
$$v_T = (8kT/\pi m_n)^{1/2} = 4.6 \times 10^7\,\mathrm{cm/s}.$$

From here we get

$$j_s = \frac{1}{4} env_T \exp(-e\varphi_d/kT) = 3 \times 10^{-22}\,\mathrm{A/cm^2}.$$

Finally, from

$$j = j_s\left(\exp(eV/kT) - 1\right)$$

we obtain $j(0.5\,\mathrm{V}) = 1.5 \times 10^{-13}$ A/cm$^2$ and $j(-5\,\mathrm{V}) = j_s$.

6. $\varphi_d = 0.74$ V, $n = 2.8 \times 10^{17}$ cm$^{-3}$.
7. A Schottky diode loses its rectifying characteristics when de Broglie wavelength, $\lambda$, of electron becomes comparable with the depletion width, $\omega$, of the diode. Since $\lambda = 2\pi\hbar/\sqrt{2m_0E}$ and $\omega = \sqrt{\epsilon_{\mathrm{Si}} E_g/4\pi e^2 n}$ from the condition $\lambda \ll \omega$ we obtain that

$$n \ll \frac{3\epsilon_{\mathrm{Si}} m_0 kT E_g}{16\pi^3 e^2 \hbar^2}$$

Here, we assumed that the mean energy of electron is $E = 3kT/2$ and the potential barrier at the contact is $\varphi_d = E_g/2e$. Substituting numerical values in the above expression we get that for proper functioning of the Schottky diode, electron concentration must be significantly less than $2 \times 10^{19}$ cm$^{-3}$.

## 4.3 Ideal *p-n* Junction

1. By definition, $e\varphi_d = F_n - F_p$. Concentrations of the free carriers are given by

$$n = N_c \exp\left(-\frac{E_g - F_n}{kT}\right),\ p = N_v \exp\left(-\frac{F_p}{kT}\right).$$

From here we get that

$$e\varphi_d = E_g + kT\ln\left(\frac{n}{N_c}\right) + kT\ln\left(\frac{p}{N_v}\right) = E_g + kT\ln\left(\frac{np}{N_cN_v}\right).$$

Since,

$$n_i^2 = N_cN_v \exp\left(-\frac{E_g}{kT}\right),$$

we obtain that

$$\varphi_d = \frac{kT}{e}\ln\left(\frac{np}{n_i^2}\right).$$

From $n = 1/e\rho\mu_n$ and $p = 1/e\rho\mu_p$, we finally get $\varphi_d = 0.68$ V.

2. Taking into account that at room temperature all donors and acceptors are ionized, i.e. $n = N_d$ and $p = N_a$, from the values found in the previous problem and

$$\omega = \left(\frac{\epsilon(\varphi_d - V)}{2\pi e}\frac{N_d + N_a}{N_dN_a}\right)^{1/2}$$

we get $\omega(-10\,\mathrm{V}) = 2\ \mu$m, $\omega(0\,\mathrm{V}) = 0.5\ \mu$m, and $\omega(+0.3\,\mathrm{V}) = 0.4\ \mu$m.

3. From the previous problem and

$$E = 2\left(\frac{2\pi e(\varphi_d - V)}{\epsilon}\frac{N_d N_a}{N_d + N_a}\right)^{1/2},$$

we obtain that $E(-10\,\mathrm{V}) = 10^5$ V/cm, $E(0\,\mathrm{V}) = 2.6\times 10^4$ V/cm, and $E(+0.3\,\mathrm{V}) = 2\times 10^4$ V/cm.

The electric field within a shallow donor is, in turn, $E \approx 3.4\times 10^5$ V/cm, that is, comparable to that of the $p$-$n$ junction.

4. Since

$$C = \frac{\epsilon S}{4\pi\omega},$$

we get $C(-10\,\mathrm{V}) = 0.5$ nF, $C(0\,\mathrm{V}) = 2$ nF, and $C(+0.3\,\mathrm{V}) = 2.6$ nF.

5. From the conditions of the problem $\omega_a = 0.01\omega$ and $\omega_d = 0.99\omega$. Since

$$\omega_a/\omega_d = N_d/N_a\,,$$

we get that $N_a = 99N_d$. Because $N_d = 1/e\rho\mu_n = 4.5\times 10^{15}$ cm$^{-3}$, we get $N_a = 4.4\times 10^{17}$ cm$^{-3}$.

6. $j_s = 1.66\,\mathrm{mA}\exp(-eV/kT) = 4$ $\mu$A.

7. For a $p^+$-$n$ junction

$$j_s = \frac{eD_p p}{L_p} = \frac{eD_p n_i^2}{N_d L_p} = \frac{e n_i^2}{N_d}\left(\frac{D_p}{\tau_p}\right)^{1/2}.$$

Taking into account that $\mu = eD/kT$, we finally get $\tau_p = 4.5\times 10^{-9}$ s.

8. Since

$$j_s \sim n_i^2 \sim T^3\exp\left(-E_g/kT\right),$$

we get

$$j_s(T_2)/j_s(T_1) = (T_2/T_1)^3\exp\left(-\frac{E_g}{kT_2} + \frac{E_g}{kT_1}\right).$$

From here the ratios of the reverse currents in the $p$-$n$ junctions made of Ge and Si are 15 and 82, respectively.

9. $p$-$n$ junction stops working when concentrations of electrons and holes equalize. It happens when $N_d(N_a) \approx n_i = \sqrt{N_c N_v}\exp(-E_g/2kT) \sim T^{3/2}\exp(-E_g/2kT)$. From here and the parameters given we get that the maximum temperatures are $T_{\mathrm{Ge}} \approx 400$ K, $T_{\mathrm{Si}} \approx 650$ K, and $T_{\mathrm{GaN}} \approx 1700$ K. That is, only wide band gap semiconductors are suitable for extremal applications.

## 4.4 Nonideal *p-n* Junction

1. By definition

$$G_n = -R_n = \frac{n_i^2 - pn}{\tau_p(n + n_i) + \tau_n(p + p_i)},$$

where $\tau_n^{-1} = \tau_p^{-1} = N_t \sigma_n v_t = 10^7$ sec$^{-1}$. In the first case $n$ and $p$ are less than $n_i$. Thus, $np < n_i^2$ and hence

$$G_n = \frac{n_i^2}{\tau_n(n_i + p_i)} = \frac{n_i}{2\tau_n} = 5.3 \times 10^{16}\ \mathrm{cm}^{-3}/\mathrm{s}.$$

In the second case $n = N_d \gg n_i$, whereas $p < n_i$, hence

$$G_n = \frac{n_i^2}{\tau_n(n + n_i + p_i)} = \frac{n_i^2}{\tau_n n} = \frac{n_i^2}{\tau_n N_d} = 1.6 \times 10^{11}\ \mathrm{cm}^{-3}/\mathrm{s}.$$

2. In equilibrium, the generation $G = 10^{21}$ cm$^{-3}$/s and recombination $R$ rates are equal,

$$G = R = \frac{np - n_i^2}{\tau_p(n + n_i) + \tau_n(p + n_i)} \approx \frac{np}{\tau_p n + \tau_n p} = \frac{np}{\tau(n + p)}.$$

Here we used $\tau_n = \tau_p = \tau = (N_t v_t \sigma_n)^{-1} = 10^{-6}$ sec.

In $n$-type Si under illumination, $n = N_d + \Delta n$, $p \approx \Delta p = \Delta n$. Thus,

$$G\tau = \frac{(N_d + \Delta n)\Delta n}{N_d + 2\Delta n}.$$

Solving this equation with respect to $\Delta n$ we obtain $p = \Delta n = 1.1 \times 10^{15}$ cm$^{-3}$ and $n = 1.1 \times 10^{16}$ cm$^{-3}$.

3. Generation current in the space charge region $w$ is given by

$$j_g = \frac{e n_i w}{2\tau}.$$

Here, $\tau^{-1} = N_t \sigma_n v_t = 10^7$ s$^{-1}$. The width $w$ of the space charge region for a $p^+$-$n$ junction under reverse bias is

$$w = \left(\frac{\epsilon(\varphi_d - V)}{2\pi e N_d}\right)^{1/2} \approx \left(\frac{\epsilon |V|}{2\pi e N_d}\right)^{1/2} = 3.6\,\mu\mathrm{m}.$$

Here we used relation $|V| \gg \varphi_d$. From here we obtain that $j_g = 3$ $\mu$A/cm$^2$.

4. Using the formulae of the previous problem and relation

$$\varphi_d = \frac{kT}{e} \ln\left(\frac{np}{n_i^2}\right),$$

we get $j_s = 1.6$ nA/cm$^2$.

5. From the parameters given, we find $\lambda_n = 2.5 \times 10^{-6}$ cm and $\lambda_p = 8.5 \times 10^{-7}$ cm. If $N_d = N_a = N$ the width of the space charge region is

$$w = \left(\frac{\epsilon \varphi_d}{\pi e N}\right)^{1/2}.$$

By definition, $w = \lambda$. Substituting

$$\varphi_d = \frac{kT}{e} \ln\left(\frac{N^2}{n_i^2}\right)$$

into the expression above and after some simplifications, we get

$$N = \frac{\epsilon}{\pi e \lambda^2} \frac{2kT}{e} \ln\left(\frac{N}{n_i}\right).$$

Solving the above equation numerically, we obtain $N = 6.8 \times 10^{18}$ cm$^{-3}$ and $6.2 \times 10^{19}$ cm$^{-3}$ for electron and holes, respectively.

6. Since $C = \epsilon/4\pi w_0$, and under strong reverse bias $w_0 \approx (\epsilon V/2\pi e N_d)^{1/2}$, we obtain $N_d = 1.1 \times 10^{15}$ cm$^{-3}$.

   Maximum electric field is at the interface and for a $p^+$-$n$ junction equals $E \approx 4\pi e N_d w_1/\epsilon$. From conditions of the problem we find that at the breakdown $w_1 = 18$ $\mu$m and, hence, the breakdown voltage is 273 $V$.

7. The width of the space charge region is $w \approx (\epsilon V/2\pi e N_d)^{1/2} = 2$ $\mu$m. From here we get that the maximum electric field at the breakdown is

$$E = \frac{4\pi e N_d}{\epsilon} w \approx 3 \times 10^5 \,\text{V/cm}.$$

## 4.5 Solar Cells

1. To find the maxima of the photon flux density and the incoming energy one has to calculate $g'(\omega_{\text{flux}}) = 0$ and $(\omega_{\text{energy}} g(\omega_{\text{energy}}))' = 0$. Denoting $x = \hbar\omega/kT$ we obtain

$$x_{\text{flux}} = 2\left(1 - e^{-x_{\text{flux}}}\right), x_{\text{energy}} = 3\left(1 - e^{-x_{\text{energy}}}\right).$$

   Solving these equations numerically we obtain that $x_{\text{flux}} = 1.59$ and $x_{\text{energy}} = 2.82$, which for $T = 5800$ K corresponds to 0.8 and 1.4 eV, respectively.

   After replacing $\omega$ with $2\pi c/\lambda$ and taking into account that $d\omega = 2\pi c\, d\lambda/\lambda^2$ we obtain that in terms of wavelength the maxima are to find from

$$x_{\text{flux}} = 4\left(1 - e^{-x_{\text{flux}}}\right), x_{\text{energy}} = 5\left(1 - e^{-x_{\text{energy}}}\right).$$

   Here, $x = 2\pi c\hbar/\lambda kT$. The solutions of the above equations are $x_{\text{flux}} = 3.92$ and $x_{\text{energy}} = 4.97$, which corresponds to $\lambda = 0.63$ and 0.5 $\mu$m, respectively.

2. The open-circuit voltage is obtained from

$$0 = I_s\left(\exp\left(\frac{eV_{oc}}{kT}\right) - 1\right) - I_L, \Rightarrow V_{oc} \approx \frac{kT}{e} \ln\left(\frac{I_L}{I_s}\right).$$

   With the parameters given we find $I_s = 2 \times 10^{-12}$ A and hence $V_{oc} = 0.61$ V at room temperature.

   The maximum power operating voltage $V_m$ we find from $dP/dV = 0$, where the operating power is

$$P = I_L V - I_s V\left(\exp\left(\frac{eV}{kT}\right) - 1\right).$$

   From here, we obtain

$$V_m \approx V_{oc} - \frac{kT}{e} \ln\left(1 + \frac{eV_m}{kT}\right).$$

   Solving the above equation numerically we get $V_m = 0.53$ V and finally obtain the maximum operating power $P_m = 48$ mW.

3. Since at room temperature $kT/e = 0.025$ V, from

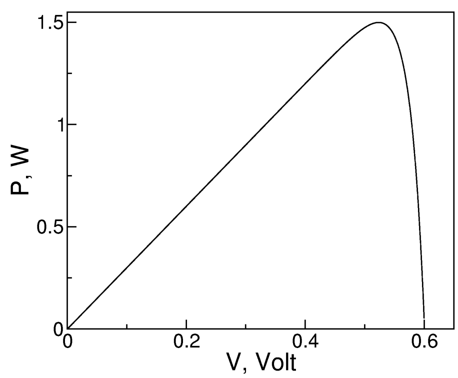
$$I_s \approx I_L \exp\left(-\frac{eV_{oc}}{kT}\right),$$

we find that $I_s = 1.1 \times 10^{-10}$ A.

The power output is

$$P(V) = IV = I_L V - I_s V\left(\exp\left(\frac{eV}{kT}\right) - 1\right).$$

From the plot $P$ vs. $V$, we find that the maximum power output is $P_m = 1.5$ W.




By definition, the fill factor is

$$F_F = \frac{P_m}{I_L V_{oc}}$$

In our case, the fill factor equals 0.83.

4. Since the flux density depends as $r^{-2}$ from the distance to the Sun, the short-circuit current is $I_L^{\mathrm{Mars}} = I_L^{\mathrm{Earth}}/1.5^2 = 1.33$ A. The open-circuit voltage is then

$$V_{oc}^{\mathrm{Mars}} = \frac{kT}{e} \ln\left(\frac{I_L^{\mathrm{Mars}}}{I_s}\right) = 0.58\,\mathrm{A}.$$

From here, we find $V_m = 0.5$ V and, thus, $P_m = V_m I_m = 0.61$ W.

5. From the paremeters given, we find that $I_s \approx I_L \exp(-eV_{oc}/kT) = 4.1 \times 10^{-9}$ A. Therefore, $V_{oc} = 0.48$, 0.46, and 0.44 V, for $I_L = 1$, 0.4, and 0.2 A, respectively.

6. From the values of $V_{oc}$ and $I_L$, we find that at 300 K $I_s \approx I_L \exp(-eV_{oc}/kT) = 4.1 \times 10^{-9}$ A. Employing the results of the previous problems, we find that the maximum output power at 300 K is equal to 0.8 W.

   In the case of a *p*-*n* junction $I_s \sim n_i^2 \sim \exp(-E_g/kT)$. Since for Si $E_g = 1.12$ eV, we obtain that $I_s = 0.2$ mA when the temperature raises to 400 K. The new value of $I_s$ corresponds to $V_{oc} = 0.3$ V.

   Finally, we get that the maximum output power of the Si solar cell at 400 K drops down to 0.4 W.

## 4.6 Bipolar Transistor

1. From the concentrations given we obtain the built-in potential of the junctions emitter-base ($\varphi_{EB}$) and base-collector ($\varphi_{BC}$)

$$\varphi_{EB} = \frac{kT}{e} \ln \left( \frac{N_E N_B}{n_i^2} \right) = 0.84\,\text{V},$$

$$\varphi_{BC} = \frac{kT}{e} \ln \left( \frac{N_B N_C}{n_i^2} \right) = 0.63\,\text{V}.$$

   Knowing the built-in potentials and the voltages applied to the emitter-base and base collector junctions, from the theory of the $p$-$n$ junction we get the appropriate widths of the space charge regions and accordingly—the neutral base width $x_B = 0.5\ \mu$m.

   The minority carrier concentraton at the emitter-base junction is

$$n_B = n_{B0} \exp\left( \frac{eV_{EB}}{kT} \right) = \frac{n_i^2}{N_B} \exp\left( \frac{eV_{EB}}{kT} \right) = 5 \times 10^{12}\,\text{cm}^{-3}.$$

2. By definition,

$$\gamma = \frac{j_{nE}}{j_{nE} + j_{pE}} = \frac{1}{1 + \frac{D_E}{D_B}\frac{N_B}{N_E}\frac{x_B}{x_E}}.$$

   Since $x_E = x_B$ and $D_E = D_B$,

$$\gamma = \frac{1}{1 + N_B/N_E} = 0.998.$$

3. For a thin base ($x_B \ll L_B$),

$$\alpha_T = \frac{1}{\cosh\left(x_B/L_B\right)} \approx \frac{1}{1 + \frac{1}{2}\left(x_B/L_B\right)^2}.$$

   From the first problem we know that $x_B = 0.5\ \mu$m. Thus, we obtain that $\alpha_T = 0.99$.

4. From the previous problem we obtain that

$$x_B = L_B \ln\left( \frac{1 + \sqrt{1 - \alpha_T^2}}{\alpha_T} \right).$$

   Substituting the proper values in this formula we get that $x_B = 0.57$, $1.87$, and $5.3\ \mu$m for $\alpha_T = 0.99$, $0.9$, and $0.5$, respectively.

5. Punch-through occurs when the neutral base width becomes zero. Employing the theory of an ideal $p$-$n$ junction, after somewhat tiresome but straightforward calculation we obtain that for the given parameters the punch-through voltage is 13.6 V.

6. For certainty, we assume that we have a $n^+$-$p$-$n$ transistor. The injection efficiency is

$$\gamma = \frac{1}{1 + \frac{D_E}{D_B}\frac{p_{E0}}{n_{B0}}\frac{x_B}{x_E}} = \frac{1}{1 + \frac{p_{E0}}{n_{B0}}} = \frac{1}{1 + \frac{N_B}{N_E}\frac{n_{Ei}^2}{n_{Bi}^2}}.$$

Here, $n_{Ei}$ and $n_{Bi}$ are the intrinsic concentrations of free carriers in emitter and collector, respectively.

Since $n_i^2 \sim \exp(-E_g/kT)$, we obtain that

$$\frac{n_{E_i}^2}{n_{Bi}^2} = \exp\left(\frac{\Delta E_{Eg} - \Delta E_{Bg}}{kT}\right).$$

From here, we get that $\gamma = 0.64$ and $0.76$ for the emitter dopings of $10^{19}$ and $10^{20}$ cm$^{-3}$, respectively.

7. For certainty, we assume that the base is $n$-type. Without applied voltage $j = en\mu_n\mathcal{E}(x) + eD_n dn/dx = 0$. Since $\mu_n = eD_n/kT$, from here we obtain an equation to determine the concentration profile

$$\frac{dn}{dx} = -\frac{e\mathcal{E}(x)}{kT}n.$$

Because $\mathcal{E}(x) = const$, from the above equation follows that the concentration profile should be exponential, $n \sim \exp(-x/l)$, where $l = kT/e\mathcal{E}$.

8. The drift time of minority carriers through the base is $\tau_{drift} = x_B/v_{drift} = x_B/\mu\mathcal{E}$. The diffusion transport time is approximately $\tau_{diff} = x_B^2/2D$. From here we get the ratio of the two values

$$\frac{\tau_{diff}}{\tau_{drift}} = \frac{x_B^2}{2D}\frac{\mu\mathcal{E}}{x_B} = \frac{ex_B\mathcal{E}}{2kT} = 10.$$

That is, the drift transistor can operate at about one order of magnitude higher frequencies.

9. In the case of common-emitter configuration, the frequency is determined by the life time of the minority carriers in the base $f \approx \tau_B^{-1} = D_B/L_B^2 = 0.4$ GHz. For the common-base configuration the frequency depends on the diffusion rate through the base, that is, $f \approx \tau_{diff}^{-1} = 2D_B/x_B^2 = 40$ GHz.

## 4.7 MIS/MOS Capacitor and MOSFET

1. The voltage ($V$) applied to a MOS capacitor appears across an insulator ($V_i$) and a semiconductor ($\psi_s$). Thus

$$V = V_i + \psi_s.$$

The voltage across the insulator is

$$V_i = \frac{|Q_s|}{C_i}, \quad \text{where} \quad C_i = \frac{\epsilon_i}{4\pi d}.$$

Here, $|Q_s|$ is the charge stored in the semiconductor, $\epsilon_i$ is the dielectric permittivity of the insulator, and $d$ is the thickness of the insulator.

If the voltage across the semiconductor is less than the one needed to bring about a strong inversion the charge stored in the semiconductor is

$$|Q_s| = eN_a w_d = eN_a \left(\frac{\epsilon_s \psi_s}{2\pi e N_a}\right)^{1/2},$$

where $\epsilon_s$ is the dielectric permittivity of the semiconductor and $w_d$ is the width of the depletion layer. From here we obtain that

$$V = \psi_s + \frac{2d}{\epsilon_i}\sqrt{2\pi e \epsilon_s N_a \psi_s}.$$

The value of $\psi_s$ to make the surface intrinsic is

$$\psi_s = \psi_{pB} = \frac{kT}{e}\ln\left(\frac{N_a}{n_i}\right),$$

whereas a strong inverstion occurs at $\psi_s = 2\psi_{pB}$.

Thus, from the parameters given we obtain that $V = 1.24$ and 2.0 V result in the intrinsic surface of Si and the strong inverstion, respectively.

2. From the results obtained in the previous problem we find that $\psi_s = 0.32$ V and $V_i = 0.68$ V.

3. By definition, the flat-band capacitance is

$$C_{FB} = \frac{\epsilon_i \epsilon_s S}{4\pi(\epsilon_s d + \epsilon_i L_D)},$$

where $L_D$ is the Debye length. Substituting in this formula the parameters given we obtain that $C_{FB} = 1.2$ nF.

4. By definition, the turn-on voltage ($V_T$) is the voltage at which the strong inversion occurs, that is $\psi_s = 2\psi_{pB}$. Based on the results of Problem 1 we obtain that

$$V_T = 2\psi_{pB} + + \frac{2d}{\epsilon_i}\sqrt{2\pi e \epsilon_s N_a (2\psi_{pB})}.$$

From the parameters given we find that $V_T = 0.9$ V.

The minimum capacitance under high-frequency regime is

$$C'_{min} = \frac{\epsilon_i \epsilon_s S}{4\pi(\epsilon_s d + \epsilon_i w_d)},$$

where $w_d$ is the width of the depletion layer under $\psi_s = 2\psi_{pB}$. From here we find that $C'_{min} = 0.3$ nF.

5. $C'_{min} = 0.32$ nF.

6. The turn-on voltage $V_T = 2\psi_{pB} = 2kT/e\ln(N_a/n_i) = 0.69$ V. The maximum width of the depletion layer is

$$w_d = \left(\frac{\epsilon_s V_T}{2\pi e N_a}\right)^{1/2} = 0.3\ \mu\text{m}.$$

Thus, the number of negatively charged acceptors per unit area in the depletion layer is $\sigma_a = N_a w_d = 3 \times 10^{11}$ cm$^{-2}$.

To find the total charge stored in the semiconductor we have to solve a nonlinear equation (see the solution of Problem 1)

$$V = \psi_s + V_i = \psi_s + \frac{|Q_s|}{C_i} = \psi_s + \frac{4\pi d}{\epsilon_i}|Q_s|. \tag{1}$$

The charge $|Q_s|$ is given by

$$|Q_s| = \frac{\sqrt{2}\epsilon_s kT}{4\pi e L_D} F(\psi_s, n_{p0}/p_{p0}).$$

Under strong inverstion

$$F \approx \left(\frac{n_{p0}}{p_{p0}}\right)^{1/2} \exp\left(\frac{\psi_s}{2kT}\right).$$

Substituting this expression into Eq. (1) we obtain an equation to determine $\psi_s$. Numerical solution gives $\psi_s = 0.85$ V. We see that $\psi_s > V_T$, which justifies our suggestion that the capacitor is in the strong inversion regime.

Knowing the value of $\psi_s$ we find $|Q_s|$ and finally determine the number of electrons per unit area in the inversion layer of the capacitor

$$\sigma_e = \frac{|Q_s|}{e} - \sigma_a = 1.1 \times 10^{12}\,\mathrm{cm}^{-2}.$$

7. The voltage shift is given by

$$\Delta V = \frac{Q}{C_i},$$

whereas the number of fixed charges in the oxide is $\sigma = Q/e$. From the parameters given we obtain that $\sigma = 1.1 \times 10^{12}$ cm$^{-2}$.

## 4.8 Low-dimensional Structures

1. Schrödinger equation for an electron in the triangle potential is

$$\widehat{H}\psi = -\frac{\hbar^2}{2m^*}\psi'' + e\mathcal{E}z\psi = E\psi. \tag{2}$$

To find the ground state energy we have to calculate

$$\langle\psi|\widehat{H}|\psi\rangle = -\frac{\hbar^2}{2m^*}\int_0^\infty \psi\psi'' dz + e\mathcal{E}\int_0^\infty z|\psi|^2 dz$$

and

$$\langle\psi|\psi\rangle = \int_0^\infty |\psi|^2 dz.$$

With the probe function $\psi = Az\exp(-z/a)$ we obtain that

$$\langle\psi|\widehat{H}|\psi\rangle = A^2\left(\frac{\hbar^2}{8m^*}a + \frac{3e\mathcal{E}}{8}a^4\right) \text{ and } \langle\psi|\psi\rangle = A^2\frac{a^3}{4}.$$

As follows from Eq. (2), energy of the ground state is a minimum of

$$E = \frac{\langle\psi|\widehat{H}|\psi\rangle}{\langle\psi|\psi\rangle} = \frac{\hbar^2}{2m^*a^2} + \frac{3}{2}e\mathcal{E}a.$$

From $E' = 0$ we obtain that $E_{min}$ occurs at $a = (2\hbar^2/3me\mathcal{E})^{1/3}$ and equals

$$E_{min} = \frac{9}{4}\left(\frac{2\hbar^2e^2\mathcal{E}^2}{3m^*}\right)^{1/3}.$$

With the values given, we get $E_{min} = 61$ meV. Note that $E_{min}$ in bigger than $kT$ at room temperature.[2]

2. Denoting the hight of the potential well as $V_0$ we get that the well width $a$ should be equal

$$a = \frac{\pi\hbar}{2\sqrt{m^*V_0}} \approx 80 \text{ Å}.$$

3. The ground state energy $E_0$ of a particle in a square box is found from

$$\left(\frac{V_0 - E_0}{E_0}\right)^{1/2} = \tan\left(\frac{\sqrt{2m^*E_0}a}{2\hbar}\right).$$

By definition $E_0 = V_0 - 5kT$. From the above equation we find that $V_0 = 1.24 \times 5kT = 155$ meV and, hence, $x = 0.16$.

4. The ground level of an electron trapped between the barriers is found from

$$\left(\frac{V_0 - E_0}{E_0}\right)^{1/2} = \tan\left(\frac{\sqrt{2m^*E_0}a}{2\hbar}\right).$$

Here, $V_0 = 0.1$ eV is the barrier hight, and $a = 5$ nm is the distance between the barriers. From here we find that $E_0 = 55$ meV.

Since the resonance tunneling diode is symmetric, to line up the ground level of an electron in the wall with the conduction band minimum of the emitter the voltage to be applied is $V \approx 2E_0/e$. Thus, the first maximum on the I/V curve occurs at approximately 110 mV.

5. Probability of tunneling for an electron with the energy $E_0 = 55$ meV through the square barrier with the hight of $V = 0.1$ eV and the width of $b = 20$ nm is

$$D = \frac{1}{1 + \frac{1}{4}\left(\sqrt{\frac{E_0}{V_0 - E_0}} + \sqrt{\frac{V_0 - E_0}{E_0}}\right)^2 \sinh^2\left(\frac{\sqrt{2m^*(V_0 - E_0)}}{\hbar}b\right)} \approx 7 \times 10^{-5}.$$

"Attempt frequency" $\nu$ can be estimated from $\nu \approx E_0/2\pi\hbar = 1.3 \times 10^{13}$ s$^{-1}$. Thus, the lifetime of an electron between the barriers is

$$\tau \approx (\nu D)^{-1} \approx 10^{-9} \text{ s}.$$

[2] The exact value of the ground state energy is about 94 % of $E_{min}$.

## 4.9 LEDs and Lasers

1. Differentiating the expression given and equalizing it to zero we find that the maximum of the emission spectrum occurs at $E_{max} = E_g + kT/2$. The FWHM is found from the condition

$$(E - E_g)^{1/2} \exp(-E/kT) = \frac{1}{2}(E_{max} - E_g) \exp(-E_{max}/kT).$$

   Numerical solution gives $\Delta E = 1.8\,kT$.

2. Based on the results of the previous problem we find that the FWHM of the spontaneous emission in wavelength is

$$\Delta\lambda = \frac{1.8\,kT\lambda^2}{2\pi\hbar c} = 12 \text{ nm}.$$

   Note that $\Delta\lambda/\lambda = 0.02$.

3. Per definition,

$$f_T = \frac{1}{2\pi\tau} = \frac{1}{2\pi\tau_r} + \frac{1}{2\pi\tau_{nr}}.$$

   Substituting in this relation the parameters given we obtain that $f_T = 1.6$ GHz.

4. Since

$$\Delta\lambda = \frac{\lambda^2}{2nL}\Delta m,$$

   for the neighboring modes ($\Delta m = 1$) we obtain $\Delta\lambda = 8.3$ Å.

5. Differentiating the condition for the resonance modes

$$m\frac{\lambda}{2n} = L,$$

   we obtain that

$$\Delta\lambda = \frac{\lambda_0^2}{2L(n_0 - \lambda_0 dn/d\lambda)} = 9.7\,\text{Å}.$$

6. The minimum gain for lasing is

$$g_{min} = \alpha + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right).$$

   Here, $R_1 = 0.9$. The reflectivity of the noncoated facet is

$$R_2 = \left(\frac{n-1}{n+1}\right)^2 = 0.35.$$

   Finally, we get that $g_{min} = 29.3 \text{ cm}^{-1}$.