# Programming with TI-Nspire 

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## 1. Introduction to the Program Editor

Functions and Programs are found in the menu of the Calculator application. We will only be working with Programs in this introduction.

Let us construct our first simple program where we will simulate a throw of a dice. To get started choose New from the Program Editor in the menu.


In the pop-up window we are able to name our program Dice. We choose Program under Type and None under Library access (the possibilities are None, LibPriv, and LibPub). If we chose LibPub the program would be accessible in other documents from the Catalog. We will return to this later but will not be working with LibPriv in this introduction:


The page is split into two with a Calculator application to the left and a Program Editor to the right:


In the Program Editor we notice that the cursor is placed just behind the name of our program in the parenthesis. Our first program does not depend on any given input and we therefore jump to the line under Prgm. This is the first and only line, which is reflected in the top of the Program Editor: 1/1.

Here we define a local variable, $\boldsymbol{a}$, via Local from the menu Define Variables. Write, a, after Local and press enter to insert a new line:


Notice the star (*) in front of the name of the program. This reflects a change of the program since it was last stored. We choose Check syntax \& store from the menu Check syntax \& store and the star disappears. In the top to the right we now see that the cursor is placed in the second line out of two: $2 / 2$.

With the command randint $(\mathbf{1}, \mathbf{6})$ a random number between 1 and 6 is given. We can use this command to simulate a throw of a dice and the command is found in Catalog:


In the second line we define our local variable as $\boldsymbol{a}:=\boldsymbol{r a n d I n t}(\mathbf{1}, \mathbf{6})$ and press enter to insert a line:


Our program is almost finished. We just need to ensure that the program will display our output. In the third line we choose Disp from the menu: I/O and after the command Disp we write: "The dice shows the number", a. The syntax is checked and the program is stored. We are now ready to test it in the Calculator application:

| dice | $3 / 3$ |
| :--- | ---: |
| Define dice $(\hat{\\|}=$ |  |
| Prgm |  |
| Local $a$ |  |
| $a:=$ randInt $(1,6)$ |  |
| Disp "The dice shows the number",$a$ |  |
| EndPrgm |  |

In the calculator we write dice():

| dice() |  | dice | 3/3 |
| :---: | :---: | :---: | :---: |
|  | The dice shows the number 6 | $\begin{aligned} & \text { Define dice }()= \\ & \text { Prgm } \\ & \text { Local } a \\ & a:=\text { randInt }(1,6) \\ & \text { Disp "The dice shows the number", } a \\ & \text { EndPrgm } \end{aligned}$ |  |
|  | Done |  |  |
| I |  |  |  |
|  |  |  |

Press enter a couple of times to check that our simulation works. Our first program is only a trivial expansion of the command randInt():

| dice() |  |
| :---: | :---: |
|  | The dice shows the number 6 |
|  | Done |
| dice() |  |
|  | The dice shows the number 6 |
|  | Done |
| dice() |  |
|  | The dice shows the number 1 |
|  | Done |
| randint( 1,6 ) | 4 |
| randint( 1,6 ) | 3 |
| randint( 1,6 ) | 5 |

In the first line of the program we insert a comment by choosing Insert comment from the menu Actions. After the symbol © we write Simulating a throw with a dice as description of our program:

| dice |
| :--- |
| Define dice $(\\|=$ |
| Prgm |
| © Simulating a throw with a dice |
| Local $a$ |
| a:=randint $(1,6)$ |
| Disp "The dice shows the number", $a$ |
| EndPrgm |

For now we can only use the program in the current problem. Let us change this so that the program is accessible in every new TI-Nspire document from the Catalog. From the menu Actions we can select LibPub from Change library access:

| dice | $4 / 4$ |
| :--- | ---: |
| Define LibPub dice ()$=$ |  |
| Prgm |  |
| © Simulating a throw with a dice |  |
| Local $a$ |  |
| $a:=$ randInt $(1,6)$ |  |
| Disp "The dice shows the number", $a$ ) |  |
| EndPrgm |  |

We need to store the file in the folder Mylib in the directory: Documents > TI-Nspire > MyLib. Finally we must update the library by pressing the icon: (or using Refresh Libraries from the menu Tools).

If we now open a new document we should be able to access the program from the Catalog:


Notice our description of the program is included.

### 1.1 Exercise: Sum of the throw of two dices

Construct a program that simulates the throw of two dices. The program shall display the outcome of each dice and the sum of the dices.

## 2. Programming with "if... then... endif" and "elseif... then..."

Let us now construct a more interesting and relevant program. We will be working with second degree polynomial functions $f(x)=a \cdot x^{2}+b \cdot x+c$. The graph is called a parabola which can have up to two intersection points with the $x$-axis:


If we solve the second degree equation $0=a \cdot x^{2}+b \cdot x+c$ we are able to determine the intersection points. A solution to the equation is called a root. The number of roots can be determined by calculating the discriminant $d=b^{2}-4 \cdot a \cdot c$. If $d<0$ there are no roots, if $d=0$ there is a double root and if $d>0$ there are two distinct roots.

We will construct a program that determines the roots for any given second degree polynomial function. The program is called roots and we want to add it to the Catalog:

| roots | $2 / 2$ |
| :--- | ---: |
| Define LibPub roots $(a, b, c)=$ |  |
| Prgm |  |
| © The roots of a 2.nd degree polynomial function |  |
| EndPrgm |  |

Notice that the program as input depends on the coefficients $a, b$ and $c$.

The program will be constructed step vice. Let us first calculate the discriminant where we define a local variable $\boldsymbol{d}$ :

| roots |
| :--- |
| Define LibPub roots $(a, b, c)=$ |
| Prgm |
| © The roots of a 2.nd degree polynomial function |
| Local $d$ |
| $d:=b^{2}-4 \cdot a \cdot d$ |
| EndPrgm |

We now need an If-command to separate the three possible numbers of roots. We choose If...Then...endif from the menu. After If we write: $d>0$. In this case there are two distinct roots $\boldsymbol{r} \mathbf{1}$ and $\boldsymbol{r} \mathbf{2}$ which we define as local variables:

| roots | $6 / 6$ |
| :--- | ---: |
| Define LibPub roots $(\mid a, b, c)=$ |  |
| Prgm |  |
| © The roots of a 2.nd degree polynomial function |  |
| Local $d$ |  |
| $d:=b^{2}-4 \cdot a \cdot c$ |  |
| If $d>0$ Then |  |
| EndIf |  |
| EndPrgm |  |

The two roots are given as either $r 1=\frac{-b+\sqrt{d}}{2 a}$ or $r 2=\frac{-b-\sqrt{d}}{2 a}$ which will be calculated in the program. Finally we want the program to display the discriminant and the roots:

| roots $(1,-8,15)$ |  | roots | $9 / 10$ |
| :---: | :---: | :---: | :---: |
|  | The discriminant is $\mathrm{d}=4$ | Define LibPub roots $(a, b, c)=$ <br> Prgm <br> © The roots of a 2 .nd degree polynomial function <br> Local $d, r 1, r 2$ $d:=b^{2}-4 \cdot a \cdot c$ <br> If $d>0$ Then $\left\{\begin{array}{l} r 1:=\frac{-b-\sqrt{d}}{2 \cdot a} \\ r 2:=\frac{-b+\sqrt{d}}{2 \cdot a} \end{array}\right.$ <br> Disp "The discriminant is $\mathrm{d}={ }^{\prime}, d$ <br> Disp "The first root is $\mathrm{r} 1={ }^{\prime \prime}, r 1$ <br> Disp "The second root is $\mathrm{r} 2={ }^{\prime \prime}, r 2$ <br> EndIf <br> EndPrgm |  |
|  | The first root is r1 $=3$ |  |  |
|  | The first root is ri $=3$ |  |  |
|  | The second root |  |  |
|  | Done |  |  |
| $\left\lvert\, \begin{aligned} & \text { If } d>0 \text { Then } \\ & r 1:=\frac{-b-\sqrt{d}}{2 \cdot a} \\ & r 2:=\frac{-b+\sqrt{d}}{2 \cdot a} \end{aligned}\right.$ <br> Disp "The discriminant is $\mathrm{d}={ }^{\prime}, d$ <br> Disp "The first root is $\mathrm{r} 1={ }^{\prime \prime}, r 1$ <br> Disp "The second root is $\mathrm{r} 2={ }^{\prime \prime}, r 2$ <br> EndIf <br> EndPrgm |  |  |  |
|  |  |  |  |
|  |  |  |  |
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We add the case where $d=0$. In line 10 (just above Endif) we insert Elseif...then which can be chosen from the menu Control. After Elseif we write $d=0$ :

| roots |
| :--- |
| Define LibPub roots $(a, b, c)=$ |
| Prgm |
| © The roots of a 2.nd degree polynomial function |
| Local $d, r 1, r 2$ |
| $d:=b^{2}-4 \cdot a \cdot c$ |
| If $d>0$ Then |
| $r 1:=\frac{-b-\sqrt{d}}{2 \cdot a}$ |
| $r 2:=\frac{-b+\sqrt{d}}{2 \cdot a}$ |
| Disp "The discriminant is $\mathrm{d}={ }^{\prime \prime}, d$ |
| Disp "The first root is $\mathrm{r} 1={ }^{\prime}, r 1$ |
| Disp "The second root is $\mathrm{r} 2={ }^{\prime}, r 2$ |
| ElseIf $d=0$ Then |
| EndIf |
| EndPrgm |

As mentioned earlier there is a double root in this case and it is given by calculating $r=\frac{-b}{2 a}$ which is written into the program. The discriminant and the root are displayed while we must remember to add $\boldsymbol{r}$ to the list of local variables:

| roots $(1,-8,15)$ |  | roots | 13/14 |
| :---: | :---: | :---: | :---: |
|  | The discriminant is $\mathrm{d}=4$ | Define LibPub root $(a, b, c)=$ <br> Prgm <br> O The roots of a 2.nd degree polynomial function Local $d, r 1, r 2, r$ |  |
|  | The first root is $\mathrm{rl}=3$ |  |  |
|  | The second root is $\mathrm{r} 2=5$ |  |  |
|  | Done | $\left\lvert\, \begin{aligned} & d:=b^{2}-4 \cdot a \cdot c \\ & \text { Tf } \end{aligned}\right.$ |  |
| roots $(1,-8,16)$ |  | $\left\{\begin{array}{l} \text { If } d>0 \text { Then } \\ x,=-b-\sqrt{d} \end{array}\right.$ |  |
|  | The discriminant is $\mathrm{d}=0$ <br> The double root is $r=4$ | $\left\lvert\, \begin{aligned} & r_{1}^{1}=\frac{1}{2 \cdot a} \\ & 2:=\frac{-b+\sqrt{d}}{} \end{aligned}\right.$ |  |
|  |  |  |  |
|  | Done | (2isp "The discriminant is $\mathrm{d}=$ =,$d$ |  |
| I |  | Disp "The first root is $\mathrm{r} 1=\mathrm{=}, r 1$ <br> Disp "The second root is $\mathrm{r} 2={ }^{\prime \prime}, r^{2}$ |  |
|  |  |  |  |
|  |  | ${ }_{-\quad \text { Elseff }}^{-b} d=0$ Then |  |
|  |  | 2.a |  |
|  |  | Disp "The discriminant is $\mathrm{d}=", d$ Disp "The double root is $\mathrm{r}=\mathrm{N}, r$ |  |
|  |  | ${ }_{\text {Endif }}^{\text {Disp }}$ Ene double root is $\mathrm{r}=$ ", |  |
|  |  | EndPrgm |  |

Now, the last case where $d<0$. Again we add the command Elseif...then in a line above
Endif and we write $d<0$ after Elseif. We want the program to display the message that there are no roots together with the discriminant:

| roots $(1,-8,15)$ |  | roots | 16/17 |
| :---: | :---: | :---: | :---: |
|  | The discriminant is $\mathrm{d}=4$ | Define LibPub roots $(a, b, c)=$ |  |
|  | The first root is r1 $=3$ | Prgm |  |
|  | The second root is $r 2=5$ | The roots of a $2 . n$ degree polynomial function |  |
|  | - | $d:=b^{2}-4 \cdot a \cdot c$ |  |
|  | Done | If $d>0$ Then |  |
| roots ( $1,-8,16$ ) |  | $\\| \begin{aligned} & \text { If } d>0 \text { Then } \\ & r 1:=\frac{-b-\sqrt{d}}{} \end{aligned}$ |  |
| The discriminant is $\mathrm{d}=0$ |  | $\begin{gathered} 2 \cdot a \\ -b+\sqrt{d} \end{gathered}$ |  |
|  | The double root is $r=4$ |  |  |
|  | Done | Disp "The discriminant is $\mathrm{d}=11, d$ |  |
| $\operatorname{roots}(1,-8,17)$ |  | Disp "The discriminant is $\mathrm{d}={ }^{\prime \prime}, d$ <br> Disp "The first root is $\mathrm{r} 1={ }^{\prime}, r 1$ |  |
| The discriminant is $\mathrm{d}=-4$ |  | Disp "The second root is $\mathrm{r} 2={ }^{\prime}, r 2$ |  |
|  | There are no roots | ElseIf $d=0$ Then |  |
|  | Done |  |  |
|  |  | Disp "The discriminant is $\mathrm{d}={ }^{\prime \prime}, d$ |  |
| \| |  |  |  |
|  |  | ElseIf $d<0$ Then |  |
|  |  | Disp "The discriminant is $\mathrm{d}=", d$ |  |
|  |  | Disp "There are no roots" |  |
|  |  | EndPrgm |  |

### 2.1 Exercise: The minimum/maximum of the parabola

Add the determination of the minimum/maximum to the program roots.

### 2.2 Exercise: Heads or tails

Construct a program that simulates a flip of a coin $\boldsymbol{n}$ times. Input must be the number of times the coin is flipped. The program must display the different outcomes together with the number of both heads and tails.

## 3. Number theory with TI-Nspire

We will not cover the number theory needed to understand the affine cryptosystem but only discuss the use of TI-Nspire in this connection. This material can be used as supplement to a mathematical learning of the relevant number theory.

Greatest common divisor gcd is a built-in command in TI-Nspire:

| $\operatorname{gcd}(6,10)$ | 2 |
| :--- | ---: |
| factor $(10)$ | $2 \cdot 5$ |
| factor $(6)$ | $2 \cdot 3$ |
| $\operatorname{gcd}(6,35)$ | 1 |
| factor $(6)$ | $2 \cdot 3$ |
| factor $(35)$ | $5 \cdot 7$ |

We see that 2 is a factor in both $10=2 \cdot 5$ and $6=2 \cdot 3$ and it is the greatest common factor. We also see that the numbers 6 and 36 are relatively prime to each other as their greatest common divisor is the unity 1 .

By modulus calculation we see that 7 is a divisor in 35 as the remainder is zero and we see that the principal remainder of 67 divided by 8 is 3 :

| $\bmod (35,7)$ | 0 |
| :--- | :--- |
| $\bmod (67,8)$ | 3 |

In this way we can show that two numbers are congruent:

| $\bmod (57-1,8)$ | 0 |
| :--- | :--- |
| $\bmod (35-5,6)$ | 0 |

Here 57 is congruent with 1 modulus 8 as 8 is a divisor in $57-1$. This is written: $57 \equiv 1$ $(\bmod 8)$ as $8 \mid(57-1)$.

With a sequence we can determine the set of different remainders when dividing with 27 which typically is written $\mathbb{Z}_{27}$ :

```
\(\operatorname{seq}(\bmod (i, 27), i, 0,26)\)
    \(\{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26\}\)
```

Using Eulers formula we can determine the number of remainders that are prime to 27 in $\mathbb{Z}_{27}$ (this corresponds to the Euler $\varphi$-function $\varphi(n)=n \cdot\left(1-\frac{1}{p_{1}}\right) \cdot\left(1-\frac{1}{p_{2}}\right) \cdot \ldots \cdot\left(1-\frac{1}{p_{k}}\right)$, where $p_{1}, p_{2}, \ldots, p_{\mathrm{k}}$ are the different prime factors associated with the number $n$ ):

| factor $(27)$ | $3^{3}$ |
| :---: | :---: |
| $27 \cdot\left(1-\frac{1}{3}\right)$ | 18 |

First we make a factorization of the number 27 and see that the only distinct prime number is 3 . By calculation $\varphi(27)=18$.

Using Eulers theorem $a^{\varphi(n)} \equiv 1(\bmod n)$ provided $a$ and $n$ are relatively prime, the different inverse elements to the remainders that are relatively prime to 27 can be determined:

| $\operatorname{seq}(\bmod (i, 27), i, 0,26)$ |  |
| :--- | ---: |
| $\{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26\}$ |  |
| $\operatorname{seq}\left(\bmod \left(i^{18-1}, 27\right), i, 0,26\right)$ |  |
| $\{0,1,14,0,7,11,0,4,17,0,19,5,0,25,2,0,22,8,0,10,23,0,16,20,0,13,26\}$ |  |
| $\bmod \left(2^{18-1}, 27\right)$ | 14 |
| $\bmod (14 \cdot 2,27)$ | 1 |
| $\bmod \left(4^{18-1}, 27\right)$ | 7 |
| $\bmod (7 \cdot 4,27)$ | 1 |

We see that 14 is inverse element to 2 modulus 27 because $14 \cdot 2 \equiv 1(\bmod 27)$. Here the inverse element to 2 is determined by the calculation: $2^{\varphi(27)-1} \equiv 2^{18-1} \equiv 2^{17} \equiv 14(\bmod 27)$.

And the inverse element to 4 modulus 27 is 7 .
From the sequence command we see that 3 has no inverse elements because $3^{18-1} \equiv 0$ (mod 27) while 3 is not relatively prime to 27 . The same accounts for $6,9,12 \ldots$

### 3.1 Exercise: Number theory

a) Determine the principal remainder of 674 divided by 13
b) Determine the greatest common divisor of the numbers 349 og 154
c) Show that -9 is congruent to 3 modulus 4 and that 54 is congruent to 11 modulus 6
d) Determine $\mathbb{Z}_{30}$ and $\varphi(30)$
e) Determine if 16 has an inverse element modulus 27. If so, what is the inverse element?

### 4.1 Affine cryptosystems with TI-Nspire

The starting point for our treatment of the affine cryptosystem is the English alphabet and messages in English. Characters are written in capital and the symbol @ is used for space between words. Therefore $26+1$ signs are available and we are working in $\mathbb{Z}_{27}$ with 18 remainders that are relatively prime to 27 . Before we start encrypting messages we will look at some commands.

The command str21st(streng) converts a string to a list:

| str2lst(streng):=seq(mid(streng, $x, 1), x, 1, \operatorname{dim}($ streng $))$ |
| :--- |
| str2lst("STRING@TO@LIST") |
| $\{" \mathrm{~S} ", " \mathrm{~T} ", " \mathrm{R} ", " \mathrm{I} ", " \mathrm{~N} ", " \mathrm{G} ", " @ ", " \mathrm{~T} ", " \mathrm{O} ", " @ ", " \mathrm{~L} ", " \mathrm{I} ", " \mathrm{~S} ", " \mathrm{~T} "\}$ |

The above command is used in the command chr2num(string) which converts a string to a list of numbers so that @ is assigned the value $\mathbf{0}, \mathbf{A}$ the value $\mathbf{1}, \mathbf{B}$ the value $\mathbf{2}$, and so on:

$$
\begin{array}{|l|}
\hline \text { chr2num(streng): }=\text { ord(str2lst(streng))-64 } \\
\hline \operatorname{chr} 2 n u m(\text { ('STRING@TO@NUMBERS") } \\
\{19,20,18,9,14,7,0,20,15,0,14,21,13,2,5,18,19\} \\
\hline
\end{array}
$$

The command 1st2str(liste) converts a list to a streng:


The command 1st2str(liste) is used in the command num2chr(liste) to convert a list of numbers to a streng:

| num2chr $($ liste $):=$ lst2str $(\operatorname{char}($ liste +64$))$ | Udfort |
| ---: | ---: |
| num2chr $(\{14,21,13,2,5,18,19,0,20,15,0,19,20,18,9,14,7\})$ |  |
| "NUMBERS@TO@STRING" |  |

### 4.1.1 Exercise

Investigate the commands str21st(streng) and chr2num(string) and determine how they work.

### 4.2 Affine encryption og decryption with TI-Nspire

In this part we are able to number the characters in the English alphabet from 0 to 26 corresponding to the order of the characters. Spaces between words are symbolized by the sign @ and given the number 0.

Encryption with the affine cryptosystem is determined by linear functions:

$$
E(x) \equiv a \cdot x+b(\bmod 27) \text { where } a, b \in \mathbb{Z}_{27}
$$

Decryption is possible if $a$ is prime to 27 , that is if $a \in \mathbb{Z}_{27}^{*}$, because $a$ then has an inverse element $a^{-1}$. By solving the equation, where $E(x)=y$, we get the following:

$$
\begin{gathered}
y \equiv a \cdot x+b(\bmod 27) \\
y-b \equiv a \cdot x(\bmod 27) \\
a^{-1} \cdot(y-b) \equiv a^{-1} \cdot a \cdot x(\bmod 27) \\
x \equiv a^{-1} \cdot(y-b)(\bmod 27)
\end{gathered}
$$

Therefore the decryption function is given by:

$$
D(y) \equiv a^{-1} \cdot(y-b)(\bmod 27)
$$

### 4.2.1 Example

Let us watch an example of an affine encryption function:

$$
E(x) \equiv 2 \cdot x+3(\bmod 27)
$$

The starting point is the English alphabet, where every character is represented by a number from 0-26:

|  | A xdata | B ydata | C plaintxt | D cryptotxt |
| :---: | :---: | :---: | :---: | :---: |
| - | =seq(n, n, 0,26) | $=\bmod \left(2^{*} \times d a t a+3,27\right)$ | =char(xdata+64) | =char(ydata+64) |
| 1 | 0 |  | @ | C |
| 2 | 1 |  | A | E |
| 3 | 2 |  | B | G |
| 4 | 3 |  | C | I |
| 5 | 4 | 11 | D | K |
| 6 | 5 | 13 | E | M |
| 7 | 6 | 15 | F | $\bigcirc$ |
| 8 | 7 | 17 | G | Q |

The xdata are the numbers 0-26 that represent the characters @, A, B, C, ..., $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ from the plaintext (plaintrt). The ydata are determined by modulus calculation and represents the cryptotext (cryptrt).

Notice that we here use the char command to convert at number to a character.

We make a scatter plot of our xdata and ydata. Then we draw the graph of our encryption function:


Now we are going to graphically determine the decryption function, which can be used as a method in crypto analysis. We do this by interchanging the independent and dependent variables by drawing a scatter plot with ydata and xdata along the $\mathbf{x}$ - and $\mathbf{y}$ axis:


We are searching for a linear decryption function:

$$
D(x) \equiv a_{d} \cdot x+b_{d}(\bmod 27) \text { where } a_{d}, b_{d} \in \mathbb{Z}_{27}
$$

We observe that when ydata equals 0 then xdata equals 12 which correspond to the intersection between the linear line and the y -axis. Therefore $b_{d}=12$. The slope is determined by connecting two neighboring points with a line segment and measuring the slope. Therefore $a_{d}=14$.

Our guess of an affine decryption function is

$$
D(x) \equiv 14 \cdot x+12(\bmod 27)
$$

We now draw the graph of our idea of a decryption function together with ydata and xdata:


As mentioned this method to determine a decryption function can be used in crypto analysis in the attempt to break an encrypted message. Let us be sure that the decryption function equals the one that could be determined analytic if the encryption function was known:

$$
E(x) \equiv 2 \cdot x+3(\bmod 27)
$$

From the theory we know that the decryption function is given by:

$$
D(y) \equiv 2^{-1} \cdot(y-3)(\bmod 27)
$$

From the part about number theory we know that by calculation we get:
$\bmod \left(2^{18-1}, 27\right) \quad 14$

Therefore we have: $\quad D(y) \equiv 14 \cdot(y-3)(\bmod 27)$
And by multiplication we get:

$$
D(y) \equiv 14 \cdot(y-3) \equiv 14 \cdot y-14 \cdot 3 \equiv 14 \cdot y+12(\bmod 27)
$$

Because:

$$
\bmod (-14 \cdot 3,27)
$$

This equals the decryption function we determined graphically.

### 4.2.2. Exercise

Determine graphically the decryption function associated with:

$$
E(x) \equiv 5 \cdot x+8(\bmod 27)
$$

Compare with the analytically determined decryption function.

### 4.3 A program for affine encryption

This program can be used for affine encryption of English plain text. The program receives a plaintext as a string a slope, and a constant of the linear encryption function:

| affinencryp | 23/2 |
| :---: | :---: |
| Define LibPub affinencryp $(t x t, a, b)=$ |  |
| Prgm |  |
| © AffinEncryp $($ txt,, , ) recieves a plaintext, a slope and a constant |  |
| Local $t 1, t 2, t 3$ |  |
| $t 4:=a$ |  |
| $t 5:=b$ |  |
| If $\operatorname{dim}($ tx $t)>1000$ Then |  |
| Disp "The plaintext is to long it must be less than 1000 charac |  |
|  |  |
| If mod $\left(t 4^{18}, 27\right)=0$ Then |  |
| Disp "The slope is not allowed because $\mathrm{a}=$ ",t4," is not prime to 27 " |  |
| Else |  |
|  |  |
|  |  |
| chr2num(streng) $=$ =ord $($ str 2 lst $($ streng $)$ ) -64 |  |
| num $2 \mathrm{chr}($ liste $):=1 s t 2 s t r \mid$ (char (liste+64$)$ ) |  |
| enkrypt $(x)=\bmod (t 4 \cdot x+t 5,27)$ |  |
| $t 1:=\operatorname{chr} 2 \mathrm{num}(t x t)$ |  |
| $t 2:=$ enkrypt $(t z)$ |  |
| $t 3:=n u m 2 c h r(t 2)$ |  |
| Disp "Affine encryption ( $\mathrm{a}=$ ", $t 4$," og $\mathrm{b}=$ ", $t 5$, ") " |  |
| Disp "Cryptotxst:", 3 \% |  |
| Endif |  |
| EndIf |  |
| DelVar $t 1, t 2, t 3, t 4, t 5, t 6$,str 2lst,lst2str,chr2num,enkrypt,num $2 c h r$ EndPrgm |  |
|  |  |

Let us imagine that ALICE wants to send a secret massage to BOB by using the affine crypto system. She wants to send this message:

## DEAR BOB I MISS YOU SO MUCH MEET ME UNDER THE BRIDGE AT TEN PM TOMORROW LOVE ALICE

Alice writes the plaintext as a string where the spaces are represented by @ and she decides to use this encryption function:

$$
E(x) \equiv 12 \cdot x+7(\bmod 27)
$$

| $t:=$ "DEAR@BOB@I@MSS@YOU@SO@MUCH@MEET@ME@UNDER@THE |
| :--- |
| "DEAR@BOB@I@MSS@YOU@SO@MUCH@MEET@ME@UNDER@THE@F |
| affinencryp $(t, 12,7)$ |
| The slope is not allowed because a = 12 is not prime to 27 |

She realizes that this function does not work because the slope does not have an inverse element modulus 27 .

With the function:

$$
E(x) \equiv 11 \cdot x+7(\bmod 27)
$$

it goes much better:

| affinencryp $(t, 11,7)$ |
| :--- |
| Affine encryption ( $\mathrm{a}=11 \mathrm{og} \mathrm{b}=7$ ) |
| Cryptotxst: |
| "XHRPGBJBGYGOY@@GLJVG@JGOVMNGOHHKGOHGVZXHPGKNHGBPY' |
|  |

The crypto text that she can send to BOB is:

## XHRPGBJBGYGOY@@GLJVG@JGOVMNGOHHKGOHGVZXHPGKNHGBPYXC HGRKGKHZGUOGKJOJPPJQGDJFHGRDYMH

Via a secure communication line BOB is told which encryption function ALICE has used.
Notice that the program will give you a warning if the plaintext exceeds 1000 characters.

### 4.3 A program for affine decryption

This program can be used to make an affine decryption of an English encrypted plain text. The program receives a crypto text as a string, a slope and a constant of the linear encryption function:


Imagine that BOB has received the message from Alice which is encrypted by the affine system. He receives this crypto text:

## XHRPGBJBGYGOY@@GLJVG@JGOVMNGOHHKGOHGVZXHPGKNHGBPYXC HGRKGKHZGUOGKJOJPPJQGDJFHGRDYMH

BOB is by a secure communication line told that ALICE has used the encryption function:

$$
E(x) \equiv 11 \cdot x+7(\bmod 27)
$$

Now it is easy for BOB to decrypt the message from ALICE:

```
t:="XHRPGBJBGYGOY@@GLJVG@JGOVMNGOHHKGOHGVZXHPGKNHGBPYXCHGRKGKHZGUOGKJOJPPJQGDJFHGRDYMEP
    "XHRPGBJBGYGOY@@GLJVG@JGOVMNGOHHKGOHGVZXHPGKNHGBPYXCHGRKGKHZGUOGKJOJPPJQGDJFHGRDYMH"
affinedecryp}(t,11,7
Affine decryption (a=11 og b=7)
Plain text:
"DEAR@BOB@I@MISS@YOU@SO@MUCH@MEET@ME@UNDER@THE@BRIDGE@AT@TEN@PM@TOMORROW@LOVE@f
```

