# Tolerance Analysis using an alternative to Monte Carlo simulation

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*Abstract*— **Monte Carlo is an efficient tool to simulate complex physical and mathematical systems.  Large sample sizes are usually required to obtain acceptable accuracy.  This obviously exaggerates the slowness and expensiveness problem inherent part of the method.  The motivation to this research is to tackle such disadvantage and identify a proper alternative to this method. The aim of this work is to conduct a comparative study of tolerance analysis of mechanical assemblies using two different techniques, namely Monte Carlo method (MC) and Orthogonal Arrays (OAs).  In particular, standard and small orthogonal experiments are utilized to replace the huge sample sizes needed in case of Monte Carlo method.  The mean and variance of design functional requirements are determined using both methods.  Then, a comparison between the results, that were obtained from both methods, were assessed to evaluate the performance of the Orthogonal Array (OA) method as an alternative of Monte Carlo simulation (MCS) tool. In this paper, disk-drive assembly, knuckle joint with three arms assembly, one-way clutch assembly, and helical spring have been used as case studies of mechanical assemblies to highlight the strength of the alternative method. Based on the achievable results one can argue that Monte Carlo method can be replaced by Orthogonal Arrays gives acceptable results in a shorter time.  The first and second moments of examples revealed that both methods give close results. However, the second moment not completely matched that obtained using Monte Carlo method at 3-levels; it matched that obtained using Monte Carlo method at 2-levels. These discrepancies are challenging issues for further investigation in future work. Apart from this imperfection, it is recommended to play a significant role, particularly to simulate a wide range of engineering applications.**

*Keywords—Tolerance analysis; Monte Carlo Simulation; Orthogonal Arrays; Mechanical assemblies.*

## I.   INTRODUCTION

Imani B.M. and Hosseini S.A.[1] proposed Direct Linearization Method (DLM) for tolerance analysis which helps to optimize the performance of assembly before manufacturing. The results showed that DLM helped to recognize the importance and impacts of manufacturing tolerance to add early in the design process and identify the critical source of variation on assemblies at manufacturing process.

Otsuka and Nagata [2] proposed an optimization allocation approach of statistical tolerance indices. The proposed approach using genetic algorithms. They applied the proposed approach to a product model consist of five parts assembled in linear combination. The results showed that the proposed method resulted an optimal allocation of statistical tolerance indices into parts of product.

Bruyere j. et al.[3] proposed a tolerance analysis technique of bevel gear to estimate the effect of tolerance on gear quality and cost. The proposed technique depends on a vertical dimensioning and tolerancing model by using Monte Carlo simulation to estimate the possibility of product assembling and its good performance under given individual tolerances. The results revealed about having a direct correlation between the precision and number of samples.

Jawad et al. [4] proposed a new technique to study the impacts of geometrical deviations for nonlinear over-constrained mechanism. That technique helps the designers to identify the control elements in a model by adding a qualitative and quantitative nature to the tolerance analysis procedure. This work has been supported using optimization and Monte Carlo simulation to determine the probability of the functional operation of the assembly.

Gadalla M.[5] proposed an alternative to Monte Carlo simulation to solve tolerance analysis problem in mechanical assembly. The proposed method aimed to replace the huge sample size by small orthogonal iterations. The results showed the proposed method matches the first moment only in a minute percentage of time used by Monte Carlo but the second, third and fourth moments needed some improvements.

## II.   THE USED METHODS

The objective of this work is to use the orthogonal arrays (OAs) method as an alternative to Monte Carlo simulation (MCS) in solving the tolerance analysis especially in mechanical assemblies' problems. Achieving the goal is necessary at first to identify the assembly problem with an available and explicit assembly function which linked the resultant variables of interest to the contributing variables or dimensions in an assembly. Then apply both methods for solving the assembly problem and determining the mean and standard deviation of critical design requirements. And finally, compare the results of first and second moments of both methods.
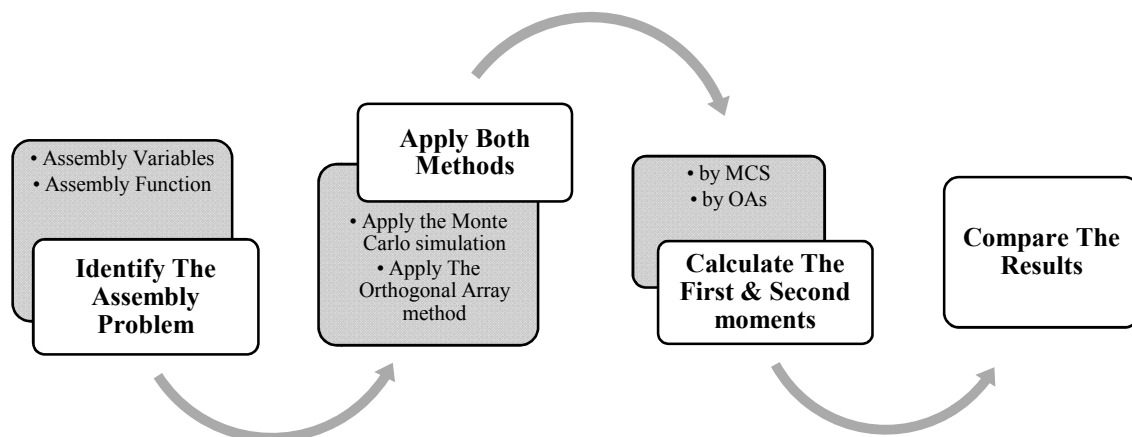


Fig. 1. The methodology of this work

Before presenting the cases study of this work, we will briefly review some of the available method for performing statistical tolerance analysis. These include:

- The Direct Linearization Method (DLM).
- Quadratic Method.
- System Moments
- Monte Carlo Simulation.
- Taguchi Method.

Monte Carlo method and orthogonal arrays are especially selected to attempt to combine the precision and simplicity at the same time at lower cost. Monte Carlo simulation is one of the most accurate methods to perform the statistical tolerance analysis but requires a huge sample size which making it highly expensive of the commonly used methods. As the result, the orthogonal array method is selected cause of the ability of replacing the large sample sizes by small orthogonal experiments and also reducing the loss function.

### A. Orthogonal Arrays (OAs)

The proposed method is Orthogonal Arrays or what is known as Taguchi method. It is one of the most powerful methods for improving engineering productivity and quality and reducing the cost [6, 7]. Orthogonal Arrays method was selected for its ability to gather dependable information about design parameters with small number of experiments. Orthogonal arrays are a set of tables of numbers, the variables can be assigned to their columns and the rows express to a number of experimental situations. Tolerance domain can be turned into numbers of points[5]. For example, the tolerances of two variables A and B were +- 0.04 and +- 0.02 respectively. The tolerance domain would be {-0.04, 0.0, +0.04} as 1st, 2nd, and 3rd levels of A and similarly {-0.02, 0.0, +0.02} for variable B. Then the combinations between that variables will be (-0.04, -0.02), (-0.04, 0.0), (-0.04, +0.02), (0.0, -0.02), (0.0, 0.0), (0.0, +0.02), (+0.04, -0.02), (+0.04, 0.0), (+0.04, +0.02). Where the number of variables=2 and number of their Levels=3, Orthogonal Arrays can be represented by $L_{Run}$ (levels$^{Factors}$). The average and standard deviations are calculated accordingly.

### B. Monte Carlo simulation (MCS)

The Monte Carlo simulation for statistical tolerance analysis is mainly depends on a random number generator to evaluate the variations of dimension tolerances for individual assemblies. Selecting value of each manufactured dimensions based on their statistical distribution as shown in Fig.2. An assembly function is combined these dimensions to calculate the value of the assembly variables for each simulation assembly. The results of set values of assembly function are used to calculate the first and second moments to study the behavior of the assembly. Although Monte Carlo method is easily tool for tolerance analysis problems, it is too expensive in terms of computational time [8-11].
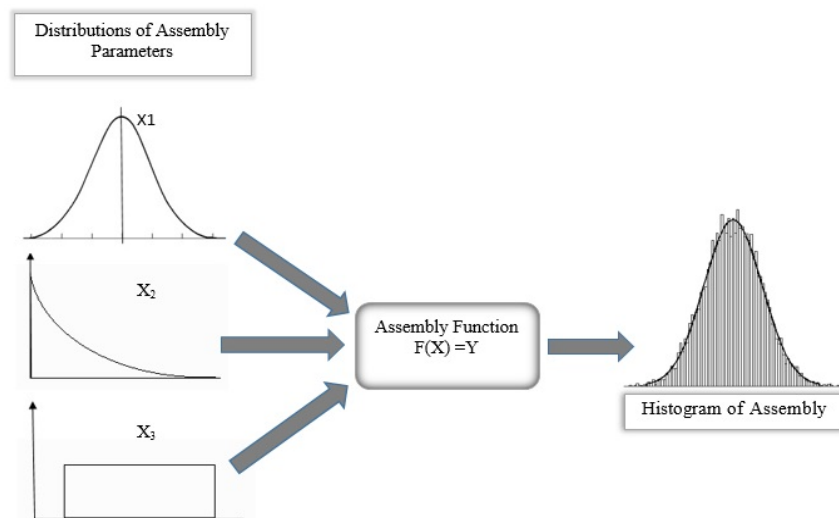


Fig. 2. Monte Carlo Simulation

### III. EXAMPLES STUDIED

Four mechanical assemblies studied herein presented to conduct a comparative study of tolerance analysis of this examples using two different techniques, namely Monte Carlo method (MC) and Orthogonal Arrays (OAs). In those assemblies, the mean and standard deviation will be computed for their design functions by the two methods. The probability distribution of inputs variables is assumed as normal distribution. In this work, the samples size 10000, 100000, and 1000000 are used.
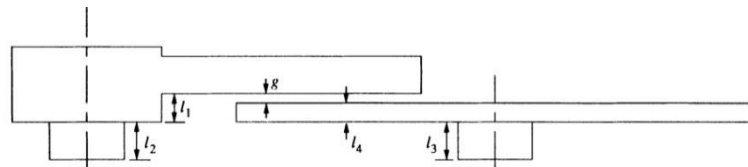
### A. Example 1: Disk-drive assembly.

Fig. 3. Disk-drive assembly

A disk-drive assembly was solved by O'Connor and Srinivasan [12] as a linear problem. The disk-drive assembly as shown in Fig. 3 consists of arm, arm_bearing, disk_bearing, and disk. The critical assembly characteristic is the spacing g between the arm and disk which assosciated with the part dimensions $l_1$, $l_2$, $l_3$, and $l_4$, shown in Table 1, by linear relation known as gap function as in (1).

$$g = l_1 + l_2 - l_3 - l_4 \quad (1)$$

TABLE I.     NOMINAL DIMENSIONS AND TOLERANCE OF  DISK-DRIVE ASSEMBLY.

| Dimension(s) | Nominal Values | Tolerances ($t_i$) ± |
|---|---|---|
| Arm  $l_1$ | 1.75 | 0.05 |
| Arm Bearing  $l_2$ | 2.00 | 0.07 |
| Disk Bearing  $l_3$ | 2.00 | 0.07 |
| Disk  $l_4$ | 1.00 | 0.03 |

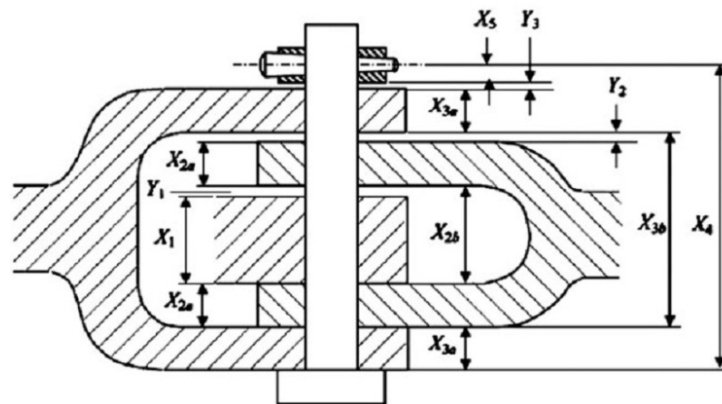*B. Example 2: knuckle joint with three arms assembly.*



Fig.4. Knuckle joint assembly

The knuckle joint assembly as shown in Fig.4 , which presented as a case study in several works [13-16] , consists of three arms. There are  three design functions $Y_1$, $Y_2$, $Y_3$ involve seven individual dimensions $X_i=(X_1, X_{2a}, X_{2b}, X_{3a}, X_{3b}, X_4, X_5)$ in product assembly as shown in Table 2. The design tolerance associated to $X_{2a}$ and $X_{2b}$ is $t_2$, and that associated to $X_{3a}$ and $X_{3b}$ is $t_3$ because all similar dimensions are manufactured in same machine.

Design Functions :

$$Y_1 = X_{2b} - X_1 \quad (2)$$

$$Y_2 = X_{3b} - (2X_{2a} + X_{2b}) \quad (3)$$

$$Y_3 = X_4 - (2X_{3a} + X_{3b} + X_5) \quad (4)$$

These design functions (2), (3), and (4) were solved by Monte Carlo Simulation (MCS) and orthogonal arrays method and then obtained the first and second moments by the both methods. A comparison between MC method and the proposed method will be clarified in the result section.

TABLE II.     DIMENSIONS AND TOLERANCE OF  KNUCKLE JOINT ASSEMBLY.

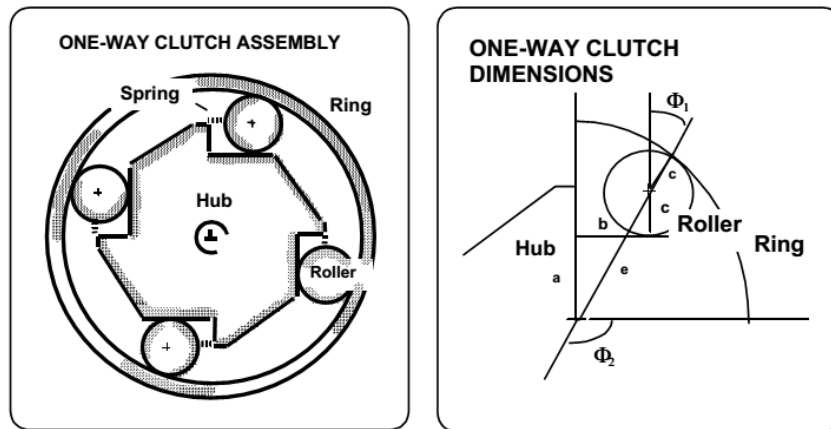| Dimension(s) | Nominal Values | Tolerances ($t_i$) ± |
|---|---|---|
| $X_1$ | 100 | 0.1478 |
| $X_{2a}$ | 50 | 0.0495 |
| $X_{2b}$ | 105 | 0.0495 |
| $X_{3a}$ | 50 | 0.0512 |
| $X_{3b}$ | 210 | 0.0512 |
| $X_4$ | 340 | 0.0214 |
| $X_5$ | 25 | 0.0241 |

## C. Example 3: One-Way Clutch assembly.



Fig. 5. One-Way Clutch assembly

The second example taken for illustration is a one-way clutch assembly which is presented as a case study in several papers [10, 17-19]. The one-way clutch consist of three important components:hub, roller, and outer ring shown in Fig. 4. The important functional dimensions and tolerances are given in table 2. The contact angle Φ must be controlled can be expreseed by nonlinear function (5).

$$\emptyset = \cos^{-1}\left(\frac{a+c}{e-c}\right) \qquad (5)$$

TABLE III.    COMPONENTS AND DIMENSIONS OF ONE-WAY CLUTCH ASSEMBLY.

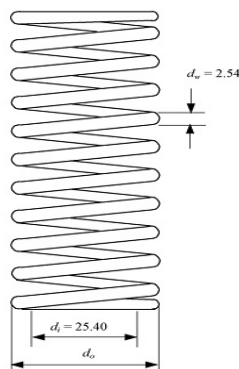| Dimension(s) | Nominal Values | Tolerances ($t_i$) ± |
|---|---|---|
| Height of hub a | 27.645 | 0.0125 |
| Radius of roller c | 11.43 | 0.01 |
| Radius of ring e | 50.8 | 0.05 |

## D. Example 4: Helical Spring.



Fig. 6. Helical Spring

The helical spring shown in Fig. 6 has three decision variables in all [13, 15]. There are two interrelated dimension chains corresponding to respective design functions; one linear and other nonlinear, giving rise to two constraints given below (6) and (7). All length dimensions of helical spring are in mm and force in N, when G is a constant value equal 100000.

Design Functions:

$$K = \frac{G.d_w^4}{\{8(d_i+d_w)^3 N\}} \qquad (6)$$

$$d_o = d_i + 2d_w \qquad (7)$$

TABLE IV.     DIMENSIONS AND TOLERANCE OF HELICAL SPRING EXAMPLE.

| Dimension(s) | Nominal Values | Tolerances ($t_i$) ± |
|---|---|---|
| $d_i$ | 25.4 | 0.02 |
| $d_w$ | 2.54 | 0.4645 |
| N | 10 | 0.2054 |

## IV.   RESULTS

The presented resutls are for samples size: 10000, 100000, and 1000000 of Monte Carlo simulation (MCS). All the design functions of case studies used contain at most four variables, so the resulting array shoud be for 2-level $2^4 = 16$, and for 3-level $3^4 = 81$. This is the full factorial array. So we focused our attention on both of L64 (2-level) and L81 (3-level) of Orthogonal Arrays method (OAs).

TABLE V.     THE RESULTS OF MCS VIA OAS.

| Examples | Obictive Functions | Moments | Monte Carlo Simulation Sample size | | | Orthogonal Arrays Experiments | |
|---|---|---|---|---|---|---|---|
| | | | *10000* | *100000* | *1000000* | *L64 (2-levels)* | *L81 (3-levels)* |
| **Disk-drive Assembly** | g | Mean | 0.7504 | 0.7500 | 0.74998 | 0.75 | 0.75 |
| | | STD | 0.1144 | 0.1149 | 0.1149 | 0.1157995 | 0.0943928 |
| **Knuckl joint with three arms assembly** | Y1 | Mean | 5 | 5 | 5 | 5 | 4.9982 |
| | | STD | 0.1556 | 0.1559 | 0.1559 | 0.1571 | 0.1263 |
| | Y2 | Mean | 4.9995 | 5 | 5 | 5 | 5 |
| | | STD | 0.12207 | 0.122 | 0.12199 | 0.1229 | 0.1002 |
| | Y3 | Mean | 5.0003 | 5 | 5 | 5 | 5 |
| | | STD | 0.1189 | 0.1187 | 0.1189 | 0.1199 | 0.0977 |
| **One-way clutch Assembly** | $\varphi$ | Mean | 6.98618 | 6.98597 | 6.98622 | 6.9869 | 6.9973 |
| | | STD | 0.66549 | 0.66517 | 0.66531 | 0.66466 | 0.54101 |
| **Helical spring** | K | Mean | 0.14588 | 0.14588 | 0.1459 | 0.1459 | 0.1458 |
| | | STD | 0.00792 | 0.0079 | 0.0079 | 0.0080 | 0.0065 |
| | $d_o$ | Mean | 30.48093 | 30.47996 | 30.47988 | 30.48 | 30.48 |
| | | STD | 0.46552 | 0.46601 | 0.46617 | 0.4699 | 0.3830 |

## V.   DISCUSSION

In this section, discussion of the proposed method as an alternative to MCS and accompanied results are given. OAs are used for planning experimentation with small number of experiments to replace the large samples size of MCS. Two and three-level of OAs are employed. From the results, the OAs method is found on a relative close to MCS espcialy at L64 2-levels for all case studies considered, i.e. disk-drive assembly, knuckle joint with three arms assembly, one-way clutch assembly, and helical spring. Table V shows that the results  obtained using MCS and OAs methods for the tolerance analysis

problems. Each problem is run 10 times for each sample size in MCS to obtain the average value of first and second moments. The 1000000 sample size can be choosen as the most accurate to compare the standard deviation (STD) for each mechanical assemblies to its counterpart of OAs method.   In the following figuers, we attempted to provide a comparison of the STD results between the both methods for each case study. A closer look at this results of each example:
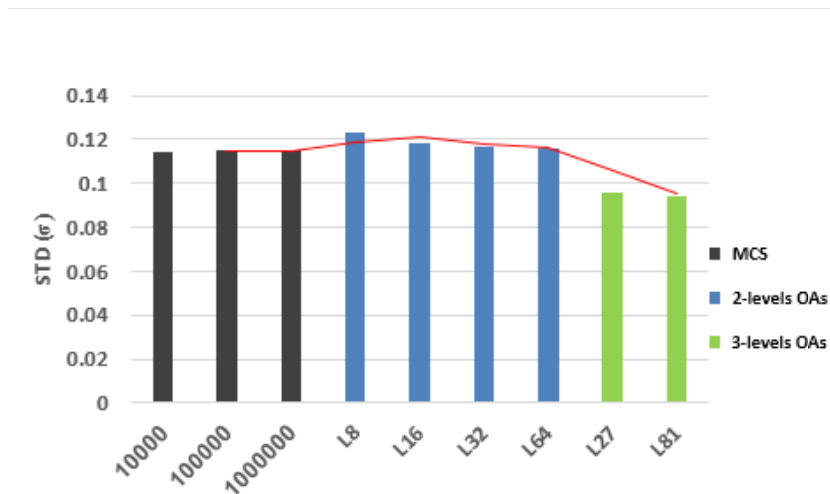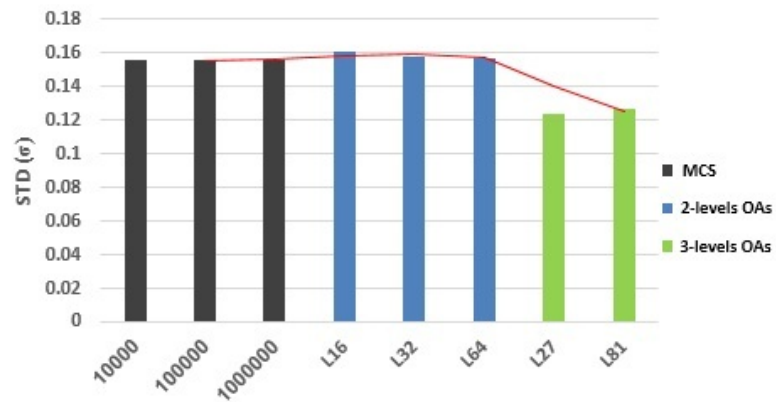
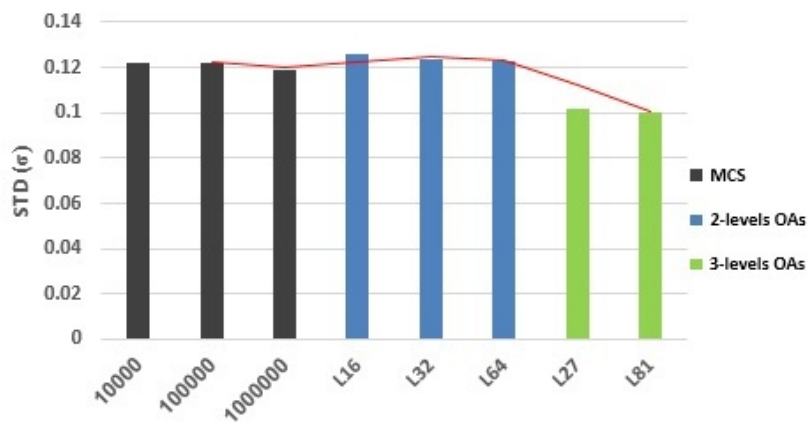*A.   Disk-drive assembly.*



Fig. 7 . A comparison of  STD  between  MCS and OAs of disk-drive assembly.

The Fig. 7 shows, the L64 (2-level) of OAs is closer to MCS than  the three-level.
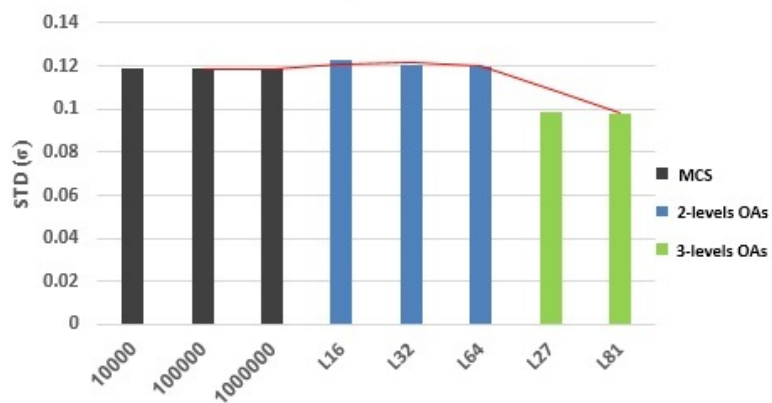
B.  knuckle joint with three arms assembly .



(a) $Y_1$ design function.



(b)  $Y_2$ design function.



(c) $Y_3$ design function.

Fig. 8. A comparison of  STD  between  MCS and OAs of knuckle joint with three arms assembly

The knuckle joint with three arms assembly has multi-design functions namely: $Y_1$, $Y_2$, and $Y_3$. The Fig. 8 shows, the L64 (2-level) of OAs is closer to MCS than the three-level.
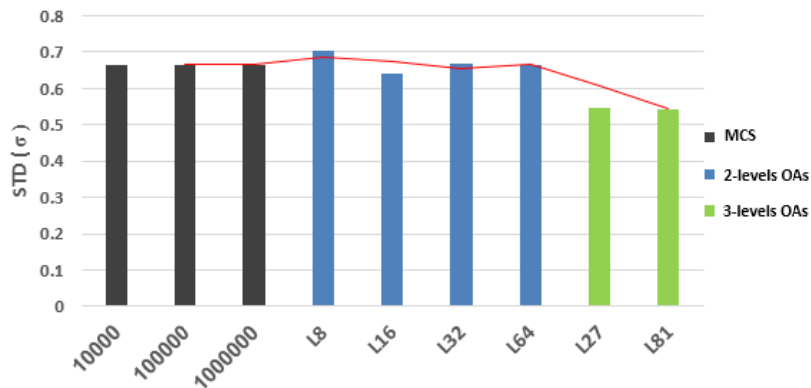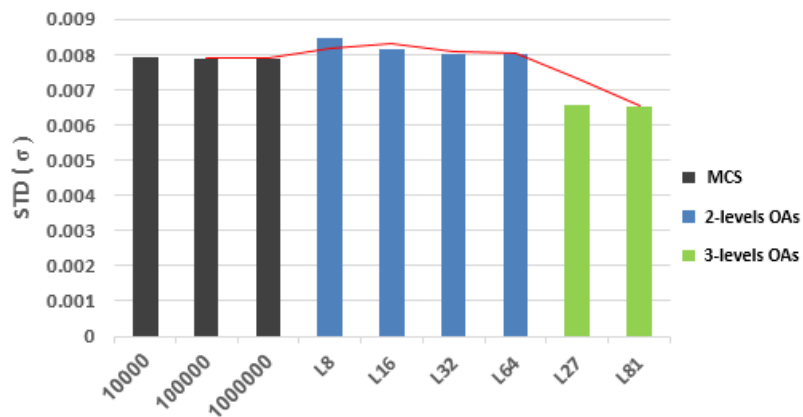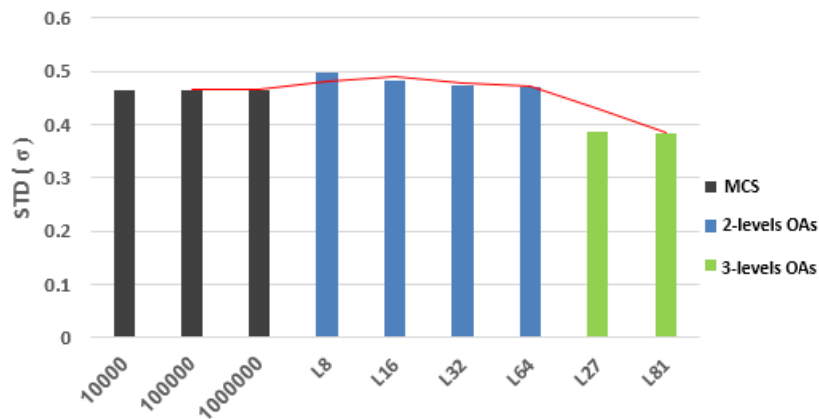
### C. One-way clutch assembly.



Fig. 9 . A comparison of STD between MCS and OAs of one-way clutch assembly.

The Fig. 8 shows, the L64 (2-level) of OAs is closer to MCS than the three-level.

### D. Helical spring example.



(a)  K design Function

(b) $d_o$ design finction

Fig. 9 . A comparison of  STD  between  MCS and OAs of of helical spring

The helical spring example has multi-design functions namely: K and do. The Fig. 9 shows, the L64 (2-level) of OAs is closer to MCS than  the three-level.

Based on the achievable results one can argue that expensive Monte Carlo method can be replaced by Orthogonal Array technique that gives acceptable results in a shorter time. In particular, from computational experience for mechanical assemblies, the first and second moments at 2-levels matched that of MCS. However, at 3-levels the first moment matches that of MCS but the second needs more improvements.

## VI.    CONCLUSION

Since the tolerance analysis on of the critical issues in the manufacturing, this paper considered four differnet problems for solving this issue of four machain elements: (i) disk-drive assembly, (ii) knuckle joint with three arms assembly, (iii) one-way clutch assembly, and (iv) helical spring. For tolerance analysis at lower cost higher accurcy for a mechanical assembly, a proposed method , called orthogonal arrays (OAs) or Taguchi method, is considerd as an alernative to Monte Carlo simulation.Out of the four problems considered, thefirst and third are a single-objective problem and the second and fourth are the multi-objective problems and these problems are used to evaluate the strength of the OAs method. The results obtained using OAs method is compared with those obtained by using the MC method. The Taguchi method has been presenterd a fairly acceptable results as an alternative of Mont Carlo simulation to solve tolerance analysis problems. Several points can be concluded:

- In this study, two-and three-level OAs have been used; other high level arrays could have been employed as four and five-level. It is expected that the higher the number of levels, the better the model error convergence. On the other hand, the expense of experimentation increases as the number of experiments increase.
- Results show that the proposed method at 2-levels matches the first and second moments of Monte Carlo except in a minute percentage of time. Some proposels can help to improve the second moment at 3-levels of orthogonal arrays method.
- We aim to achieve the optimal design of experiments with minimum number of experiments by selecting the optimal number of experiments with optimal domain.

## ACKNOWLEDGMENT

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