Project: Dimensional Analysis

How big do you think one of the blocks that make up the Cheops Pyramid at Giza is?

This question actually came up in conversation with some friends at our house a few years back. Our conversation had turned to climbing, then climbing ridiculous things, then the pyramids at Giza...well, you get the picture. And if you can't, then there's Khufu's pyramid at right.

Someone there wanted to know how big the stones were. I remembered reading that they were limestone, and they weighed around 2 and a half tons each. I ran inside our house and Googled the density of limestone. After some rough and ready back-of-the-envelope calculations, I told my friends that the blocks, on average, were approximately 1.5 cubic yards in volume. To visualize this, imagine a space a little bit smaller than an F-150 cargo bed…it's in roughly the same size.

So now, maybe, you're wondering: how did I do that conversion? Well, that's what this project is all about. As I move through life, I'm amazed at how often people need to do this kind of math, but sometimes feel stymied by it. It's very, very simplistic math (in fact, it's not even algebra!), but it's one of the most powerful types. It allows you to do everything from calculating how much money COCC could save by turning off all of our computers at night (around $45,000 ± $10,000 annually) to finding out how much oil flowed out of the Deepwater horizon explosion's hole in the Gulf of Mexico (roughly 5 million barrels)[a]. This math is called **dimensional analysis**. My stat classes routinely use it to check the "facts" they hear on the news every day.

Dimensional analysis gives you the method needed to switch between one unit of measure and another. People in the medical field have to do this all of the time. For example, a friend, a Doctor of Osteopathy in San Diego, was once trying to publish a paper and needed to convert micromoles per liter of creatinine to milligrams per milliliter. I'll show you how later. Let's start our journey down this badass road!

Note: in the beginning of this project, I'm going to focus on the "pencil and paper" way to do these. Later, in the problems you do, we'll hit Google up and se how it can help us do (some of) the heavy lifting.

* * * * * * * * * * * *

Example 1: Suppose you have to convert my height, 72 inches, to feet. How would you do that? I like to use a method involving multiplication of fractions[b], as follows:

$$72 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}}$$

Start with the quantity you have...

...multiply by a fraction with the units you have on the bottom and the ones you want on the top.

---

[a] Some of you might calculate these numbers in MTH 244, for those of you making that journey.

[b] You might have your own ways! For example, many students who come from chemistry classes do this differently.

Two things to notice here:

1) The fraction you have on the right is equal to 1, since 1 foot is equal to 12 inches. So, since you're multiplying by 1, you're not distorting the original quantity.

2) Since you have inches in the numerator with the 72, and in the denominator with the 12, the inches can be "divided off" (if you took MTH 058 and/or MTH 098, you did this lots!), and only feet will remain.

$$72 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} = 72 \cancel{\text{ inches}} \cdot \frac{1 \text{ foot}}{12 \cancel{\text{ inches}}} = \frac{72 \text{ feet}}{12} = 6 \text{ feet}$$

* * * * * * * * * * * *

<u>Example 2</u>: OK, so that one was pretty simple, right? So let's pretend we're European now. I want to now know how tall I am, in meters. This is fundamentally difficult for Americans, since the metric system is something we fight against pretty readily. We need a conversion, though. One meter is equivalent to roughly 3.28 feet[c]. Let's give 'er a go!

$$6 \text{ feet} \cdot \frac{1 \text{ meter}}{3.28 \text{ feet}} = 6 \cancel{\text{ feet}} \cdot \frac{1 \text{ meter}}{3.28 \cancel{\text{ feet}}} = \frac{6 \text{ meters}}{3.28} \approx 1.83 \text{ meters}$$

<u>A note</u>: the " $\approx$ " means "approximately equal to". Since we rounded off our answer, we're only *close* to the exact number...but close ("ish") is good enough, since the 3.28 was rounded, anyway.

* * * * * * * * * * * *

<u>Example 3</u>: Now, suppose we wanted to go straight from inches to meters:

$$72 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} \cdot \frac{1 \text{ meter}}{3.28 \text{ feet}} = 72 \cancel{\text{ inches}} \cdot \frac{1 \cancel{\text{ foot}}}{12 \cancel{\text{ inches}}} \cdot \frac{1 \text{ meter}}{3.28 \cancel{\text{ feet}}} = \frac{72 \text{ meters}}{12 \cdot 3.28} \approx 1.83 \text{ meters}$$

So you see, you can string together any number of these fractions that you like. Since they're all equal to one, you're only multiplying by 1…so the only thing changed is the unit.

* * * * * * * * * * * *

<u>Example 4</u>: OK, onto weight. I weigh 180(ish) pounds. How much do I weigh in kilograms?

You may have noticed, if you've ever been in a gym with weights, that a pound is about a half of a kilogram. It's actually a little less; a good estimate is that 1 pound is 0.45 kilograms:

$$180 \text{ pounds} \cdot \frac{0.45 \text{ kilograms}}{1 \text{ pound}} = 180 \cancel{\text{ pounds} \cdot} \frac{0.45 \text{ kilograms}}{1 \cancel{\text{ pound}}} = 180 * 0.45 \text{ kg} = \mathbf{81 \text{ kg}}$$

* * * * * * * * * * * *

---

[c] The reason for the goofy number is that the metric system is based on the speed of light (a constant), while the English system is based on random things like king's feet and horse's behinds and such.

At this point, I'm going to share an artifact with you…but not without good reason. Here's a list of fairly common conversions that are often used when math and careers intersect.

Why "artifact", you might ask? Well, you can Google all this stuff now. The only reason I included it here is to make sure you see the dimensionality of some of the conversions; for example ***in³*** is "cubic inches" (a measure of volume), while ***m²*** is "square meters" (a measure of area). To visualize volume, think of how much water you need to fill it. That's its volume. To visualize area, think of how much paint you'd need to paint it.

<u>Units within the Metric system:</u>

Length 1 meter (m) = 100 centimeters (cm) = 1000 millimeters (mm)

Volume 1 Liter (L) = 1000 milliliters (ml)
1 ml = 1 cubic centimeter (cc)

Weight 1 gram (g) = 1000 milligrams (mg)
1 mg = 1000 micrograms (μg)

<u>Units within the English system:</u>

Length 1 foot (ft) = 12 inches (in)
1 yard (yd) = 3 feet (ft)

Volume 1 cubic foot (ft^3) = 1728 cubic inches (in^3)
1 ft^3 = 7.481 US gallons

Weight 2000 pounds (lbs) = 1 ton

<u>Conversions between systems</u>

	Metric	English
Length	2.54 cm	1 in
	0.3048 m	1 ft
	1.609 kilometers (km)	1 mile
Volume	16.3871 cc	1 in^3
	3.785 L	1 US gallon
Weight	1 kilogram (kg)	2.2 lbs
	28.35 g	1 ounce (oz)
Area	0.0929 square meters (m^2)	1 square foot (ft^2)

Also, 1 gallon of water weighs approximately 8 pounds. Of course, there are many, many others. Let's try some more...

* * * * * * * * * * * *

<u>Example 5</u>: If a patient received 3 L of intravenous fluid at a hospital, how many cc's of fluid did she receive?

$$3\ \cancel{L} \cdot \frac{1000\ \cancel{ml}}{1\ \cancel{L}} \cdot \frac{1\ cc}{1\ \cancel{ml}} = 3000\ cc$$

So, she received 3000 cc's.

* * * * * * * * * * * *

<u>Example 6</u>: A man has a body surface area (BSA) of 1.8 m^2. Convert this to square feet.

$$1.8\ m^2 . \left(\frac{1\ ft}{0.3048\ m}\right)^2 = 1.8\ \cancel{m^2} . \frac{1\ ft^2}{0.0929\ \cancel{m^2}} \approx 19.38\ ft^2$$

I had to square the conversion from meters to feet to get square meters to square feet...

* * * * * * * * * * * *

Example 7: Now, we can get a little more challenging...our backyard pond has a volume of about 500 cubic feet. If I were to fill it all the way up, how much would the water weigh, in pounds?

$$500\ \cancel{\text{ft}^3\ \text{H}_2\text{0}} \cdot \frac{7.481\ \cancel{\text{gallons}}}{1\ \cancel{\text{ft}^3\ \text{H}_2\text{0}}} \cdot \frac{8\ \cancel{\text{pounds}}}{1\ \cancel{\text{gallon}}} \cdot \frac{1\text{ ton}}{2000\ \cancel{\text{pounds}}} \approx 15\text{ tons}$$

Cool, huh?

* * * * * * * * * * * *

We can also convert rates using this method. Rates, as you may remember, are also called "speeds". For example, suppose you are driving along at 50 miles per hour. How many feet do you cover each second?

Example 8: We use the same procedure, but keep in mind that there is now a numerator as well as a denominator. Let's try it:

$$\frac{50\ \cancel{\text{miles}}}{1\ \cancel{\text{hour}}} \cdot \frac{1\ \cancel{\text{hour}}}{60\ \cancel{\text{minutes}}} \cdot \frac{1\ \cancel{\text{minute}}}{60\text{ seconds}} \cdot \frac{5280\text{ feet}}{1\ \cancel{\text{mile}}} = 73\text{ feet per second}$$

That's pretty fast. See why you should always obey the 4-second rule?

* * * * * * * * * * * *

Example 9: Another from home…our garden hose provides the perfect flow rate for a small tadpole pond we built. I wanted to buy a small pond pump that flows at the same rate. The hose filled a 1 – quart Nalgene bottle with water in 2.2 seconds. What size pump did I purchase to match the flow of the hose?

$$\frac{1\ \cancel{\text{quart}}}{2.2\ \cancel{\text{seconds}}} \cdot \frac{1\text{ gallon}}{4\ \cancel{\text{quarts}}} \cdot \frac{60\ \cancel{\text{seconds}}}{1\ \cancel{\text{minute}}} \cdot \frac{60\ \cancel{\text{minutes}}}{1\text{ hour}} = \frac{3600\text{ gallons}}{8.8\text{ hours}} \approx 400\text{ GPH}$$

* * * * * * * * * * * *

Example 10: Here's the one my friend the D.O. needed...convert 2.3 micromoles/Liter to mg/ml:

$$\frac{2.3\ \cancel{\text{micromoles}}}{\cancel{\text{Liter}}} \cdot \frac{1\ \cancel{\text{Liter}}}{1000\text{ milliliters}} \cdot \frac{880\text{ milligrams}}{1\ \cancel{\text{micromole}}} = 20.24\text{ mg/ml}$$

I had to look this conversion up in a medical manual.

* * * * * * * * * * * *

OK...now it's your turn! **Do each of the following conversions. Round to two decimal places in each, if needed, unless otherwise noted. Remember that answers and hints are given in** **red** **are there for your reference; you still need to show me how to get the answers! And, as always – start up a new document before answering!**

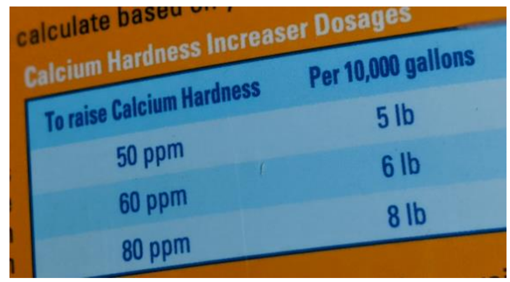
1. (**2 points**) (**w**) A thousand seconds is how many minutes?

2. (**2 points**) (**w**) A million seconds is how many ***days***? **Between a week and two weeks!**

3. (**3 points**) (**w**) A billion seconds is...well, how many ***what***? Pick a good time unit to discuss "1 billion seconds" in, and then convert it for me![d]

   Now, we'll work in using Google (combined with pencil and paper work) to help with conversions!

4. For all you pre-medical field folks, here's a chem example (don't worry; you don't have to know any chemistry to do this problem. ☺) We have a hot tub and, every time we fill it, we have to raise the Calcium Hardness level of the water (apparently, Bend has "soft water"). We have a 5-pound bottle of Calcium Hardness Increaser, and it shows the chart at right (this size bottle is designed to use with huge pools, and not tiny hot tubs).



   a. (**2 points**) (**w**) Figure out how many ***pounds*** of calcium hardness increaser we would have to add to our **300 gallon** hot tub to raise it **50 PPM** ("parts per million"). **You're only going to be using the first line of the that chart here!**

   b. (**3 points**) Let's work on your Google Ninja skills – do some research and figure out a conversion between ***pounds*** of calcium hardness increaser and ***cups*** of calcium hardness increaser. **Note**: you need to find this conversion ***for calcium hardness increaser***! Pounds is a measure of ***weight***, and cups is a measure of ***volume*** (a cup of, say, feathers would weigh a ***LOT*** less than a cup of rocks!) Please include a screen grab of the source you used!

   c. (**2 points**) (**w**) Convert that weight you got in part **a** to cups, using your conversion factor from ***b***. **I get about ¼ of a cup for this, but if you find a different conversion, you might get a slightly different number.**

5. Recently (as I type this in 2020), I read about the first human being to run a marathon in less than 2 hours. His name is Eliud Kipchoge. Watch this video next where we break down just how fast he was moving!

   OK! ***Your*** turn now! In that video, I walked you through how to change a total marathon time into miles per (one) hour, feet per second and minutes per mile. Now, you get to show3 me you know your stuff, but figuring out ***my*** fastest marathon numbers! You might have to go back into the video to find my time again.

---

[d] I like to tell folks that I cried through our son's thousandth second, likely was napping with him during his millionth, and will hopefully be on a hike with him for his billionth. ☺ You can figure your billionth second here!

a. (**2 points**) (**w**) Start by converting my total marathon time to decimal hours (like we did with Kipchoge's 1:59:40). For (**w**), either show me a screengrab of what you did in Google (or your calculator) or work it out by hand. Include the unit!

b. (**2 points**) (**w**) What was my average mile per hour rate, based on what you found in **5a**? Same note about "work".

c. (**3 points**) Contextual question: why was your last answer an "average"? Write a sentence or two explaining! sentences!

d. (**2 points**) (**w**) On average, how many feet was I traveling per second?

e. (**2 points**) (**w**) How many *minutes per mile* is that last speed?

Take ***that***, Eliud. 😂