Production inventory model with exponential demand rate and exponentially declining deterioration

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Abstract. In this paper, a production inventory model with an exponential demand rate and exponentially declining deterioration is considered. The production rate of the model is expected to be proportional to the demand rate. The optimal total inventory cost per cycle, the optimal length of the cycle and optimal production length are determined. The proposed model has eight parameters. Numerical example of the proposed model is presented. Finally, the sensitivity analysis of the developed model is demonstrated.
Keywords: production inventory model, exponential demand rate, exponentially declining deterioration, optimal production cycle, total inventory cost per cycle.

1. Introduction

Inventory refers to goods or materials that are held by an organization for forthcoming usage. It is the common thread that ties all the functions and departments of the organization together. An inventory system is the set of policies and controls that monitors levels of inventory, so as to minimise the total inventory cost per cycle and to guarantee a smooth operation of the organisation. One main reason that an organisation maintains inventory that it is rarely possible to forecast / predict sale levels, production times, demand and usage requirements exactly. An inventory model can be either deterministic, probabilistic or imprecise, according to the predictability of demand involved. The demand of an inventory model depends upon many factors like market conditions, availability

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of substitutes etc., and hence, it is not under the control of the decision maker. If the demand is more than the supply, the shortage may arise and hence, the stock-out cost is incorporated.

In modelling inventory problem, two factors of the problem have been of growing interest to researchers, one being the deterioration of items and the other being the variation in the demand rate. Usually in most of all developed inventory models, it is assumed that the products during storage have endless life time. This means that an item once stocked will always remain in perfect condition and will be ideal to satisfy the customer's future requirement. But this assumption is not true for all types of products. There are many products such as vegetables, food products, fruits and pharmaceutical products in which deterioration occurs. After a certain period, the entire unsold lot will deteriorate completely and hence, they cannot be sold to customers. So, deterioration of physical goods is one of the important factors in the vendor-buyer system. Perishable inventory theory was developed in which products are to be deteriorated. Whitin [25] initiated and studied inventory of deteriorating fashion goods at the end of prescribed storage period. Ghare and Schrader [6] developed and analysed an inventory model for exponentially decaying perishable items. After then, many researchers have developed perishable inventory models. In such perishable models, the rate of deterioration is an important feature of consideration and these models were studied using exponential distribution and weibull distribution in general.

The economic order quantity (EOQ) model to assist organisations in minimizing total inventory costs was originated by Harris [9]. It balances inventory holding and setup costs and derives the optimal order quantity. The EOQ model is still applied in industry because of its simplicity. Covert and Philip [5] presented an inventory model for deteriorating items, in which the items deteriorate at the rate of weibull distribution function. Goswami and Chaudhuri [7] developed an inventory model for deteriorating items with finite production rate proportional and the time dependent demand rate. Two inventory models in which deterioration rate and demand rate are taken as a linear function of time were analysed by Bhunia and Maiti [2]. Liang-Yuh Ouyang et al. [12] proposed an EOQ inventory model for deteriorating items with exponential decreasing demand and partial backlogging. A deterministic inventory model for quadratic demand rate, quadratic production and weibull deterioration rate with shortages was developed by Kalam et al. [11]. Gour Chandra Mahata [8] constructed an order level inventory model for deteriorating items with instantaneous replenishment, exponential decay rate and a time varying linear demand without shortages under permissible delay in payments. Jhuma Bhowmick and Samanta [10] studied a continuous production inventory model for deteriorating items with shortages in which two different rates of production are considered.

An EOQ inventory mathematical model for deteriorating item having time dependent demand when delay in payment is permissible was constructed by Singh and Pattanayak [23]. Chandrasekhara and Ranganathan [4] discussed the EOQ for an infinite horizon model when the demand increases exponentially under two levels of storage. An inventory model for deteriorating items with linear demand rate and traditional parameter of holding cost is linearly increasing functions of time was developed by Sachin et al. [18]. Shital and Raman [21] presented an EOQ model with demand rate is linear function of time, deterioration is two parameter weibull distribution and inventory holding cost is linear function of time under inflationary conditions with permissible delay in payments. An inventory model for deteriorating items with exponential time dependent demand rate was studied by Shukla et al. [22]. Palanivel and Uthayakumar [16] presented the economic production lot size model for determining the optimal production length and the optimal total cost for deteriorating items. Niketa et al. [15] analysed a deterministic inventory model with time dependent quadratic demand and time varying holding cost where units have expiry date.

An order level inventory model for deteriorating items with general ramp-type demand rate, partial backlogging of unsatisfied demand and time varying holding cost has been studied by Biplab and Karabi [3]. Ajay and Kuldeep [1] presented a continuous production control inventory model for deteriorating items under permissible delay in payments with variable demand rate and small deterioration rate. A production inventory model for exponential dependent demand and cost reduction delivery policy was proposed by Neeraj [14]. Sharmila and Uthayakumar [19] analysed the fuzzy inventory model for deteriorating items for power demand under fully backlogged conditions. Mishra et al. [13] investigated the inventory system for perishable items with time proportional deterioration rate, time dependent demand rate and linear holding cost with shortages. A deterministic inventory model for deteriorating items assuming exponentially declining demand was developed by Preeti and Malhotra [17]. Venkateswarlu and Reddy [24] proposed a production inventory model for deteriorating items which follow weibull distribution, variable holding cost and time dependent quadratic demand rate. A production inventory model for deteriorating items with price and inventory level dependent demand with different deterioration rates have been developed by Shital [20].

The rest of this article is arranged as follows: In Section 2., the assumptions and notations which are used throughout this article, are described. A production inventory model with exponential demand rate and exponentially declining deterioration items is considered in the Section 3 and the total inventory cost per cycle and the production cycle are optimized. Section 4 presents numerical example and sensitivity analysis. This is followed by conclusion in Section 5.

In the proposed model, we assume that the demand rate is an exponential function and the declining deterioration items follow exponential distribution. Because of the exponential nature, the demand is increasing smoothly which helps us to reduce the holding cost. The deterioration of the items is declining gradually because of exponential distribution which supports us to reduce the setup cost. The proposed model has an application in all organizations having non-zero demand at the starting time of the production, smoothly increasing demand and gradually decreasing deterioration.

2. Assumptions and notations

In the proposed inventory model, the following assumptions and notations are being made.

2.1 Assumptions

A single item is considered over an infinite planning horizon.

The demand rate follows an exponential distribution and is given as

$$D\left(t\right)=ae^{bt},\ a>0,0<b<1$$

Production rate is proportional to the demand rate and is given as

$$P(t)=\lambda D(t),\lambda>1.$$

The deterioration function follows an exponential distribution with probability density function,

$$f\left(t\right)=\begin{cases}\theta e^{-\theta t}, & t\geq 0\\ 0, & t<0\end{cases},\left(0<\theta<1\right)$$

The rate of deterioration, $h\left(t\right)=\theta$ is a constant fraction θ $\left(0<\theta<1\right)$ of the on hand inventory deteriorates per unit time. It is assumed that no repair or replacement of the deteriorated items takes place during a given cycle.

Shortages are not allowed.

Demand is non- zero at $t=0$.

2.2 Notations

$D\left(t\right)$is the demand rate.

$P(t)$ is the production rate.

$h\left(t\right)$ is the rate of deterioration.

C_h is the holding cost per unit per unit time.

C_d is the deteriorating cost per unit per unit time.

C_o is the set up cost per cycle.

$I\left(t\right)$ is the inventory level at time t.

Q is the maximum inventory level.

T is the fixed duration of a production cycle.

T_1 is the time at which the maximum inventory level occurs.

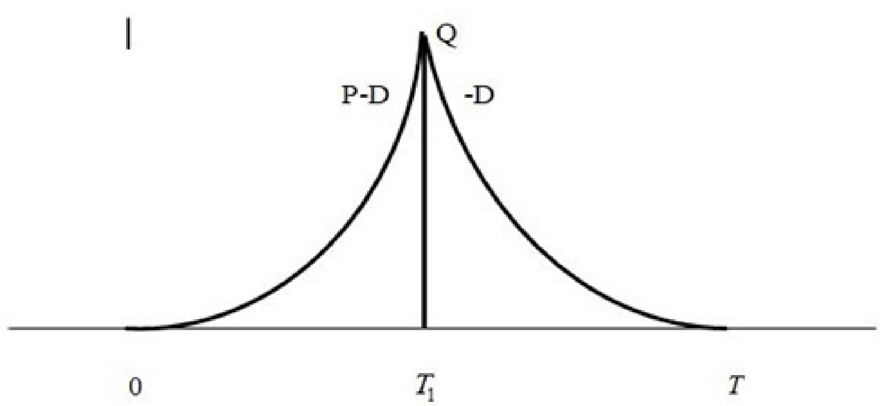
TC is the total inventory cost per cycle.

3. Model description

The proposed inventory model is developed on the basis of exponential market demand and production capacity of the organization with exponential decay. At the beginning, $t = 0$, the production starts with zero inventory. In this model, the production rate $P(t)$ is proportional to the demand rate $D(t)$. The demand exponentially decreases time to time and is given as $D(t) = ae^{bt}, a > 0, 0 < b < 1$. During the time $t = 0$ to $t = T_1$, due to the limited shelf–life and market demand, deterioration occurs. The inventory attains the maximum level Q at $t = T_1$ and the production is stopped at $t = T_1$.

Now, due to demand and deterioration from T_1 to T, the inventory level reduces and becomes zero at time $t = T$. The cycle repels itself after time T. Now, governing differential equation with boundary condition for the above said

Figure 1: The production inventory model



inventory model are given below:

$$\frac{d}{dt}I(t) + \theta I(t) = \begin{cases} (\lambda - 1)\, ae^{bt}, & 0 \le t \le T_1 \\ -ae^{bt}, & T_1 \le t \le T \end{cases} \tag{1}$$

with boundary conditions $I(0) = 0$, $I(T_1) = Q$ and $I(T) = 0$.
The solution of equation (1) is given below:

$$I(t) = \begin{cases} \frac{a(\lambda-1)}{b+\theta}\left(e^{bt} - e^{-\theta t}\right), & 0 \le t \le T_1 \\ \frac{a}{b+\theta}\left(e^{(b+\theta)T-\theta t} - e^{bt}\right), & T_1 \le t \le T. \end{cases} \tag{2}$$

Now, the sum of the holding cost, HC and the deteriorating cost, DC is given by

$$\begin{aligned} HC + DC &= C_h\left\{\int_0^T I(t)\,dt\right\} + C_d\left\{\int_0^T h(t)I(t)\,dt\right\} \\ &= C_h\left\{\int_0^T I(t)\,dt\right\} + C_d\left\{\int_0^T \theta I(t)\,dt\right\} \\ &= (C_h + \theta C_d)\left\{\int_0^T I(t)\,dt\right\} \\ &= (C_h + \theta C_d)\left\{\int_0^{T_1} \frac{a(\lambda-1)}{b+\theta}\left(e^{bt} - e^{-\theta t}\right)dt + \int_{T_1}^T \frac{a}{b+\theta}\left(e^{(b+\theta)T-\theta t} - e^{bt}\right)dt\right\} \\ &= (C_h + \theta C_d)\left\{\frac{a(\lambda-1)}{b+\theta}\left(\frac{e^{bt}}{b} + \frac{e^{-\theta t}}{\theta}\right)_0^{T_1} + \frac{a}{b+\theta}\left(\frac{e^{(b+\theta)T-\theta t}}{-\theta} - \frac{e^{bt}}{b}\right)_{T_1}^{T}\right\} \\ &= (C_h + \theta C_d)\left\{\begin{array}{l} \frac{a(\lambda-1)}{b+\theta}\left(\frac{e^{bT_1}-1}{b}\right) + \frac{a(\lambda-1)}{b+\theta}\left(\frac{e^{-\theta T_1}-1}{\theta}\right) \\ +\frac{a}{b+\theta}\left(\frac{e^{bT+\theta(T-T_1)}-e^{bT}}{\theta}\right) - \frac{a}{b+\theta}\left(\frac{e^{bT}-e^{bT_1}}{b}\right) \end{array}\right\}. \end{aligned}$$

Expanding the exponential function and neglecting the third and higher powers of b and θ, we get.

$$HC + DC = (C_h + \theta C_d)\left\{\begin{array}{l} \frac{a(\lambda-1)}{2}T_1^2 + \frac{a}{(b+\theta)}\{bT(T-T_1) \\ +\frac{\theta}{2}(T-T_1)^2 - \frac{b}{2}(T^2 - T_1^2)\} \end{array}\right\}. \tag{3}$$

Now, total inventory cost per cycle, TC = set up cost + inventory holding cost + deterioration cost, that is,

$$\begin{aligned} TC &= \frac{1}{T}\{C_o + HC + DC\} \\ &= \frac{C_o}{T} + \frac{(C_h + \theta C_d)}{T}\left\{\begin{array}{l} \frac{a(\lambda-1)}{2}T_1^2 + \frac{a}{(b+\theta)}\{bT(T-T_1) \\ +\frac{\theta}{2}(T-T_1)^2 - \frac{b}{2}(T^2 - T_1^2)\} \end{array}\right\}. \ (by (3)) \end{aligned} \tag{4}$$

Let us assume that $T_1 = pT$,where $0 < p < 1$. Now, the expression 4 becomes

$$TC = \frac{C_o}{T} + (C_h + \theta C_d)\left\{\begin{array}{l} \frac{a(\lambda-1)p^2}{2} + \frac{a}{(b+\theta)}\{b(1-p) \\ +\frac{\theta(1-p)^2}{2} - \frac{b(1-p^2)}{2}\} \end{array}\right\}T. \tag{5}$$

To find optimal values of T and TC. Now, since TC is continuous for all $T\ in\ (0,\infty)$ and $\frac{d^2}{dT^2}(TC) = \frac{2C_o}{T^3} > 0$, for all $T\ in\ (0,\infty)$, TC is convex on $(0,\infty)$.

Now, solving $\frac{d}{dT}(TC) = 0$, we obtain a critical point of TC, T^* where

$$T^* = \sqrt{\frac{2C_o(b+\theta)}{a(C_h+\theta C_d)\left\{(\lambda-1)p^2(b+\theta)+\left\{2b(1-p)+\theta(1-p)^2-b(1-p^2)\right\}\right\}}}$$

Now, since TC is convex on $(0,\infty)$ and $T^*$is a critical point of TC, TC attains its minimum value at T^* and $T^*$is the optimum value of T.

Now, the optimum of TC, $TC^*$is obtained from 5 by replacing T by $T^*$and the optimum of T_1, $T_1^*$is obtained from $T_1 = pT$ by replacing T by T^*

Remark 1. With the help of the parameter p, the length of the production period $(0, T_1)$can be modified according to the wish of the production department of the organization.

4. Numerical example and sensitivity analysis

Consider the production inventory model for exponential deteriorating items with exponential demand rate where the parameters are given as $C_o = 25$, $C_h = 30$, $C_d = 20$, $\theta = 0.6$, $\lambda = 1.5$, $a = 100$, $b = 0.8$ and $p = 0.4$.

Now, using MATLAB, we obtain the optimum value of Tas $T^* = 0.1645$, the optimum value of T_1 as $T_1^* = 0.0658$ and the optimum total inventory cost as $TC^* = 303.9737$.

In the developed inventory model, the total cost per cycle TC is a real valued function of T in which eight model parameters are assumed to be static values. Therefore, the sensitivity of each one of the model parameters in the developed inventory model is examined.

Case (i). Set up cost $C_o$varies from 20 to 70 and the other parameters are fixed, that is, $C_h = 30$, $C_d = 20$, $\theta = 0.6$, $\lambda = 1.5$, $a = 100$, $b = 0.8$ and $p = 0.4$. Using MATLAB, we obtain two graphs (Figure 2) for time verse set up cost and total inventory cost per cycle verse set up cost and from the Figure 2, we observe that T_1 , T and TC increases as C_0 increases.

Case (ii). Holding cost $C_h$varies from 30 to 70 and the other parameters are fixed, that is, $C_o = 25$, $C_d = 20$, $\theta = 0.6$, $\lambda = 1.5$, $a = 100$, $b = 0.8$ and $p = 0.4$. Using MATLAB, we get two graphs (Figure 3) for time verse holding cost and total inventory cost per cycle verse holding cost and from the Figure 3, we observe that T_1 and T decreases and TC increases as C_h increases.

Case (iii). Deterioration cost C_d varies from 20 to 60 and the other parameters are fixed, that is, $C_o = 25$, $C_h = 30$, $\theta = 0.6$, $\lambda = 1.5$, $a = 100$, $b = 0.8$ and $p = 0.4$. Using MATLAB, we find two graphs (Figure 4) for time verse deteriorating cost and total inventory cost per cycle verse deteriorating cost and from the Figure 4, we observe that T_1 and T decreases and TC increases as C_d increases.

Figure 2: Set up cost

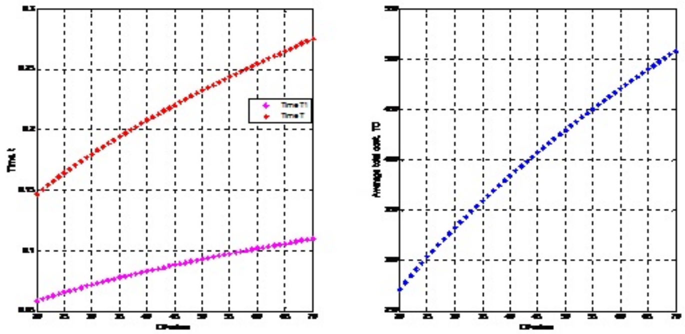
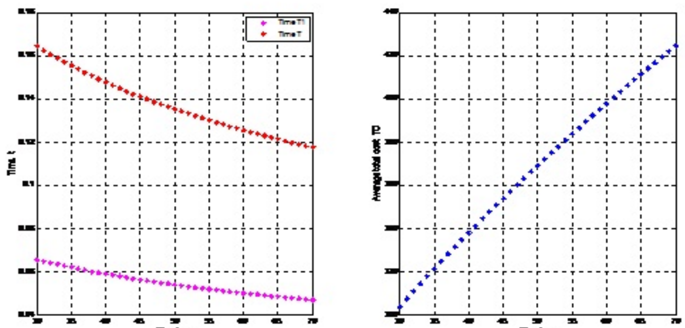


Figure 3: Holding cost



Case (iv). The rate of deterioration θ varies from 0.1 to 0.9 and the other parameters are fixed, that is, $C_o = 25$, $C_h = 30$, $C_d = 20$, $\lambda = 1.5$, $a = 100$, $b = 0.8$ and $p = 0.4$. Two graphs (Figure 5) for time verse rate of deterioration and total inventory cost per cycle verse rate of deterioration are obtained using MATAB and from the Figure 5, we observe that T_1 and T decreases and TC increases as θ increases.

Case (v). The production proportional λ varies from 1.2 to 5 and the other parameters are fixed, that is, $C_o = 25$, $C_h = 30$, $C_d = 20$, $\theta = 0.6$, $a = 100$, $b = 0.8$ and $p = 0.4$. Using MATLAB, two graphs (Figure 6) for time verse production proportional and total inventory cost per cycle verse production proportional are found and from the Figure 6, we observe that $T_1$and T decreases and TC increases as λ increases.

Case (vi). The demand constant a varies from 99 to 150 and the other parameters are fixed, that is, $C_o = 25$, $C_h = 30$, $C_d = 20$, $\theta = 0.6$, $\lambda = 1.5$, $b = 0.8$ and $p = 0.4$. Using MATLAB, two graphs (Figure 7) for time verse demand constant and total inventory cost per cycle verse demand constant are

Figure 4: Deterioration cost

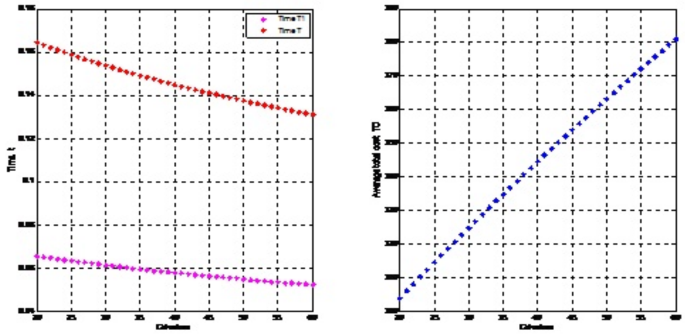
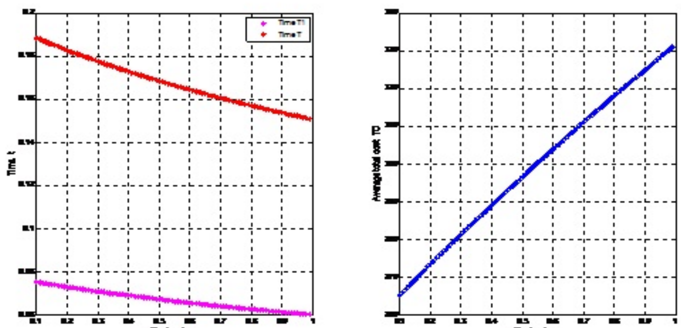


Figure 5: Rate of deterioration



got and from the Figure 7, we observe that T_1 and T decreases and TC increases as a, the demand coefficient, increases.

Case (vii). The exponential constant b in demand function varies from 0.1 to 0.9 and the other parameters are fixed, that is,$C_o = 25$, $C_h = 30$, $C_d = 20$, $\theta = 0.6$, $\lambda = 1.5$, $a = 100$and $p = 0.4$. Using MATLAB, we obtain two graphs (Figure 8) for time verse exponential constant and total inventory cost per cycle verse exponential constant and from the Figure 8, we observe that T_1 , T and TC are constants as b, demand coefficient increases.

Case (viii). The time period constant p varies from 0.1 to 0.9 and the other parameters are fixed, $C_o = 25$, $C_h = 30$, $C_d = 20$, $\theta = 0.6$, $\lambda = 1.5$, $a = 100$and $b = 0.8$. Using MATLAB, we get two graphs (Figure 9) for time verse time period constant, p and total inventory cost per cycle verse time period constant, p and from the Figure 9, we observe that T_1 increases as p increases, T increases as p increases from 0.001 to 0.68 (nearly) and decreases as p increases from 0.68 (nearly) to 0.999 and TC decreases as p increases from 0.001 to 0.68 (nearly) and increases as p increases from 0.68 (nearly) to 0.999.

Figure 6: Production proportional

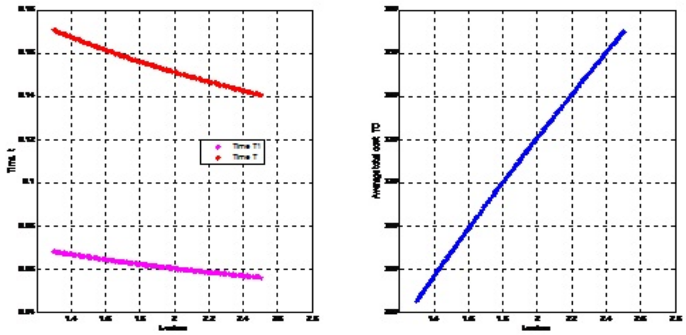
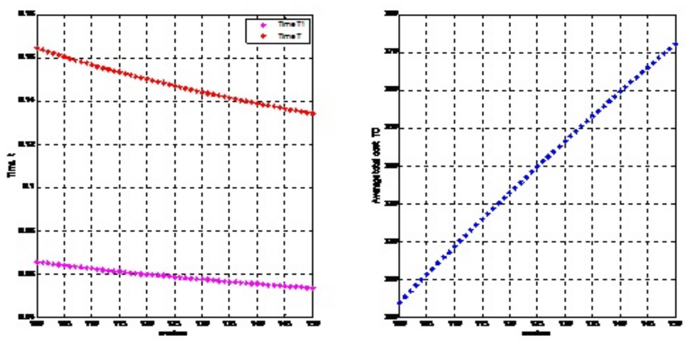


Figure 7: Demand constant



5. Conclusion

In this paper, a production inventory model for exponentially declining deterioration with an exponential demand rate is presented in which the production rate of the model is assumed to be proportional to the demand rate. The optimal total inventory cost per cycle and the optimal length of the cycle and optimal production length are determined. The developed model is illustrated with a numerical example. The proposed production inventory model having eight model parameters has been analysed sensitively with respect to each one of its parameters using MATLAB.

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Figure 8: Exponential constant

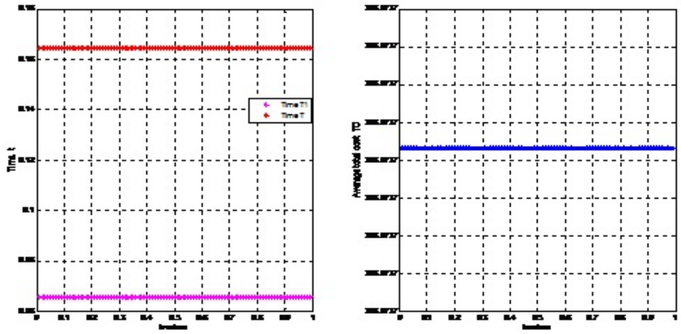
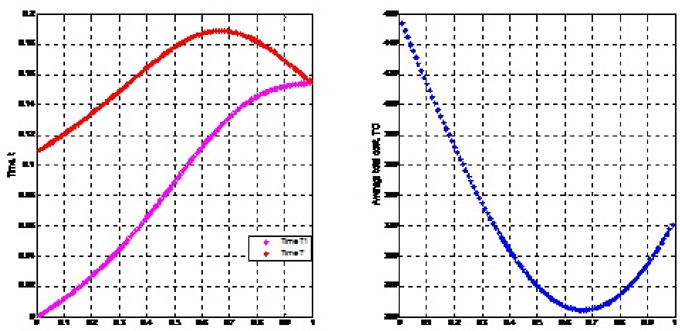


Figure 9: Time period constant



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