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Québec Education Program
Progression of Learning in Secondary School

Progression of Learning in Secondary School

Mathematics

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Update of the CST Option in Secondary V Mathematics

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Progression of Learning in Secondary School

The progression of learning in secondary school constitutes a complement to each school subject, providing further information on the knowledge that the students must acquire and be able to use in each year of secondary school. This tool is intended to assist teachers in planning both their teaching and the learning that their students are to acquire.

The role of knowledge in learning

The knowledge that young people acquire enables them to better understand the world in which they live. From a very early age, within their families and through contact with the media and with friends, they accumulate and learn to use an increasingly greater body of knowledge. The role of the school should be to progressively broaden, deepen and structure this knowledge.

Knowledge and competencies must mutually reinforce each other. On the one hand, knowledge becomes consolidated when it is used and, on the other hand, the exercise of competencies entails the acquisition of new knowledge. Helping young people acquire knowledge raises the challenging question of how to make this knowledge useful and durable, and thus evokes the notion of competency. For example, we can never be really assured that a grammar rule has been assimilated until it is used appropriately in a variety of texts and contexts that go beyond the confines of a repetitive, targeted exercise.

Intervention by the teacher

The role of the teacher in knowledge acquisition and competency development is essential, and he or she must intervene throughout the learning process. In effect, the *Education Act* confers on the teacher the right to "select methods of instruction corresponding to the requirements and objectives fixed for each group or for each student entrusted to his care." It is therefore the teacher's responsibility to adapt his or her instruction and to base it on a variety of strategies, whether this involves lecture-based teaching for the entire class, individualized instruction for a student or a small group of students, a series of exercises to be done, a team activity or a particular project to be carried out.

In order to meet the needs of students with learning difficulties, teachers should encourage their participation in the activities designed for the whole class, although support measures should also be provided, when necessary. These might involve more targeted teaching of certain key elements of knowledge, or they might take the form of other specialized interventions.

As for the evaluation of learning, it serves two essential functions. Firstly, it enables us to look at the students' learning in order to guide and support them effectively. Secondly, it enables us to verify the extent to which the students have acquired the expected learning. Whatever its function, in accordance with the *Policy on the Evaluation of Learning*, evaluation should focus on the acquisition of knowledge and the students' ability to use this knowledge effectively in contexts that draw upon their competencies.

Structure

The progression of learning is presented in the form of tables that organize the elements of knowledge similarly to the way they are organized in the subject-specific programs. In mathematics, for example, learning is presented in fields: arithmetic, geometry, etc. For subjects that continue on from elementary school, the *Progression of Learning in Secondary School* has been harmonized with the *Progression of Learning in Elementary School*. Every element of learning indicated is associated with one or more years of secondary school during which it is formally taught.

A uniform legend is used for all subjects. The legend employs three symbols: an arrow, a star and a shaded box. What is expected of the student is described as follows:

→	Student constructs knowledge with teacher guidance.
★	Student applies knowledge by the end of the school year.
	Student reinvests knowledge.

An **arrow** indicates that teaching must be planned in a way that enables students to begin acquiring knowledge during the school year and continue or conclude this process in the following year, with ongoing systematic intervention from the teacher.

A **star** indicates that the teacher must plan for the majority of students to have acquired this knowledge by the end of the school year.

A **shaded box** indicates that the teacher must plan to ensure that this knowledge will be applied during the school year.

Mathematics

Introduction

Mathematics is a science that involves abstract concepts and language. Students develop their mathematical thinking gradually through personal experiences and exchanges with peers. Their learning is based on situations that are often drawn from everyday life. In elementary school, students take part in learning situations that allow them to use objects, manipulatives, references and various tools and instruments. The activities and tasks suggested encourage them to reflect, manipulate, explore, construct, simulate, discuss, structure and practise, thereby allowing them to assimilate concepts, processes and strategies[1] that are useful in mathematics. Students must also call on their intuition, sense of observation, manual skills as well as their ability to express themselves, reflect and analyze. By making connections, visualizing mathematical objects in different ways and organizing these objects in their minds, students gradually develop their understanding of abstract mathematical concepts. With time, they acquire mathematical knowledge and skills, which they learn to use effectively in order to function in society.

In secondary school, learning continues in the same vein. It is centred on the fundamental aims of mathematical activity: interpreting reality, generalizing, predicting and making decisions. These aims reflect the major questions that have led human beings to construct mathematical culture and knowledge through the ages. They are therefore meaningful and make it possible for students to build a set of tools that will allow them to communicate appropriately using mathematical language, to reason effectively by making connections between mathematical concepts and processes, and to solve situational problems. Emphasis is placed on technological tools, as these not only foster the emergence and understanding of mathematical concepts and processes, but also enable students to deal more effectively with various situations. Using a variety of mathematical concepts and strategies appropriately provides keys to understanding everyday reality. Combined with learning activities, everyday situations promote the development of mathematical skills and attitudes that allow students to mobilize, consolidate and broaden their mathematical knowledge. In Cycle Two, students continue to develop their mathematical thinking, which is essential in pursuing more advanced studies.

This document provides additional information on the knowledge and skills students must acquire in each year of secondary school with respect to arithmetic, algebra, geometry, statistics and probability. It is designed to help teachers with their lesson planning and to facilitate the transition between elementary and secondary school and from one secondary cycle to another. A separate section has been designed for each of the above-mentioned branches, as well as for discrete mathematics, financial mathematics and analytic geometry. Each section consists of an introduction that provides an overview of the learning that was acquired in elementary school and that is to be acquired in the two cycles of secondary school, as well as content tables that outline, for every year of secondary school, the knowledge to be developed and actions to be carried out in order for students to fully assimilate the concepts presented. A column is devoted specifically to learning acquired in elementary school.[2] Where applicable, the cells corresponding to Secondary IV and V have been subdivided to present the knowledge and actions associated with each of the options that students may choose based on their interests, aptitudes and training needs: *Cultural*, *Social and Technical* option (CST), *Technical and Scientific* option (TS) and *Science* option (S).

1. Examples of strategies are provided in the Appendix.
2. Information concerning learning acquired in elementary school was taken from the *Mathematics program* and the document *Progression of Learning in Elementary School - Mathematics*, to indicate its relevance as a prerequisite and to define the limits of the elementary school program. Please note that there are no sections on vocabulary or symbols for at the secondary level, these are introduced gradually as needed.

Mathematics

Arithmetic

In elementary school,[1] students developed their understanding of numbers and operations involving natural numbers less than 1 000 000, fractions and decimals up to thousandths. They identified the properties of operations as well as the relationships between them and learned to follow the order of operations in simple sequences of operations involving natural numbers. They were introduced to the concept of integers and performed operations with natural numbers and decimals mentally, in writing and using technological tools. They also used objects and diagrams to perform certain operations involving fractions.

In Secondary Cycle One, students continue to develop their number sense, to perform operations on written numbers in decimal and fractional notation, and to further their understanding of the processes associated with these operations. The numbers are positive or negative, without restrictions as to the order of magnitude. Students also develop proportional reasoning, an essential concept that has many applications both within and outside mathematics. For example, students use percentages (calculating a certain percentage of a number and the value corresponding to 100 per cent) in various situations involving discounts, taxes, increases, decreases, etc. They also make scale drawings and represent data using circle graphs. They look for unknown values in algebraic or geometric situations involving similarity transformations, arc lengths, sector areas or unit conversions.

In Secondary Cycle Two, students assimilate the concept of real numbers (rational and irrational), particularly in situations involving exponents, radicals or logarithms.

The following tables present the learning content associated with arithmetic. By basing themselves on the concepts and processes targeted, students develop the three competencies of the program, which in turn enable students to better integrate the mathematical concepts and processes presented.

- Understanding real numbers
- Understanding operations involving real numbers
- Operations involving real numbers
- Understanding and analyzing proportional situations

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1. Given the scope of this branch in elementary school, we recommend that you consult the document *Progression of Learning in Elementary School — Mathematics* for more information on the concepts and processes acquired by students.

Mathematics

Arithmetic

Understanding real numbers

Understanding real numbers

→	Student constructs knowledge with teacher guidance.
★	Student applies knowledge by the end of the school year.
■	Student reinvests knowledge.[1]

		Secondary				
		Cycle One		Cycle Two		
	6	1	2	3	4	5
1. Natural numbers less than 1 000 000						
a. Reads and writes any natural number	★	■				
b. Represents natural numbers in different ways	★	■				
c. Composes and decomposes a natural number in a variety of ways and identifies equivalent expressions	★	■				
d. Approximates a natural number	★	■				
e. Compares natural numbers or arranges natural numbers in increasing or decreasing order	★	■				
f. Classifies natural numbers in various ways, based on their properties (e.g. even numbers, composite numbers)	★	■				
2. Fractions						
a. Represents a fraction in a variety of ways (using objects or drawings)	★	■				
b. Identifies the different meanings of fractions: part of a whole, division, ratio, operator, measurement	→	→	★			
c. Verifies whether two fractions are equivalent	★	■	■			
d. Compares a fraction to 0, $\frac{1}{2}$ or 1	■	■	■			
e. Orders fractions with the same denominator or where one denominator is a multiple of the other or with the same numerator	■	■	■			
3. Decimals up to thousandths						
a. Represents decimals in a variety of ways (using objects or drawings) and identifies equivalent representations	★	■	■			
b. Reads and writes numbers written in decimal notation	★	■	■			
c. Approximates a number written in decimal notation	★					
d. Composes and decomposes a number written in decimal notation and recognizes equivalent expressions	★	■	■			
e. Compares numbers written in decimal notation or arranges them in increasing or decreasing order	★	■	■			
4. Integers						
a. Represents integers in a variety of ways (using objects or drawings)	★	■	■			
b. Reads and writes integers	★	■	■			
c. Compares integers or arranges integers in increasing or decreasing order	★	■	■			

5. Expresses numbers in a variety of ways (fractional, decimal percentage notation)	★						
6. Represents, reads and writes numbers written in fractional or decimal notation		★					
7. Approximates, in various contexts, the numbers under study (e.g. estimates, rounds off, truncates)		★					
8. Distinguishes rational numbers from irrational numbers in the set of real numbers Note : Although students do not systematically study sets of numbers in Secondary Cycle One, they should still be encouraged to use the proper terms learned in elementary school (natural numbers, integers, decimals).				★			
9. Represents, in different types of notation, various subsets of real numbers (discrete or continuous): interval, list/roster, on a number line Note : In TS and S, set builder notation may be introduced as needed.				★			
10. Defines the concept absolute value in context (e.g. difference between two numbers, distance between two points) Note : In Cycle One and Secondary III, the concept of *absolute value* is introduced informally, using examples.		→	→	→	★		
11. Represents and writes							
a. the power of a natural number	★						
b. squares and square roots		→	★				
c. numbers in exponential notation (integral exponent)		→	★				
d. numbers in scientific notation				★			
e. cubes and cube roots				★			
f. numbers in exponential notation (fractional exponents)				★			
g. numbers using radicals or rational exponents							CST
					★		TS
						★	S
h. numbers in logarithmic notation using the equivalence $\log_a x = n \Leftrightarrow a^n = x$, if necessary						★	CST
					→	★	TS
						★	S
12. Estimates the value of the power of an exponential expression with respect to its components: base (between 0 and 1, greater than 1), exponent (positive or negative, integral or fractional) Note : The same applies for a logarithmic expression in TS and S.					★		CST
				→	★		TS
						★	S
13. Estimates the order of magnitude of a real number in different contexts	→	→	→	★			
14. Estimates the order of magnitude of a real number using scientific notation				★			
15. Compares and arranges in order							
a. numbers written in fractional or decimal notation		★					
b. numbers expressed in different ways (fractional, decimal, exponential [integral exponent], percentage, square root, scientific notation) Note : Scientific notation is introduced in Secondary III.		→	★				

1. Mathematical knowledge is constructed using prerequisites or by making connections between concepts and processes. The elements described in the tables will be reinvested and further developed as students progress through secondary school. When actions are included as part of other actions carried out in subsequent years, the shading in the table is not extended to cover all five years of secondary school.

Mathematics

Arithmetic

Understanding operations involving real numbers

Understanding operations involving real numbers

→ Student constructs knowledge with teacher guidance.

★ Student applies knowledge by the end of the school year.

Student reinvests knowledge.

		Secondary				
		Cycle One		Cycle Two		
	6	1	2	3	4	5
1. Natural numbers less than 1 000 000						
a. Determines the operation(s) to perform in a given situation	★					
b. Uses objects, diagrams or equations to represent a situation and, conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of the four operations)	★					
c. Establishes equality relations between numerical expressions (e.g. 3 + 2 = 6 – 1)	★					
d. Determines numerical equivalencies using relationships between operations, the commutative and associative properties of addition and multiplication, the distributive property of multiplication over addition or subtraction	★					
e. Translates a situation using a sequence of operations in accordance with the order of operations	★					
2. Fractions						
a. Uses objects, diagrams or an operation to represent a situation and, conversely, describes a situation represented by objects, diagrams or an operation (use of different meanings of addition, subtraction and multiplication by a natural number)	★					
b. Uses an operation to represent a situation (use of different meanings of operations)	→	★				
3. Decimals						
a. Uses objects, diagrams or equations to represent a situation and, conversely, describes a situation represented by objects, diagrams or equations (use of different meanings of the four operations)	★					
b. Determines numerical equivalencies using relationships between operations (inverse operations), the commutative and associative properties of addition and multiplication, the distributive property of multiplication over addition or subtraction	★					
c. Translates a situation using a sequence of operations in accordance with the order of operations	★					
4. Chooses an appropriate way of writing numbers for a given context **Note** : Over the years, new notation systems such as scientific notation are added to the students' repertoire.	★					
5. Looks for equivalent expressions: decomposing (additive, multiplicative, etc.), equivalent fractions, simplifying and reducing, factoring, etc.		★				
6. Translates (mathematizes) a situation using a sequence of operations (no more than two levels of parentheses)		★				
7. Anticipates the results of operations		★				
8. Interprets the results of operations in light of the context		★				

Mathematics

Arithmetic

Operations involving real numbers

Operations involving real numbers	Elementary	Secondary				
→ Student constructs knowledge with teacher guidance. ★ Student applies knowledge by the end of the school year. ■ Student reinvests knowledge.		Cycle One		Cycle Two		
	6	1	2	3	4	5
1. Natural numbers less than 1 000 000						
a. Approximates the result of an operation	★	■				
b. Using personal processes, mentally computes operations	★	■				
c. Determines in writing – the sum of two natural numbers of up to 4 digits – the difference between two natural numbers of up to 4 digits whose result is greater than 0 – the product of a three-digit number by a two-digit number – the quotient of a four-digit number and a two-digit number and expresses the remainder of a division as a decimal that does not go beyond the second decimal place – the result of a sequence of operations in accordance with the order of operations	★	■				
2. Fractions (using objects or diagrams)						
a. Generates a set of equivalent fractions	★	■				
b. Reduces a fraction to its simplest form	★	■				
c. Adds and subtracts fractions when the denominator of one fraction is a multiple of the other fraction	★	■				
d. Multiplies a natural number by a fraction and a fraction by a natural number	★	■				
3. Decimal numbers up to thousandths						
a. Approximates the result of an operation	★	■				
b. Mentally computes – operations (addition, subtraction, multiplication, division by a natural number) – multiplications by 10, 100, 1000	★	■				
c. Computes in writing – additions and subtractions of numbers whose result does not go beyond the second decimal place – multiplications of numbers whose product does not go beyond the second decimal place – divisions of a decimal by a natural number less than 11	★	■				
4. Properties of divisibility						
a. Determines the divisibility of a number by 2, 3, 4, 5, 6, 8, 9 and 10	★	■				
b. Uses, in different contexts, the properties of divisibility: 2, 3, 4, 5 and 10		★	■	■	■	■
5. Approximates the result of an operation or sequence of operations		→	★	■	■	■

6. Mentally computes the four operations, especially with numbers written in decimal notation, using equivalent ways of writing numbers and the properties of operations		→	★				
7. Computes, in writing, the four operations[1] with numbers that are easy to work with (including large numbers), using equivalent ways of writing numbers and the properties of operations							
a. numbers written in decimal notation, using rules of signs		★					
b. positive numbers written in fractional notation, with or without the use of objects or diagrams		→	★				
c. numbers written in fractional notation				★			
8. Computes, in writing, sequences of operations (numbers written in decimal notation) in accordance with the order of operations, using equivalent ways of writing numbers and the properties of operations (with no more than two levels of parentheses)		★					
9. Computes, using a calculator, operations and sequences of operations in accordance with the order of operations		★					
10. Switches, as needed, from one way of writing numbers to another: from fractional to percentage notation, from decimal to fractional notation, from decimal to percentage notation, and vice versa	★						
11. Switches, as needed, from one way of writing numbers to another **Note** : In Secondary Cycle One, the students should use positive numbers when switching from one type of notation to another. In Secondary Cycle Two, new types of notation are introduced: exponential, scientific notation, etc.		→	★				
12. Calculates the power of a natural number	★						
13. Decomposes a natural number into prime factors	★						
14. Manipulates numerical expressions involving							
a. integral exponents (rational base) and fractional exponents **Note** : When manipulating numerical expressions, students learn to deduce the properties of powers.				★			
b. powers of bases (change of base), exponents, radicals (*n*th root), using their properties **Note** : In CST, radicals and their properties are not covered. For base changes in TS in Secondary IV, students use bases 2 and 10. In S, students learn to deduce the properties of radicals.						★	CST
					→	★	TS
						★	S
c. logarithms							
i. definition and change of base						★	CST
					★		TS
						★	S
ii. properties							CST
						★	TS
						★	S
d. absolute values							CST
							TS
						★	S

1. Students use technological tools for operations in which the divisors or multipliers have more than two digits; however, for written computation, the understanding and mastery of the processes is more important than the ability to do complex calculations.

Mathematics

Arithmetic

Understanding and analyzing proportional situations

Understanding and analyzing proportional situations

→	Student constructs knowledge with teacher guidance.
★	Student applies knowledge by the end of the school year.
	Student reinvests knowledge.

		Secondary				
		Cycle One		Cycle Two		
	6	1	2	3	4	5
1. Calculates						
a. a certain percentage of a number	→	★				
b. the value corresponding to 100 per cent		→	★			
2. Recognizes ratios and rates		→	★			
3. Interprets ratios and rates		→	★			
4. Describes the effect of changing a term in a ratio or rate		→	★			
5. Compares						
a. ratios and rates qualitatively (equivalent rates and ratios, unit rate)		→	★			
b. ratios and rates quantitatively (equivalent rates and ratios, unit rate)		→	★			
6. Translates a situation using a ratio or rate **Note** : Situations involving ratios and rates are enriched in Secondary Cycle Two (similarity ratio, metric relations, etc.).		→	★			
7. Recognizes a proportional situation using the context, a table of values or a graph		→	★			
8. Represents or interprets a proportional situation using a graph, a table of values or a proportion		→	★			
9. Solves proportional situations (direct or inverse variation) by using different strategies (e.g. unit-rate method, factor of change, proportionality ratio, additive procedure, constant product [inverse variation])		→	★			
10. Establishes relationships between first-degree or rational functions and proportional situations (direct or inverse variation)				★		

Mathematics

Algebra

Through their various mathematical activities in elementary school, students were introduced to prerequisites for algebra, such as finding unknown terms using properties of operations and relationships between these operations, developing an understanding of equality and equivalence relationships, following the order of operations and looking for patterns in different situations.

In Secondary Cycle One, students move from arithmetic thinking to algebraic thinking. They use and further develop their understanding of numbers, operations and proportionality. For example, in studying patterns, elementary school students learned to determine rules for constructing number sequences between terms, whereas in secondary school, students learn to establish the relationship between a term and its rank. Algebraic expressions are added to known registers (types) of representation to observe situations from different perspectives. Students refine their ability to switch from one register of representation to another in order to analyze situations in the register(s) of their choice. Thus, they learn to manipulate algebraic expressions with or without technological aids, and interpret tables of values and graphs. The use of technology makes it easier to explore and examine these relationships in greater depth and makes it possible to describe and explain them more fully. Lastly, students learn to search for mathematical models representing various situations.

In Secondary Cycle Two, students hone their ability to evoke a situation by drawing on several registers of representation and switching from one register to another, without any restrictions. For example, functions may be represented using graphs, tables or rules, and each of these representations conveys a specific point of view and is complementary or equivalent to other types of representation. Students learn to analyze and deal with situations that involve a set of algebraic concepts and processes. They establish dependency relationships between variables; model, compare and optimize situations, if necessary; and make informed decisions about these situations, depending on the case.

The following tables present the learning content associated with algebra. By basing themselves on the concepts and processes targeted, students develop the three competencies of the program, which in turn enable students to better integrate the mathematical concepts and processes presented.

- Understanding and manipulating algebraic expressions
- Understanding dependency relationships

Mathematics

Algebra

Understanding and manipulating algebraic expressions

Understanding and manipulating algebraic expressions

→	Student constructs knowledge with teacher guidance.
★	Student applies knowledge by the end of the school year.
	Student reinvests knowledge.

		Secondary				
		Cycle One		Cycle Two		
A. Algebraic expressions	**6**	**1**	**2**	**3**	**4**	**5**
1. Describes, using his/her own words and mathematical language, numerical patterns						
2. Describes, using his/her own words and mathematical language, series of numbers and family of operations	★					
3. Adds new terms to a series when the first three terms or more are given	★					
4. Describes the role of components of algebraic expressions:						
a. unknown **Note** : This concept was introduced in elementary school (although not named as such) when students were asked to find a missing term.	→	→	★			
b. variable, constant		→	★			
c. parameter **Note** : The concept of parameter is introduced intuitively (although not named as such) in Secondary I, II and III.		→	→	→	CST: TS: → S: ★	CST: TS: ★ S:
d. coefficient, degree, term, constant term, like terms		→	★			
5. Constructs an algebraic expression using a register (type) of representation		→	★			
6. Interprets an algebraic expression in light of the context		→	★			
7. Recognizes or constructs equivalent algebraic expressions		→	★			
8. Recognizes or constructs						
a. equalities and equations	→	→	★			
b. inequalities				★		
B. Manipulating algebraic expressions	**6**	**1**	**2**	**3**	**4**	**5**
1. Calculates the numeric value of an algebraic expression		→	★			
2. Performs the following operations on algebraic expressions, with or without objects or diagrams: addition and subtraction, multiplication and division by a constant, multiplication of first-degree monomials		→	★			
3. Factors out the common factor in numerical expressions (distributive property of multiplication over addition or subtraction)		→	★			
4. Multiplies						
a. algebraic expressions of degree less than 3				★		

b. algebraic expressions							CST
					★		TS
					★		S
5. Divides							
a. algebraic expressions by a monomial				★			
b. a polynomial by a binomial (with or without a remainder)							CST
					★		TS
							S
c. a polynomial by another polynomial (with or without a remainder)							CST
							TS
					★		S
6. Factors polynomials by							
a. finding the common factor				★			
b. factoring by grouping (polynomials including decomposable second-degree trinomials)							CST
					★		TS
					★		S
c. completing the square (factoring and switching from one type of notation to another)							CST
						★	TS
					★		S
d. using formulas for trinomials of the form $ax^2 + bx + c$: $x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$							CST
						★	TS
					★		S
e. substituting second-degree algebraic identities (perfect square trinomial and difference of two squares)							CST
					★		TS
					★		S
7. Manipulates rational expressions Note : Rational expressions (algebraic fractions) are part of the algebraic expressions to be covered. When finding the common denominator in order to add two rational expressions, students in TS will deal only with cases in which the denominator of one fraction is the multiple of the denominator of the other fraction.							CST
					★		TS
					★		S

C. Analyzing situations using equations or inequalities	6	1	2	3	4	5
1. Recognizes whether a situation can be translated by						
a. an equation		→	★			
b. an inequality				★		
2. Recognizes or constructs						
a. relations or formulas		→	★			
b. inequality relations and first-degree inequalities in one variable				★		
3. Manipulates relations or formulas (e.g. isolates an element)		→	★			
4. Represents a situation using						
a. a first-degree equation with one unknown		→	★			
b. a first-degree inequality with a variable				★		
5. Represents						
a. an equation using another register (type) of representation, if necessary		→	★			
b. an inequality using another register (type) of representation, if necessary				★		

	6	1	2	3	4	5	
6. Determines the missing term in an equation (relations between operations) :[1] $a + b = \square$, $a + \square = c$, $\square + b = c$, $a - b = \square$, $a - \square = c$, $\square - b = c$, $a \times b = \square$, $a \times \square = c$, $\square \times b = c$, $a \div b = \square$, $a \div \square = c$, $\square \div b = c$	★						
7. Transforms arithmetic equalities and equations to maintain equivalence (properties and rules for transforming equalities) and justifies the steps followed, if necessary		→	★				
8. Transforms inequalities to maintain equivalence (properties and rules for transforming inequalities) and justifies the steps followed, if necessary				★			
9. Uses different methods to solve first-degree equations with one unknown of the form $ax + b = cx + d$: trial and error, drawings, arithmetic methods (inverse or equivalent operations), algebraic methods (balancing equations or hidden terms)		→	★				
10. Solves first-degree inequalities in one variable				★			
11. Solves the following types of equations or an inequalities in one variable							
a. second-degree							CST
Note : In TS, this is taught over two years using the functional models under study.					→	★	TS
					★		S
b. exponential, logarithmic or square root, using the properties of exponents, logarithms and radicals						★	CST
Note : In CST in Secondary V, students use the definitions of logarithm and change of base to solve exponential and logarithmic equations, but they are not required to solve square root equations or study the properties of radicals and logarithms. In TS, this is taught over two years, using the functional models under study.					→	★	TS
						★	S
c. rational							CST
						★	TS
						★	S
d. absolute value							CST
							TS
						★	S
e. first-degree trigonometric involving a sine, cosine or tangent expression							CST
						★	TS
							S
f. trigonometric that can be expressed as a sine, cosine or tangent function							CST
							TS
						★	S
12. Solves a second-degree equation in two variables							CST
Note : In TS, this is taught over two years, using the functional models under study.					→	★	TS
					★		S
13. Validates a solution, with or without technological tools, by substitution		→	★				
14. Solves an inequality graphically and checks the feasible region of a							
a. first-degree inequality in two variables						★	CST
					★		TS
					★		S
b. second-degree inequality in two variables							CST
Note : In TS, this is taught over two years using the functional models under study.					→	★	TS
					★		S
15. Interprets solutions or makes decisions, if necessary, depending on the context		→	★				
D. Analyzing situations using systems of equations or inequalities	**6**	**1**	**2**	**3**	**4**	**5**	
1. Determines whether a situation may be translated by a system of							
a. equations				★			
b. inequalities						★	

	6	1	2	3	4	5	
2. Translates a situation algebraically or graphically using a system of							
a. equations				★			
b. inequalities						★	
3. Solves a system							
a. of first-degree equations in two variables of the form $y = ax + b$ by using tables of values, graphically or algebraically (by comparison), with or without technological tools				★			
b. of first-degree equations in two variables **Note** : The student chooses the method.					★		
c. composed of a first-degree equation in two variables and a second-degree equation in two variables **Note** : In TS, these systems are solved using graphic representations, with or without the use of technological tools.							CST
						★	TS
					★		S
d. of second-degree equations in relation to conics using changing variables, if applicable							CST
							TS
						★	S
e. involving various functional models (mostly graphical solutions)							CST
						★	TS
							S
4. Solves a system							
a. of first-degree inequalities in two variables						★	
b. involving various functional models (mostly graphical solutions)							CST
						★	TS
							S
5. Validates the solution, with or without technological tools				→	★		
6. Interprets the solution or makes decisions if necessary, depending on the context				→	★		
E. Linear programming	**6**	**1**	**2**	**3**	**4**	**5**	
1. Analyzes a situation to be optimized – mathematizing the situation using a system of first-degree inequalities in two variables – drawing a bounded or unbounded polygon of constraints to represent the situation – determining the coordinates of the vertices of the bounded polygon (feasible region) **Note** : In TS, the coordinates of points of intersection may be determined algebraically, using matrices, or approximated based on a graph. – recognizing and defining the function to be optimized						★	
2. Optimizes a situation by taking into account different constraints and makes decisions with respect to this situation – determining the best solution(s) for a particular situation, given a set of possibilities – validating and interpreting the optimal solution, depending on the context – justifying the solution(s) chosen – changing certain conditions associated with the situation to provide a more optimal solution, if necessary						★	

1. Students were not shown this symbolic notation in elementary school. They did learn, however, to determine the value of a missing term, among other things, in situations that call for additive or multiplicative structures, within the limits of the elementary-level Mathematics program.

Mathematics

Algebra

Understanding dependency relationships

Understanding dependency relationships

→ Student constructs knowledge with teacher guidance.

★ Student applies knowledge by the end of the school year.

■ Student reinvests knowledge.

	Elementary	Secondary Cycle One		Secondary Cycle Two			
A. Relations, functions and inverses	**6**	**1**	**2**	**3**	**4**	**5**	
1. Identifies patterns in various situations and in various forms	■	■	■	■	■	■	
2. Analyzes situations using different registers (types) of representation	→	→	★	■	■	■	
3. Represents a situation generally using a graph		→	★	■	■	■	
4. Chooses the dependent variable and the independent variable				★	■	■	
5. Recognizes relations, functions and inverses				★	■	■	
6. Describes, in the functions under study, the role of							
a. multiplicative parameters							CST
					★	■	TS
					★	■	S
b. additive parameters							CST
						★	TS
					★	■	S
7. Performs operations on functions (including composition) **Note** : In TS, operations on functions can be approached intuitively as of Secondary IV. In Secondary V, they are studied using concrete situations.							CST
						★	TS
						★	S
B. Analyzing situations using real functions[1]	**6**	**1**	**2**	**3**	**4**	**5**	
Note : Statements 1 to 9 apply to the functions listed below. 1. Models a situation verbally, algebraically, graphically, using a table of values or a scatter plot 2. Finds the rule of a function or its inverse, depending on the context 3. Represents and interprets the inverse 4. Interprets parameters (multiplicative or additive) and describes the effect of changing their value, if necessary 5. Describes the properties of real functions: domain, range, interval within which the function is increasing or decreasing, sign, extrema, x-intercept and y-intercept **Note** : In Secondary III, students are informally introduced to the study of properties, always in relation to a context. In CST, students use a graphical representation to describe the context. 6. Determines values or data by solving equations and inequalities 7. Interpolates and extrapolates data, if applicable 8. Compares situations or graphical representations 9. Makes decisions, if necessary, depending on the context							
a. Polynomial functions of degree 0 or 1				★	■	■	

b. Second-degree polynomial functions							
i. $f(x) = ax^2$					★		CST
					★		TS
							S
ii. $f(x) = (bx)^2$ or $f(x) = a(bx)^2$							CST
					★		TS
							S
iii. $f(x) = ax^2 + bx + c$, $f(x) = a(b(x - h))^2 + k$, $f(x) = a(x - x_1)(x - x_2)$							CST
						★	TS
					★		S
c. Square root functions							
i. $f(x) = a\sqrt{bx}$ Note : This function is introduced in connection with the second-degree function as inverse (relation expressed as two square root functions).							CST
					★		TS
							S
ii. $f(x) = a\sqrt{b(x - h)} + k$							CST
						★	TS
						★	S
d. Rational functions							
i. $f(x) = \frac{k}{x}$ or $xy = k$, $k \in \mathbb{Q}^+$				★			
ii. $f(x) = a\left(\frac{1}{b(x-h)}\right) + k$ and $f(x) = \frac{ax+b}{cx+d}$							CST
						★	TS
						★	S
e. Exponential functions							
i. $f(x) = ac^x$					★		CST
							TS
							S
ii. $f(x) = ac^{bx}$ Note : In CST, students are able to manipulate this type of function, but are not required to determine the rule.							CST
					★		TS
							S
iii. $f(x) = ac^{b(x-h)} + k$ Note : The study of these functions should focus on bases 2, 10 and							CST
						★	TS
						★	S
f. Logarithmic functions							
ii. $f(x) = a \log_c bx$ Note : This function is introduced in connection with exponential functions (as an inverse).							CST
					★		TS
							S
iii. $f(x) = a \log_c b(x - h) + k$ Note : The study of these functions should focus on bases 2, 10 and e.							CST
						★	TS
						★	S
g. Piecewise functions Note : In Secondary III, students are introduced to this type of function informally.					★		CST
				→	★		TS
						★	S
h. Absolute value functions : $f(x) = a\lvert b(x - h)\rvert + k$ Note : In TS, this function is treated mainly as a piecewise function.							CST
							TS
						★	S
i. Step functions					★		
j. Greatest integer functions							
i. $f(x) = a[bx]$							CST
					★		TS
							S
ii. $f(x) = a[b(x - h)] + k$							CST
						★	TS
					★		S

k. Functions							
i. Modelling periodic occurences (e.g. natural phenomena such as tides or sound, medical or electrical phenomena) **Note** : The analysis is based on a graphical representation. In this context, students are not required to determine the rule.					★		CST
					★		TS
						★	S
ii. sinusoidal : $f(x) = a \sin b(x - h) + k$, $f(x) = a \cos b(x - h) + k$							CST
						★	TS
						★	S
iii. tangent : $f(x) = a \tan b(x - h) + k$							CST
						★	TS
						★	S

1. Functions are introduced using contexts adapted to Secondary III and the various options, with or without the use of technological tools.

Mathematics

Probability

By acquiring probabilistic reasoning skills, students avoid the confusion between probability and proportion and demystify certain false preconceptions about odds or chance, such as the bias associated with equiprobability, availability and representativeness. This prepares them to exercise their critical judgment in various situations.

In elementary school, students conducted experiments related to chance. They made qualitative predictions about outcomes using concepts related to outcome (certainty, possibility and impossibility) and the probability that an event will occur (more likely, just as likely and less likely). They listed the outcomes of a random experiment using tables or tree diagrams and compared the actual outcomes with theoretical probabilities.

In Secondary Cycle One, students go from using subjective, often arbitrary, reasoning to reasoning based on various calculations. They further develop the concept of probability of an event—the cornerstone in calculating probabilities—and are introduced to the language of sets. They learn to enumerate possibilities using different registers (types) of representation, to calculate probabilities and to compare experimental and theoretical probabilities. With this knowledge and skills, students are able to make predictions and informed decisions in various types of situations.

In Secondary Cycle Two, students continue to build on what they learned in Cycle One. They use the results of combinatorial analysis (permutations, arrangements and combinations) and add the ability to calculate probabilities in certain measurement contexts to their repertoire of knowledge and skills. Depending on the option, students learn to distinguish between subjective probabilities and experimental or theoretical probabilities. They interpret or distinguish between various relationships (e.g. the probability of an event and the *odds for* or the *odds against*). They use the concept of mathematical expectation to determine whether a game is fair or the possibility of a gain or a loss. Lastly, they analyze situations and make decisions based on conditional probability.

The following tables present the learning content associated with probability. By basing themselves on the concepts and processes targeted, students develop the three competencies of the program, which in turn enable students to better integrate the mathematical concepts and processes presented.

Understanding data from random experiments	Elementary	Secondary				
→ Student constructs knowledge with teacher guidance. ★ Student applies knowledge by the end of the school year. ■ Student reinvests knowledge.		Cycle One		Cycle Two		
A. Processing data from random experiments	6	1	2	3	4	5
1. Simulates random experiments with or without the use of technological tools	★	■	■	■		
2. Experiments with activities involving chance, using various objects (e.g. spinners, rectangular prisms, glasses, marbles, thumb tacks, 6-, 8- or 12-sided dice)	★	■	■	■		
3. In activities involving chance						
a. recognizes variability in possible outcomes (uncertainty)	★	■	■	■		
b. recognizes equiprobability (e.g. quantity of objects, symmetry of an object such as a cube)	★	■	■	■		
c. becomes aware of the independence of events (e.g. rolling dice, tossing a coin, drawing lots)	★	■	■	■		
4. Uses tables or diagrams to collect and display the outcomes of an experiment	★	■	■	■		
5. Compares the outcomes of a random experiment with known theoretical probabilities	★	■	■	■		
6. Distinguishes between prediction and outcome	★	■	■	■		
7. Conducts or simulates random experiments involving one or more steps (with or without replacement, with or without order)		→	★	■		
8. Identifies the type of random variable: discrete or continuous				★	CST: TS: ■ S:	

9. Enumerates the possible outcomes of a random experiment using							
a. tables, tree diagram	★						CST TS S
b. networks, tables, diagrams, Venn diagrams **Note** : In developing their probabilistic thinking skills, students are introduced to the language of sets, which is considered to be a comprehension and communication tool.		→	★				CST TS S
c. geometric figures				★			CST TS S
10. Defines the sample space of a random experiment		→	★				CST TS S
11. Recognizes certain, probable, impossible, simple, complementary, compatible, incompatible, dependent, independents events		→	★				CST TS S
12. Distinguishes between mutually exclusive and non-mutually exclusive, and between dependent and independent events					★	★	CST TS S
13. Uses fractions, decimals or percentages to quantify a probability	★						CST TS S
14. Recognizes that a probability is always between 0 and 1	★						CST TS S
15. Predicts qualitatively an outcome or several events using a probability line, among other things							
a. certain, possible or impossible outcome	★						
b. more likely, just as likely, less likely event	★						
16. Uses factorial notation, if necessary **Note** : This notation may be introduced in CST.					★		CST TS S
17. Recognizes, depending on the context, different types of probabilities: experimental, theoretical, subjective					★	★	CST TS S
18. Defines or interprets the concept of odds/chance (*odds for* and *odds against*) (e.g. makes connections between odds and probabilities)					★	★	CST TS S
19. Defines or interprets the concept of mathematical expectation (e.g. makes connections between mathematical expectation and weighted mean)					★	★	CST TS S
B. Analyzing probability situations	**6**	**1**	**2**	**3**	**4**	**5**	
1. Represents an event using different registers (types of representation))	★						CST TS S
2. Compares qualitatively the theoretical or experimental probability of an event occurring	★						
3. Distinguishes between theoretical and experimental probability		→	★				
4. Calculates the probability of an event		→	★				CST TS S

5.	Calculates the probability of outcomes of random experiments related to situations involving arrangements, permutations or combinations **Note** : Calculations are based on reasoning, not on counting formulas. The terminology (permutation, arrangement, combination) may be introduced in the first year of Secondary Cycle Two.							CST
					★			TS
								S
6.	Associates the type of probability to a situation: experimental, theoretical, subjective						★	CST
						★		TS
								S
7.	Calculates probabilities, including geometric probabilities, in measurement contexts							CST
					★			TS
								S
8.	Calculates conditional probabilities						★	CST
						★		TS
								S
9.	Interprets probabilities and makes appropriate decisions							CST
			→	★				TS
								S
10.	Chooses and applies the concept of odds/chance (*odds for* and *odds against*) or probability, depending on the context						★	CST
						★		TS
								S
11.	Determines the *odds for* or *odds against*						★	CST
						★		TS
								S
12.	Interprets and makes decisions with respect to the odds obtained						★	CST
						★		TS
								S
13.	Calculates mathematical expectation						★	CST
						★		TS
								S
14.	Modifies, if necessary, the parameters of a situation in order to make it fair, attain an objective or optimize a gain or loss						★	CST
						★		TS
								S
15.	Interprets the resulting mathematical expectation and makes appropriate decisions						★	CST
						★		TS
								S

Mathematics

Statistics

Statistics, which is used to gather, process and analyze data regarding a population,[1] is a valuable decision-making tool in many fields. This branch of mathematics is based on concepts and processes related to probability, particularly as regards sampling.

In elementary school, students were introduced to descriptive statistics, which allowed them to summarize raw data in a clear and reliable way. They conducted surveys, i.e. they learned how to formulate questions, gather data and organize it using tables, and interpreted and displayed data using bar graphs, pictographs and broken-line graphs. They also obtained relevant information using circle graphs, and calculated and interpreted the arithmetic mean of a distribution.

In Secondary Cycle One, students carry out studies using sample surveys and censuses. They acquire the tools they need to process the data they may or may not have gathered and extract information from these data. They learn about circle graphs as a possible method of data representation. They choose the graph(s) that will best illustrate a situation. They learn to highlight information such as minimum value, maximum value, range and mean and look for potential sources of bias.

In Secondary Cycle Two, descriptive statistics is used to introduce students to inferences. The situations dealt with allow students to gather, organize and represent data using the most appropriate graph, and determine certain statistical measures such as measures of central tendency, of position and of dispersion. They interpret data by observing their distribution (type, range, centre, groups), and check whether the distribution includes outliers that could influence certain measures or conclusions. They compare distributions, using appropriate measures of central tendency and of dispersion. They learn how to interpret a correlation qualitatively, using an approximate value of the correlation coefficient or quantitatively, by calculating its exact value using technological tools, if necessary.

The following tables present the learning content associated with statistics. By basing themselves on the concepts and processes targeted, students develop the three competencies of the program, which in turn enable students to better integrate the mathematical concepts and processes presented.

Analyzing and making decisions about one- or two-variable distributions, using statistical tools

→ Student constructs knowledge with teacher guidance.

★ Student applies knowledge by the end of the school year.

Student reinvests knowledge.

	Elementary	Secondary Cycle One		Secondary Cycle Two			
A. One-variable distributions	6	1	2	3	4	5	
1. Conducts a survey or a census							
a. Formulates questions for a survey **Note** : The questions become more refined over the years.	★						CST
							TS
							S
b. Chooses a sampling method :							
i. simple random, systematic		→	★				
ii. stratified, cluster				★			CST
							TS
							S
c. Chooses a representative sample		→	★				
d. Collects, describes and organizes data (classifies or categorizes) using tables	★						CST
							TS
							S
2. Recognizes possible sources of bias **Note** : In CST in Secondary IV, students learn to correct the source of bias, if applicable.		→	★				
3. Interprets data presented in a table or a bar graph, a pictograph, a broken-line graph or a circle graph	★						

4. Distinguishes different types of statistical variables: qualitative, discrete or continuous quantitative		→	★				
5. Chooses appropriate register(s) (types) of representation to organize, interpret and present data							
6. Organizes and presents data using							
a. a table, a bar graph, a pictograph and a broken-line graph	★						
b. a table presenting variables or frequencies, or using a circular graph		→	★				
c. a table of condensed data or data grouped into classes, a histogram, or box-and-whisker plot				★			CST
							TS
							S
d. a stem-and-leaf diagram					★		CST
							TS
							S
7. Compares one-variable distributions		→	★				
8. Understands and calculates the arithmetic mean	★						
9. Describes the concept of arithmetic mean (levelling or balance point)		→	★				
10. Calculates and interprets an arithmetic mean **Note** : In Secondary Cycle One, the arithmetic mean is calculated using positive or negative numbers written in decimal notation or using positive numbers written in fractional notation.		→	★				
11. Determines and interprets							
a. measures of central tendency: mode, median, weighted mean				★			
b. measures of dispersion :							
i. range		→	★				
ii. range of each part of a box-and whisker plot, interquartile range				★			
iii. mean deviation					★		CST
					★		TS
							S
iv. standard deviation							CST
					★		TS
							S
c. measures of position :							
i. minimum, maximum		→	★				
ii. percentile **Note** : Percentile is determined using a sufficient number of data. Students can also determine corresponding data from a percentile.					★		CST
							TS
							S
12. Chooses the appropriate statistical measures for a given situation		→	★				CST
							TS
							S
B. Two-variable distributions	**6**	**1**	**2**	**3**	**4**	**5**	
1. Compares experimental and theoretical data **Note** : In Secondary III, the study of linear and rational functions is introduced through the use of scatter plots.				★			
2. Represents data using a scatter plot or a double-entry (two-variable) distribution table					★		

3. Associates the most appropriate functional model with a scatter plot :							
a. first-degree polynomial function					★		CST
					★		TS
					★		S
b. functions under study **Note** : In TS, technological tools should be used for models that are not linear.							CST
					★		TS
							S
4. Describes and interprets the relationship between two variables, if any					★		
5. Gives a qualitative assessment of a linear correlation **Note** : In TS, qualitative assessment should be used for nonlinear models.					★		
6. Approximates and interprets the linear correlation coefficient **Note** : If necessary, technological tools can be used to determine the value of the correlation coefficient for the models under study.					★		
7. Draws a curve associated with the chosen model **Note** : In Secondary V, scatter plots are studied with functions.					★		CST
					★		TS
					★		S
8. Represents a regression line algebraically or graphically **Note** : In addition to drawing this line freehand, students may use other methods, such as the median-median line or the Mayer line method.					★		
9. Interpolates or extrapolates values using							
a. a regression line					★		CST
					★		TS
					★		S
b. the functional model best suited to the situation							CST
					★		TS
						★	S
10. Compares two-variable distributions					★		CST
					★		TS
					★		S

1. A population is a set of entities (e.g. individuals of a species, facts) included in a statistical study.

Mathematics

Geometry

As part of their mathematical training, students go from using intuitive, inductive geometry, based on observation, to using deductive geometry. They discover the properties of figures by constructing and observing them. Little by little, they stop relying on measuring and start to use deduction as the basis for their reasoning. By referring to data, initial hypotheses or accepted properties, students prove statements that they believe are true (known as conjectures), which are then used to prove new ones.

In elementary school, students developed their measurement sense[1] by making comparisons and estimates and taking various measurements using conventional and unconventional units of measure. They designed and built measuring instruments and used invented and conventional ones. They calculated direct and indirect measurements.[2] They also located numbers on an axis and in a Cartesian plane. They constructed and compared different solids, focusing on prisms and pyramids. They learned to recognize the nets of convex polyhedrons and tested Euler's theorem. They described circles and described and classified quadrilaterals and triangles. They observed and produced frieze patterns and tessellations, using reflections and translations. Lastly, they estimated and determined different measurements: lengths, angles, surface areas, volumes, capacities, masses, time and temperature.

In Secondary Cycle One, students construct and manipulate relations or formulas, particularly when calculating the perimeter and area of geometric figures,[3] using arithmetic and algebraic concepts and processes. They learn the concept of similar figures, look for unknown figures resulting from a similarity transformation, determine arc measurements and calculate the area of segments, using the concept of proportionality. By studying lines, plane figures and solids, students identify properties and relationships between measurements. Lastly, they are introduced to deductive reason, in which they use different statements (definitions, properties, axioms, previously proven conjectures) to justify the steps in their approach or validate conjectures.

In Secondary Cycle Two, students construct and manipulate relations or formulas when calculating the area and volume of solids and determining unknown measurements in right triangles or other triangles, using metric and trigonometric relations. If necessary, they convert various units of measure. They refine their understanding of congruence or similarity, particularly by studying the conditions that allow them to conclude that triangles are congruent or similar. They analyze and optimize situations using the concept of equivalent geometric figures. The concept of vectors is introduced and builds on what students have learned about linearity in the previous cycle. In these various contexts, students use different types of reasoning, particularly deductive reasoning, to validate conjectures.

The following tables present the learning content associated with geometry. By basing themselves on the concepts and processes targeted, students develop the three competencies of the program, which in turn enable students to better integrate the mathematical concepts and processes presented.

- Spatial sense and analyzing situations involving geometric figures
- Analyzing situations involving measurements

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1. Unlike in elementary school, in secondary school, measurement is part of geometry.
2. Calculating a perimeter or area and graduating a ruler are examples of direct measurements. Reading a scale drawing, making a scale drawing, measuring an area of a figure by decomposing it, calculating the thickness of a sheet of paper based on the thickness of several sheets are examples of indirect measurements.
3. In a geometric space of a given dimension (0, 1, 2 or 3), a geometric figure is a set of points representing a geometric object such as a point, line, curve, polygon or polyhedron.

Geometry

Spatial sense and analyzing situations involving geometric figures

Spatial sense and analyzing situations involving geometric figures

→ Student constructs knowledge with teacher guidance.

★ Student applies knowledge by the end of the school year.

Student reinvests knowledge.

	Elementary	Secondary Cycle One		Secondary Cycle Two		
A. Plane figures	**6**	**1**	**2**	**3**	**4**	**5**
1. Describes convex and nonconvex polygons						
2. Describes and classifies quadrilaterals						
3. Describes and classifies triangles	★					
4. Describes circles: radius, diameter, circumference, central angle	★					
5. Recognizes and names regular convex polygons		★				
6. Decomposes plane figures into circles (sectors), triangles or quadrilaterals		→	★			
7. Describes circles and sectors		→	★			
8. Recognizes and draws main segments and lines						
a. diagonal, altitude, median, perpendicular bisector, bisector, apothem, radius, diameter, chord		→	★			
b. leg, hypotenuse				★		
9. Identifies the properties of plane figures using geometric transformations and constructions **Note** : See the Secondary Cycle One Mathematics program, p. 219.		→	★			
10. Justifies statements using definitions or properties[1] of plane figures		→	★			
B. Solids	**6**	**1**	**2**	**3**	**4**	**5**
1. Matches the net of a convex polyhedron to the corresponding convex polyhedron	★					
2. Determines the possible nets of a solid		→	★			
3. Names the solid corresponding to a net		→	★			
4. Describes solids :						
a. vertex, edge, base, face	★					
b. altitude, apothem, lateral face		→	★			
5. Tests Euler's relation on convex polyhedrons **Note** : In CST in Secondary V, this relation can be put to use (planar graph). See Avenues of Exploration, Secondary Cycle Two Mathematics program, p. 124.	★					CST TS S
6. Recognizes solids that can be split into						
a. right prisms, right cylinders, right pyramids		→	★			

b. right cones and spheres				★			
7. Represents three-dimensional figures in the plane, using different procedures : – net – projection and perspective (e.g. orthogonal projections [different views], parallel projections [cavalier and axonometric perspectives] or central projections [with one or two vanishing points])				★			
C. Geometric constructions and transformations in the Euclidian plane[2]	**6**	**1**	**2**	**3**	**4**	**5**	
1. Observes and produces frieze patterns and tessellations using reflections and translations	★						
2. Identifies properties and invariants resulting from geometric constructions and transformations		→	★				
3. Identifies congruence (translation, rotation and reflection) between two figures		→	★				
4. Constructs the image of a figure under a translation, rotation and reflection		→	★				
5. Recognizes dilatation with a positive scale factor		→	★				
6. Constructs the image of a figure under a dilatation with a positive scale factor		→	★				
D. Congruent, similar or equivalent figures	**6**	**1**	**2**	**3**	**4**	**5**	
1. Identifies congruent figures in frieze patterns and tessellations	★						
2. Recognizes congruent or similar figures		→	★				
3. Recognizes the geometric transformation(s) linking a figure and its image		→	★				
4. Determines the properties and invariants of congruent or similar figures		→	★				
5. Determines the minimum conditions required to conclude that triangles are congruent or similar **Note** : See Avenues of Exploration in Appendix E of the Secondary Cycle Two Mathematics program.					★		
6. Demonstrates the congruence or similarity between triangles or finds unknown measurements using minimum conditions					★		
7. Recognizes equivalent figures (plane figures or solids)					 ★	★ ★ 	CST TS S
8. Justifies statements using definitions or properties of congruent, similar or equivalent figures, depending on the cycle and year		→	★				

1. In all statements involving justification, the properties used were identified through exploration or have been proven.
2. Geometric transformations in the Cartesian plane are not covered in Secondary Cycle One.

Mathematics

Geometry

Analyzing situations involving measurements

Analyzing situations involving measurements[1]

→ Student constructs knowledge with teacher guidance. ★ Student applies knowledge by the end of the school year. Student reinvests knowledge.	Elementary	Secondary					
		Cycle One		Cycle Two			
A. Mass	**6**	**1**	**2**	**3**	**4**	**5**	
1. Chooses the appropriate unit of mass for the context							
2. Estimates and measures mass using unconventional units: grams, kilograms	★						
3. Establishes relationships between units of mass	★						
B. Time	**6**	**1**	**2**	**3**	**4**	**5**	
1. Chooses the appropriate unit of time for the context							
2. Estimates and measures time using conventional units							
3. Establishes relationships between units of time: second, minute, hour, day, daily cycle, weekly cycle, yearly cycle	★						
4. Distinguishes between duration and position in time **Note** : This includes the concept of negative time, where the start time $t = 0$ is arbitrarily chosen.		→	★				
C. Angles	**6**	**1**	**2**	**3**	**4**	**5**	
1. Compares angles: acute angle, right angle, obtuse angle							
2. Estimates and determines the degree measure of angles	★						
3. Describes the characteristics of different types of angles: complementary, supplementary, adjacent, vertically opposite, alternate interior, alternate exterior and corresponding		→	★				
4. Determines measures of angles using the properties of the following angles: complementary, supplementary, vertically opposite, alternate interior, alternate exterior and corresponding		→	★				
5. Finds unknown measurements using the properties of figures and relations							
a. measures of angles in a triangle		★					
b. degree measures of central angles and arcs		→	★				
6. Defines the concept of radian						 ★ ★	CST TS S
7. Determines the correspondence between degrees and radians						 ★ ★	CST TS S
8. Justifies statements using definitions or properties associated with angles and their measures		→	★				

D. Length	6	1	2	3	4	5	
1. Chooses the appropriate unit of length for the context							
2. Estimates and measures the dimensions of an object using conventional units: millimetre, centimetre, decimetre, metre and kilometre	★						
3. Establishes relationships between							
a. units of length: millimetre, centimetre, decimetre, metre and kilometre	★						
b. measures of length of the international system (SI)		★					
4. Constructs relations that can be used to calculate the perimeter or circumference of figures		→	★				
5. Finds the following unknown measurements, using properties of figures and relations							
a. perimeter of plane figures							
b. a segment in a plane figure, circumference, radius, diameter, length of an arc, a segment resulting from an isometry or a similarity transformation		→	★				
c. segments in a solid resulting from an isometry or a similarity transformation				★			
d. segments or perimeters resulting from equivalent figures					 ★	★ ★ 	CST TS S
6. Justifies statements concerning measures of length		→	★				

E. Area	6	1	2	3	4	5	
1. Chooses the appropriate unit of area for the context							
2. Estimates and measures surface areas using conventional units: square centimetre, square decimetre, square metre	★						
3. Establishes relationships between SI units of area		→	★				
4. Constructs relations that can be used to calculate the area of plane figures: quadrilateral, triangle, circle (sectors) **Note** : Using relations established for the area of plane figures and the net of solids, students identify relationships to calculate the lateral or total area of right prisms, right cylinders and right pyramids.		→	★				
5. Uses relations that can be used to calculate the area of a right cone and a sphere				★			
6. Finds unknown measurements, using properties of figures and relations							
a. area of circles and sectors		→	★				
b. area of figures that can be split into circles (sectors), triangles or quadrilaterals		→	★				
c. lateral or total area of right prisms, right cylinders and right pyramids		→	★				
d. lateral or total area of solids that can be split into right prisms, right cylinders or right pyramids		→	★				
e. area of figures resulting from an isometry		→	★				
f. area of figures resulting from a similarity transformation **Note** : In similar plane figures, the ratio of the areas is equal to the square of the similarity ratio.			→	★			
g. area of a sphere, lateral or total area of right cones and decomposable solids				★			
h. area of equivalent figures					 ★	★ ★ 	CST TS S
7. Justifies statements concerning measures of area		→	★				

F. Volume	6	1	2	3	4	5	
1. Chooses the appropriate unit of volume for the context							
2. Estimates and measures volume or capacity using conventional units: cubic centimetre, cubic decimetre, cubic metre, millilitre, litre	★						
3. Establishes relationships between SI units of volume				★			
4. Establishes relationships between							
a. capacity units : millilitre, litre							
b. measures of capacity				★			
c. measures of volume and of capacity				★			
5. Constructs relations that can be used to calculate volumes: right cylinders, right pyramids, right cones and spheres				★			
6. Finds unknown measurements using properties of figures and relations							
a. volume of right prisms, right cylinders, right pyramids, right cones and spheres				★			
b. volume of solids that can be split into right prisms, right cylinders, right pyramids, right cones and spheres				★			
c. volume solids resulting from an isometry or a similarity transformation **Note** : In similar solids, the ratio of the volumes is equal to the cube of the similarity ratio.				★			
d. volume of equivalent solids						★	CST
						★	TS
					★		S
7. Justifies statements concerning measures of volume or capacity				★			

G. Metric or trigonometric relations	6	1	2	3	4	5	
1. Determines, through exploration or deduction, different metric relations associated with plane figures		→	→	→	→	→	
2. Finds unknown measurements in various situations							
a. in a right triangle rectangle using							
i. Pythagorean relation				★			
ii. the following metric relations – The length of a leg of a right triangle is the geometric mean between the length of its projection on the hypotenuse and the length of the hypotenuse. – The length of the altitude to the hypotenuse of a right triangle is the geometric mean between the lengths of the segments of the hypotenuse. – The product of the lengths of the legs of a right triangle is equal to the product of the length of the hypotenuse and the length of the altitude to the hypotenuse.					★		
iii. trigonometric ratios: sine, cosine, tangent **Note** : In TS and S, students also use cosecant, secant and cotangent in Secondary V.					★		
b. in any triangle using							
i. sine law					★		CST
						★	TS
					★		S
ii. cosine law						★	CST
						★	TS
					★		S

iii. Hero's formula **Note** : In TS and S, this formula may be provided and used, if necessary.					★		CST
							TS
							S
c. in a circle: measure of arcs, chords, inscribed angles, interior angles and exterior angles **Note** : See Avenues of Exploration, Secondary Cycle Two Mathematics program, p. 127.							CST
						★	TS
							S
3. Calculates the area of a triangle given the measure of an angle and the lengths of two sides or given the measures of two angles and the length of one side					★		
4. Proves trigonometric identities by using algebraic properties, definitions (sine, cosine, tangent, cosecant, secant, cotangent), Pythagorean identities, and the properties of periodicity and symmetry **Note** : Formulas for finding the sum or difference of angles are compulsory in S only.							CST
						★	TS
						★	S
5. Justifies statements concerning							
a. Pythagorean relation				★			
b. metric or trigonometric relations					★		
H. Vectors in the Cartesian or Euclidian plane	**6**	**1**	**2**	**3**	**4**	**5**	
1. Defines a vector: magnitude (length or norm), direction, sense **Note** : In Secondary Cycle One, vectors are used in translations.							CST
						★	TS
						★	S
2. Represents a vector graphically (arrow in a plane or pair in a Cartesian plane) **Note** : In TS, students may use a matrix with geometric transformations.							CST
						★	TS
						★	S
3. Identifies properties of vectors							CST
						★	TS
						★	S
4. Performs operations on vectors **Note** : In TS, operations on vectors are performed in context.							
a. determination of the resultant or projection of a vector							CST
						★	TS
						★	S
b. addition and subtraction of vectors							CST
						★	TS
						★	S
c. multiplication of a vector by a scalar							CST
						★	TS
						★	S
d. scalar product of two vectors							CST
						★	TS
						★	S
e. linear combination of vectors							CST
							TS
						★	S
f. application of Chasles relation							CST
							TS
						★	S
5. Justifies statements using properties associated with vectors							CST
							TS
						★	S
6. Analyzes and models situations using vectors (e.g. displacements, forces, speeds or velocities)							CST
						★	TS
						★	S

1. Depending on the context, measurement prefixes (e.g. *nano*, *micro*, *milli*, *deca*, *kilo*, *mega*, *giga*) are introduced.

Mathematics
Analytic Geometry

Analytic geometry provides a link between geometry and algebra. It allows students to represent geometric objects using equations and inequalities. Students therefore work on representations in a Cartesian plane.

In Secondary Cycle One, students perfect their ability to locate points in the Cartesian plane, using the types of numbers under study. They learn to represent a situation generally, using a graph.

In Secondary Cycle Two, students learn to model and analyze situations using a Cartesian reference point. They calculate distances, determine the coordinates of a point of division and study geometric loci. Depending on the option, they use coordinates to perform geometric transformations and determine results in a standard unit circle.

The following tables present the learning content associated with analytic geometry. By basing themselves on the concepts and processes targeted, students develop the three competencies of the program, which in turn enable students to better integrate the mathematical concepts and processes presented.

Analyzing situations using analytic geometry

→ Student constructs knowledge with teacher guidance.

★ Student applies knowledge by the end of the school year.

Student reinvests knowledge.

	Elementary	Secondary					
		Cycle One		Cycle Two			
A. Locating	6	1	2	3	4	5	
1. Locates objects/numbers on an axis, based on the types of numbers studied **Note** : In Secondary Cycle One, students locate positive or negative numbers written in decimal or fractional notation.	★	→	★				
2. Locates points in a Cartesian plane, based on the types of numbers studied (x- and y-coordinates of a point)	★	→	★				
B. Straight lines and half-planes	6	1	2	3	4	5	
1. Uses the concept of change to							
a. calculate the distance between two points **Note** : In Secondary III, students are introduced to the concept of distance between two points while studying the Pythagorean relation. In Secondary IV, the distance between two parallel lines or from a point to a line or segment is studied using concepts and processes associated with distance and equations systems.				→	★		
b. determine the coordinates of a point of division using a given ratio (including the coordinates of a midpoint) **Note** : In S, students can also determine the coordinates of a point of division using the product of a vector and a scalar.					★		CST
					★		TS
						★	S
c. calculate and interpret a slope **Note** : In Secondary III, students are introduced informally to the concept of slope while studying the rate of change of functions (degree 0 and 1).				→	★		
2. Determines the relative position of two straight lines using their respective slope (intersecting at one point, perpendicular, non-intersecting parallel or coincident) **Note** : In Secondary III, students are introduced to the concept of relative position between two lines when comparing the rate of change and graphs of functions (degree 0 and 1). The same is true for solving systems of linear equations in two variables.				→	★		
3. Models, with or without technological tools, a situation involving							
a. straight lines: graphically and algebraically **Note : In Secondary III, students are introduced informally to the concept of lines when they study functions of degree 0 and 1. The different forms of equations of a line (standard, general and symmetric) are explored in the various options. The symmetric form of the equation of a line is not covered in CST; it is optional in TS and compulsory in S. The general form of the equation of a line is optional in CST.**				→	★		
b. a half-plane: graphically and algebraically						★	CST
					★		TS
					★		S
c. parallel lines and perpendicular lines							

	6	1	2	3	4	5	
4. Determines the equation of a line using the slope and a point or using two points **Note** : The general form of the equation of a line is optional in CST.					★		
5. Determines the equation of a line parallel or perpendicular to another **Note** : The general form of the equation of a line is optional in CST.					★		
C. Geometric transformations	**6**	**1**	**2**	**3**	**4**	**5**	
1. Identifies, through observation, the characteristics of geometric transformations in the Cartesian plane: translations, rotations centred at the origin, reflections with respect to the x-axis and y-axis, dilatations centred at the origin, scaling (expansions and contractions)							CST
						★	TS
							S
2. Defines algebraically the rule for a geometric transformation **Note** : In TS, students may also use a matrix to define a geometric transformation.							CST
						★	TS
							S
3. Constructs, in the Cartesian plane, the image of a figure using a transformation rule **Note** :In TS, students also determine the vertices of an image using a matrix.							CST
						★	TS
							S
4. Anticipates the effect of a geometric transformation on a figure							CST
						★	TS
							S
D. Geometric loci	**6**	**1**	**2**	**3**	**4**	**5**	
1. Describes, represents and constructs geometric loci in the Euclidian and Cartesian planes, with or without technological tools **Note** : In S, the study of geometric loci is limited to conics.							CST
						★	TS
						★	S
2. Analyzes and models situations involving geometric loci in the in the Euclidian and Cartesian planes **Note** : In TS, geometric loci also include plane loci, i.e. geometric loci involving lines or circles only. In S, the study of geometric loci is limited to conics.							CST
						★	TS
						★	S
3. Analyzes and models situations using conics – describing the elements of a conic: radius, axes, directrix, vertices, foci, asymptotes, regions – graphing a conic and its internal and external region – constructing the rule of a conic based on its definition – finding the rule (standard form) of a conic and its internal and external region – validating and interpreting the solution, if necessary							
a. parabola centred at the origin and resulting from a translation							CST
						★	TS
						★	S
b. circle, ellipse and hyperbola centred at the origin							CST
						★	TS
						★	S
c. circle, ellipse and hyperbola resulting from a translation							CST
						★	TS
							S
4. Determines the coordinates of points of intersection between							
a. a line and a conic **Note** : In TS, this is associated with solving systems involving the functional models under study and entails mostly graphical solutions (with or without the use of technological tools).							CST
						★	TS
						★	S
b. two conics (a parabola and a conic)							CST
							TS
						★	S

E. Standard unit circle	6	1	2	3	4	5	
1. Establishes the relationship between trigonometric ratios and the standard unit circle (trigonometric ratios and lines)							CST
						★	TS
						★	S
2. Determines the coordinates of points associated with significant angles using metric relations in right triangles (Pythagorean relation, properties of angles: 30°, 45°, 60°)							CST
						★	TS
						★	S
3. Analyzes and uses periodicity and symmetry to determine coordinates of points associated with significant angles in the standard unit circle							CST
						★	TS
						★	S
4. Proves Pythagorean identities							CST
						★	TS
						★	S

Mathematics

Discrete Mathematics

Discrete mathematics is a branch of mathematics that focuses mainly on situations involving finite sets and countable objects. Its focus of study involves ali areas of mathematics and numerous applications in a variety of fields: transportation, telecommunications, health, programming, etc. This section covers the concepts and processes more directly related to graphs, social choice and matrices.

- Graphs
- Social Choice Theory
- Matrices

Mathematics

Discrete Mathematics

Graphs

In learning about graph theory, students in the *Cultural*, *Social and Technical* option acquire new tools for analyzing situations and are introduced to a different way of reasoning. This theory is used to model and, if necessary, to optimize situations in different branches of mathematics (e.g. tree diagrams in probability, the representation of convex polyhedrons [planar graph]) and in various fields such as social sciences, chemistry, biology or computer science. It can be used to relate various elements associated with task planning, scheduling or inventory management, communication or distribution networks, electric or other types of circuits, incompatibilities (interactions), localizations, strategies, and so on.

To draw a graph for a given situation, students must choose the elements that will be represented by vertices and those that will be represented by edges. The terms associated with graphs are introduced as they arise in the situations presented; however, the point is not to memorize a series of definitions. The properties are also introduced during exploration activities.[1]

The following tables present the learning content associated with graphs. By basing themselves on the concepts and processes targeted, students develop the three competencies of the program, which in turn enable students to better integrate the mathematical concepts and processes presented.

Introduction to graph theory

- → Student constructs knowledge with teacher guidance.
- ★ Student applies knowledge by the end of the school year.
- (blue) Student reinvests knowledge.

	Elementary	Secondary					
		Cycle One		Cycle Two			
A. Concepts associated with graph theory	**6**	**1**	**2**	**3**	**4**	**5**	
1. Describes the basic elements of graph theory: degree distance, path, circuit						★	CST
							TS
							S
2. Recognizes Euler paths or circuits and Hamiltonian paths or circuits						★	CST
							TS
							S
3. Constructs graphs: directed graphs, weighted graphs, coloured graphs, trees						★	CST
							TS
							S
4. Identifies properties of graphs						★	CST
							TS
							S
B. Situation analysis, optimization and decision making	**6**	**1**	**2**	**3**	**4**	**5**	
1. Determines elements of a situation associated with vertices and edges						★	CST
							TS
							S
2. Represents a situation using a graph						★	CST
							TS
							S
3. Compares graphs, if necessary						★	CST
							TS
							S
4. Finds Euler and Hamiltonian paths and circuits, the critical path, the shortest path, the tree of minimum or maximum values or the chromatic number, depending on the situation						★	CST
							TS
							S

1. See Avenues of Exploration in Appendix E of the Secondary Cycle Two Mathematics program, p. 124.

Mathematics

Discrete Mathematics

Social Choice Theory

Mathematical models are used in social, political and economic situations. Some models are used to ensure the fair distribution of individuals and goods, while other models or voting procedures involve aggregating individual preferences in order to clarify the choices to be made in satisfying as many people as possible (e.g. elections, market surveys, classifications). By using the mathematical concepts and processes already acquired, students in the Cultural, Social and Technical option can compare and analyze the different models associated with voting procedures. (*Which method is most accurate? Which method is most representative of the majority? In what way could results be influenced?*)

The following tables present the learning content associated with social choice theory. By basing themselves on the concepts and processes targeted, students develop the three competencies of the program, which in turn enable students to better integrate the mathematical concepts and processes presented.

Introduction to social choice theory							
→ Student constructs knowledge with teacher guidance. ★ Student applies knowledge by the end of the school year. Student reinvests knowledge.		**Secondary**					
		Cycle One		**Cycle Two**			
	6	**1**	**2**	**3**	**4**	**5**	
1. Makes decisions concerning social choices							
a. Counts and enumerates possibilities						★	CST
							TS
							S
b. Compares and interprets different voting procedures and their results **Note** : In cases that involve aggregating individual preferences, situations will be limited to no more than 4 "candidates." In particular, students compare and analyze majority rule, plurality voting, the Borda count, the Condorcet method, the elimination or runoff method and approval voting. See Avenues of Exploration in Appendix E of the Secondary Cycle Two Mathematics program, p. 125.						★	CST
							TS
							S

Mathematics

Discrete Mathematics

Matrices

In Secondary Cycle Two, the study of matrices is integrated into various branches of mathematics in the *Technical and Scientific* option. It is based on situations in which the use of matrices is relevant, and the terminology associated with it is introduced when required.

Matrices are a register of representation (grid, table) that can be used to interpret, process and manipulate effectively several data at the same time. Operations such as matrix addition and matrix multiplication with a scalar or another matrix (e.g. purchases/sales, inventory) form the bases of spreadsheet programs. Matrices can also be used to perform geometric transformations [reflections, translations, rotations,[1] dilatations (uniform scaling or homothety)] by using concepts and processes associated with analytic geometry and trigonometry. Solving systems of equations by using row operations on augmented matrices is another example of how matrices are used.

The following tables present the learning content associated with matrices. By basing themselves on the concepts and processes targeted, students develop the three competencies of the program, which in turn enable students to better integrate the mathematical concepts and processes presented.

Introduction to matrices

→ Student constructs knowledge with teacher guidance.

★ Student applies knowledge by the end of the school year.

(Shaded) Student reinvests knowledge.

	Elementary	Secondary Cycle One		Secondary Cycle Two			
	6	1	2	3	4	5	
1. Understands tables of numbers: lines, columns	(shaded)	(shaded)	(shaded)	(shaded)	(shaded)	(shaded)	
2. Represents, interprets data using matrices							CST
						→	TS
							S
3. Performs operations on matrices: addition and subtraction, multiplication with a scalar and with another matrix							CST
						→	TS
							S
4. Performs geometric transformations (transformation matrices)							CST
						★	TS
							S
5. Solves systems of equations (augmented matrix)							CST
						→	TS
							S

1. Rotation could be done with measures of significant angles.

Mathematics

Financial Mathematics

During the last year of the Cultural, Social and Technical option, students are introduced to financial mathematics and become familiar with the related vocabulary. Because it is only an introduction, all calculations are performed using previously studied formulas. The following tables present the learning content associated with financial mathematics.

By basing themselves on the concepts and processes targeted, students develop the three competencies of the program, which in turn enable students to better integrate the mathematical concepts and processes presented.

Introduction to financial mathematics

→ Student constructs knowledge with teacher guidance.

★ Student applies knowledge by the end of the school year.

Student reinvests knowledge.

	Elementary	Secondary					
		Cycle One		Cycle Two			
	6	1	2	3	4	5	
1. Describe the concepts related to financial mathematics							
a. Interest rates (simple and compound interest)						★	CST
							TS
							S
b. Interest period						★	CST
							TS
							S
c. Discounting (present value)						★	CST
							TS
							S
d. Compounding (future value)						★	CST
							TS
							S
2. Models financial situations						★	CST
							TS
							S
3. Calculates compounding using the following formula : $C_n = C_0(1+i)^n$ **(where C_n = future value, C_0 = present value, *i* = interest rate and *n* = interest period)** Note : Student may use technological tools.						★	CST
							TS
							S
4. Calculates discounting using the following formula : $C_0 = C_n(1+i)^{-n}$ **or** $C_0 = \frac{C_n}{(1+i)^n}$ **(where C_n = future value, C_0 = present value, *i* = interest rate and *n* = interest period)** Note : Student may use technological tools.						★	CST
							TS
							S
5. Determines values or data by solving equations						★	CST
							TS
							S
6. Compares financial situations						★	CST
							TS
							S
7. Makes decisions, if necessary, depending on the context						★	CST
							TS
							S

Mathematics

Examples of Strategies[1]

The strategies that are helpful for the development and use of the three mathematics competencies are integrated into the learning process. It is possible to emphasize some of these strategies, depending on the situation and educational intent. Since students must build their own personal repertoire of strategies, it is important to encourage them to become independent in this regard and help them learn how to use these strategies in different contexts. Students can be encouraged to explore strategies associated with other subject areas, such as reading strategies, as these can be very useful to fully understand all aspects of a question or situation. Please note that the strategies listed below can be used in any order or sequence.

Cognitive and metacognitive strategies	
Planning	▪ What is the task that I am being asked to perform? ▪ What concepts and processes do I need to use? ▪ What information is relevant, implicit or explicit? ▪ Is some information missing? ▪ Do I need to break the task down? ▪ How much time will I need to perform this task? ▪ What resources will I need? ▪ What do I need to establish a work plan?
Comprehension and discrimination	▪ Am I able to extract the information contained in the registers (type) of representation involved? ▪ Which terms seem to have a mathematical meaning different from their meaning in everyday language? ▪ What is the purpose of the task? Am I able to explain it in my own words? ▪ Do I need to find a counterexample to prove that what I am stating is false? ▪ Is all the information pertaining to the situation relevant? Is some information missing? ▪ Is there any way I can illustrate the steps involved in the task?
Organization	▪ Should I group, list, classify, reorganize or compare data? Should I use diagrams to show the relationships between objects or data? ▪ Can I use objects or technological tools to simulate the situation? ▪ Can I use a table or chart? Should I draw up a list? ▪ Are the main ideas in my approach well represented? ▪ What concepts and mathematical processes should I use? ▪ What registers (types) of representation (words, symbols, figures, graphs, tables, etc.) could I use to translate this situation?

Development	▪ Can I represent the situation mentally or in written form?
	▪ Have I solved a similar problem before?
	▪ What additional information could I find using the information I already have?
	▪ What mathematical concepts could apply? What related properties or processes could I use?
	▪ Have I used the information that is relevant to the task? Have I considered the unit of measure, if applicable?
	▪ Can I see a pattern?
	▪ Which of the following strategies could I adopt?
	    ▪ Use trial and error
	    ▪ Work backwards
	    ▪ Give examples
	    ▪ Make suppositions
	    ▪ Break the task down
	    ▪ Change my point of view or strategy
	    ▪ Eliminate possibilities
	    ▪ Simplify the task (e.g. reduce the number of data, replace values by values that can be manipulated more readily, rethink the situation with regard to a particular element or case)
	    ▪ Translate (mathematize) a situation using a numeric or algebraic expression

Regulation and control	■ Is my approach effective and can I explain it? ■ Can I check my solution using reasoning based on an example or a counterexample? ■ What have I learned? How did I learn it? ■ Did I choose an effective reading strategy and take the time I needed to fully understand the task? ■ What are my strengths and weaknesses? ■ Did I adapt my approach to the task? What was the expecte result? ■ How can I explain the difference between the expected result and the actual result? ■ What strategies used by my classmates or suggested by the teacher can I add to my repertoire of strategies? ■ Can I use this approach in other situations?
Generalization	■ In what ways are the examples similar or different? ■ Which models can I use again? ■ Can the observations made in a particular case be applied to other situations? ■ Are the assertions I made or conclusions I drew always true? ■ Did I identify examples or counterexamples? ■ Did I see a pattern? ■ Am I able to formulate a rule? ■ Am I able to interpolate or extrapolate?
Retention	■ Is what I learned connected in any way to what I already know? ■ Which concepts are the most important for identifying other concepts? ■ Under what conditions does a certain process work? On what properties is it based? ■ Am I able to illustrate or modify the concepts and processes I know? ■ What characteristics would a situation need in order for me to reuse the same strategy? ■ Would I be able to repeat the task again on my own? ■ What methods did I use (e.g. repeated something several times to myself or out loud; highlighted, underlined, circled, recopied important concepts; made a list of terms or symbols)?
Development of automatic processes	■ Did I find a solution model and list the steps involved? ■ Did I practise enough in order to be able to repeat the process automatically? ■ Am I able to effectively use the concepts learned? ■ Did I compare my approach to that of others?
Communication	■ Did I show enough of my work so that my approach was understandable? ■ What registers (types) of representation (e.g. words, symbols, figures, diagrams/graphs, tables) did I use to interpret a message or convey my message? ■ Did I experiment with different ways of conveying my mathematical message? ■ What method could I use to convey my message? ■ What methods would have been as effective, more effective or less effective? ■ Did I follow the rules and conventions of mathematical language? ■ Did I adapt my message to the audience and the communication intent? How can I adapt it?

Other strategies	
Affective strategies	■ How do I feel? ■ What do I like about this situation? ■ Am I satisfied with what I am doing? ■ What did I do particularly well in this situation? ■ What methods did I use to overcome difficulties and which ones helped me the most to reduce my anxiety? stay on task? control my emotions? stay motivated? ■ Am I willing to take risks? ■ What did I succeed at? ■ Do I enjoy exploring mathematical situations?
Resource management strategies	■ Whom can I turn to for help and when should I do so? ■ Did I accept the help offered? ■ What documentation (e.g. glossary, ICT) should I use? Will it be helpful? ■ What manipulatives can help me in my task? ■ Did I estimate correctly the time needed for the activity? ■ Did I plan my work well (e.g. planned short, frequent work sessions; set goals to attain for each session)? ■ What methods should I use to stay on task (e.g. appropriate environment, available materials)?

1. These examples are based on strategies developed by the students in elementary school. They are considered to be necessary, if not indispensable, regardless of the level.