Productive Struggle: Solving Problems One Step at a Time

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A Problem for You…

A snail is at the bottom of a well that is 10 feet deep. Each day he crawls up 3 feet and each night he slides back 2 feet. How many days will it take him to reach the top of the well? Show your work to defend your answer.

Reflect…

- What struggles did you have in solving this problem?
- What struggles would your students have in solving this problem?
- What would you as the teacher do to support students in solving this problem?

Agenda

- What is productive struggle?
- What does productive struggle look like in the classroom?
- What are some strategies we can use to encourage the development of productive struggle?
- What are some resources that we can use to encourage the development of productive struggle?

Time to think……

How do you define productive struggle?

Productive Struggle

Productive Persistence = Tenacity + Good Strategies

Carnegie Foundation

The effort to make sense of something, to figure something out that is not immediately apparent.

Hiebert & Grouws, 2007

Opportunities for delving more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions.

NCTM, *Principles to Actions*

Effortful practice that goes beyond passive reading, listening, or watching – that builds useful, lasting understanding and skill.

Hiebert & Grouws, 2007

Mathematics Teaching Practices …

1) Establish mathematics goals to focus learning.
2) Implement tasks that promote reasoning and problem solving.
3) Use and connect mathematical representations.
4) Facilitate meaningful mathematical discourse.
5) Pose purposeful questions.
6) Build procedural fluency from conceptual understanding.
7) **SUPPORT PRODUCTIVE STRUGGLE IN LEARNING MATHEMATICS.**
8) Elicit and use evidence of student thinking.

NCTM, *Principles to Actions*

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Productive Struggle

"Teachers sometimes perceive student frustration or lack of immediate success as indicators that they have somehow failed their students. As a result, they jump in to 'rescue' students by breaking down the task and guiding students step by step through the difficulties. Although well-intentioned, such 'rescuing' undermines the efforts of students, lowers the cognitive demand of the task, and deprives students of opportunities to engage fully in making sense of mathematics."

NCTM, *Principles to Actions*

Productive Struggle

What skills/abilities
do “productive strugglers”
exhibit?

Productive Struggle

- Students are accustomed to explaining their ideas and questioning solutions that don't make sense to them.
- Students are not afraid to take risks and know that it is acceptable to struggle with some ideas and to make mistakes.
- Students recognize that mistakes are a means to learning and not an end.

--NCTM

Productive Struggle

- Fixed Mindset
  - Believe intelligence (math ability) is an innate trait
- Growth Mindset
  - Believe intelligence can be developed through effort
  - More likely to persevere through challenge – view as opportunity to learn and grow

-- Dweck, 2008

Productive vs. Destructive Struggle

Productive Struggle

- Leads to understanding
- Makes learning goals feel attainable and effort seem worthwhile
- Yields results
- Leads students to feelings of empowerment and efficacy
- Creates a sense of hope

Destructive Struggle

- Leads to frustration
- Makes learning goals feel hazy and out of reach
- Feels fruitless
- Leaves students feeling abandoned and on their own
- Creates a sense of inadequacy

ASCD, 2012

Rethinking Success

- **What does it mean to be a successful learner of mathematics?**
- **What does it mean to be a successful teacher of mathematics?**

Rethinking Success

- **Smith, 2000**
- **Redefining success**
- **Sort the cards to tell 5 stories of productive struggle and success including:**
    - **an expectation for students (orange)**
    - **a related teacher action (gold)**
    - **an indicator of success (white)**

Powerful Practices:
Fostering Productive Struggle

Strategy 1: Anticipate student struggles and misconceptions that might occur.

- **Plan ways to support students without removing the opportunities for students to develop deeper understanding of the mathematics.**

Powerful Practices:
Fostering Productive Struggle

Back to the snail problem

- **Look at common issues/misconceptions**
- **Look at questions/prompts**

Ray and Waggoner, 2012

A Problem for You…

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Powerful Practices:
Fostering Productive Struggle

Your turn – Shopping Trip Task

- **Work as a student**
- **Identify common issues/misconceptions**
- **Identify questions/prompts**

- NCTM, *Principles to Actions*

Shopping Trip Task

Joseph went to the mall with his friends to spend the money he had received for his birthday. When he got home, he had $24 remaining. He had spent 3/5 of his birthday money at the mall on video games and food. How much money did he spend? How much money did he get for his birthday?

- NCTM, *Principles to Actions*

- Work the task as a student.
- Identify common issues/misconceptions.
- Suggest questions or prompts.

Powerful Practices:
Fostering Productive Struggle

Strategy 2: Process vs Person Praise:

Instead of praising students for their talent or smarts, teachers should praise students for:

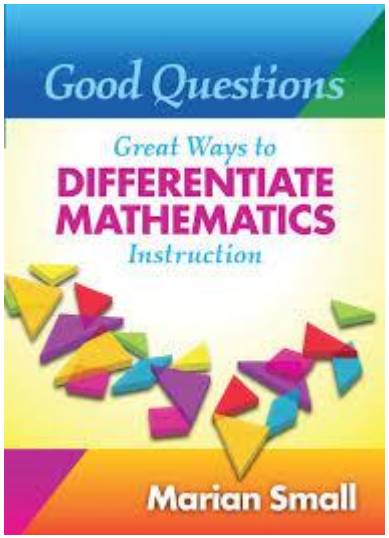
- ✓The strategies they use
- ✓The specific work they do
- ✓Their persistence or effort

Powerful Practices: Fostering Productive Struggle

Strategy 3: Use open questions.

- **Framed in a way that multiple approaches and answers are expected and accepted.**
- **Promotes productive struggle because of multiple entry points.**

Open Question Example



Create a set of data in which the mean is greater than the median.

Powerful Practices: Fostering Productive Struggle

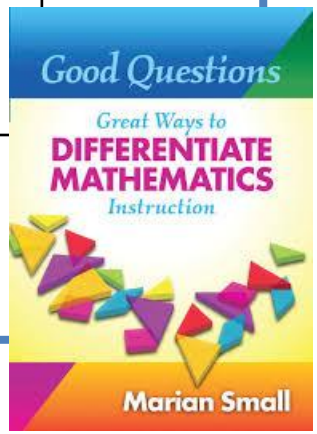
Strategy 4: Utilize parallel tasks.

- **Two or more tasks that are designed to meet the needs of students working at different levels but address the same concept and use a similar context so that they can be discussed simultaneously.**

Parallel Task Example

What is a * b if the following statements are true?

Option 1	Option 2
2 * 4 = 8 2 * 5 = 9 2 * 6 = 10 3 * 4 = 10 3 * 5 = 11 3 * 6 = 12	2 * 4 = 20 3 * 4 = 25 5 * 6 = 61 8 * 9 = 145



Powerful Practices:
Fostering Productive Struggle

Strategy 5: Attend to Student-Teacher Interaction

- **The way we respond to students when they struggle is vitally important.**
- **Support without reducing demand.**
- **Support enough to avoid frustration.**
- **Work the Rain Barrel problem**
- **Compare student-teacher interactions**

Warshauer, 2015

Rain Barrel Problem

Suppose we have a 48 gallon rain barrel containing 24 gallons of water and a 5 gallon water jug containing 3 gallons of water. Which container is said to be the fuller? If we drain a gallon of water from each container, does this change your answer about which container is fuller? Explain.

Warshauer, 2015

Powerful Practices: Fostering Productive Struggle

Strategy 6: Incorporate ongoing formative assessment

- **Requires students to express what they understand about new materials and allows them to pinpoint and correct their knowledge gaps and misconceptions**

Powerful Practices:
Fostering Productive Struggle

Strategy 7: Use Spaced/Distributed practice

- Don’t have to understand everything the first time. Come back to topics.
- Produces lasting learning because long-term memory of material is strengthened each time information is actively retrieved.

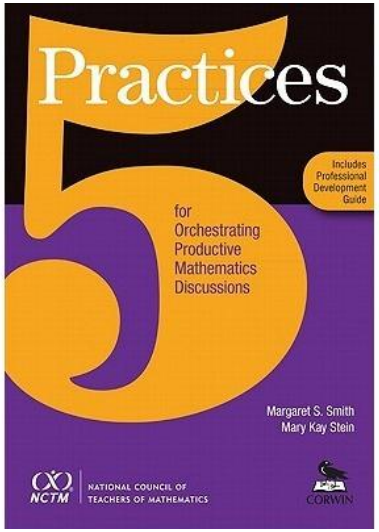
Powerful Practices: Fostering Productive Struggle

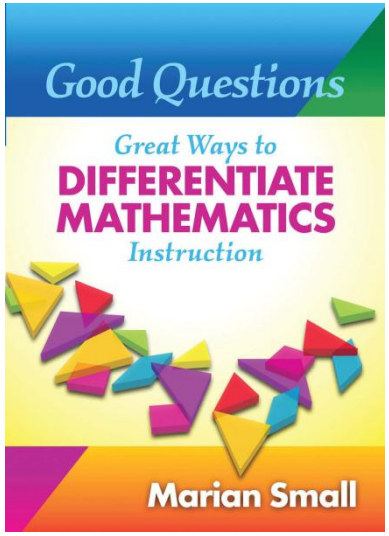
- **Strategy 8: Incorporate Mixed practice**
  - **Problems don't come to students with labels. Practicing different types of questions and problems builds learning-for-transfer more effectively.**

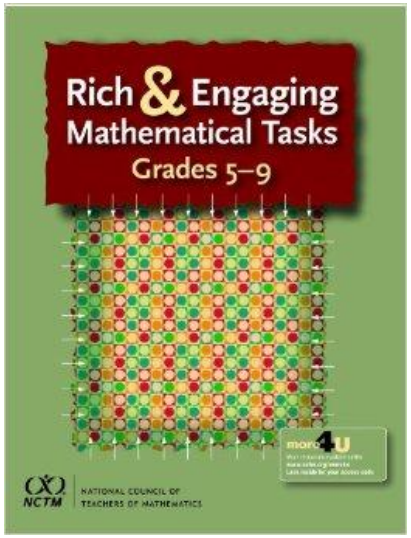
Powerful Practices:

Resources and Strategies from the Classroom

NCTM Publications







Powerful Practices:
Resources and Strategies from the Classroom
Digital Tool

Challenges faced by our school:

- **Struggling students**
- **ELL students**
- **Lack of perseverance**
- **Limited in strategy use and exposure to Common Core content and types of questions posed**

Powerful Practices:
Resources and Strategies from the Classroom
Digital Tool

LearnBop

- Presents problem and then guides students and scaffolds
- Academic language
- Misconceptions identified and hints to address
- Multiple strategies
- Opportunities for class discourse
- Mathematical practices
  - Problem solving
  - Modeling
  - Representations

Powerful Practices:
Resources and Strategies from the Classroom
Digital Tool

LearnBop

– **Results**

- **More student confidence with word problems**
- **Better student engagement in accountable talk**
- **Improved performance and grades**
- **Improved critical thinking skills**
- **Increased independence**

www.learnbop.com

Productive Struggle

- Reflect on your definition of productive struggle.
- Reflect on what success in mathematics teaching and learning entails.
- What ideas do you have for your own classroom?

Questions?
Thank you!!

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