CASIO Education Workbook Series

ALGEBRA II

with the

Casio fx-9750GII

Author: *Judy Johnson*
Editor: *Amber M. Branch*

This workbook is a product of the:

CASIO TEACHER ADVISORY COUNCIL (CTAC)

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Topic Area: Data Analysis and Probability

NCTM Standard:

- For bivariate measurement data, be able to display a scatterplot, describe its shape, determine regression coefficients, regression equations, and correlation coefficients using technological tools.

Objective

Given a set of data, the student will be able to use the Financial Menu (TVM), the Statistical Menu, and the Table Menu to solve a problem involving calculations of future payments for a mortgage payment that includes real estate tax.

Getting Started

Discuss with students what a mortgage loan involves, including the definition of interest, principal, and escrow accounts. Include a discussion on budgets, and the effects an increase in a payment can make on a budget.

Prior to using this activity:

- Students should be able to calculate loan payments using the TVM Menu.
- Students should be able to enter data into lists using the STAT Menu, perform operations on the list, set up the calculator to draw a scatterplot, find the equation for the line of best fit, and copy the equation to the TABLE Menu.
- Students should be able to find values using the TABLE Function.

Ways students can provide evidence of learning:

- The students will be able to discuss graphs relating to the problem and explain its appearance.
- The students will be able to discuss details regarding data in statistical lists and the TABLE Function, including the meaning of the values and their importance.

Common mistakes to be on the lookout for:

- Students might incorrectly enter data in the table or list menus. When using percentages, students might have issues with decimal placement.

Definitions

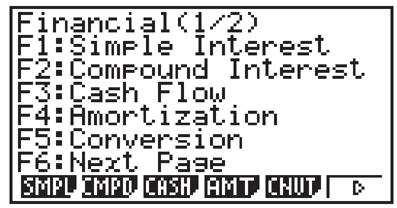
- Interest
- Principal
- Escrow Accounts

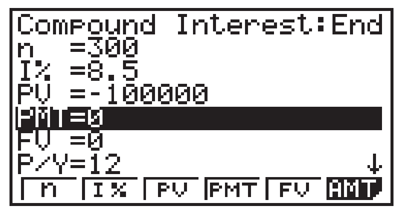
The following will demonstrate how to find the payment amount for a loan given the principal, rate of interest, an number of years using the TVM Menu of the Casio *fx-9750GII*. This will also demonstrate how to find the amount of tax for a column of values using the STAT menu, find the equation for the line of best fit using the STAT Menu, use the line of best fit (linear regression) and use the TABLE Menu to make predictions of future values.

Find the payment for a $100,000 mortgage for 25 years at a rate of 8.5%.

To enter the above set of data using the TVM Menu:

1. Select the TVM Menu and press [EXE] or [X,θ,T].
2. Press [F2] for Compound Interest.
3. For *n* (number of payments), enter the number of years x 12 by inputting: [2] [5] [×] [1] [2] [EXE].
4. For I% (interest rate), enter the rate as a percent by inputting: [8] [•] [5] [EXE].
5. For PV (Principal Value), enter the principal amount of the loan as a negative number by inputting: [(–)] [1] [0] [0] [0] [0] [0] [EXE].
6. To calculate the payment amount, press [F4].

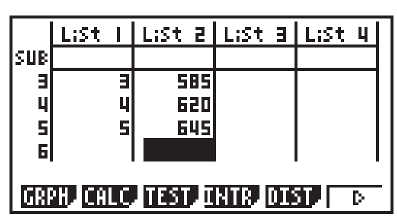




Steps for Using the STAT Menu:

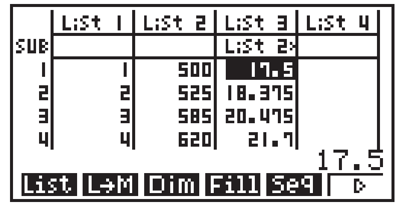
Property	1	2	3	4	5
Value	$500	$525	$585	$620	$645

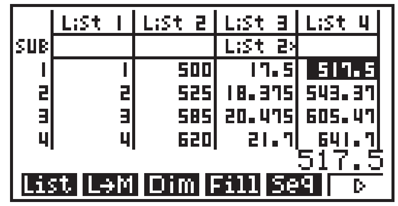
1. Press [MENU] to return to the Main Menu. Use the arrow keys to highlight the STAT Menu and [EXE].
2. To enter the property numbers into List 1, move the cursor to the left of SUB 1 and begin entering each number, pressing [EXE] after each entry.
3. Press the [▶] to move to the beginning of List 2. Enter List 2 values as in the previous step.



To find 3.5% tax for the property values in List 2:

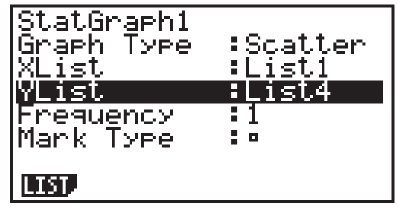
1. Press ▶ once and ▲ twice to highlight **List 3**.
2. Press [OPTN] [F1] [F1] [2] [×] [•] [0] [3] [5] [EXE].
3. To add the tax to the property value in **List 2** to the tax in **List 3**, press ▶ once and then ▲ twice to highlight **List 4**.
4. Press [OPTN] [F1] [F1] [2] [+] [OPTN] [F1] [F1] [3] [EXE].

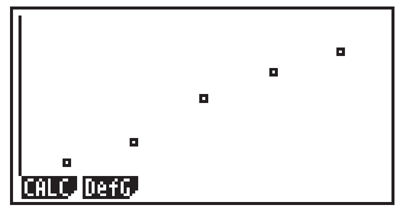




To graph a scatterplot for the data:

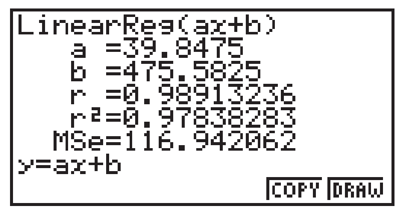
1. Press [EXIT] twice to return to the List home screen. Press [F1] (GRPH) then [F6] (SET).
3. To change your **YList** to List 4, press ▼ three times then [F1] [4] [EXE].
6. Press [EXE] [F1] to view the scatterplot.

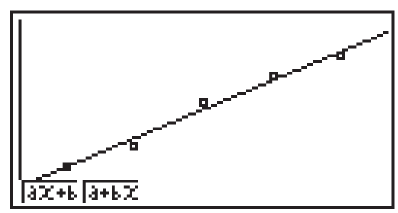


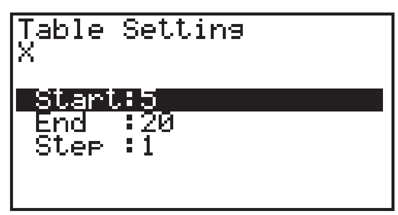


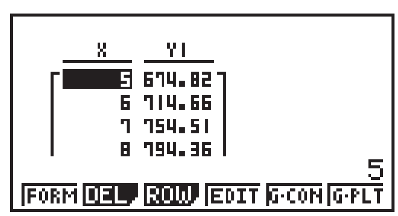
To find the equation for the line of best fit:

1. Press [F1] (CALC), [F2] (X), [F1] (ax+b) to view the linear regression. To copy the equation for use in the Table or Graph menu, press: [F5] [EXE] [VARS].
2. Press [F6] to draw.
3. Press [MENU], highlight [TABLE] and press [EXE].
4. Press [F5] (SET).
5. Enter [5] [EXE] for **Start:** and enter [2] [0] [EXE] for **End:**
6. To view the table, press [EXE], then [F6]. Use ◀ ▶ ▲ ▼ to move through the table.









Thought it Was Fixed Activity

When someone takes out a mortgage on a piece of property, part of the payment may include an escrow account that will be used to pay the real estate taxes on the property. The fixed rate advertised with many mortgages applies only to the percent of interest. The monthly payment can increase with the increase in taxes. Since property is seen as an investment, it is hoped that the value of the property will increase. This increase in property value will cause an increase in the amount of taxes owed which, in turn, causes an increase in the mortgage payment. If a person is budgeting for their mortgage payment, they need to be able to calculate the increase of taxes and how this would affect the mortgage payment.

In this activity, you will calculate the total mortgage payment (principal plus interest), the amount of real estate tax due and the monthly payment. You will also calculate the increase in the mortgage payment due to an increase in property value.

The Jones' took out a 30-year mortgage in the amount of $125,000 for a home valued at $150,000 five years ago. The interest rate on the mortgage is 6.5%. Part of their payment goes into an escrow account that is used to pay their real estate tax; their current tax rate is 1.5%. The property values for the first five years are:

Year	1	2	3	4	5
Property Value	$150,000	$152,000	$155,000	$159,000	$161,000

Questions

1. Find the amount of the monthly mortgage payment that covers principal plus interest.

   ____

2. During the first year, what were the Jones' escrowing for monthly real estate taxes?

   ____

3. What was the monthly amount of real estate taxes that the Jones' family was charged during the second year?

   ____

4. How much had the monthly real estate tax amount changed in the first 5 years?

   ____

5. Find the total amount paid per month for both the mortgage and real estate taxes for the first year?

   ____

6. Find the total amount paid per month for both the mortgage and real estate taxes for the fifth year? Justify your answer.

   ____

   ____

7. What was the yearly dollar amount needed to pay both the mortgage and the real estate taxes during the first year?

8. What was the yearly dollar amount needed to pay both the mortgage and the real estate taxes during the fifth year?

9. The Jones' are trying to plan ahead for their budget. They need to find an equation that could be used to estimate their monthly payment for both the mortgage and real estate taxes. What equation should they use? How did you come up with your answer?

10. Why would it be important for the Jones' family to estimate their future payment?

11. For the tenth year, how much would they need per month?

12. What would be the total amount paid by the Jones' during the final year of their mortgage?

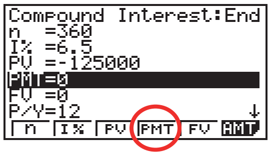
13. If their yearly income was $68,000 during the first year of the mortgage, what percent of their income went to paying the mortgage and real estate taxes?

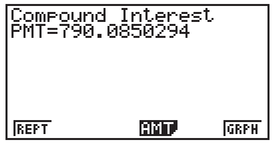
14. If they estimate their yearly income to be $83,500 after ten years, what percent of their income would have to be budgeted during that year for the mortgage payment and real estate taxes?

15. Describe the significance between your answers for question 13 and 14.

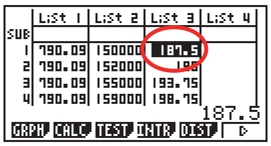
Solutions

1. $790.09

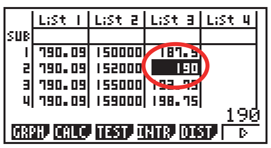




2. $187.50

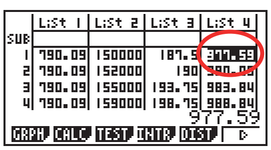


3. $190.00

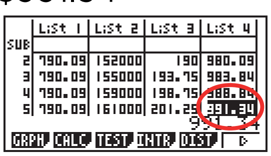


4. $201.25 - $187.50 = $13.75

5. $977.59



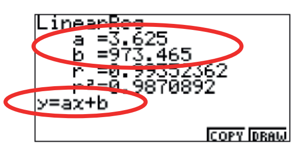
6. $991.34



7. $11,731.08

8. $11,896.08

9. y = 3.625x + 973.465



10. Answers will vary based on student experience.

11. $1009.72

12. $1045.90

13. $1082.20 (12) = $12,986.40

14. 17.5%

15. 14.5%

16. Answers will vary based on student experience. Hopefully, they note that after 10 years, they are spending a smaller percentage of their yearly income.

Topic Area: Patterns and Functions – Algebraic Thinking

NCTM Standard:

- Understand patterns, relations, and functions by interpreting representations of functions of two variables using symbolic algebra to explain mathematical relationships.

Objective

Given a set of formulas, the student will be able to use the GRAPH Menu and G-Solve Function to solve problems involving the investment of money.

Getting Started

Discuss the importance of saving money and the difference between simple and compound interest.

Prior to using this activity:

- Students should have a working knowledge of using the calculator to enter various formulas and display the corresponding graph.
- Students should be able to use the G-Solve Function to find specific x- and y-values.

Ways students can provide evidence of learning:

- The students will be able to discuss the results of the activity and justify their answers to specific questions.
- The students will be able to discuss how the graphical results correlate to their answers.

Common mistakes to be on the lookout for:

- When there are multiple formulas used, students could utilize the wrong formula or substitute the incorrect information for a particular variable.

Definitions

- Interest
- Principal
- Rate of Interest
- Compound Interest
- Future value

Formulas

One-Time Investment: $A = \frac{P}{i}\left[(1+i)^n - 1\right]$ Number of Payments: $N = \frac{\log(1 + iF \div P)}{\log(1+i)}$

Monthly Payment for a Desired Future Value: $P = \frac{iF}{(1+i)^n - 1}$

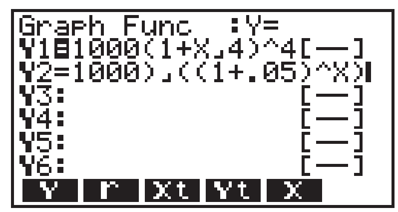
The following will demonstrate how to enter a given formula into the GRAPH module the Casio *fx-9750GII*, graph the data, and use G-Solve to find x- and y-values.

Example Formulas:

$$A = P\left(\frac{1+r}{n}\right)^n \quad \text{where } n = 4,\ P = 100, \text{ and } r = x$$

$$A = \frac{nP}{(1+r)^n} \quad \text{where } n = x,\ P = 1000, \text{ and } r = 0.05$$

Steps for Using the GRAPH Menu:

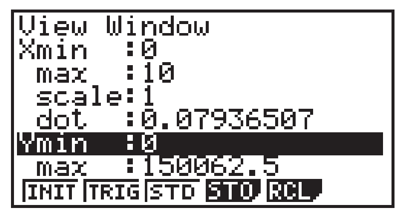


1. From the Main Menu, press [3] for the graph menu.
2. Enter the first formula into **Y1**: by inputting:

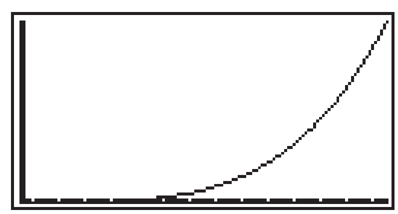
   [1] [0] [0] [0] [(] [1] [+] [X,θ,T] [a b/c] [4] [)] [^] [4] [EXE]
3. Enter the second formula into **Y2**: by inputting:

   [(] [X,θ,T] [×] [1] [0] [0] [0] [)] [÷] [(] [(]

   [1] [+] [•] [0] [5] [)] [^] [X,θ,T] [)] [EXE].



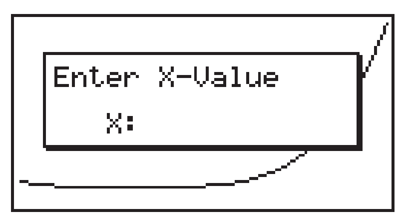
To select the viewing window for the graph of this data:



1. Press the [▲] once, then press [F1] to deselect **Y2**:
2. Press [F6] to display the graph of **YI**:.
3. Press [SHIFT] [F2] (V-Window), then [F5] (AUTO).
4. To see only the first quadrant, press [SHIFT] [F3] [0] [EXE]

   [▼] three times, [0] then [EXE] three times.

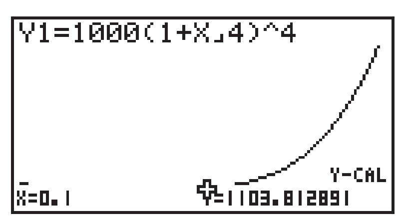
Steps for Using G-Solve:



1. Press [SHIFT] [F5] (G-Solv), [F6] (▷)and [F1] (Y-CAL).
2. To find the y-value when x = 0.1, input the following:

   [•] [1] [EXE].
3. Press [SHIFT] [F5] (G-Solv), [F6] (▷) and [F2] (X-CAL).
4. To find the x-value when y = 2000, input the following:

   [2] [0] [0] [0] [EXE].



Investing for the future is usually the last thing on a person's mind when they are just entering the workforce. Paying bills, buying groceries, and purchasing a home are usually at the top of the list. However, putting money away in some form of savings should be the number one priority of every budget. Social security and retirement plans are often not enough to allow a person to continue living as they have in the past.

In this activity, you will investigate how much a single investment will earn, calculate the balance of an account with a given monthly payment, and determine the investment amount that is needed to reach a specific financial goal.

Questions

The amount of income earned from a onetime investment can be calculated using the following formula.

$$A = \frac{P}{i}\left[(1+i)^n - 1\right]$$

Where *A* is the ending balance, *P* is the principal, *i* is the rate of interest, and *n* is the number of times the interest is calculated.

1. What would be the amount of income for a principal of $1000, compounded annually for 5 years at 2.5% APR?

____

2. What would be the amount of income for a principal of $1000, compounded annually for 5 years at 4% APR?

____

3. What would be the amount of income for a principal of $1000, compounded annually for 5 years at 10% APR?

____

4. What would be the amount of income for a principal of $10,000, at 2.5% for 5 years, compounded annually?

____

5. What would be the amount of income for a principal of $10,000, at 4% for 5 years, compounded annually?

____

6. What would be the amount of income for a principal of $10,000, at 10% for 5 years, compounded annually?

____

7. What is the difference between the amount earned at 2.5% and the amount of income earned at 10%?

____

8. How long would it take for an investment of $500, earning 4% APR, compounded annually, to earn $1,000?

____

9. How long would it take for an investment of $500, earning 4% APR, compounded annually, to earn $2,000?

____

10. How long would it take for an investment of $500, earning 4% APR, compounded annually, to earn $3,000?

____

11. How long would take to earn $5,000?

____

The formula for finding the number of payments, at a given percent, for a particular annual investment, to reach a specified goal is:

$$N = \frac{\log(1 + iF \div P)}{\log(1 + i)}$$

Where *N* is the number of payments, *i* is the interest rate, *F* is the future value of the investment, and *P* is the monthly amount invested.

12. Calculate the future value of an investment of $100 a year, at 5% APR, for 10 years?

____

13. Calculate the future value of an investment of $100 a year, at 5% APR, for 30 years?

____

14. Calculate the difference between #12 and #13?

____

15. Calculate the future value of an investment of $500 a month, at 5% APR, for 10 years?

____

16. Calculate the future value of an investment of $500 a month, at 5% APR, for 30 years?

____

17. Calculate the difference between #15 and #16?

____

The smart move is to start investing early and put aside a set amount each year. The formula for finding the amount of money earned from an annual investment at a given rate is:

$$P = \frac{iF}{(1+i)^n - 1}$$

Where *P* is the annual amount invested, *i* is the rate of interest, *n* is the number of times the interest is calculated, and *F* is the future values of the investment.

18. Calculate the annual investment needed at 10% APR to earn $100,000 in 10 years?

____

19. What is the difference between the investment for 10 years and the investment at 30 years?

____

20. What is the benefit of starting to invest early?

____

____

Extensions

1. Credit cards charge interest, however, that interest is compounded daily. What changes would need to be made to the compound interest formula to be able to calculate credit card interest that is compounded daily? Explain your thinking.

____

____

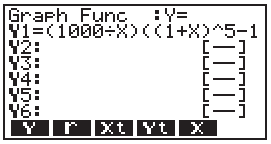
____

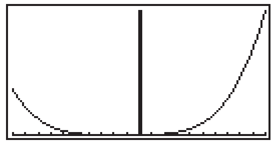
____

____

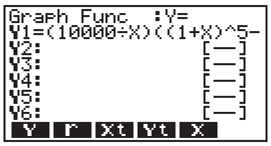
Solutions

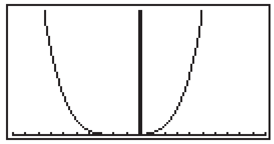
1. $5256.33
2. $5416.32
3. $6105.10



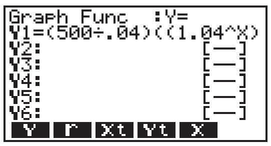


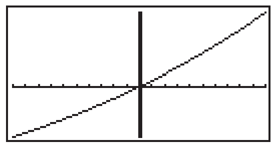
4. $52,563.28
5. $54,163.22
6. $61,051.00



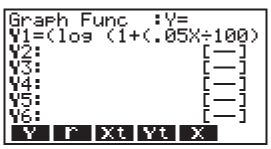


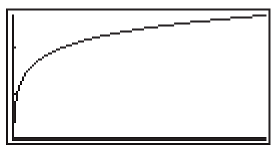
7. $8,487.72
8. 1.9 years
9. 3.8 years
10. 5.5 years
11. Answers will depend on student experience



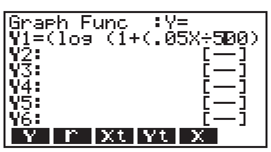


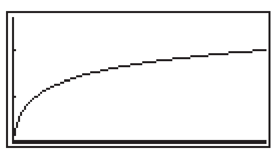
12. $1,257.79
13. $6,643.88
14. $5,386.09





15. $6,288.95
16. $33,219.42
17. $26,930.47





18. $627.45
19. $566.66
20. Answers will vary depending on student experience but should involve the fact that the longer money is invested, the more money you have.

Extensions

1. Answers will vary. Things of note would be the interest rate is being charged not received and that *n* would be changed to a compounded daily rate.

Topic Area: Absolute Value Equations

NCTM Standards:

- Use symbolic algebra to represent and explain mathematical relationships.
- Use trigonometric relationships to determine lengths and angle measures. Specify locations and describe spatial relationships using coordinate geometry and other representational systems

Objective

The student will be able to find the slope of a line in a three-dimensional model, the value of the tangent for an angle between two non-vertical lines, and find the measure of the angle between the lines in degrees.

Getting Started

Have the students work in pairs or in small groups to discuss how lights are used to enhance displays of objects in museums and art galleries. Provide them with a small flashlight, have them change the focus of the light, and discuss how this can be applied to displays. Have students think of other areas that would use this type of technology.

Prior to using this activity:

- Students should have an understanding of finding the slope of a line on a coordinate plane.
- Students should have an understanding of how to find the measure of an angle given the value of its tangent.

Ways students can provide evidence of learning:

- The student will be able to create a three dimensional model of a room and show how they would set up lights to enhance items displayed in the room.
- The student will be able to draw a three-dimensional drawing of a room with coordinates and write problems that can be solved by other students.

Common mistakes to be on the lookout for:

- Students may use the wrong formula for finding slope.
- Students may find the measure of the angle in radians instead of degrees.

Definitions

- Slope
- Tangent

Formulas

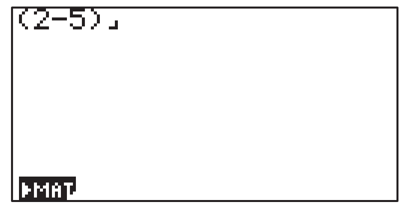
Slope: $m=\frac{y_2 - y_1}{x_2 - x_1}$ Tangent Between Lines: $\tan\alpha=\left|\frac{m_2 - m_1}{1+m_1 m_2}\right|$

The following will demonstrate how to use the fraction key to find the slope of a line and use the tangent key to find the measure of an angle given the value of the tangent using the Casio fx-*9750GII*.

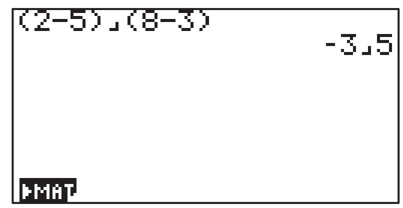
Find the slope of a line whose coordinates are (3, 5) and (8, 2).
Find the measure of an angle whose tangent value is 5.3417.

To find the slope of a line:

1. Highlight the RUN•MAT icon in the Main Menu and press [EXE]. Enter the numerator (difference of the y-coordinates) in parentheses, press [a b/c]. Using parentheses, enter the denominator and press [EXE]. Since the slope was entered as a fraction, the answer will be displayed as a fraction.

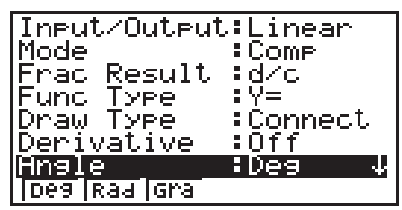


2. To see the decimal equivalent of $-\frac{3}{5}$, press [F↔D].

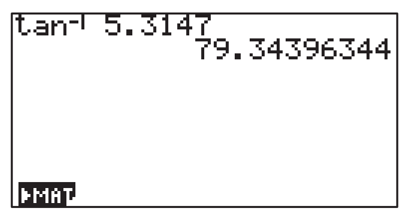


To find the measure of an angle given its tangent:

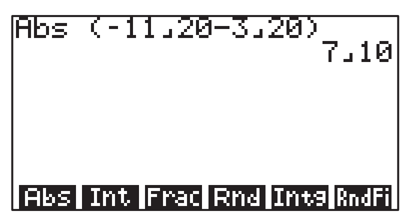
1. To find the measure of an angle, the calculator must be in Degree mode. To do this, press [SHIFT] [MENU] (Setup), [▼] to highlight the word Angle. If Deg is not selected, press [F1] (Deg) and [EXE].



2. Press [EXIT] to return to the Run screen. Press [SHIFT] [tan] and enter the value for the tangent. Press [EXE] to see the measure of the angle as shown at the right.



To find absolute value using the calculator:



1. Press [OPTN] [F6] [F4] (Num), then [F1] (Abs).

2. Enter the problem and press [EXE].

Many museums and galleries set up spectacular displays that are enhanced by spot lighting. The lights are set at a specific angle to highlight items within the display. One such use would be to highlight a particular piece of clothing or tool that was used by an ancient culture.

In this activity, you will use the slope of a line and a formula to find the angle of a spot light beam directed at different objects.

Questions

1. A spotlight is being directed at a painting on the opposite side of a 20 ft. wide room. The spotlight is located at the top of a 12 ft. wall. The bottom of the painting is 5 ft. from the floor and the top of the painting is 8 ft. from the floor. Draw a diagram to illustrate this spot light.

   What is the slope from the spot light to the top of the painting?

   ____

   What is the slope from the spot light to the bottom of the painting?

   ____

2. Use the formula to find the measure of the angle for the spot light.

   ____

3. In another room, a sculpture is being spot lighted from the floor 3 ft. from the base. The bottom of the light hits the sculpture 2 ft. above the floor and the top of the light hits the sculpture 8 ft. from the floor. What is the angle of this light?

   ____

4. In a gallery displaying ancient buildings, two spot lights are directed at a model of the Tower of London. The first light is 1 ft. off the floor and 5 ft. away from the model. It shows a spot ranging from 3 ft. to 5 ft. off the floor. The second spot is 2 ft. off the floor and 2 ft. away from the model. It shows a spot ranging from 4 ft. to 8 ft. off the floor. What is the angle of each light?

   First Light: ______________________ Second Light: ______________________

Extension

1. Take several flashlights of different sizes and find the angle of their light. Does the angle change if the flashlight is pointed at different angles? Justify your answer.

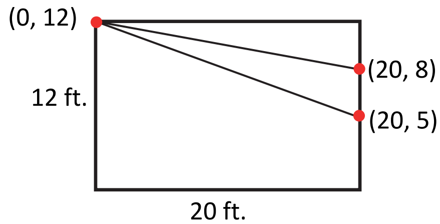
   ____

   ____

2. Create a display that illustrates the use of a spot light on an object and show the measurements including the angle of the light.

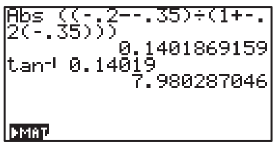
Solutions

1.

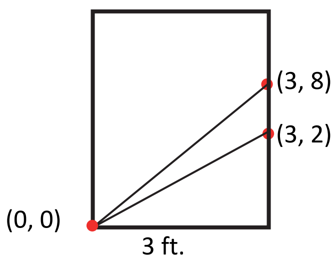


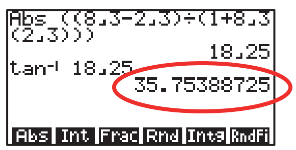
Slope to Top = -.2
Slope to Bottom = -.35

2. $\alpha \approx 8^0$

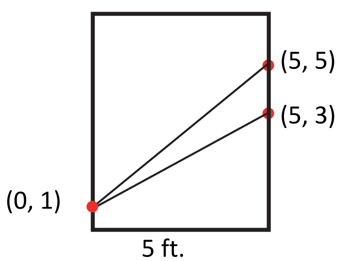


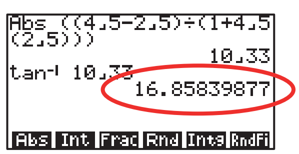
3. $\alpha \approx 35.8^0$



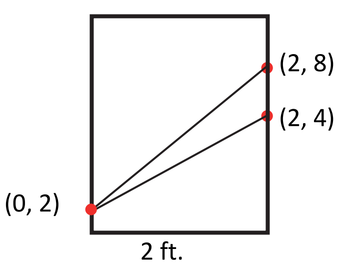


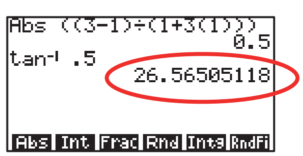
4. $\alpha \approx 16.9^0$





$\alpha \approx 26.6^0$





Maximize Space for Minimum Price — Teacher Notes

Topic: Represent and Analyze Mathematical Situations

NCTM Standard:

- Represent and analyze mathematical situations and structures using algebraic symbols, and use mathematical models to represent and understand quantitative relationships.

Objective

The student will be able to use the Casio *fx-9750GII* to write an objective function with inequalities, graph the function and find solutions to the problem.

Getting Started

Discuss with students the use of linear programming and how to interpret the graph. Explain what is meant by objective functions and constraints. Review how to write an inequality and how to graph the results. Discuss situations in which constraints would be put on a product or item and let the students give examples of their own.

Prior to using this activity:

- Students should be able to enter inequalities on the calculator and know how to change the inequality sign, when necessary.
- Students should be able to set up the view window, and find intersections of the vertices.

Ways students can provide evidence of learning:

- If given an objective function, students can discuss the resulting graph.
- If given an objective function, students can determine the intersection.
- If given an objective function, students can discuss what each intersection represents.
- If given an objective function, students can discuss the outcome.

Common mistakes to be on the lookout for:

- Students may choose the wrong inequality.

Definitions

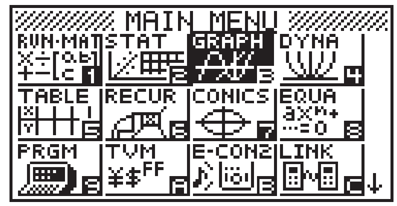
- Objective Function
- Constraints
- Inequalities
- Vertices

The following will demonstrate how to enter an objective function with the inequalities for the constraints, graph it, set the appropriate view window, and calculate a specific value for an objective function on the Casio *fx-9750GII*.

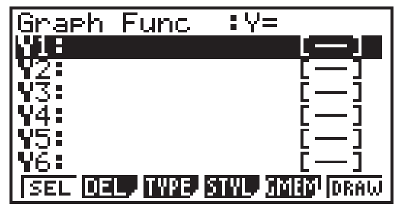
Inequalities to graph:

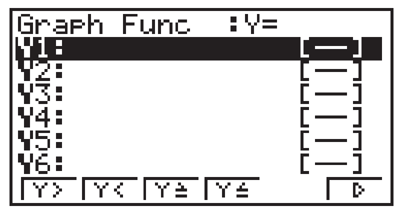
$y \geq -x + 6 \quad y \geq 2x - 3 \quad y \leq 5 \quad x \geq 0 \quad y \geq 0$

To graph an inequality:

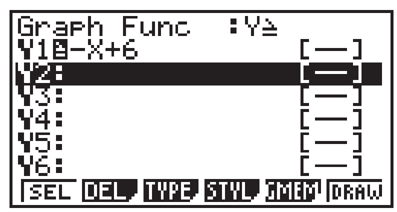


1. From the main menu, highlight the GRAPH icon and press [EXE] or press [3].

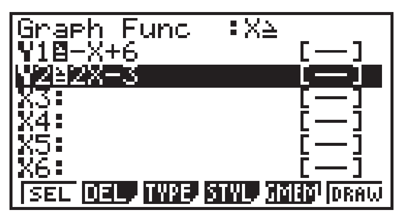




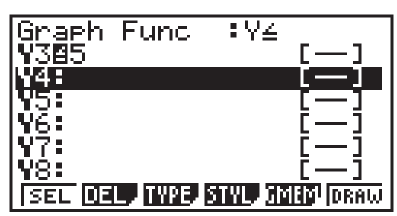
2. Press [F3] (TYPE) to select the type of graph. Press [F6] (▷) for more options. The inequality options are displayed across the bottom of the screen. To graph the first inequality, we will choose [F3].



3. The calculator now shows Y≥ at the top of the screen. Press [(−)] [X,θ,T] [+] [6] [EXE] to enter the first inequality.

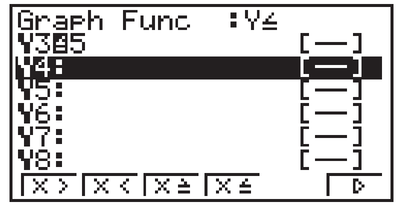


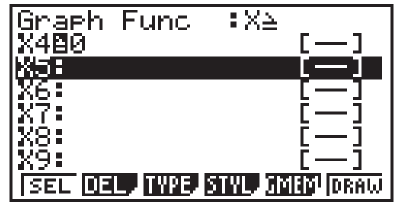
4. To enter the second inequality, input: [2] [X,θ,T] [−] [3] [EXE].



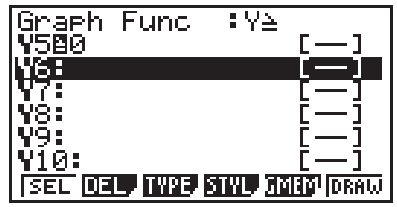
5. The third inequality does not have the same sign, so you will need to change the type of graph again. Press [F3] (TYPE), then press [F6] for more options, and finally, press [F4] for the Y≤ inequality followed by [5] [EXE].

6. The fourth inequality starts with x. (The Casio *fx-9750GII* can graph this inequality).Press F3 (TYPE), F6 for more options, F6 again the choices for an x-inequality will display. Press F3 for the X ≥ inequality followed by 0 EXE.

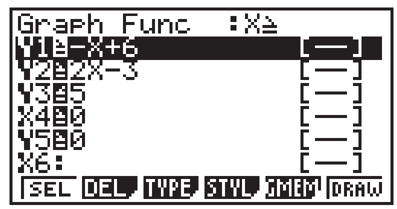




7. To enter the last inequality, press F3 (TYPE), F6 for more options, and then F4 0 EXE.

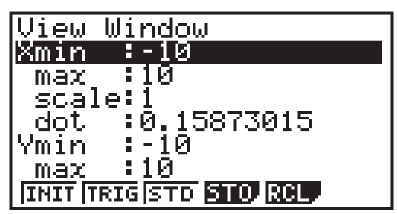


8. ▲ to see all the inequalities entered. The screen should look like the one to the right. The highlighted inequality sign indicates it will be graphed. To deselect an inequality, press F1 (SEL).



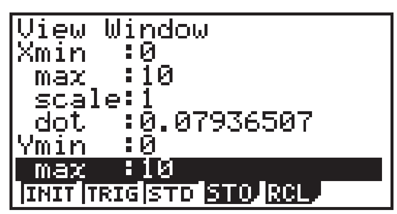
To set up the View Window and Graph:

1. Press SHIFT and then F3 (V-Window).



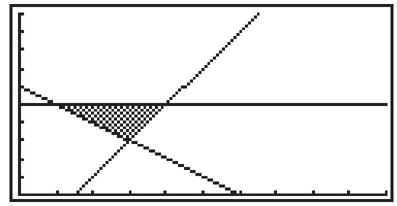
2. F3 (STD) will set the window to a Standard viewing window.

3. Since the graph will be located in the first quadrant, change the minimum x- and y-values to 0. Enter 0 for Xmin and press EXE then ▼ three times to change the Ymin. Enter 0 and press EXE.



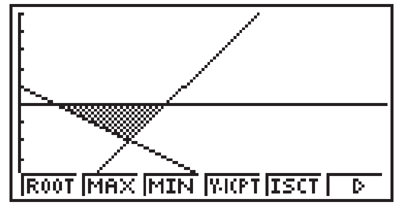
4. Press EXIT to return to the graphing home screen.

5. Press F6 (DRAW) to view the graph.

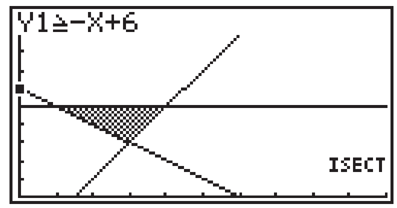


To find the intersections:

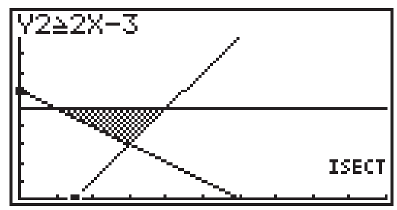
1. Press SHIFT F5 (G-Solv), then F5 (ISCT) to find the intersection of two of the inequalities.



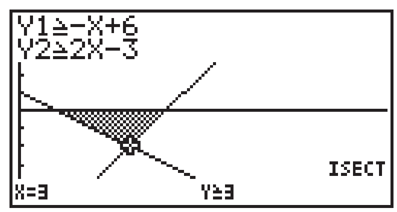
2. Press ▲ or ▼ until the desired inequality is displayed in the upper left corner and press EXE.



3. Use the same process to select the second inequality.



4. The calculator will calculate the intersection of the two inequalities. Notice the two inequalities displayed in the upper left corner. The x- and y-value will be displayed at the bottom of the screen. In the right hand corner, the calculator displays the letters ISCT to remind you that you are finding an intersection point.



5. To find the intersection of two different inequalities, repeat the steps from the beginning of this section.

With the population ever increasing, the need for homes increases as well. Developers design new housing areas to maximize the number of homes, minimize costs, and increase profits. Large sections of land are divided up into lots that are then sold. The size of the lot often determines the type of home that can be built on the property. In this activity, you will complete a linear programming problem that will take a piece of land, determine the number of lots to be developed, and the profit for the company according to specific requirements.

In this activity, you will write inequalities to represent the conditions for sectioning a piece of land, graph the inequalities to determine the best combination of homes to be built, and calculate the profit to be gained.

Type of Home	Number of Lots	Lot Size	Cost per Lot
One Story	x	0.75 acres	$30,000
Two Story	y	1 acres	$50,000
Maximum Number	150	130 acres	$6,200,000

Questions

1. Write the inequality with the constraint on the number of lots.

____

2. Write the inequality with the constraint on the size of the lots.

____

3. Write the inequality with the constraint on the cost per lot.

____

4. What is the possible combination of types of homes built, if only the number of lots and the size of the lots are considered?

____

5. What is the possible combination of types of homes built, if only the number of the lots and the cost per lot are considered?

____

6. What is the possible combination of types of homes built, if only the size of the lots and the cost per lot are considered?

____

7. Write the objective function for the profit from the sales of the lots if each 0.75 acre lot will yield $3,500 and each acre lot will yield $4,375.

____

8. What will the profit be if only the number of lots and the size of the lots are considered?

____

9. What will the profit be if only the number of lots and the cost per lot are considered?

____

10. What will the profit be if only the size of the lots and the cost per lot are considered?

____

11. Which combination will give the highest profit?

____

Extensions

1. The number of one-story houses will be divided into 1,500 square feet and 2,000 square feet homes. The number of 2,000 square feet homes will be one-and-a-half times that of the 1,500 square feet homes. Write an inequality to show the total number of one-story houses. Write another inequality to show the relationship between the two sizes of one-story homes.

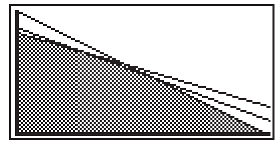
____

2. How many of each type of home should be built?

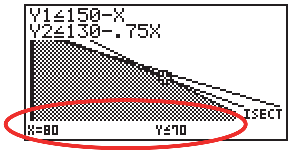
____

Solutions

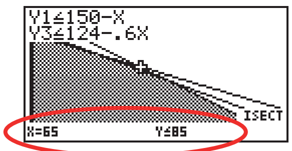
1. $x + y \le 150$ or $y \le 150 - x$



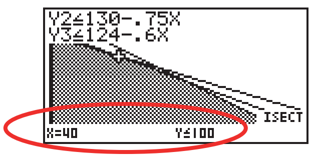
2. $0.75x + y \le 130$ or $y \le 130 - 0.75x$
3. $30{,}000x + 50{,}000y \le 6{,}200{,}000$ or $y \le 124 - 0.6x$
4. 80 One-Story Houses and 70 Two-Story Houses. $x = 80$, $y = 70$



5. 65 One-Story Houses and 85 Two-Story Houses. $x = 65$, $y = 85$



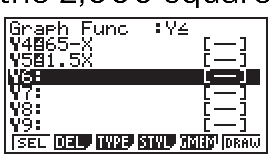
6. 40 One-Story Houses and 100 Two-Story Houses. $X = 40$, $y = 100$



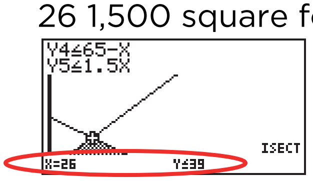
7. Profit = $3500x + 4375y$
8. Profit = $586,250. Plug $x = 80$ and $y = 70$ into the Profit equation
9. Profit = $599,375
10. Profit = $577,500
11. 65 one-story homes and 85 two-story homes

Extensions

1. $x + y \le 65$ or $y \le 65 - x$ (*x* represents 1,500 square foot homes and *y* represents the 2,000 square foot homes.); $y \le 1.5x$



2. 26 1,500 square foot homes and 39 2,000 square foot homes



Topic Area: Three Variable Equations

NCTM Standards:

- Use symbolic algebra to represent and explain mathematical relationships.
- Develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.

Objective

Given a set of equations in three variables, the student will be able to solve the equations for the unknown values using inverse matrices, then use the information to solve other problems.

Getting Started

As a class, discuss how someone in the catering business would determine the price of an item they are selling. What factors would the seller take into account when selecting the items? Could these factors change during the year? How does location affect their decision?

Prior to using this activity:

- Students should have an understanding of how to set up a matrix.
- Students should be able to solve a system of equations with two unknown values.
- Students should understand what is meant by an inverse matrix.

Ways students can provide evidence of learning:

- The student will be able to create a matrix for a set of equations with three unknowns.
- The student will be able to demonstrate how to use a calculator to solve a system of equations using inverse matrices.

Common mistakes to be on the lookout for:

- Students may set up the matrices using the wrong values.
- Students may interchange the order of the matrices when entering them into the calculator.

Definitions:

- System of Equations
- Matrix
- Inverse Matrix

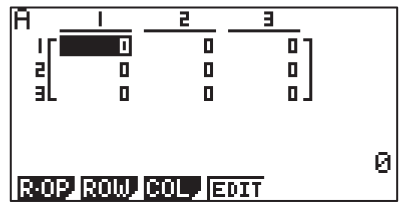
The following will demonstrate how to enter a three variable system of equations into the Casio *fx-9750GII* as two separate matrices, and then find the value of the variables using an inverse matrix.

Solve the given system for x, y, and z.

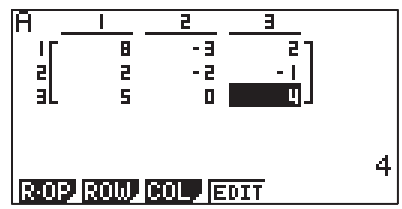
$$\begin{cases} 8x - 3y + 2z = 5 \\ 2x - 2y - z = 6 \\ 5x + 4z = 10 \end{cases}$$

To set up the matrices:

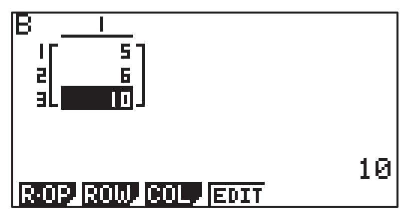
1. Highlight the RUN•MAT icon and press [EXE].
   Press [F1] (Mat) to enter the matrix editor.
   To set up the matrix for the variable terms, press [▶] [3] [EXE] [3] [EXE] [EXE]. The screen should look like the one at the right.



2. Enter the coefficient values for each variable into its appropriate cell followed by [EXE]. The screen at the right shows the results.

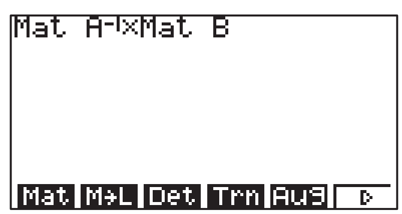


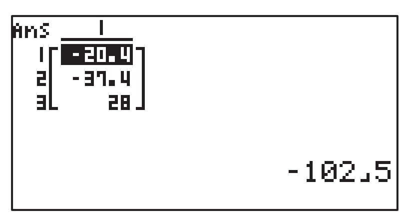
3. Press [EXIT] to set up the matrix for the constant terms. To set up a 3x1 matrix, press:
   [▼] [▶] [3] [EXE] [1] [EXE] [EXE]. Enter the constant values into its appropriate cell followed by [EXE].
   Press [EXIT] twice to return to the RUN•MAT Menu.



To find the unknown values:

1. To enter the inverse of Matrix A, press:
   [OPTN] [F2] [F1] [ALPHA] [X,θ,T] [SHIFT] [)]
   To see the values of the variables press:
   [×] [F1] [ALPHA] [log] [EXE]





The Silver Spoon Catering Company offers several different party trays. Four of the categories are listed below:

Meat	Seafood	Cheese	Veggies
Roast Beef	Shrimp	American	Peppers
Turkey	Crab	Cheddar	Carrots
Ham	Lobster	Swiss	Celery

In determining the possible pounds of each item used for the trays, the owners created three possible combinations for each type of tray using past experience.

1. The following are three possible meat trays and their price according to cost per pound. Find the cost per pound for each of the different meat choices.

Meat Trays	Roast Beef	Turkey	Ham	Price
Large Meat Tray	5 lb.	4.5 lb.	4 lb.	$51.10
Small Meat Tray	3 lb.	2.5 lb.	3 lb.	$32.20
Small Two Meat Combo	4.5 lb.	4.5 lb.	-	$34.65

Roast Beef: ______________________

Turkey: ______________________

Ham: ______________________

2. The possibilities for the seafood trays are as follows:

Seafood Trays	Shrimp	Crab	Lobster	Price
Shrimp and Crab Combo	3 lb.	4.5 lb.	-	$25.90
Seafood Medley	3 lb.	2.5 lb.	2.5 lb.	$47.75
Shrimp and Lobster Combo	4.5 lb.	3 lb.	-	$48.15

Find the cost per pound for each of the different seafood choices.

Shrimp: ______________________

Crab: ______________________

Lobster: ______________________

3. For appetizers, the Silver Spoon Catering Company offers a variety of items including a cheese tray and veggie tray. Find the cost per pound for each of these items.

Cheese Trays	American	Cheddar	Swiss	Price
Large Cheese Tray	3 lb.	2.5 lb.	2 lb.	$21.85
All American	2.5 lb.	2.5 lb.	-	$14.00
Small Cheese Tray	1.5 lb.	1.5 lb.	1 lb.	$11.65

Veggie Trays	Peppers	Carrots	Celery	Price
Large Dipping Tray	2.5 lb.	3 lb.	4 lb.	$16.05
Classic Dipping Tray	-	2.5 lb.	2.5 lb.	$6.75
Small Dipping Tray	.5 lb.	2 lb.	2 lb.	$6.75

American: ______________________________

Cheddar: ______________________________

Swiss: ______________________________

Peppers: ______________________________

Carrots: ______________________________

Celery: ______________________________

4. A client is requesting a special combination of meat and cheese trays for a reception. Use the price per item found in problems 1 and 3 to determine the price of each tray.

Meat Tray: 4 lb. Roast Beef 2.5 lb. Turkey 2.5 lb. Ham

Cost: ______________________________

Cheese Tray: 2 lb. American 2 lb. Cheddar 1.5 lb. Swiss

Cost: ______________________________

5. A hostess has ordered 3 Large Meat Trays for an upcoming event. It is recommended that there is at least 0.5 pounds of meat for each guest. If the party will have 50 guests, will this be enough food for the party? Why or Why not?

____

____

6. If it is also recommended that a hostess plan for 0.25 pounds of cheese for each guest, how many Large Meat Trays and Large Cheese Trays are needed for an event in which 150 guests will attend?

   Meat Trays: ______________________ Cheese Trays: ______________________

7. If there is a 9% sales tax, how much will the total cost be for the trays in problem 6?

   ____

Extensions

1. A client is planning a special party and needs two Large Meat Trays with the following amounts: 5 lb. roast beef, 3 lb. turkey, and 4 lb. of ham. Using the same prices, find the cost of each tray and the total bill if there is a 8% sales tax.

   Cost per Tray: ____

   Total Cost: ____

2. The same client needs two Seafood Medley Trays with the following amounts: 4 lb. shrimp, 3 lb. crab, and 4 lb. lobster. How much would each of these trays cost a piece and what is the total cost?

   Cost per Tray: ____

   Total Cost: ____

3. Mrs. Smith is in charge of planning a reception for a visiting speaker. She has a budget of $500 for food not including the condiments. There are 75 guests expected to attend. Decide how many of each tray she will order and the total cost to include a 6% sales tax.

   ____

   ____

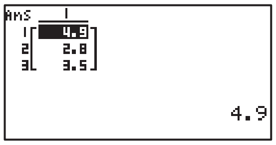
   ____

Solutions

1. Roast Beef: $4.90

   Turkey: $2.80

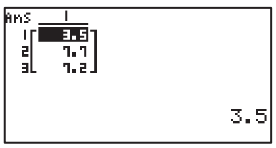
   Ham: $3.50



2. Shrimp: $3.50

   Crab: $7.70

   Lobster: $7.20



3. American: $2.90

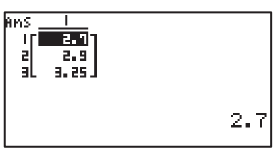
   Cheddar: $2.70

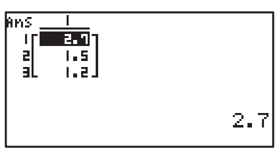
   Swiss: $3.25

   Peppers: $2.70

   Carrots: $1.50

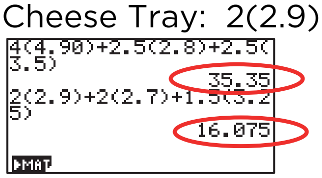
   Celery: $1.20





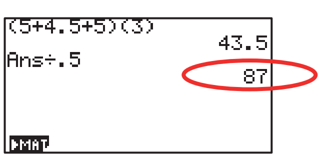
4. Meat Tray: 4(4.90) + 2.5(2.8) + 2.5(3.5) = $35.35

   Cheese Tray: 2(2.9) + 2(2.70) + 1.5(3.25) = $16.08

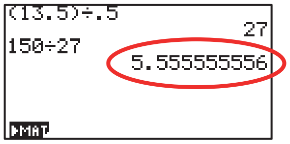


5. (13.5 lb.)(3)/.5 = 87 servings;

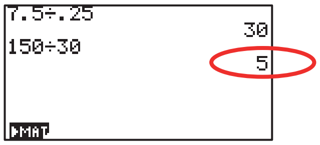
   Yes, there will be extra for 37 people.



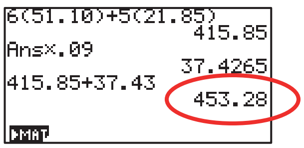
6. Meat Trays: 6



Cheese Trays: 5



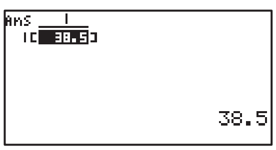
7. $415.85 + 37.43 = $453.28



Extensions

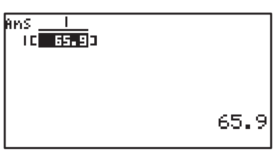
1. Cost: $38.50

   Total: $83.16



2. Cost: $65.90

   Total: $142.34



3. Answers will vary according to choices.

Topic Area: Matrices

NCTM Standard:

- Develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.
- Develop an understanding of properties of, and representations for, the addition and multiplication of vectors and matrices.

Objective

Given several data tables, the student will be able to create a matrix to represent the data, perform operations using the matrices, and apply the results to problem solving tasks.

Getting Started

Have the students work in pairs or in small groups to determine what information would be needed to calculate the cost of manufacturing an item for retail sales and what would influence these expenses.

Prior to using this activity:

- Students should have a basic understanding of the properties of matrices.
- Students should be able to determine if two matrices can be multiplied and perform the operation.
- Students should be able to understand the meaning of the resulting matrix.
- Students should be able to understand and calculate percent in relationship to profit and loss.

Ways students can provide evidence of learning:

- Given a table of data, the student will be able to create an appropriate matrix to represent the data.
- Given two matrices, the student will be able to multiply the matrices and analyze the results.

Common mistakes to be on the lookout for:

- Students may create a matrix that does not accurately represent the data.
- Students may multiply matrices in the wrong order, resulting in a single number rather than a list of numbers.

Definitions:

- Matrix
- Cell
- Percent

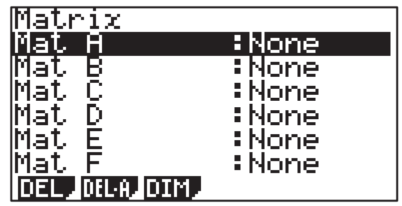
The following will demonstrate how to create a matrix and enter the values of the cells into the Casio *fx-9750GII*, recall a matrix, and perform operations with matrices.

Orchard	Apples	Pears	Peaches
Farm 1	125	110	135
Farm 2	205	95	185
Farm 3	158	82	170

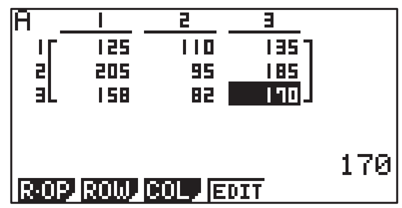
Price/Box
$29
$22
$27

To create a matrix for the table above:

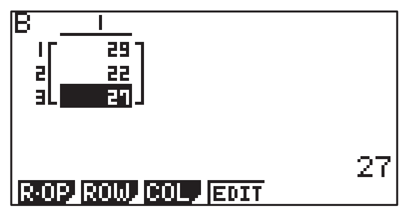
1. From the main menu, highlight the RUN•MAT icon and press [EXE] or press [1].
   Press [F1] to access the matrix editor.



2. To clear any data from previous matrices, press [F2] (DEL-A), then [F1] (Yes).
3. To create a 3 x 3 matrix, press:
   [▶] [3] [EXE] [3] [EXE] [EXE]
   Enter the values for each cell and press [EXE].
   The screen should look like the one to the right.

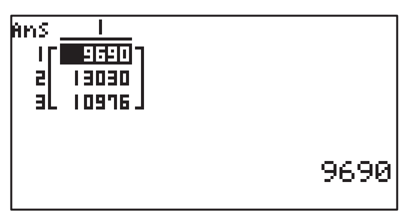


4. To enter the second matrix, press:
   [EXIT] [▼] [▶] [3] [EXE] [EXE] for a 3 x 1 matrix.
   Enter the values in the same way as above.
   The screen should like the one to the right.



To multiply the two matrices:

1. Press [EXIT] twice to return to the RUN-MAT screen.
2. Press [OPTN] [F2] [F1] [ALPHA] [X,θ,T] [×] [F1] [ALPHA] [log] [EXE] to multiply the two matrices. The results are shown on the screen to the right.



What Happened to the Cost? Activity

Ever wonder why it cost so much to buy your favorite sport jersey? The amount of material it would take to make the shirt does not seem to match the price. Some talented people can make a great jersey and save themselves money while others are willing to spend the money and save the time for other things. In this activity, you will explore the costs involved in manufacturing an athletic jersey and discover why it costs so much more to buy one than make it yourself.

In this activity, you will use the given data tables and matrices to calculate the cost of materials, production, labor, and advertising and apply these costs to the final price of a football, basketball, and baseball jersey. The company is manufacturing 2000 football jerseys, 1500 basketball jerseys, and 1200 baseball jerseys. Using your calculations you will then determine how much each shirt must be sold for in order to make a profit.

Production Costs			
Sport	Cutting	Assembly	Packaging
Football	$0.25/hr.	$1.75/hr.	$0.08/hr.
Basketball	0.10/hr.	0.50/hr.	0.05/hr.
Baseball	0.15/hr.	0.75/hr.	0.05/hr.

Labor Costs			
Sport	Cutting	Assembly	Packaging
Football	$1.88/hr.	$10.31/hr.	$0.58/hr.
Basketball	0.75/hr.	4.13/hr.	0.36/hr.
Baseball	1.13/hr.	6.19/hr.	0.36/hr.

Advertising Costs			
Sport	Photography	Copy	Model
Football	$2.25/hr.	$0.38/hr.	$22.50/hr.
Basketball	3.00/hr.	0.50/hr.	24.00/hr.
Baseball	3.75/hr.	0.63/hr.	20.83/hr.

Questions

1. Create a matrix for the production costs, the labor costs, the advertising costs, and the number of each type of jersey to be manufactured. Multiply each of the matrices for the various costs times the number of jerseys. Fill in the resulting matrices. What is the meaning for each of the values in the matrices?

Production Costs   Labor Costs   Advertising

$\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}$ $\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}$ $\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}$

____

____

2. Calculate the total cost for material for each jersey. Explain why this cannot be done using matrices.

____

____

3. Calculate the total cost for manufacturing a football jersey. What are some reasons why production and labor costs are higher for this jersey than for the others?

____

____

4. Find the cost of manufacturing each baseball jersey; is it more or less than the football jersey? Does this seem like a reasonable cost? What would account for the difference?

____

____

5. Calculate the total cost for manufacturing a basketball jersey. What are some of the reasons why advertising costs are higher for this jersey than for the others?

6. If each jersey sells for $55.00, create a matrix that can be used to find the product of the cost per jersey and the number of jerseys to be sold in the space below. Multiply this times the matrix for the total number being manufactured. What is the projected total from the sales of the jerseys? What will be the percentage of profit from the sales?

[ ]

Extensions

1. Explain how the configuration of two matrices determines whether they can be multiplied? Given a 2 x 3 matrix and a 4 x 2 matrix, tell which one should be first and give the dimensions of the resulting matrix.

2. The school store is selling shirts with short sleeves, long sleeves, and hoods. The price of the shirts are $15.00 for short sleeve, $17.50 for long sleeve, and $20.00 for hooded shirts. Below are the matrices that were set up to calculate the total sales.

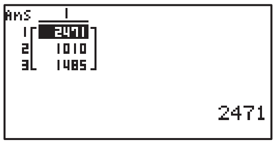
Number of Shirts	Cost per Shirt
[200 155 125]	[15.00 17.50 20.00]

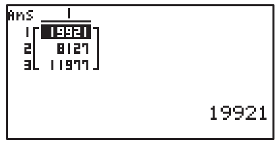
a) When these matrices were multiplied using a calculator, an error message was given; explain why.

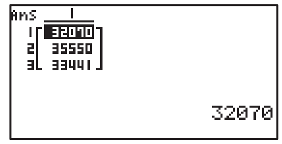
b) Explain how to work the problem in the calculator, in order to obtain a correct answer for total sales.

Solutions

1. **Production Cost** | **Labor Cost** | **Advertising Cost**







For each matrix, row 1 is the total cost for football jerseys, row 2 is the total cost for basketball jerseys, and row 3 is the total cost for baseball jerseys.

2. Football: 2000 ($5.99) = **$11,980.00**
Basketball: 1500 ($3.99) = **$5,985.00**
Baseball: 1200 ($4.99) = **$5,988.00**

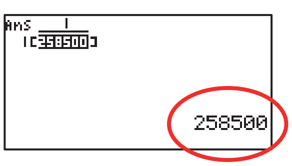
Multiplying a 3 x 1 matrix by a 1 x 3 matrix would result in a 3 x 3 matrix which would not give a correct response. Multiplying a 1 x 3 matrix by a 3 x 1 matrix would result in giving the total cost of materials for all of the jerseys.

3. -The total cost is $2,471 + $19,921 + $32,070 + $11,980 which equals **$66,442**.
-Answers may vary. Some reasons would include the cost of materials are higher, the amount of time for construction is higher, and the cost of advertising is more due to endorsements by athletes who are paid higher salaries.

4. -The total cost is $1,485 + 11,977 + 33,441 + 5,988 which equals **$52,891**.
-This cost is less than a football jersey.
-Answers will vary according to experience.
-Some of the difference is the number of jerseys being made and the extra materials.

5. -The total cost is $1,485 + 8,127 + 35,550 + 5,985 which equals **$51,147**.
-One answer may be that although endorsements may not cost as much due to popularity, the cost per jersey is more since there are less being manufactured.

6. [55 55 55]



Percentage of Profit:
[258,500 – (66,442 + 52,891 + 51,147)] /258,500 = .34 or **34%**

Extensions

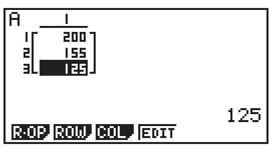
1. The dimensions of the matrices must be such that the number of columns in the first matrix is the same as the number of rows in the second matrix.

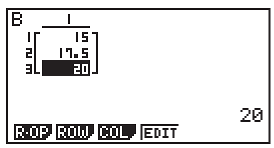
   The 4 x 2 matrix would be entered first followed by the 2 x 3 matrix. The resulting dimensions would be a 4 x 3 matrix.

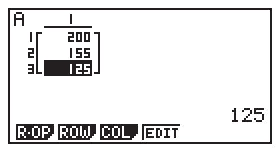
2. a) Matrix A = [200 155 125] Matrix B = [15.00 17.50 20.00]

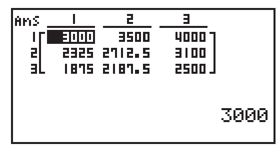
   Matrix AB = [200(15) + 155(?) 200(17.50) 200(20.00)]

   b) Since the first cell of AB must contain the product of $\text{cell}_{1,1}$ + $\text{cell}_{2,2}$ and there is not a second row, these matrices will not work. The second matrix would need to be rewritten as a 3 x 1 matrix in order to calculate the total.









Do You Have Tunnel Vision? Teacher Notes

Topic Area: Radical Equations

NCTM Standards:

- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations.

Objective

The student will be able to write the equation of a circle or ellipse given a diagram on a coordinate plane and solve the formula for a circle or ellipse for *y*. The student will be able to write the equation for finding area and perimeter of a rectangular area under a curve, and determine maximum and minimum values for the area and perimeter.

Getting Started

Discuss with students the various shapes of tunnels they have driven thru. Have the students discuss what shape a tunnel should be to ensure that traffic can travel along the roadway efficiently.

Prior to using this activity:

- Students should be able to solve equations for a specified variable.
- Students should have an understanding of how to find the area and perimeter of a rectangle.

Ways students can provide evidence of learning:

- The student will be able to write an equation for finding the area in terms of x-values.
- The student will be able to graph the formulas, find a range of values and the maximum value for the area and perimeter.

Common mistakes to be on the lookout for:

- Students may use the wrong formula for a circle or ellipse.
- Students may use incorrect formulas for finding the area and perimeter of a rectangle.

Definitions

Area Perimeter Semicircle

Formulas

Circle: $x^2 + y^2 = r^2$ Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Do You Have Tunnel Vision? “How-To”

The following will demonstrate how to enter a formula into the Casio *fx-9750GII*, trace the graph, and find the value of a specified variable.

Enter the formula $Y1 = 2x\sqrt{(16 - x^2)}$ into the calculator, graph the equation and find the maximum point. Find the x-coordinate that corresponds to a given y-coordinate.

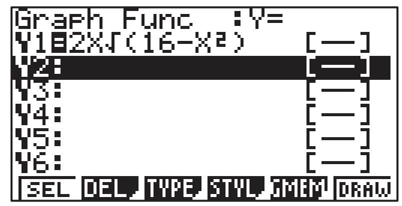
To enter a formula into the Graph Function:

1. From the main icon menu, highlight the GRAPH icon and press [EXE] or [5].

2. Select **Y=** by pressing [F3] [F1]. To enter the formula, $2x\sqrt{(16 - x^2)}$, input the following:

   [2] [X,θ,T] [SHIFT] [x^2] [(] [1] [6] [−] [X,θ,T] [x^2] [)] [EXE]

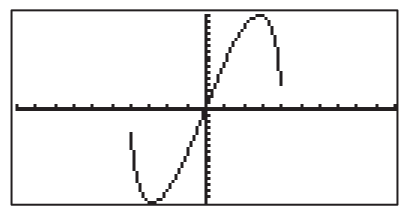
   The screen will look like the one at the right.



To set up the values for the window:

1. Press [SHIFT] [F3] (V-Window), then [F3] (STD) to display a standard 10x10 grid. Press [EXE] twice to view the graph.

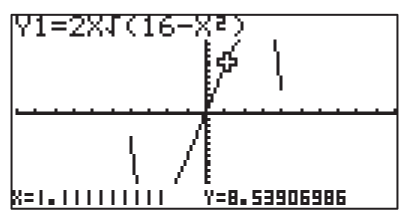
2. If you do not see the graph, press [F2] (Zoom) then [F5] (Auto) to view the graph seen at the right.



To trace a graph and locate values:

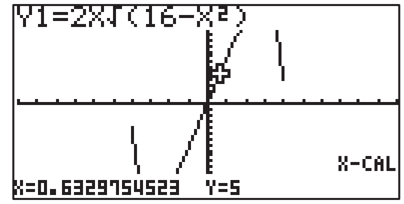
1. With the graph showing, press [F1] (Trace) and use the [◀] [▶] keys to move along the graph.

2. If there is more than one graph, use the [▲] [▼] keys to move between the graphs. The graph at the right shows the value of *y* when x = 1.1.



To find the maximum point using G-Solve:

1. After graphing, press F5 (G-Solv), then F2 (Max).



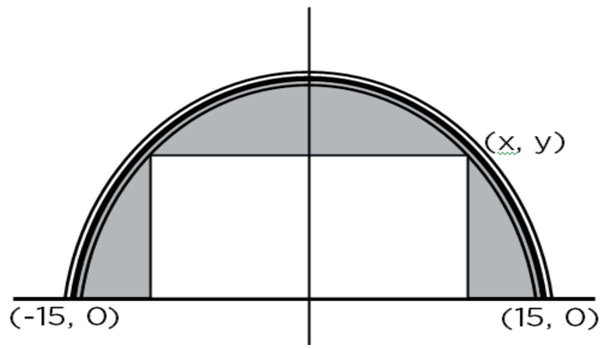
2. To find the value of x for a specified y-value, press SHIFT F5 F6 (▷) then F2 (x-cal). Enter the specified y-value followed by EXE.

   The screen displays the value of x when y = 5.

Many tunnels are built using either a circular or elliptical shape in order to distribute the stress on the walls. The stress of the ground is spread along the surface which reduces the possibility of a collapse. The tunnel opening must be large enough for the desired lanes of traffic.

Using the formula for a circle and solving for the y-value, you will explore a circular entrance for a tunnel and find its area and perimeter.

Questions



1. Given the tunnel entrance at the right, write an equation for the semicircle used in the figure.

   ____

2. What would be the length of the rectangular region in terms of x?

   ____

   What would be the height in terms of x?

   ____

3. Write the formula for finding the perimeter and area of the figure using the expressions found in question 2.

   Perimeter: ____________________ Area: ________________________

4. Graph the formula for finding the perimeter of the region. What is the range of positive values for x?

   ____

5. What is the maximum perimeter for the entrance to this tunnel?

   ____

6. What would be the value of x for a perimeter of 20? Of 30?

   ____

7. Graph the formula for finding the area of the region. What is the range of positive values for *x*?

____

8. What is the maximum area for the entrance to this tunnel?

____

9. What would be the value of *x* for an area of 50? Of 100?

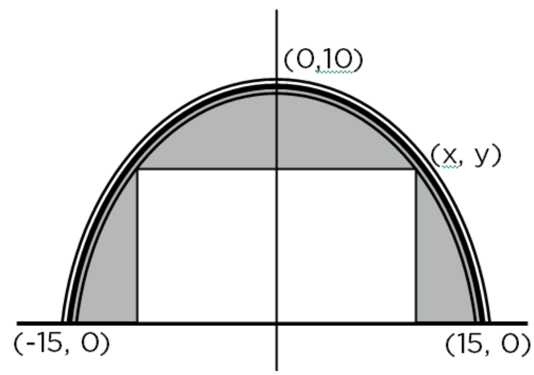
____

10. Would this be an adequate opening for a tunnel? Why or why not?

____

____

Extensions



1. Use the diagram at the right to write an equation for the elliptical entrance.

____

2. Find equations for the perimeter and area of the rectangular region.

Perimeter: ______________________

Area: ______________________

3. How does the range of x-values compare to that of the circle?

____

4. How does the maximum area compare to that of a circle?

____

5. Which shape do you feel would be better? Why?

____

____

Solutions

1. $x^2 + y^2 = 225$

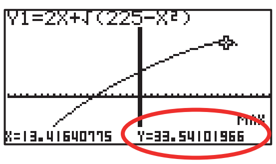
2. $2x$; $\sqrt{225 - x^2}$

3. Perimeter: $P = 2x + 2\sqrt{225 - x^2}$

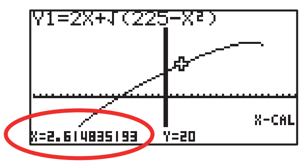
   Area: $A = 2x\sqrt{225 - x^2}$

4. $0 < x < 15$

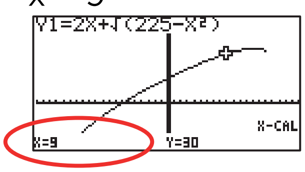
5. Pmax = 33.5



6. x = 2.6

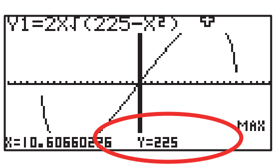


x = 9

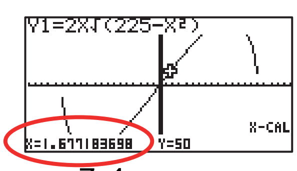


7. $0 < x < 15$

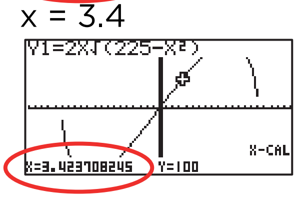
8. Amax = 225



9. x = 1.7



x = 3.4



10. Answers will vary.

Extensions

1. $\frac{x^2}{225}+\frac{y^2}{100}=1$

2. Perimeter: $P=2x+\sqrt{\left(100-0.8x^2\right)}$

   Area: $A=2x\sqrt{\left(100-0.8x^2\right)}$

3. The range is the same.

4. It is less than that of the circle.

5. Answers will vary.

Where Did That Come From? Teacher Notes

Topic Area: Circles

NCTM Standards:

- Use symbolic algebra to represent and explain mathematical relationships.
- Use geometric models to gain insights into, and answer questions in other areas of mathematics.
- Recognize and apply mathematics in contexts outside of mathematics.

Objective

The student will be able to write the equation of a circle given the center and radius, find the intersection of two circles using a graphing calculator, and apply finding the equation and intersections of circles to problem solving.

Getting Started

Discuss with students how sound and motion waves travel. Give examples such as dropping a stone in a pond and discuss how this is related to earthquakes. Discuss with them how triangulation is used to locate a particular site. Have the students create a list of places where this would be useful.

Prior to using this activity:

- Students should have an understanding of how to write an equation for a circle.
- Students should have an understanding of how to find the intersection of two functions.

Ways students can provide evidence of learning:

- The student will be able to graph and locate the intersections of two or more circles.
- The student will be able to discuss the difference between using circles and linear functions when using triangulation.

Common mistakes to be on the lookout for:

- Students may use the diameter instead of the radius in the equation of the circle.
- Students may make sign errors when using a given point as the center of a circle when writing its equation.

Definitions

- Radius
- Epicenter

Formulas

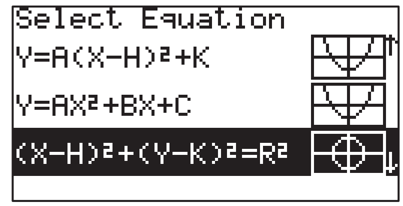
Equation of a Circle: $(x-h)^2 + (y-k)^2 = r^2$

The following will demonstrate how to graph a circle using the Conics Function, save it as the background for another graph, and use the Trace function to find the intersection for the two graphs using the Casio *fx-9750GII*.

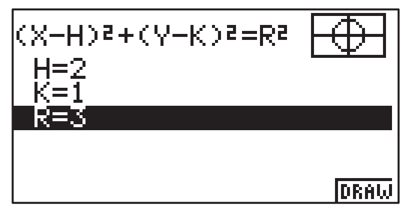
Circle 1: Center (2, 1) Radius = 3
Circle 2: Center (-1, -4) Radius = 5

Draw the two circles and find the points of intersection.

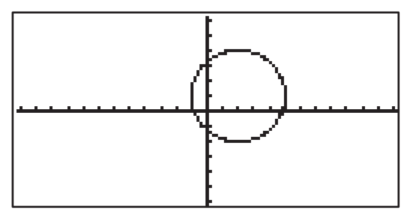
To enter values for a circle into the Conics Function:



1. Highlight the CONICS icon in the Main Menu and press [EXE]. Use [▼] to scroll to the formula for a circle given the center and radius, press [EXE].



2. Enter the values of H, K, and R for Circle 1, pressing [EXE] after each entry. The screen should look like the one to the right. Press [F6] (Draw) to view the graph. Note: The graph may appear elliptical due to the viewing window values.

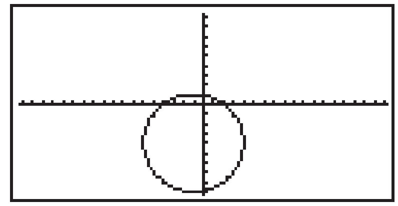


3. To change the view screen so that the circle appears round, press [F3] (V-Window) and enter the following values:
   Xmin = -18.3 Xmax = 18.3
   Ymin = -9.3 Ymax = 9.3
   Press [EXE] [F6] (Draw). The screen should look like the one to the right.

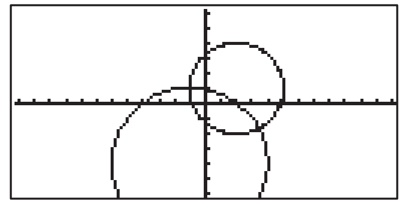
To save a graph as a background for another graph:

1. Press [OPTN] [F1] (PICT), then [F1] (Sto) [1] [EXE]. The graph can now be recalled to appear as the background of another graph.

2. Press [EXIT]; follow the steps above to enter the information for Circle 2.

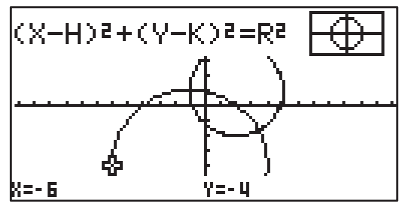


3. To recall the background, press [OPTN] [F1] (Pic), then [F2] (Rcl) [1] [EXE]. The screen at the right shows Circle 2 with Circle 1 as a background.



To trace a graph using the Trace function:

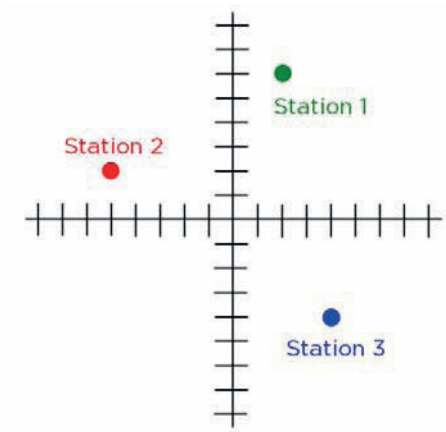
1. Press [F1] (Trace) and use ▶ to move the cursor along the graph. For tracing circles, the cursor will only move to the right.



Where Did That Come From? Activity

When an earthquake occurs, it causes waves of energy in the form of movement. These waves are picked up by devices known as seismographs. In areas where earthquakes happen frequently, scientist set up a seismic network comprised of different stations that record these waves. In order to determine the origin or epicenter of an earthquake a system called triangulation is used in which the intersection of three or more graphs is used to locate the position of the epicenter on a map.

In this activity, you are going to calculate the distance from the epicenter of an earthquake to three different stations and determine the location of the epicenter. The grid at the right shows the location of the stations with each tick mark equal to 10 km.



Questions

1. What are the coordinates for each of the stations in the grid to the right?

   Station 1: ______________________________

   Station 2: ______________________________

   Station 3: ______________________________

2. Station 1 registers an earthquake and determines that its center is 60 km from the station. Write an equation to show the possible locations of this earthquake?

   ____

3. At the same time, Station 2 registers the earthquake and determines that its epicenter is halfway between Station 1 and Station 2. How far is the epicenter from Station 2?

   ____

   Write an equation for possible locations for the earthquake from Station 2.

   ____

4. Graph the equations for questions 1 and 2. What are the intersections between Station 1 and Station 2?

   ____

5. Station 3 also registers the earthquake. The seismologist determines that from the time it started until it reached the station was 25 sec. If it is known that the waves travel at 5 km/sec in this area, how far was the epicenter from Station 3?

   ____

6. Write an equation representing the distance of the earthquake from Station 3.

7. Graph the equation in question 6 along with the other two equations. What are the intersections between Station 1 and Station 3?

8. Determine the possible location for the epicenter of the earthquake. (Hint: Draw lines through the intersections from questions 4 and 7.)

Extensions

1. Station 4 is located at (-80, -70) and registers an earthquake with an epicenter estimated to be 40 miles away. Station 1 registers the same earthquake with an epicenter that is 68 miles away after 5 min. What is the speed of the waves from the earthquake?

2. A new station is being established whose x-coordinate is halfway between Station 1 and Station 3 and whose y-coordinate is 50 miles north of Station 1. What will be the coordinates for the new station?

3. The epicenter of a minor earthquake is located at (-40, 35). If the range for registering earthquakes for each station is 80 miles, which stations will be able to register this earthquake? Justify your answer.

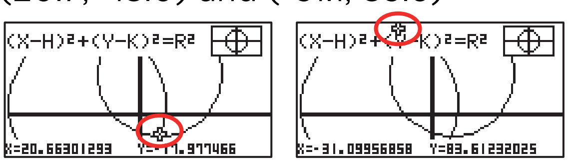
Solutions

1. Station 1: (20, 40)
   Station 2: (-50, 10)
   Station 3: (60, -40)

2. Center is (20, 40)
   Radius is 60
   $(x-20)^2+(y-40)^2=3600$

3. $D=\sqrt{(-50-20)^2+(10-40)^2}\approx 76\text{km}$
   $(x+50)^2+(y-10)^2=5776$

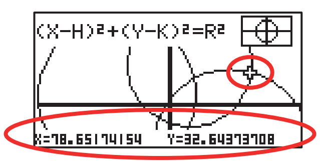
4. (20.7, -18.0) and (-31.1, 83.6)



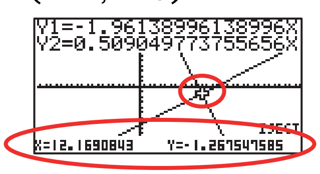
5. 75 km

6. $(x-60)^2+(y+40)^2=5625$

7. (-9.7, -12.4) and (78.7, 32.6)



8. (12.2, -1.3)



Extensions

1. (68 – 40) / 7 min. = 4 min.
2. (10, 90)
3. Stations 1, 4, and 5.
   Answers will vary.

Out to Sea Teacher Notes

Topic Area: Hyperbolas

NCTM Standards:
- Interpret representations of functions of two variables.
- Solve problems that arise in mathematics and in other contexts.
- Recognize and apply mathematics in contexts outside of mathematics.

Objective

The student will be able to write an equation for a hyperbola, evaluate the hyperbola for a specified location, and use the equation of the hyperbola to solve problems involving navigation.

Getting Started

Discuss with students the properties of a hyperbola. Use a diagram of a hyperbola to show the foci and vertices. Discuss how hyperbolas are used in locating ships via radio waves.

Prior to using this activity:
- Students should have an understanding of hyperbolas.
- Students should have an understanding of the Pythagorean Theorem.

Ways students can provide evidence of learning:
- The student will be able to write an equation given the foci and a point on the hyperbola.
- The student will be able to solve problems involving navigation of ships and apply this to other areas such as location of aircraft.

Common mistakes to be on the lookout for:
- Students may use the formula for an ellipse instead of a hyperbola.
- Students may use the wrong form of the hyperbola.

Definitions:
- LORAN

Formulas

Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Pythagorean Theorem: $a^2 + b^2 = c^2$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

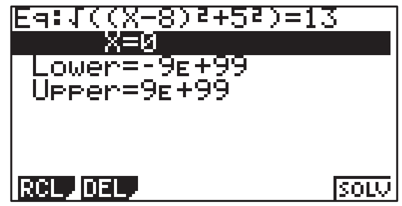
The following will demonstrate how to enter an equation and a formula into the Equation Solver Function of the Casio f*x-9750GII* and graph the resulting equation.

Enter the equation $\sqrt{(x-8)^2+5^2}$ = 13 and solve for x.

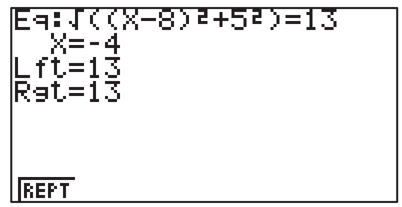
Enter the formula $a^2+b^2=c^2$ and find the value of b when a = 6 and c = 15.

To enter an equation into the Solver function:

1. Highlight the EQUA icon from the Main Menu and press [EXE]. To select Solver, press [F3].
   To enter the formula, input the following:
   [SHIFT] [x^2] [(] [(] [X,θ,T] [−] [8] [)] [x^2] [+] [5] [x^2] [)] [SHIFT] [•] [1] [3] [EXE]
   The screen should look like the one to the right.

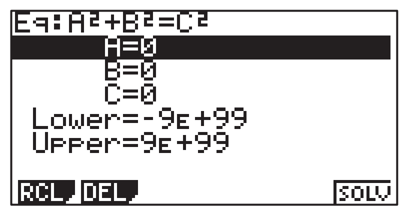


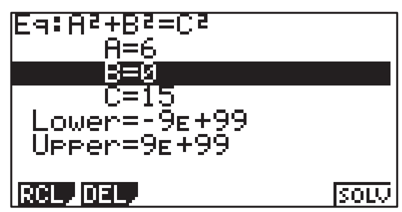
2. Press [F6] (Solv) to see the solution shown at the right.



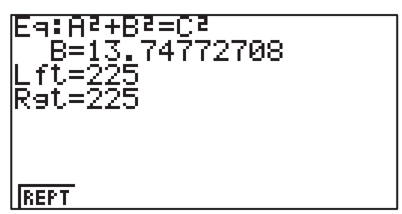
To use the Solver function with a formula:

1. Highlight the EQUA icon from the Main Menu and press [EXE]. To select Solver, press [F3]. To enter the formula input the following:
   [ALPHA] [X,θ,T] [x^2] [+] [ALPHA] [log] [x^2] [SHIFT] [•] [ALPHA] [ln] [x^2] [EXE]
   Note: To input a variable, press [ALPHA] then the key associated with the letters written in red.
   The screen should look like the one to the right.





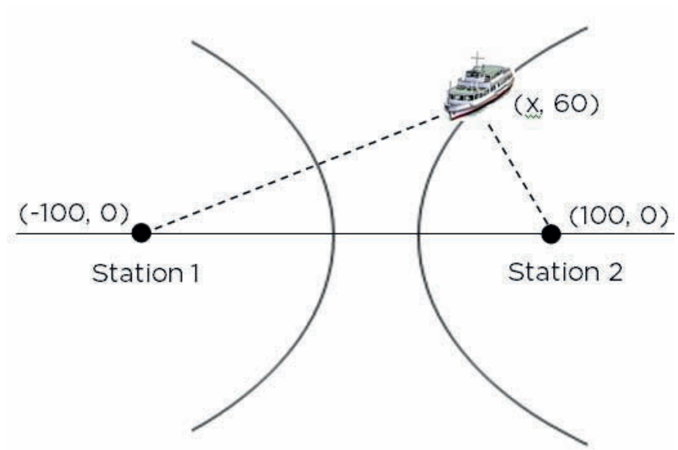
2. To solve for an unknown variable, enter each of the known values and press [EXE]. Use the arrow keys to highlight the unknown value and press [F6] (Solv).



Navigators use LORAN, which stands for long-distance radio navigation for aircraft and ships, in order to locate the position of aircraft and ships. The system uses synchronized pulses that are transmitted by widely separated transmitting stations that are located at the foci of a hyperbola. These pulses travel at the speed of light (186,000 miles per second) and represent the difference in the times of arrival of an aircraft or ship which is constant on a hyperbola.

In this activity you will be given two stations, 200 miles apart that are positioned on a rectangular coordinate system at points (-100, 0) and (100, 0) and a ship is traveling on a path with coordinates (x, 60). You will find the x-coordinate for the position of the ship if the time difference between the pulses from the transmitting stations is 1000 microseconds (0.001 second) and time travels at 186,000 miles per second. You will then use this information to determine where the ship will dock on the shore.

Questions



1. Find the difference between the pulses from the radio stations.

   ____

2. Using the distance formula, find the x-coordinate for the position of the ship.

   ____

3. Find the values of c, a, and b for this hyperbola.

   c = ______; a = _______; b = _______

4. Find the equation for the hyperbola that represents these two transmitting stations.

   ____

5. What will the x-coordinate be if the ship is 85 miles off shore?

   ____

6. How far is the ship from Station 2 at this point?

   ____

7. Find the distance from shore for the given x-coordinates.

a. $x = 150$ __________ b. $x = 95$ __________

Extensions

1. A distress call comes into Station 1 from a ship starting 45 miles off shore. What are the coordinates of the ship?

____

2. What is the distance of the ship from Station 1?

____

3. A rescue helicopter located at Station 1 can travel at 80 mph. A rescue vessel located at Station 2 can travel 75 mph. How long would it take each vessel to reach the disabled ship?

____

Solutions

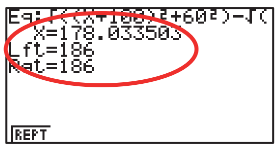
1. 186,000 (.001) = **186 miles**

2. d_1 (Station 1 to Ship) = $\sqrt{(x-100)^2+60^2}$

   d_2 (Station 2 to Ship) = $\sqrt{(x+100)^2+60^2}$

$$\sqrt{(x-100)^2+60^2} - \sqrt{(x+100)^2+60^2} = 186$$

$x = 178$

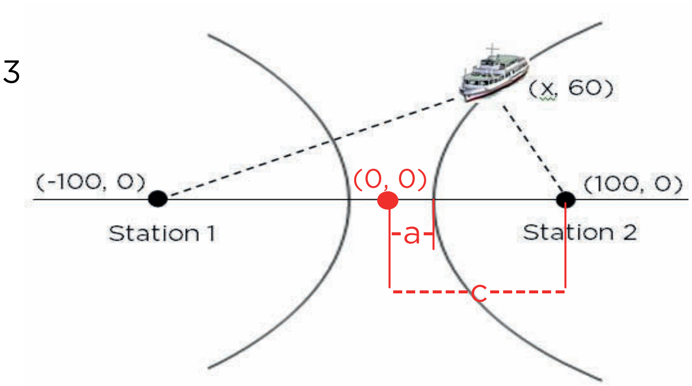


3. $c = 100$; $a = \frac{186}{2} = 93$

   $c^2 - a^2 = b^2$

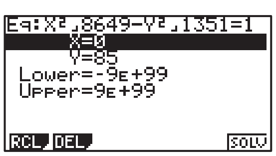
   $100^2 - 93^2 = 1351 = b^2$

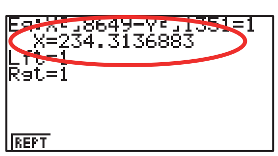
   $b = 36.8$



4. $$\frac{x^2}{8649} - \frac{y^2}{1351} = 1$$

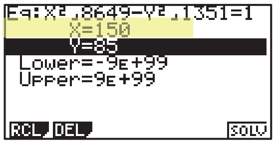
5. $x = 234.3$ mi.

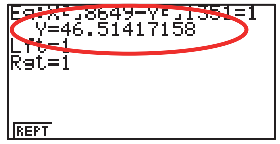




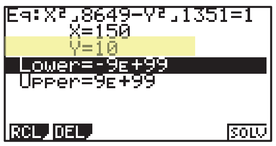
6. $d = \sqrt{(234.3-100)^2+85^2}$ = 158.9 mi.

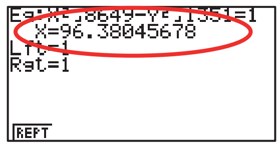
7. x = 150; y = 46.5





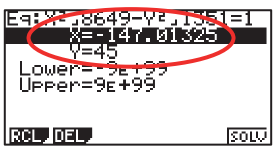
y = 10; x = 96.3



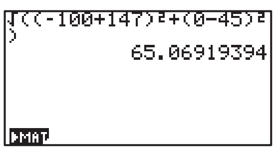


Extensions

1. Coordinates: (-147, 45)



2. Station 1; 65 miles away



3. Helicopter : 48.75 min.
Rescue Vessel: 3.3 hrs.

Keeping up the Pressure — Teacher Notes

Topic Area: Patterns and Functions – Algebraic Thinking

NCTM Standard:

- Understand patterns, relations, and functions by interpreting representations of functions of two variables, and use symbolic algebra to represent and explain mathematical relationships.

Objective

Given a set of data, the students will be able to use the GRAPH Menu and the TRACE Function to solve problems involving water pressure used by firefighters.

Getting Started

Discuss with the students what is meant by force as determined in Newton's Third Law which states that for every action there is an equal and opposite reaction. Relate this to the formulas used in the activity.

Prior to using this activity:

- Students should be able to enter various formulas into GRAPH Menu and use the TRACE Function to find specific x- and y-values.

Ways students can provide evidence of learning:

- The student will be able to discuss the results of the activity and justify their answers to the questions.
- The student will be able to discuss how the formulas relate to their corresponding graphs used to answer questions.

Common mistakes to be on the lookout for:

- Students may need to adjust the window prior to tracing so that they can follow the pointer using the trace function

Definitions

- Nozzle Pressure
- Nozzle Reaction
- Friction Loss
- Pump Discharge Pressure

Formulas

Nozzle Reaction: $NR = (1.57)(d^2)(NP)$

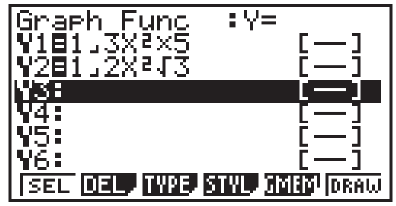
Gallons Per Minute: $GPM = (29.7)(d^2)(\sqrt{NP})$

Friction Loss: $FL = (C)(0.01 \cdot GMP)^2(0.01 \cdot L)$

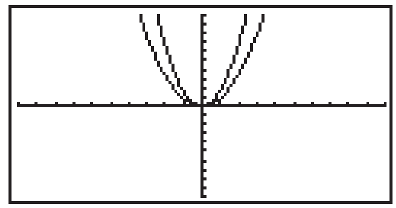
The following will demonstrate how to enter a given formula into the graph menu of the Casio *fx-9750GII*, graph the data, and trace along the graph to find x- and y-values.

$V = \frac{1}{3}b^2 h$; where h = 5 and $A = \frac{1}{2}b^2\sqrt{3}$

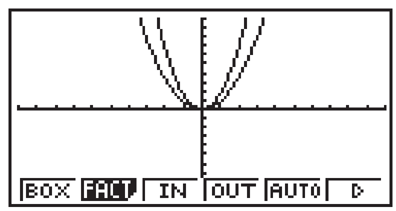
Steps for using the GRAPH Menu:



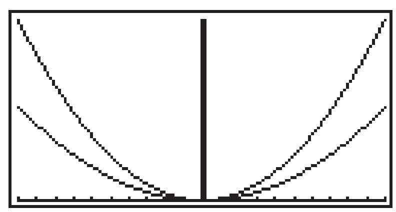
1. Press [MENU] to access the Main Menu, and then press [3] to select the GRAPH Icon.
2. Enter the first formula into **Y1:** by inputting: [1] [a b/c] [3] [X,θ,T] [x²] [×] [5] [EXE].
3. Enter the next formula into **Y2:** by inputting: [1] [a b/c] [2] [X,θ,T] [x²] [SHIFT] [x²] [3] [EXE].
4. Notice the equal signs are highlighted, this indicates they are selected to be graphed.
5. Press [F6], both of the graphs will display.



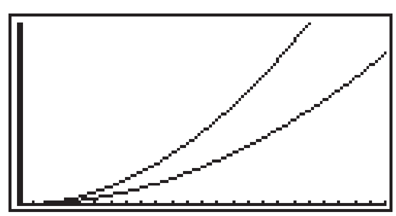
Steps for changing the size of the View Window:



1. If you are unable to view the graph, or if the graph appears too small; pressing [SHIFT] [F2] (Zoom) and [F5] (AUTO), will automatically fit the graph to the viewing window.

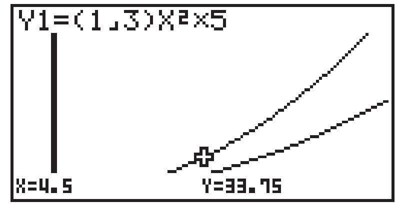


2. To change the view window so that only the first quadrant shows; press [F3] (V-Window) and enter [(−)] [1] [EXE] for Xmin, [▼] twice and enter [(−)] [1] [EXE] for Ymin.
4. Press [EXE] twice to display the graph.



Using the Trace Function:

1. Press SHIFT F1 (Trace). A small + cursor will be displayed on the graph and the x- and y-coordinates will be displayed along the bottom of the screen.



3. Press ◀ ▶ to move the cursor along the graph and ▲ ▼ to change the graph being traced.

When a building or property is burning, people call upon the fire department to put out the fire. These firefighters are a group of highly trained personnel, each having a specific duty during the incident. One job is the Driver/Operator. The Driver/Operator is responsible for initiating and maintaining the correct water pressure so that the proper amount of water is delivered to the firefighters operating the nozzle. In this activity, you will be asked to calculate the gallons per minute (GPM) for specific nozzle diameters, the amount of friction loss (FL) in pounds per square inch (psi), the nozzle reaction (NR), and the pump discharge pressure (PDP). These are all basic calculations skills that are required when a firefighter is trained as a Driver/Operator.

In this activity, you will find the NR for a smoothbore nozzle operating at both 50psi and 80 psi. This activity also helps find the GPM for both a 50 psi and 80 psi smooth bore nozzle. You will also calculate the FL for a given problem.

Questions

Nozzle reaction is the force pushing back against a nozzle when water is forced through a nozzle tip. One such nozzle used in firefighting is called a smoothbore which may be handheld or attached to a mechanical device.

1. Handheld smoothbore nozzles generally operate at 50 psi at the nozzle (NP). Find the nozzle reaction of the nozzle with a ½” diameter.

   ____

2. Find the NR of the nozzle with a ¾” diameter.

   ____

3. Find the NR of the nozzle with a 1” diameter.

   ____

4. Find the NR of the nozzle with a 1 ¼” diameter.

   ____

5. Smoothbore nozzles that create master streams generally operate at 80 psi at the nozzle (NP). Find the nozzle reaction of the nozzle with a 1 ½” diameter.

   ____

6. Find the NR of the nozzle with a 2” diameter.

   ____

7. Find the NR of the nozzle with a 2 ½” diameter.

8. Find the NR of the nozzle with a 3” diameter.

9. Look at the graph used for questions 1 – 4. What is the shape of the graph?

10. What does the graph tell you about the amount of nozzle reaction as the diameter increases?

11. How does the graph for questions 1 – 4 compare to the graph used for questions 5 – 8?

12. How does this help to explain why the larger diameter nozzles are attached mechanically to a surface?

The amount of water used varies due to the diameter of the nozzle tip.

13. Calculate the amount of water discharged at 50 psi through a ½’’ diameter nozzle tip.

14. Calculate the amount of water discharged at 50 psi through a ¾’’ diameter nozzle tip.

15. Calculate the amount of water discharged at 50 psi through a 1” diameter nozzle tip.

16. Calculate the amount of water discharged at 50 psi through a 1 ¼” diameter nozzle tip.

17. Calculate the amount of water discharged at 80 psi through a 1 ½'' diameter nozzle tip.

____

18. Calculate the amount of water discharged at 80 psi through a 2'' diameter nozzle tip.

____

19. Calculate the amount of water discharged at 80 psi through a 2 ½'' diameter nozzle tip.

____

20. Calculate the amount of water discharged at 80 psi through a 3'' diameter nozzle tip.

____

21. Compare the graph used in questions 13 – 16 and the graph used in questions 17 – 20. What do these graphs show about the amount of water that is being expelled through the nozzle?

____

Friction loss (FL) is based on the flow rate, hose length, and friction loss coefficient associated with the diameter of the hose. A $1\frac{3}{4}$" hose has a friction loss coefficient of 15.5 and a $2\frac{1}{2}$" hose has a friction loss coefficient of 2.

22. Calculate the FL for a 1 ¾ " hose with a length of 100 feet using 100 GPM.

____

23. Calculate the FL for a 1 ¾ " hose with a length of 200 feet using 100 GPM.

____

24. Calculate the FL for a 2 ½ " hose with a length of 200 feet using 250 GPM.

____

25. Calculate the FL for a 2 ½ " hose with a length of 500 feet using 250 GPM.

____

Extensions

1. Pump Discharge Pressure (PDP) is the "actual velocity pressure of water as it leaves the pump and enters the hose line" and is found using the formula:

   PDP = NP + FL.

   Using the results from question 22; find the PDP for a 100 foot hose using a nozzle pressure of 50 psi.

   ____

2. Using the results from question 23; find the PDP for a 200 foot hose using a nozzle pressure of 50 psi.

   ____

3. Using the results from question 24; find the PDP for a 200 foot hose using a nozzle pressure of 80 psi.

   ____

4. Using the results from question 25; find the PDP for a 500 foot hose using a nozzle pressure of 80 psi.

   ____

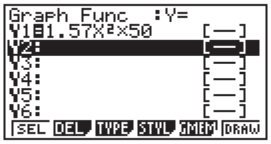
5. Using the results from question 26; find the PDP for a 1000 foot hose using a nozzle pressure of 80 psi.

   ____

Solutions

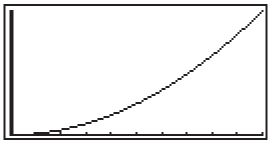
1. 19.6 lbs

2. 44.2 lbs



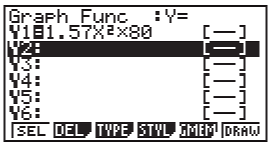
3. 78.5 lbs

4. 122.71 lbs



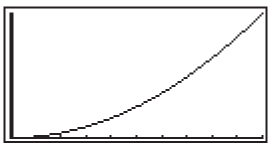
5. 282.6 lbs

6. 502.4 lbs



7. 785 lbs

8. 1130.4 lbs



9. Parabolic

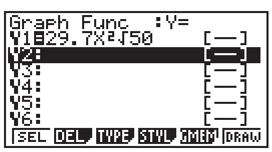
10. As the diameter increases, the amount of nozzle reaction increases rapidly.

11. The second graph is narrower than the first; increases faster.

12. Answers will vary based on student experience. Students should note that the force of water through larger diameter nozzles is too much for handheld control.

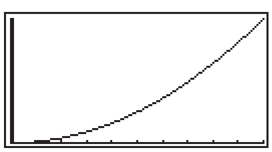
13. 52.5 gpm

14. 118.1 gpm



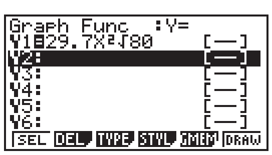
15. 210 gpm

16. 643.2 gpm



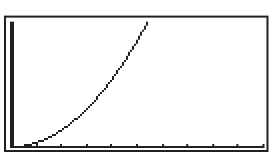
17. 597.7 gpm

18. 1062.57 gpm



19. 1660.3 gpm

20. 2390.8 gpm



21. As the nozzle diameter increases, the amount of water rapidly increases.

22. 15.5 psi

23. 300 psi

24. 25 psi

25. 62.5 psi

26. 125 psi

Extensions

1. 65.5 psi

2. 81 psi

3. 105 psi

4. 142.5 psi

5. 205 psi

Topic Area: Logarithms

NCTM Standards:

- Recognize and apply mathematics in contexts outside of mathematics.
- Use a variety of symbolic representations, including recursive and parametric equations, for functions and relations.
- Use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts.

Objective

Given two formulas for finding the magnitude of an earthquake, the student will be able to determine the intensity (energy) of an earthquake and its magnitude.

Getting Started

As a class, discuss what causes an earthquake and how the area around the earthquake is affected. Discuss the need for studying earthquakes. What industries would be affected by these studies? How would this benefit populations facing future earthquakes?

Prior to using this activity:

- Students should have a basic understanding of the properties of logarithms.
- Students should be able to enter and graph equations using a graphing calculator.
- Students should understand how to work with scientific notation.

Ways students can provide evidence of learning:

- Given the magnitude of an earthquake, the student will be able to determine its intensity and compare this with other energy levels.
- Given the intensity of an earthquake, the student will be able to determine the magnitude and types of damage associated with the earthquake.

Common mistakes to be on the lookout for:

- Students may enter the formula into the graph function of a graphing calculator incorrectly.
- Students may incorrectly write large numbers in scientific notation.

Definitions

- Richter Scale
- Intensity
- Seismic Moment
- Moment Magnitude

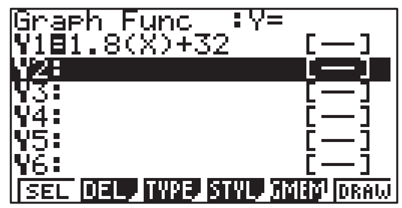
The following will demonstrate how to enter a formula into the Casio *fx-9750GII*, find a value for *y* given its corresponding x-value, and find the value for *x* given its corresponding y-value.

From a graph of the formula F = 1.8C + 32, find the temperature in degrees Celsius for a temperature of 75^0 Fahrenheit and the temperature in degrees Fahrenheit for a temperature of 18^0 Celsius.

To enter a formula into the Graph Function:

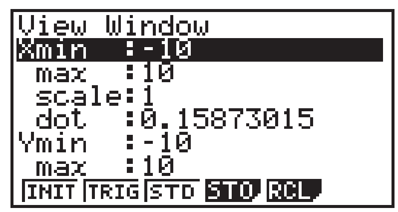
1. From the main icon menu, highlight the GRAPH icon and press [EXE] or [5].

2. Select the graph type by pressing [F3] [F1] for Y=.
   Enter the formula into the calculator by inputting:
   [1] [•] [8] [(] [X,θ,T] [)] [+] [3] [2] [EXE]
   The screen should look like the one at the right.

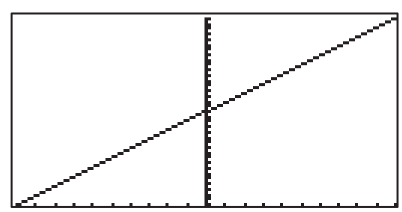


To set up the values for the window:

1. Press [SHIFT] [F3] (V-Window) then [F3] (STD) for a standard 10 by 10 grid. Press [EXE] twice to see the graph.



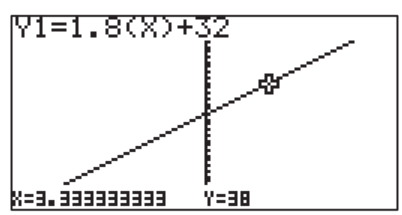
2. If you can not see the graph, press [F2] (Zoom) and [F5] (Auto) to see the graph as seen at the right.



To trace a graph and locate values:

1. With the graph showing, press [F1] (Trace) and Use the [◀] [▶] to move along the graph.

2. If there is more than one graph, use the [▲] [▼] to move between the graphs. The graph at the right shows the value of *y* when x = 3.33.



Natural changes in the Earth occur all over the world and can bring devastation to large areas. One such disaster is an earthquake. Caused by the shifting of layers of rock below the Earth's surface, these changes may or may not be felt. However, those carrying a tremendous amount of force can cause cracks in the Earth's surface resulting in collapsed structures and loss of lives. Some even can trigger other disasters such as tsunamis.

In this activity, you will explore the relationship between the intensity and magnitude of earthquakes as measured on the Richter scale and the Moment Magnitude scale. The Richter scale was developed to measure the seismic magnitude and energy of an earthquake. This was acceptable for smaller earthquakes. Later a formula to calculate moment magnitude scale was developed to measure the area of damage created by the earthquake. This scale provided a better picture of larger earthquakes. By understanding the magnitude and intensity, scientists and engineers can work together to build safer structures and possibly lessen the effect of an earthquake in populated areas.

Questions

Using Seismic Magnitude: $M_e = \frac{2}{3}\log E - 2.9$

1. In 1906, San Francisco experienced an earthquake with a magnitude (Me) of 8.3 on the Richter scale. What was the intensity of this earthquake?

   ____

2. In 2007, San Francisco experienced another earthquake with a magnitude of 5.6 on the Richter scale. What was the intensity of this earthquake?

   ____

3. How many times more intense was the earthquake in 1906 than the one in 2007?

   ____

4. One of the largest earthquakes in the world occurred in Prince William Sound, Alaska in 1964. The magnitude of the earthquake measured 8.4 on the Richter scale. What was its intensity?

   ____

5. An earthquake in Canada had an intensity that was one-fourth the intensity of the Alaskan earthquake. What was the magnitude of the Canadian earthquake?

____

Using the Moment Magnitude: $M_w = \frac{2}{3}\log M_o - 10.7$

1. Seismic moment is the measure of an earthquake according to its size rather that its energy. What is the seismic moment of the 1906 San Francisco earthquake if its moment magnitude (M_w) is 7.7?

____

2. The Alaskan earthquake of 1964 had a moment magnitude of 9.2. How much larger was this seismic moment than the one in San Francisco?

____

3. If an earthquake occurring in Peru has a seismic moment (M_o) of 5.4 and another earthquake occurring in Indonesia has a seismic moment of 6.2; what would be the difference in the magnitudes of these earthquakes?

____

4. If the seismic moment of one earthquake is 20 times that of a second, what is the difference in their magnitudes?

____

Extensions

1. One of the most active parts of the world for earthquakes is China. Make a table comparing the year with the magnitude of the earthquake for the last 10 years. Graph the results using the statistics function of a graphing calculator. What can you conclude from the graph of the data?

____

____

2. Research other magnitude scales used in measuring earthquakes. Select one of these scales and discuss what it measures. Give any formulas that are used to determine the magnitude using this scale.

____

____

3. Compare the magnitude of the Alaskan earthquake with that of a nuclear blast. What is equivalent magnitude of an earthquake compared to that of a nuclear explosion?

____

____

Solutions

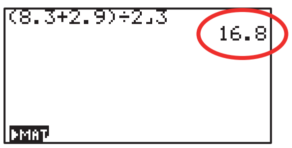
Using Seismic Magnitude: $M_e = \frac{2}{3}\log E - 2.9$

1. $M_e = 8.3$

   $8.3 = \frac{2}{3}\log E - 2\text{-}9$

   $16.8 = \log E$

   $E = 10^{16.8}$ ergs

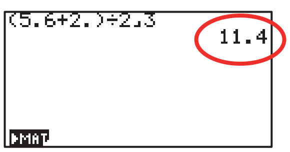
   

2. $M_e = 5.6$

   $5.6 = \frac{2}{3}\log E - 2\text{-}9$

   $11.4 = \log E$

   $E = 10^{11.4}$ ergs

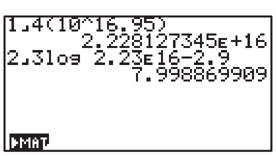
3. $10^{16.8} / 10^{11.4} = 10^{4.5}$

4. $M_e = 8.4$

   $8.4 = \frac{2}{3}\log E - 2\text{-}9$

   $16.95 = \log E$

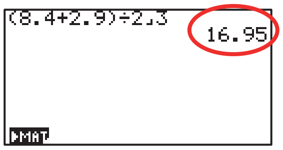
   $E = 10^{16.95}$ ergs.

5. $M_e = \frac{2}{3}\log 2.23E16 - 2.9$

$E = \frac{1}{4}(10^{16.95})$

$E = 8.0$

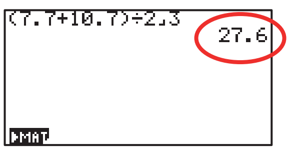


Using the Moment Magnitude: $M_w = \frac{2}{3}\log M_o - 10.7$

1. $M_w = 7.7$

$7.7 = \frac{2}{3}\log M_o - 10.7$

$M_o = 10^{27.6}$

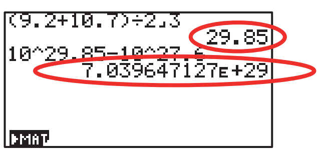


2. $M_w = 9.2$

$9.2 = \frac{2}{3}\log M_o - 10.7$

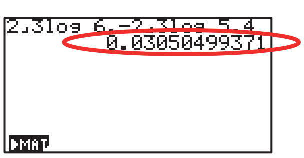
$M_o = 10^{29.85}$

$M_A - M_{SF} = 10^{29.85} - 10^{27.6} = 7.04\,E29$



3. $M_I - M_P = \frac{2}{3}\log 6.2 - 10.7 - \frac{2}{3}\log 5.4 - 10.7$

$= 0.031$



Extensions

1. Answers will vary.
2. Answers will vary.
3. 90.7 gigatons

Germs, Germs, Everywhere — Teacher Notes

Topic: Patterns and Functions- Algebraic Thinking

NCTM Standard:

- Understand patterns, relations, and functions by interpreting representations of functions of two variables and use symbolic algebra to represent and explain mathematical relationships.

Objective:

- The student will be able to use the Casio *fx-9750GII* to take a set of formulas, graph them, and find solutions, specifically to half-life.

Getting Started:

Discuss the Law of Growth and Decay with the students, including where it is used and what information it gives. Demonstrate how to find the value of the constraint, k, and write an equation to represent the Law of Decay.

Prior to using this activity:

- Students should be able to enter an equation, using e, into the calculator.
- Students should be able to set up the view window and calculate x- and y-values using the calculator.

Ways students can provide evidence of learning:

- If given an equation, students can determine if it is an equation that represents the Law of Growth or Law of Decay.
- If given an equation for Law of Growth or Decay, students can explain their findings and discuss what is happening.
- If given an equation, students will be able to interpret the graph and give alternative methods for calculating x and y.

Common mistakes to be on the lookout for:

- Students may have trouble setting up the window to the desired parameters.

Definitions

- Law of Growth
- Law of Decay
- Constant
- Natural Constant

Formula

Law of Growth and Decay: $A = A_0e^{kt}$

The following will demonstrate how to take a formula, graph it, and calculate a specific value on the Casio *fx-9750GII*.

Formula: $A = A_0e^{kt}$

A_0: Initial amount
A: Ending amount
e: Natural constant
k: Rate of growth or decay
t: Time

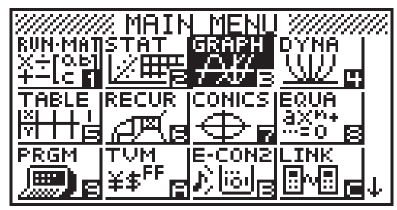
If $A_0 = 500$, $A = 250$, and $t = 4$, find k.

$250 = 500e^{4k}$
$0.5 = e^{4k}$
$\ln 0.5 = 4k$
$k = -.173$

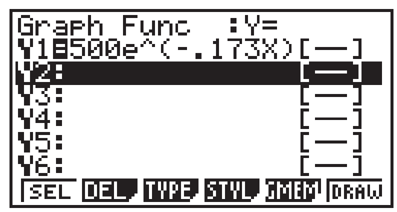
If $A = 250e^{-.173t}$, solve for A when t = 3. Solve for t, when A = 50.

To graph the equation:

1. From the main menu, highlight the GRAPH icon and press [EXE] or press [3].



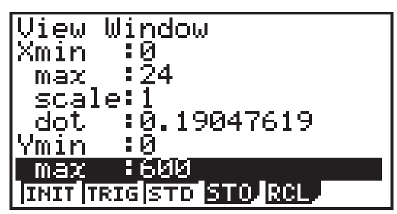
2. Enter the formula into Y1: by inputting: [5] [0] [0] [SHIFT] [ln] [(] [(−)] [•] [1] [7] [3] [X,θ,T] [)] [EXE].



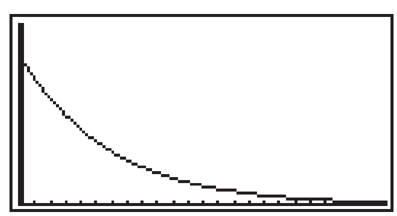
To change the viewing window:

1. Press [SHIFT] [F3] (V-Window).

2. Enter [0] [EXE] for the Xmin and [2] [4] [EXE] for the Xmax. Enter [0] [EXE] for the Ymin and [6] [0] [0] [EXE] for the Ymax.

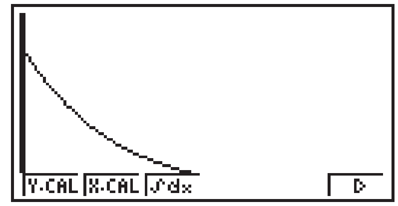


3. Press [EXIT] and then [F6] to view the graph.

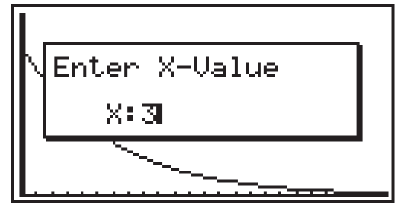


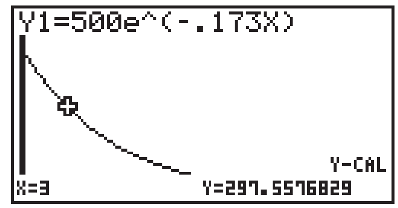
To calculate a y-value:

1. Press [SHIFT] [F5] (G-Solv). To calculate a value for *y* given "A" (x-value), press [F6] (▷) for more options.



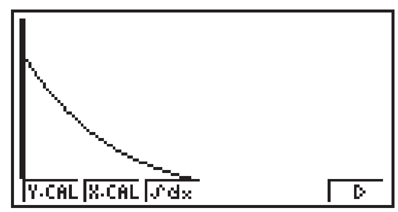
2. Select [F1] for Y-Cal, to calculate the x-value. Enter [3] [EXE] for the x-value. The equation displays in the upper left corner, the values of *x* and *y* are displayed along the bottom along with the calculation performed .



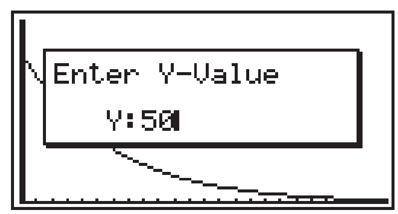


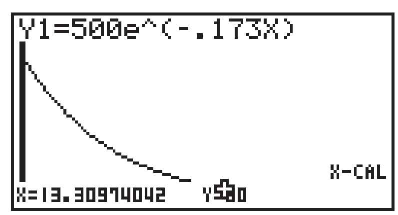
To calculate an x-value:

1. Press [SHIFT] [F5] (G-Solv). Press [F6] for more options.



2. Press [F2] for X-Cal, to calculate the value of "t" (y-value). Enter [5] [0] [EXE] for the y-value.





Medications are given to patients in a variety of forms including liquids, solids, and sprays. The form of the medication is directly related to the absorption rate by the body and how much of medicine actually gets into the system. Once in the system, the medication is metabolized within the system and then excreted from the body. In this activity, you will determine the amount of medication that is actually used by the body and how much is left after a certain amount of time.

In this activity, you will write an exponential equation to represent the Law of Decay for a given medication and calculate the amount of medication in the body's system after a given amount of time.

Reminder: $A = A_0e^{kt}$

Questions

1. Pharmaceutical Company A is marketing a medication with a half-life of 2 hours. The medication comes in 200 mg tablets. Calculate the value of the constant for the rate of decay for this medication.

   ____

2. The specified adult dose is 600 mg every 6 hours. Write an equation to represent the amount of medication in the blood stream at a given time.

   ____

3. Find the amount of medication that is remaining in the system after 1 hour.

   ____

4. Find the amount of medication that is remaining in the system after 8 hours.

   ____

5. In order to be effective, at least 10% of the medication must be in the system. What is the least amount of medication that must be in the system in order to be effective?

   ____

6. After how many hours will the system contain this amount?

7. Pharmaceutical Company B is marketing a medication with a half-life of 3 hours. The medication comes in 400 mg tablets. Calculate the value of the constant for the rate of decay for this medication.

8. The specified adult dose is 400 mg every 8 hours. Write an equation to represent this situation.

9. Find the amount of medication that is remaining in the system after 1 hour.

10. Find the amount of medication that is remaining in the system after 8 hours.

11. In order to be effective, at least 20% of the medication must be in the system. What is the least amount of medication that must be in the system in order to be effective?

12. After how many hours will the system contain this amount?

Extensions

1. Which medication do you think would be more effective? Explain your answer.

Solutions

1. $100 = 200e^{k(2)}$

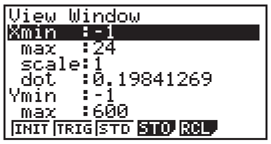
   $0.5 = e^{k(2)}$

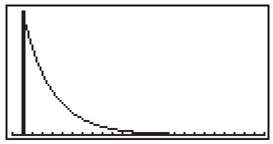
   $\ln 0.5 = 2k$

   $k = \dfrac{\ln .5}{2}$

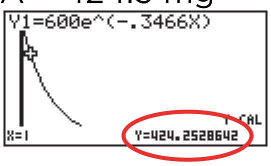
   $k = -0.3466$

2. $A = 600e^{-0.3466t}$

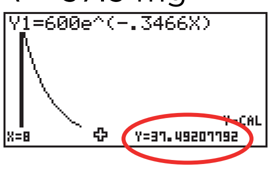




3. A = 424.3 mg

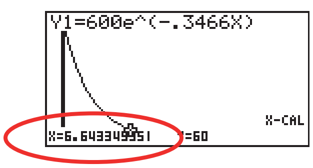


4. A = 37.5 mg



5. 600 x .1 = 60 mg

6. 6.6 hours



7. $200 = 400e^{3k}$

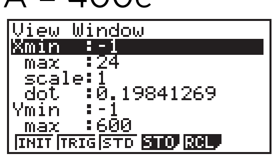
   $0.5 = e^{3k}$

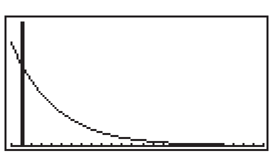
   $\ln 0.5 = 3k$

   $k = \dfrac{\ln .5}{3}$

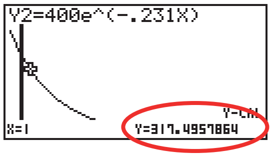
   $k = -0.2310$

8. $A = 400e^{-0.2310t}$

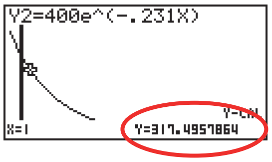




9. A = 317.5 mg

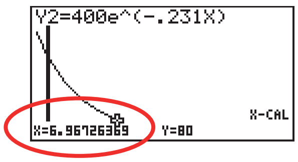


10. A = 63.0 mg



11. 400 x .2 = 80 mg

12. 7.0 hours



Extensions

1. Answers will vary.

Planning for the Future — Teacher Notes

Topic Area: Sequences and Series

NCTM Standards:

- Use a variety of symbolic representations, including recursive and parametric equations, for functions and relations.
- Use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts.

Objective

The student will be able to write a recursive function, graph the function using a graphing calculator and solve problems based on the graph of the recursive function.

Getting Started

Have the students work in pairs or in small groups to research the current retirement age in order to collect social security payments. Students should research and discuss various investment options and the corresponding annual interest rates. After completing the research and discussion, students should determine how much money they feel would be needed to supplement their retirement income per month.

Prior to using this activity:

- Students should have an understanding of sequences and series.
- Students should have an understanding of the compound interest formula.

Ways students can provide evidence of learning:

- The student will be able to create an equation of a sequence for a given situation.
- The student will be able to graph the equation and solve problems using the information derived from the graph.

Common mistakes to be on the lookout for:

- Students may use annual interest instead of monthly interest.
- Students may confuse the formulas for simple interest and compound interest.

Definitions:

- Annual Percentage Rate (APR)
- Compound Interest
- Continuous Interest

Formulas: Compound Interest: $A_1 = A_0\left(1+\frac{r}{n}\right)^{nt}$

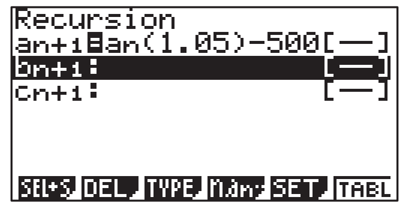
Continuous Interest: $A_1 = A_0e^{nt}$

The following will demonstrate how to enter an equation into the Recursion Function of the Casio *fx-9750GII* and graph the resulting equation.

Enter the formula $A_{n+1} = A_n(1.5) - 200$ and set up the parameters.

Graph the formula using the calculator and find the x- and y-intercepts using the Trace Function.

To enter a formula into the Recursive Function:

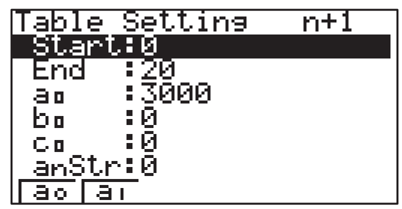


1. From the main icon menu highlight the RECUR icon and press [EXE]. Press [F4] (n a_n), then [F2] (a_n).

   Now enter:

   [(] [1] [•] [0] [5] [)] [−] [5] [0] [0] [EXE].

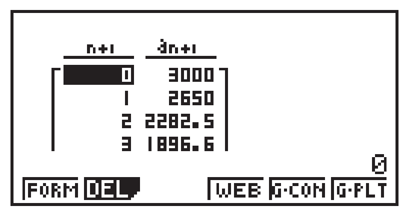
   The screen should look like the one at the right.



2. To set up the parameters for the formula, press [F5] (Set) then [F1] (a_0). To start the set up for the formula, enter the following:

   [0] [EXE] [2] [0] [EXE] [3] [0] [0] [0] [EXE].

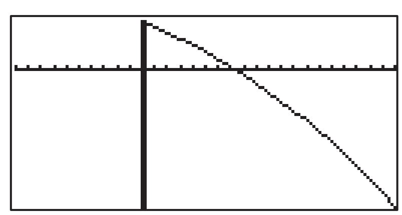
   The screen should look like the one at the right



3. Press [EXE] [F6] (TABL) to see the table of values.

To graph the recursive function:

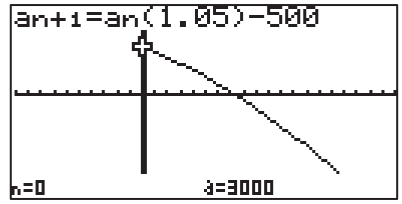
1. Press [F5] [F2] (Zoom), then [F5] (Auto) to see the graph for the table of values. The screen should look like the one at the right.



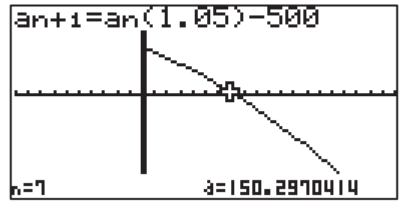
2. The [◀] [▶] [▲] [▼] keys can be used to shift the graph if need be.

To trace the graph:

1. To trace the graph, use F1 (Trace) and the arrow keys to move along the graph.



2. Move the cursor until the bottom screen display shows n = 0; this is the beginning amount.
   Now move the cursor so that n = 7. This is the last positive amount, therefore a_{n+1} will equal 0 somewhere between n = 7 and n = 8.
   See the screen shots to the right.



When you are young and just starting out on your own, you may not be thinking about retirement. However, this is the perfect time to start thinking about how much money it will take to maintain your lifestyle. Investing in an annuity is a wise decision but just how long will it last? Investing just $2,400 each year compounded continuously for 25 years at 12% APR yields approximately $88,000. Would this be enough?

In this activity, you will discover how long it would take for a balance in an annuity to reach $0 if the rate of interest for the annuity is 7.5% APR compounded monthly and $3,000 is withdrawn each month starting with the first month of retirement. You will then determine the amount that would have to be invested at 10% APR to achieve that beginning balance if interest is compounded continuously.

Questions

1. Given that an investment earns 10% compounded continuously, how much per year would one need to invest to earn $125,000 in 45 years?

   ____

   How much would be needed to invest per month?

   ____

2. Using the same investment plan, how much would one need to invest per year to earn $250,000?

   ____

   How much would be needed to invest per month?

   ____

3. How much per year would have to be saved to earn $350,000?

   ____

   How much would need to be saved per month?

   ____

4. Using the formula for compound interest, $A_1 = A_0\left(1+\frac{r}{n}\right)^{nt}$, and an APR of 7.5%, calculate the value of $\left(1+\frac{r}{n}\right)^{nt}$ if the number of interest periods, *n*, is 12 and the *t* is 1 month. (Hint: What part of a year is 1 month?)

____

5. Write a recursive equation that shows an annuity that earns 7.5% APR and includes a withdraw of $3,000 each month.

____

6. Given an annuity that has a balance of $125,000, graph the equation in Question 3 and find the number of months until the balance is near or at $0.

____

How many years does this represent?

____

7. If the same annuity has a balance of $250,000, graph the equation in Question 3 and find the number months until the balance is near or at $0.

____

How many years does this represent?

____

8. If the same annuity has a balance of $350,000, graph the equation in Question 3 and find the number of years until the balance is near or at $0.

____

9. If your monthly income were $1,400, which of the above investment amounts would be the best choice? Explain your answer choice.

____

____

10. If your monthly income were $3,300, which of the above investment amounts would be the best choice? Explain your answer choice.

____

____

11. Would it be a good idea to depend on an annuity as the only source of income for retirement? Justify your answer.

____

____

12. How does the number of years needed to accumulate the different balances relate to the number of years for available withdrawal?

____

____

Extensions

1. Using the same interest rates given in the activity, determine the amount of an annuity at which the annuity would start to gain value, even though money is being withdrawn.

____

2. Draw a graph to compare the initial annuity balances and the number of years they would be available for withdrawal. The average life expectancy is approximately 78 years or 13 years past retirement. Using the graph, determine the balance needed in an annuity in order to withdraw $3,000 a month of supplemental income from the account for at least 13 years.

____

3. What factors would affect one's ability to invest money in an annuity?

____

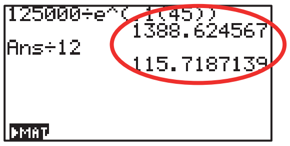
4. If it investment is not used as a supplemental retirement income, what other reasons might someone have an annuity?

____

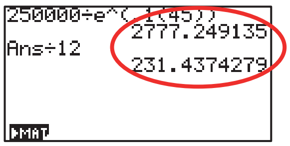
____

Solutions

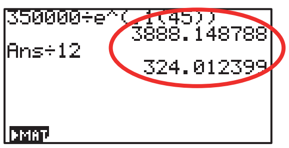
1. Yearly: $1,388.62 Monthly: $115.72



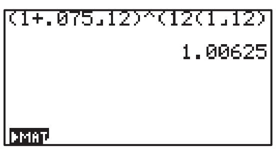
2. Yearly: $2,777.25 Monthly: $231.44



3. Yearly: $3,888.15 Monthly: $324.01

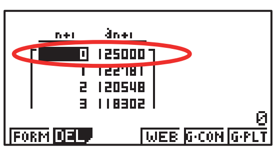


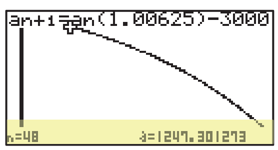
4. 1.00625



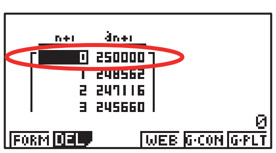
5. $A_{n+1} = A_n\,(1.00625) - 3000$

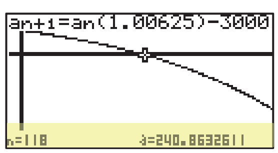
6. 48 months; 4 years



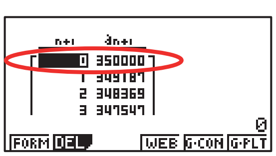


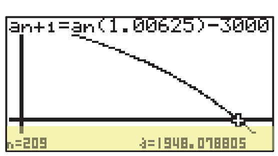
7. 118 months; 9 years 10 months





8. 17 years 5 months





9. Answers will vary according to experience.

10. Answers will vary according to experience.

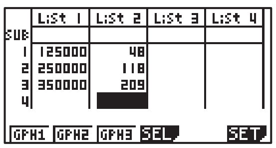
11. Answers will vary according to experience.

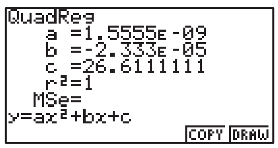
12. Answers will vary according to experience.

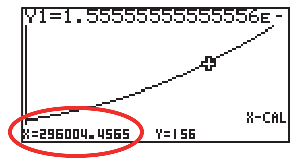
Extensions

1. At approximately $500,000, the annuity would begin gaining value even though money is being withdrawn.

2. $296,004.46





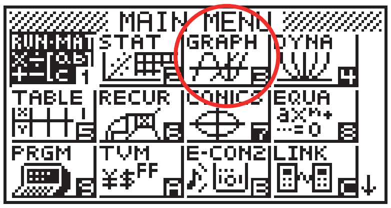


3. Answers will vary according to experience.

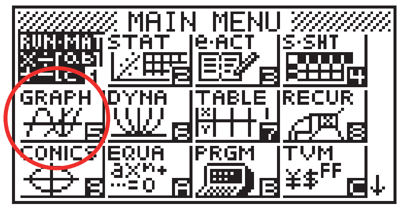
4. Answers will vary according to experience.

The *fx-9860GII* calculator is similar to the *fx-9750GII* and can be used for any activity in this workbook. Although the fx-*9860GII* uses the same Icon-based Main Menu, there are additional icons and the icons appear in a different order. For example, on the fx-*9750GII*, the Graph icon is the third icon and it's the fifth icon on the fx-*9860GII*.

fx-*9750GII* Icon-based Main Menu

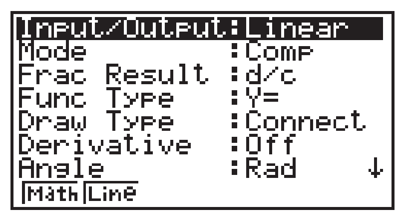


fx-*9860GII* Icon-based Main Menu



In addition to the difference in icons, the fx-*9860GII* offers a "Math" input mode in addition to the "Linear" mode used in the fx-*9750GII*. The Math mode allows for natural input and display of certain functions so that they are displayed in the calculator as they would in a textbook. Here are some basic examples of how to utilize the Math mode's natural display.

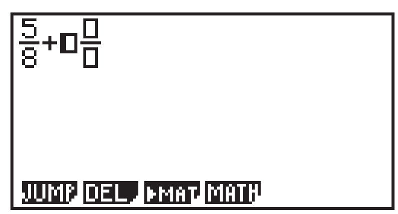
1. To access the "Math" mode, make sure you are in the RUN•MAT module and press: [SHIFT] [MENU] [F1] [EXIT].

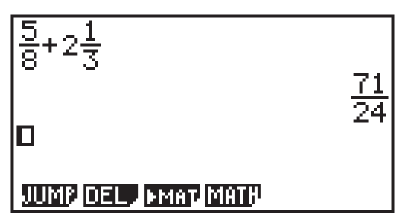


2. Press the desired function first (fractions, summations, square roots, absolute value, etc) and then fill in the displayed template.

   To solve the problem $\frac{5}{8}+2\frac{1}{3}$, input the following:

   [a b/c] [5] [▼] [8] [▶] [+] [SHIFT] [a b/c] [2] [▶] [1] [▼] [3] [EXE]

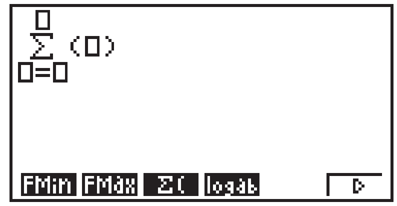


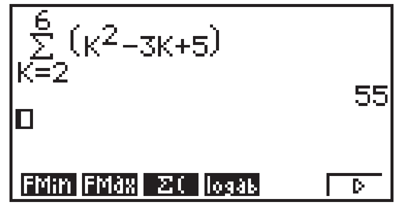


3. To calculate $\sum_{k=2}^{6}(k^2-3k+5)$; input the following:

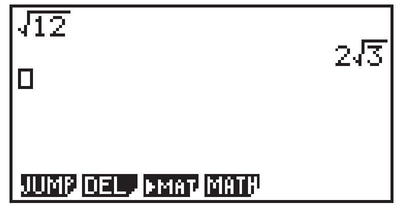
[OPTN] [F4] [F6] [F3] [ALPHA] [,] [x^2] [3] [ALPHA] [,] [+] [5] [▶]

[ALPHA] [,] [▶] [2] [▲] [6] [EXE]



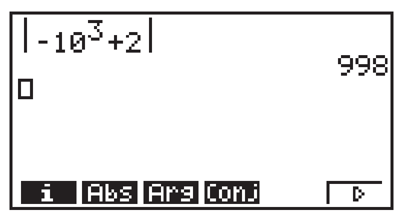


4. When calculating square roots, the answer will be displayed as the simplified form instead of the decimal form.



5. To calculate $\left|-10^3+2\right|$, input the following from the Run home screen:

[OPTN] [F3] [F2] [(–)] [1] [0] [^] [3] [▶] [+] [2] [EXE]



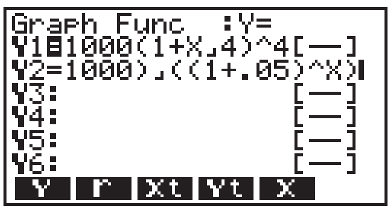
The following will show the standard Linear Mode display for the screen shots in the "How To" sections of this workbook along with the corresponding screen shots utilizing the Math Mode of the fx-*9860GII*.

Activity 1 – Thought It Was Fixed

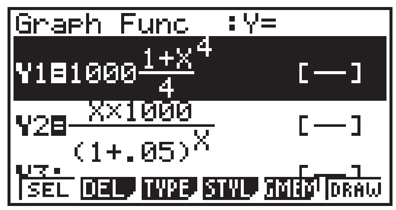
This activity uses the TVM and STAT menu; there is no difference to note for this section.

Activity 2 – Saving For a Rainy Day

Linear Mode

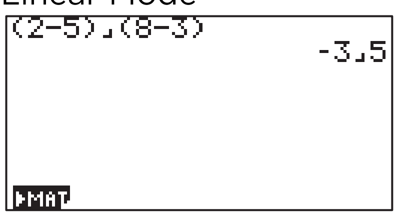


Math Mode



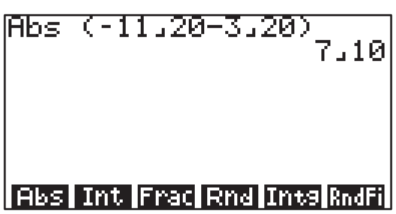
Activity 3 – Spotlight On Art

Linear Mode

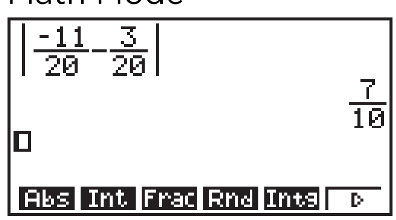


Math Mode

Linear Mode



Math Mode

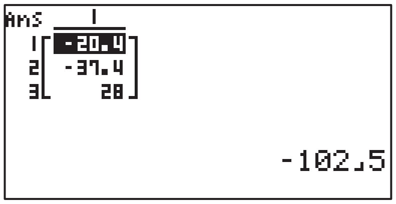


Activity 4 – Maximum Space For Minimum Money

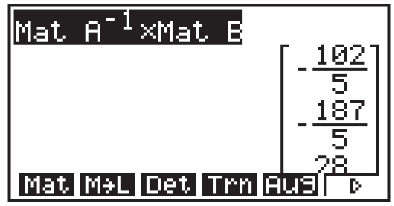
There are no differences to note for this section.

Activity 5 – What's For Dinner?

Linear Mode



Math Mode

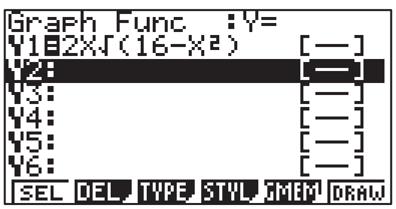


Activity 6 – What Happened to the Cost?

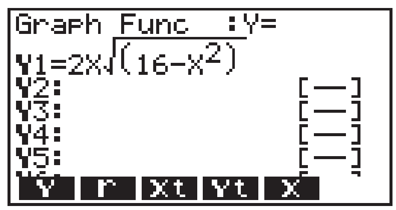
There are no differences to note for this section.

Activity 7 – Do You Have Tunnel Vision?

Linear Mode



Math Mode

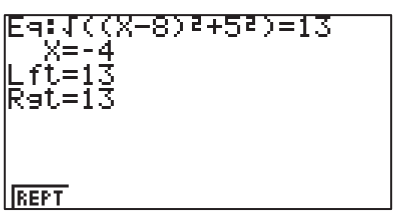


Activity 8 – Where Did That Come From?

There are no differences to note for this section.

Activity 9 – Out To Sea

Linear Mode



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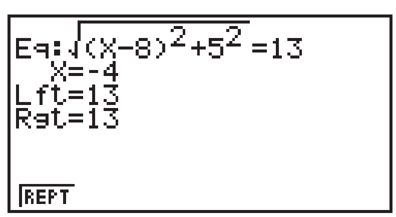
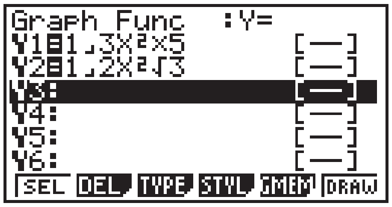
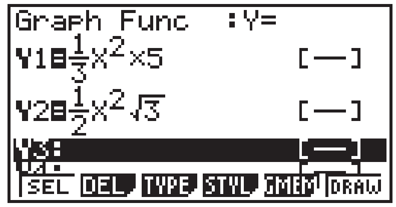


Activity 10 – Keeping Up The Pressure

Linear Mode



Math Mode

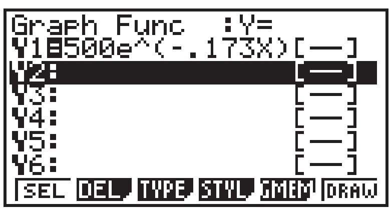


Activity 11 – That Shaky Feeling

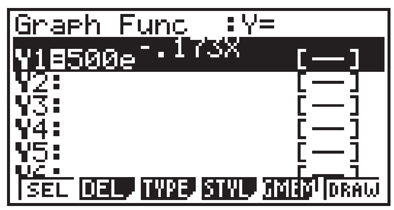
There are no differences to note for this section.

Activity 12 – Germs, Germs Everywhere

Linear Mode



Math Mode



Activity 13 – Planning For The Future

There are no differences to note for this section.