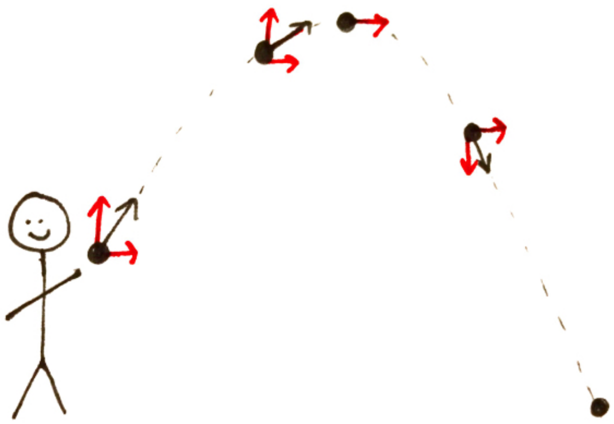
PROJECTILE MOTION

CONTENT

Projectile motion refers to the motion of a particle or object that is influenced by the acceleration due to the Earth's gravity (if we assume there is no air resistance). For example, throwing a ball in the air. Just like in kinematics, we can resolve the velocity of the projectile into its x and y components. (You can revise this in the Kinematics worksheet: Vector Components).

In the example of the ball, once the ball leaves your hands the only acceleration is downwards due to gravity. This means there is no horizontal acceleration. Since there is no horizontal acceleration, there is a constant horizontal velocity. The projectile is a parabola, as shown below. The black vector is the total velocity and the red vectors are the x and y components. Notice how the red horizontal vector doesn't change at different places despite the overall velocity changing.



We can calculate several things from the path of the projectile using the equations of motion. (You can revise these in the Kinematics worksheet: Equations of Motion).

$$s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$
$$v = u + at$$

For each of these equations, we can resolve the displacement, s, the initial velocity, u, the final velocity, v, and the acceleration, a, into their x and y components. This is how to derive the equations for projectile motion. The acceleration in the x-direction is always zero and the acceleration in the y-direction is always due to gravity. So, the acceleration doesn't change with time. This is called **uniform acceleration**.

For example, we can resolve the displacement into the x and y displacement in the first equations in the following way:

$$u_x = u\cos\theta \qquad u_y = u\sin\theta$$
$$v_x = v\cos\theta \qquad v_y = v\sin\theta$$
$$a_x = 0 \qquad a_y = -g$$

$$s = ut + \frac{1}{2}at^2 \qquad s = ut + \frac{1}{2}at^2$$
$$s_x = u\cos\theta\, t \qquad s_y = u\sin\theta\, t - \frac{1}{2}gt^2$$

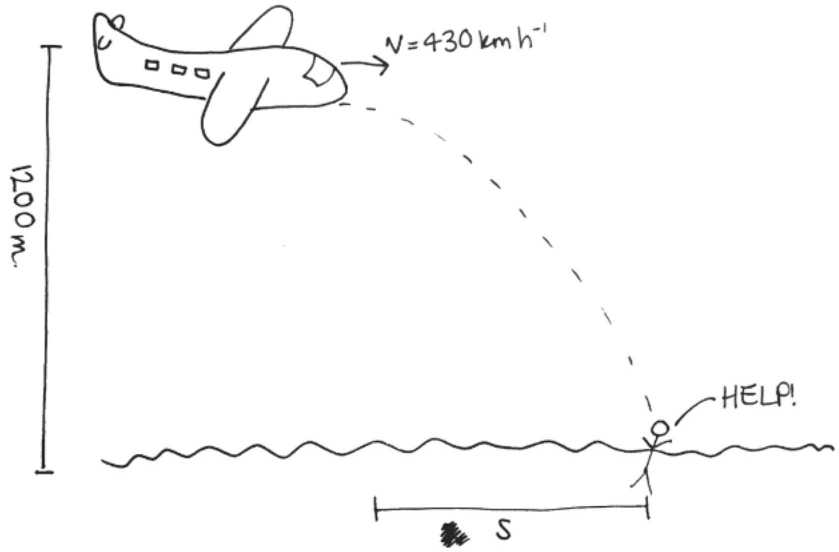
Just like in kinematics, the best way to approach problems is:

1. Draw a diagram of the problem deciding which direction is positive and which is negative
2. Write down all the variables we know and what we're looking for
3. Determine what equation to use to solve the problem

EXAMPLE

A rescue plane is flying at constant elevation of 1200 m with a speed of 430 kmh^{-1} toward a point directly above a person struggling in the water. At what distance should the pilot release a rescue capsule if it is to strike close to the person in the water?

⇒ So firstly, we will draw a diagram of the problem setting downwards as negative and upwards as positive with the origin set at the plane:



⇒ Now, to write down all the variables we have and determine which formula we should use noting, the capsule is released straight ahead so there is no angle, the only acceleration is due to gravity and we are ignoring air resistance:

Variable	Value
s_y	$-1200m$
u_x	$119.4ms^{-1}$
u_y	$0ms^{-1}$
θ	$0°$
a_y	$-9.8ms^{-1}$
s_x	?

Since we need to find s_x first we will need to solve $s = ut + \frac{1}{2}at^2$ in the y direction for t. Then we will sub that value into the same equation so solve for s_x.

⇒ So, finally calculating:

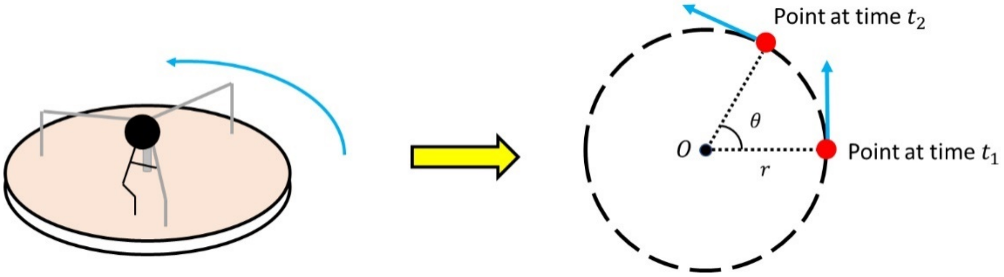
$$s = ut + \frac{1}{2}at^2$$
$$s_y = u_y t + \frac{1}{2}at^2$$
$$-1200 = 0.t + \frac{1}{2}(-9.8)t^2$$
$$-1200 = -4.9t^2$$
$$t^2 = \frac{1200}{4.9}$$
$$\Rightarrow t = 15.649....$$

$$s = ut + \frac{1}{2}at^2$$
$$s_x = u_x t + \frac{1}{2}a_x t^2$$
$$s_x = 119.4 \times 15.65 + \frac{1}{2}(0)(15.65)^2$$
$$= 1869.21...$$
$$\Rightarrow s_x = 1869m$$

CIRCULAR MOTION

CONTENT – UNIFORM CIRCULAR MOTION

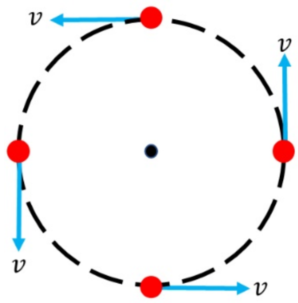
In module 1 we looked at the motion of an object with mass in a straight line. Here in module 5, we will look at the motion of an object moving in a circular path. An example where you might have experienced moving in a circular motion is the Merry-Go-Round, and we *feel* a 'force' pushing us away from the centre. Why do we *feel* this 'force'? Let's use physics to understand this problem and assume that the motion can be represented on a 2D plane with the centre located at the origin O. We will represent the person riding the Merry-Go-Round as an object with *mass* (m) positioned distance (r) away from the origin (i.e. *radius*). If the Merry-Go-Round moves anti-clockwise, then we can represent the motion as



Average speed is equal to distance divided by time, and for circular motion, the total distance travelled is equal to the circumference of the circle ($C = 2\pi r$). The total time it takes for the object to return to its original position is called the period T. Thus the average speed of the object moving in a circle with radius r is

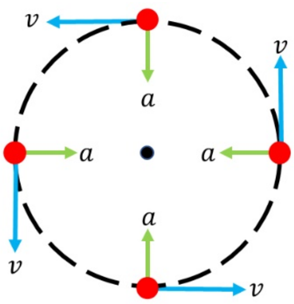
$$|v| = \frac{2\pi r}{T} \tag{1}$$

The velocity of the object at any point on the circumference is equal to the instantaneous speed at that point. The direction of the velocity follows the same path as the motion of the object. Since the motion is circular, the direction of the velocity will change continuously as shown on the right.



It is important to note that although the velocity of the object continuously changes (i.e. direction) the average speed (i.e. the magnitude of the velocity) is the same in uniform circular motion. We can quantify the angular velocity by dividing the change in angle, $\Delta\theta = \theta_2 - \theta_1$, between two points on the circular path over time. In terms of linear velocity the angular velocity is equal to the tangential velocity diveded by the radius.

$$\omega = \frac{\Delta\theta}{t} = \frac{v}{r} \tag{2}$$



For an object in uniform motion, the acceleration of an object is equal to zero as any change in acceleration will change the velocity. This is also true for uniform circular motion; if the acceleration in the direction of the velocity is not zero, then the motion will not be uniform. However, an object moving in a circle does have an acceleration called the '*centripetal acceleration*'. The centripetal acceleration points in the direction of the centre of the circle (perpendicular to the velocity vector). Since the direction is perpendicular to the velocity vector, the average acceleration does not change. This is demonstrated on the image on the left. The equation for centripetal acceleration is given by

$$\vec{a} = \frac{|\vec{v}|^2}{r} \tag{3}$$

Recall that the force of an object given by Newton's 2nd law is $F = ma$. Using this equation, we can calculate the '*centripetal force*' of an object moving in a circular motion in the same direction of the centripetal acceleration.

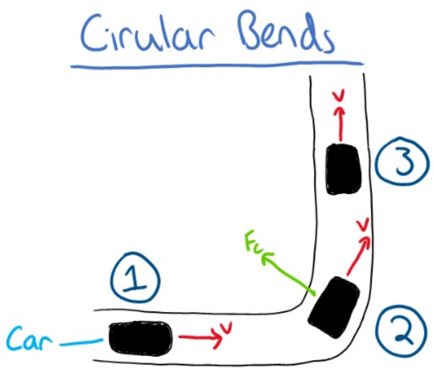
$$\vec{F} = m\frac{|\vec{v}|^2}{r} \quad (4)$$

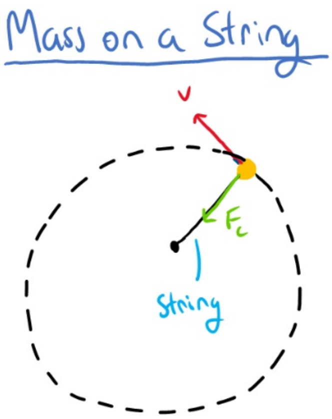
So why do we feel a 'force' pushing us away from the centre? We feel we are being pushed outwardly because the velocity vector is tangential to the circular path. Our body wants to move in a straight path, but since we are holding on the bar of the Merry-Go-Round, there is a centripetal force pointing towards the centre. This prevents us flying out of the Merry-Go-Round, unless of course until we let go of the bar.

Content – Real World Examples

For a car moving around a circular bend the car will experience certain forces. Following the labels on the diagram on the right:

1) The car moves in a straight line to the right with velocity v.
2) As the car makes a turn on the circular bend, the car experiences a centripetal force due to the friction between the tyres and the surface of the road. The passenger inside the car is pushed outwardly in the opposite direction to the centripetal force. The direction of the velocity changes.
3) After the turn the car moves in a straight path again and the centripetal force vanishes.

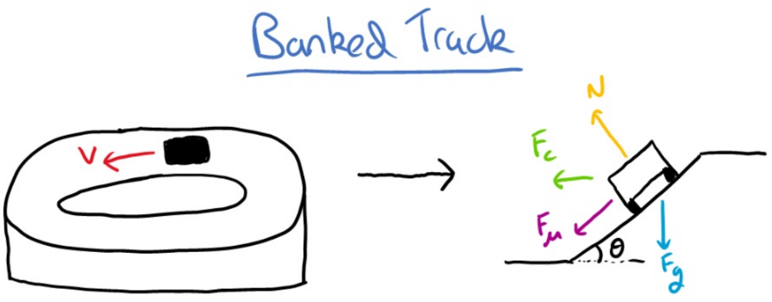




The example on the left is of a mass attached to a string is similar to the Merry-Go-Round example. As we spin the string the mass follows a circular path. The velocity is in the direction of the motion and tangential to the path. The mass at the end of the string experiences a centripetal force pointing to the other end of the string. The centripetal force is a result of the mass attached to the string. If the string breaks the object will fly off the circular path.

The last example on the right is of an object moving on a banked track. The diagram next to the track shows the forces available on the object. The centripetal force is a result of the sum of the frictional F_μ and normal F_N force.

$$F_c = F_\mu + F_N$$



CIRCULAR MOTION – ENERGY AND WORK

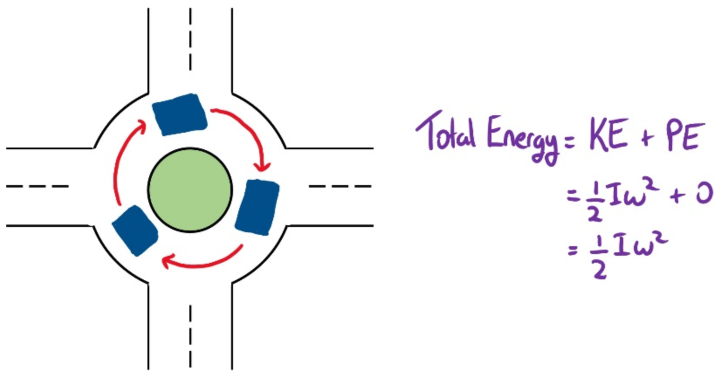
CONTENT – TOTAL ENERGY

We have learned the concept of kinetic energy in *Module 2: Dynamics* and it is given by $\frac{1}{2}mv^2$. The velocity in the equation is for linear velocity. Angular velocity is linear velocity divdied by the radius - $\omega = v/r$. Rearranging this equation to the make linear velocity on the left-hand side and substituting into the kinetic energy expression we get

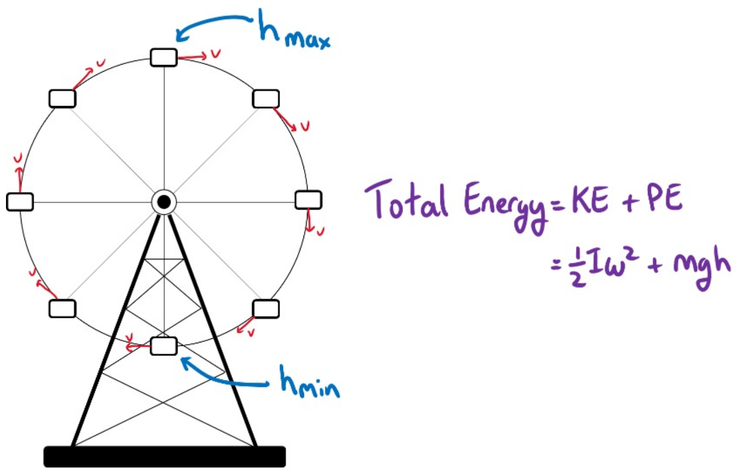
$$\begin{aligned} KE &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}m(\omega r)^2 \\ &= \frac{1}{2}m\omega^2 r^2 \\ &= \frac{1}{2}I\omega^2 \end{aligned} \tag{1}$$

In the last step we have replaced mr^2 with the variable I. This quantity is called the *moment of inertia* or *rotational inertia* and is a measure of how much an object resists rotational motion.

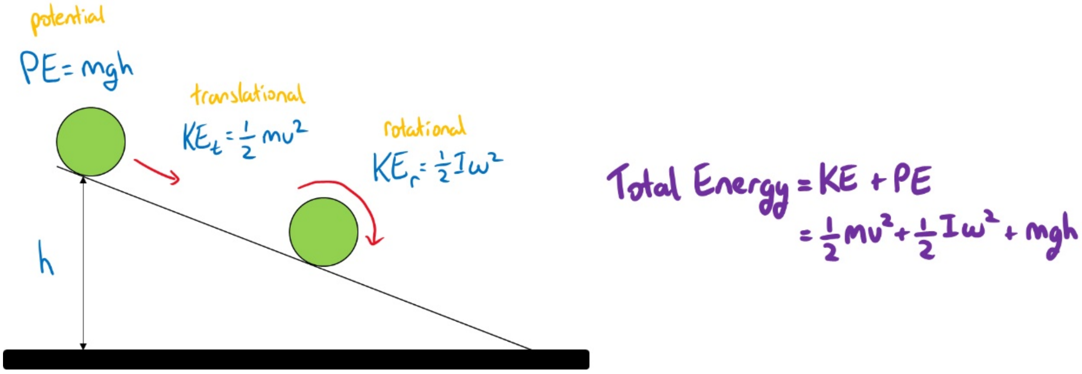
The potential energy of a body is given by $PE = mgh$, which depends on the height. For a circular motion on a flat surface, like a car turning in a roundabout, the height can be considered zero resulting in a zero potential energy. Thus, the total energy for an object in uniform circular motion on a flat horizontal surface is just the rotational kinetic energy as shown in Figure (A) below.



If the circular motion varies with height then the potential energy is not necesarrily zero. Figure (B) shows a Ferris wheel with 8 carrieges. The total energy is equal to the rotational kinetic energy plus the potential energy. The potential energy at the bottom, carriege 5, is at the minimum while at the top, carriege 1, the potential energy is at the maximum. Thus, the total energy of a carriege changes along the circular path.

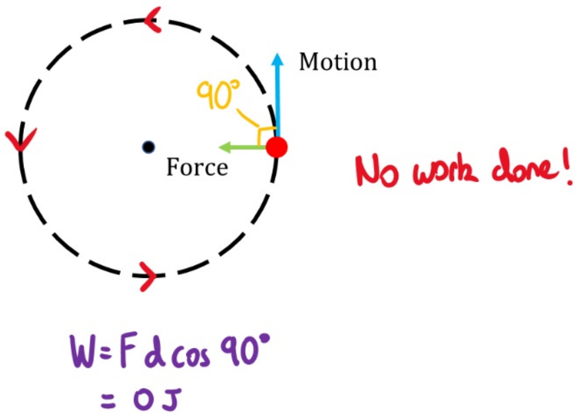


As a final example, consider a ball rolling down a frictionless inclined surface, Figure (C). The ball initially has a potential energy of mgh and as the ball rolls this potential energy is converted into kinetic energy. The ball will move with both translational kinetic energy, $\frac{1}{2}mv^2$, and rotational kinetic energy, $\frac{1}{2}I\omega^2$. Therefore, the total energy of the ball is equal to the sum of both kinetic term and the potential energy.



CONTENT – WORK

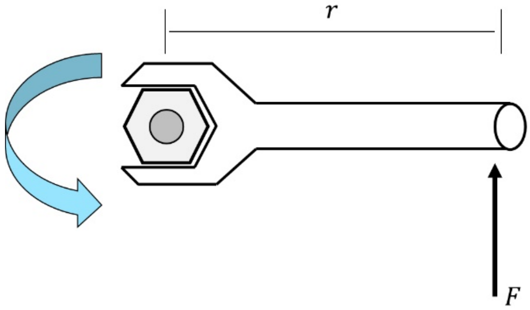
Work is defined as the amount of energy required to move an object from point A to B by a given applied force (i.e. $W = F.d\cos\theta$). If the displacement is zero then the amount of work done is zero no matter how much force is applied to the object. In uniform circular motion the centripetal force is pointing towards the center and the displacement is perpendicular to the force. This means that the angle θ is $90°$, thus the amount of work is zero (i.e $\cos 90° = 0 \rightarrow F = 0$). Alternatively, we can arrive at the same conclusion if we define the work done as the change in kinetic energy (i.e. $W = \Delta KE$). If the object moves with constant velocity around the circular path then the change in kinetic energy is zero, leading to no work being done.



CIRCULAR MOTION – ROTATION AND TORQUE

CONTENT – TORQUE

We will now look at the rotation of a mechanical object where previously we have only looked at the circular motion of a simple object. Let's say we want to loosen a bolt using a wrench. We apply a force on the wrench anti-clockwise direction. If enough force is applied the bolt will be loosen. The tendency for the applied force to cause the rotational motion of the bolt is called *torque*. Torque depends not only of the applied force but also the distance the force is applied to from a pivot. The equation for torque in vector and scalar form is

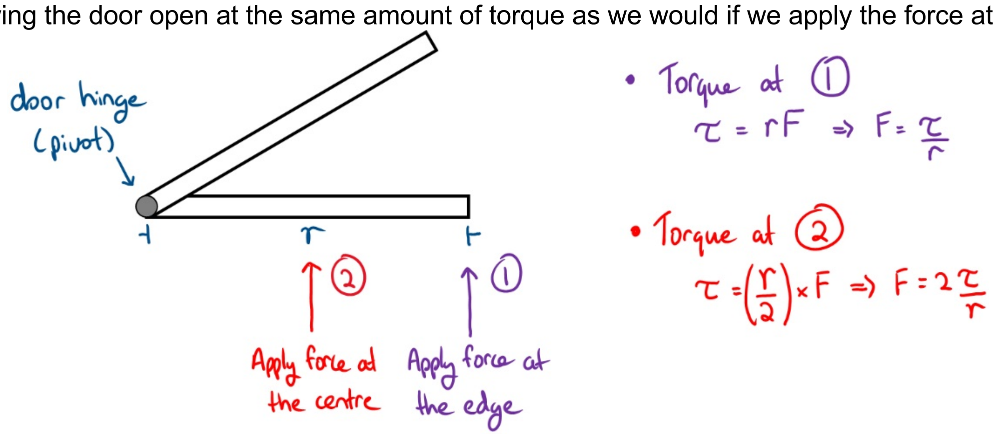


$$\tau = \vec{r}\vec{F}_{\perp} = |\vec{r}||\vec{F}|\sin\theta \qquad (1)$$

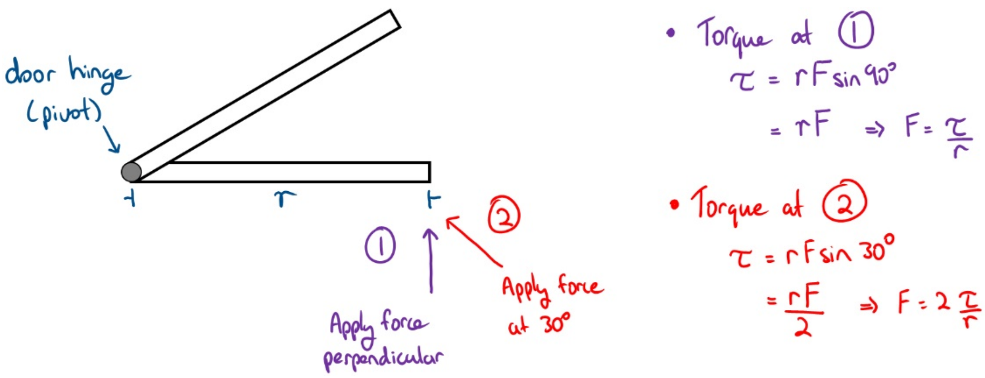
where *r* is the distance from the pivot point and *F* is the applied force. The quantity τ has units of Newton-metre *N.m* or Joule per radian *J/rad*.

REAL WORLD EXAMPLE – OPENING A DOOR

Using the definition in equation (1) we can learn a few things about the rotation of mechical objects. Let's say we want to open the door by pushing at a location half way between the hinges and the handle. We apply a perpendicular force on this point (i.e. $\theta = 90°$), which will make the sine term equal to one. The amount of force required to swing the door open at the same amount of torque as we would if we apply the force at the handle is



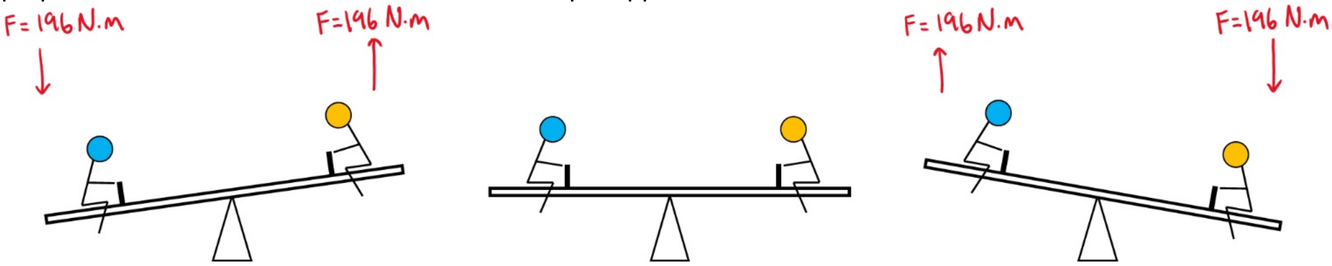
This means that we need to apply two times the amount force on the same door! Another situation we can learn about torque with opening a door is when you apply the force that is not perpendicular. Let's say we apply the force that is $30°$ to the surface of the door at the handle. The amount of force required to swing the door will be two times as well.



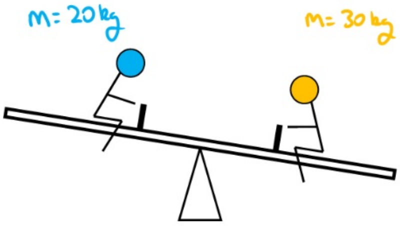
Thus, the reason why doorknobs/handles are located near the edge opposite from the hinges and apply the force perpendicularly is so we only need minimal effort to swing the door.

Real World Example – Seesaw

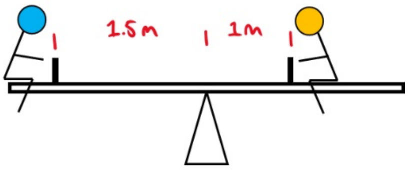
Another real world example where we see torque in action is the seesaw in the park. The seesaw is pivoted at the centre (fulcrum) and as one end is lifted the other end goes down. Suppose two children are sitting 1 *m* away from the fulcrum with masses of 20 *kg*. The amount of force applied on each end is the same (F=mg=196 *N*) and is perpendicular to the seesaw. The amount of torque applied on each side is 196 *N.m*.



Suppose one of the children is 30 *kg*, this will create an imbalance in the seesaw. To restore the balance in the seesaw the with the lower mass needs to move on the seesaw. To determine the location we make the torque of each side to be equal. Then we have one unknown in the equation, which is the distance the child must sit.



$$\tau_1 = \tau_2$$
$$r_1 F_1 = r_2 F_2$$
$$r_1 m_1 g = r_2 m_2 g$$
$$r_1 = r_2 \frac{m_2}{m_1}$$
$$= 1m \times \frac{30kg}{20kg}$$
$$= 1.5m$$



Thus, the child must sit 1.5 *m* away from the fulcrum for the seesaw to be balanced.

Question 1

What is the torque on a bolt when a 150 *N* force is applied to a wrench of length 20 *cm*. The force is applied at an angle of 40º to the wrench.

Ans: 19.28 *N.m*

Question 2

Determine the angle at which the force is applied to lever if the force applied is 400 *N* resulting in a torque of 50 *N.m*. The force applied is located at 1.2 *m* from the pivot point.

Ans: 5.98º

GRAVITATIONAL MOTION 1

CONTENT

A gravitational field is the region surrounding a mass that attracts other bodies to it due to the force of gravity. The more massive the object, the greater its gravitational attraction. For example, the Earth has a far greater gravitational pull than a tennis ball. We can calculate the strength of a gravitational field using the equation:

$$g = \frac{GM}{r^2}$$

where g is the gravitational field strength, M is the mass that is producing the gravitational field in kg, r is the distance from the mass and G is the universal gravitational constant 6.67×10^{-11}Nm^{2}kg^{-2}. This gravitational field strength is also known as the acceleration due to gravity, it has units ms^{-2}. The gravitational field is isotropic, i.e. it is the same in all directions. It depends only on how massive the object is and how far away from the object you are. So being close to a very massive object will mean there is a large gravitational attraction.

When we put two objects near each other, they both have their own gravitational field. So, they are both experiencing a force of attraction to the other masses. Just like in Newton's Second Law which states $F = ma$, we can calculate the strength of the force of attraction between two masses, M and m, due to gravity using the same formula where my acceleration is the acceleration due to gravity calculated above:

$$F = \frac{GMm}{r^2}$$

where F is the force due to gravity, M and m are the masses of the two objects, r is the distance between them and G is the universal gravitational constant again. This means that any two masses experience a force of attraction due to gravity.

But, in both cases, the magnitude of that field and the force is tiny until we get to incredibly large masses like the Moon and the Earth. The strength increases for both if we increase the masses involved and decrease the distance.

EXAMPLE

Daliah is a space explorer who is tasked with comparing the gravitational field strength in different areas around the Solar System. She compares the strength of the gravitational field due to the Earth at the orbit of the Moon and the strength of the gravitational field due to Saturn at the orbit of one of its moons Rhea. Given the mass of the Earth is 6×10^{24}kg, the mass of Saturn is 568×10^{24}kg, the radius of the moons orbit is $385{,}000$km and the radius of Rheas orbit is $527{,}000$km, what are the factors that will affect the strength of the gravitational fields? Which location/position would Daliah measure a stronger gravitational field and why?

⇒ Firstly, we will explain what factors influence the strength of a gravitational field and then we will calculate the field in both positions in order to compare the two.

⇒ So, according to the equation for gravitational field strength, $g = \frac{GM}{r^2}$, the two variables that will influence the strength of the field are the mass of the object and the distance from that object. Since Saturn has a much larger mass than the Earth this will increase the field strength. However, the radius of Rheas orbit is also much larger than the orbit of the Moon. Since the gravitational field strength is inversely proportional to the distance squared from the mass this has a larger influence on the strength of the field. In this particular case, however, while the radius of Rheas orbit is larger than the Moon's the difference is nowhere near as large as the difference between the masses of Earth and Saturn. Thus, it is likely that the mass of Saturn compared to the mass of the Earth will play the dominant role in the gravitational field strength.

⇒ Now, to test our hypothesis we shall calculate the two gravitational fields at the orbits of their moons and compare:

$$g_E = \frac{GM_E}{r_m^2} = \frac{6.67\times10^{-11}\times6\times10^{24}}{(385{,}000\times10^3)^2} = 0.0027\ Nkg^{-1} = 0.0027\ ms^{-2}$$

$$g_S = \frac{GM_S}{r_R^2} = \frac{6.67\times10^{-11}\times568\times10^{24}}{(527{,}000\times10^3)^2} = 0.14\ Nkg^{-1} = 0.14\ ms^{-2}$$

⇒ So, the gravitational field strength due to Saturn at the orbit of its moon Rhea is larger than the gravitational field strength due to the Earth at the Moons orbit.

Example

After comparing the strength of the gravitation field at Rhea's orbit and the Moon's orbit, Daliah decided it was also a good idea to determine the force between each planet and its satellite. Given the mass of the Moon is 73.5×10^20kg and the mass of Rhea is 2.31×10^{20}kg how does the force due to gravity between Saturn and Rhea compare with the force due to gravity between Earth and the Moon?

⇒ To solve this problem, first we will write down all the variables we have and then sub them into the equations. Then we will compare the two results:

Variable	Value
M_E (Mass of the Earth)	$6 \times 10^{24} kg$
M_S (Mass of Saturn)	$568 \times 10^{24} kg$
r_M (Radius of the Moons orbit)	$3.85 \times 10^{8} m$
r_R (Radius of Rheas orbit)	$5.27 \times 10^{8} m$
m_M (Mass of the Moon)	$7.35 \times 10^{22} kg$
m_R (Mass of Rhea)	$2.31 \times 10^{21} kg$

⇒ Now we can calculate the force between the two planets and their moons using the formula $F = \frac{GMm}{r^2}$:

$$F_{E\Leftrightarrow M} = \frac{GM_E m_m}{r_m^2} = \frac{6.67\times10^{-11}\times6\times10^{24}\times7.35\times10^{22}}{(3.85\times10^8)^2} = 1.98\times10^{20}\ N$$

$$F_{S\Leftrightarrow R} = \frac{GM_S m_R}{r_R^2} = \frac{6.67\times10^{11}\times568\times10^{24}\times2.31\times10^{21}}{(5.27\times10^8)^2} = 3.15\times10^{20}\ N$$

GRAVITATIONAL MOTION 2

CONTENT

The massive objects will experience a force of attraction due to their gravitational fields. This force is what keeps planets in orbits around the Sun and moons in orbit around their host planets. Just as in circular motion, we can calculate the velocity of planets orbiting around a host star using a very similar formula:

$$v = \frac{2\pi r}{T}$$

where r is the radius of the orbit, and T is the time taken for one full cycle of the orbit. This formula is for the average velocity of an object moving in a circle. While the orbits of planets are not perfectly circular, we can still approximate their orbits as circular and calculate the average velocity of their entire orbit using this formula.

Just like an object moving on the surface of the Earth, objects in orbit such as planets or satellites, have kinetic energy and gravitational potential energy. Potential energy on Earth is dependent on the mass of the object and the height/distance from the Earth. Gravitational potential energy is very similar:

$$U = -\frac{GMm}{r}$$

where M and m are the two masses in kg, r is the distance between them, or the radius of the orbit, and G is the universal gravitational constant 6.67×10^{-11} Nm^{2}kg^{-2}. While this is just the gravitational potential energy, the total energy of the satellite in orbit is a combination of the gravitational potential energy and the orbital kinetic energy. It can be found using the equation:

$$E = -\frac{GMm}{2r}$$

Both these equations for the energy of the satellite or planet are very similar and combine the mass and distance of the two objects involved. This means satellites that are more massive and orbit with a smaller radius have larger gravitational potential energy and total energy.

EXAMPLE

Given the Earth has a mass of 5.972×10^{24}kg and orbits the Sun each year at an average radius of 145×10^9m, what is the average velocity of the Earth's orbit and the total energy of the orbit, assuming the Sun's mass is 1.989×10^{30}kg?

⇒ We start by writing down all the variables we have and calculating the number of seconds in a year:

Variable	Value
m_e	5.972×10^{24}kg
M_s	1.989×10^{30}kg
r	145×10^9m
T	3.15×10^7s

⇒ Now we can sub these values into the equation for velocity and energy:

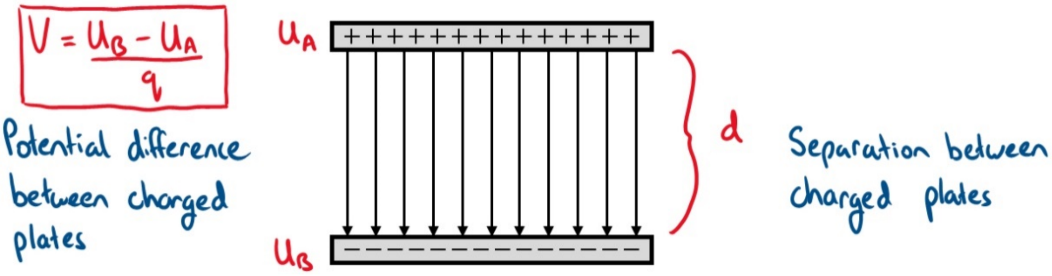
$$v = \frac{2\pi r}{T} = \frac{2 \times \pi \times 145\times10^9}{3.15\times10^7} = 28.9\times10^3 \text{ m/s}$$

$$E = -\frac{GMm}{2r} = -\frac{6.67\times10^{-11}\times1.989\times10^{30}\times5.972\times10^{24}}{2\times145\times10^9} = -2.73\times10^{33} \text{ J}$$

CHARGED PARTICLES IN AN ELECTRIC FIELD 1

CONTENT – CHARGED PARTICLES

In *module 4* we have looked at the definition of the voltage. Voltage is the difference in potential energy between two points. For parallel charged plates the potential difference is illustrated below.

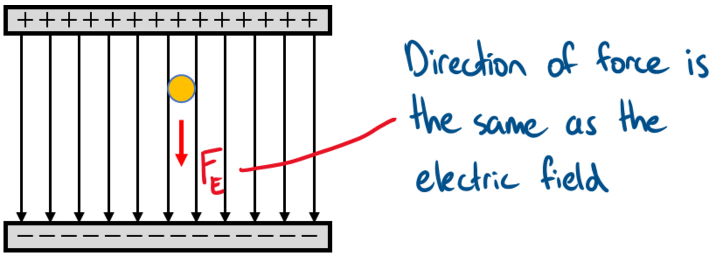


The electric field between the parallel plates is uniform and is defined as the potential difference divided by the distance between the plates.

$$|E| = -\frac{V}{d} \quad (1)$$

The units for the electric field defined above is Volts per metre (V/m).

Let's now suppose a positively charged particle is placed in between the parallel plates. The particle will experience a force resulting from the electric field, and the direction will be parallel to the field.



If the charge on the particle is q the force on the particle from the electric field is

$$F_{charge} = qE \quad (2)$$

If we equate this force to the force of a moving object (i.e. $F = ma$) we, can determine the acceleration of the charged particle when it is inside the electric field.

$$F_{net} = F_{charge}$$
$$ma = qE$$
$$a = \frac{qE}{m} \quad (3)$$

The acceleration above is the acceleration of the particle as it moves towards the negative plate and is analogous to the acceleration due to gravity (i.e. $g = 9.8\ m/s^2$). Meaning, the charged particle is moving like an object falling in a gravitational field. If the charge is negative then ,the direction will be upwardly instead.

In addition to the force, we can calculate the work done on the particle from the potential difference between the two plates

$$W = qV \quad (4)$$

We can also define the work in terms of the electric field using equation (1) and arrive with the expression below

$$W = qEd \quad (5)$$

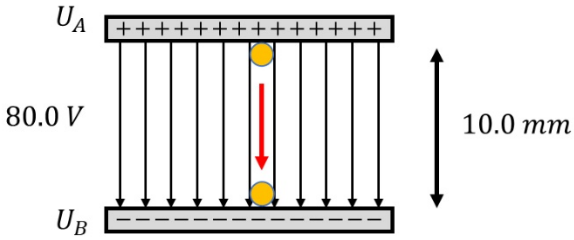
Now that we have defined the expression for work we can obtain the velocity of the particle. Recall the *work-energy theorem* that states work done on a particle is equal to the change in kinetic energy (i.e. $W = 1/2\, mv^2$). With this definition the velocity of the charged particle is

$$W = KE$$
$$qEd = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2qEd}{m}} \qquad (6)$$

Note: the terms Ed can be replaced with V in the velocity equation above.

Worked Example

A positively and negatively charged plates are separated by 10.0 mm as shown on the right. If the potential difference is 80.0 V what is the velocity of a proton as it hist the negative plate? What is the velocity if the potential difference is 1.0 V instead? (mass of proton = 1.67 x 10^{-27} kg and charge = 1.6 x 10^{-19} C).



⇒ To determine the velocity, we start with the work-energy theorem to arrive at an expression for velocity

$$W = KE$$
$$qEd = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2qEd}{m}}$$

⇒ The electric field can be written in terms of the potential difference thus,

$$V = Ed \rightarrow v = \sqrt{\frac{2qV}{m}}$$

⇒ We now have an expression for velocity where we have all the numbers.

$$v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} C \times 80.0\ V}{1.67 \times 10^{-27} kg}}$$
$$= 1.24 \times 10^5\ m/s$$

⇒ The velocity above is extremely fast and to put it in perspective it is 375x the speed of sound.

⇒ If the potential difference is 1.0 V, then the velocity is

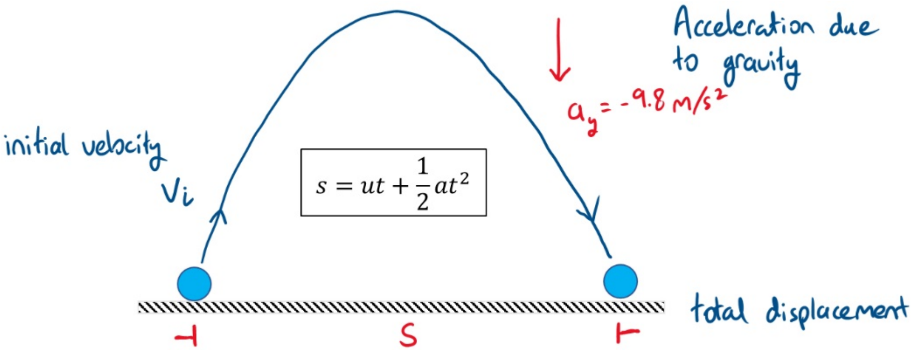
$$v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} C \times 1.0\ V}{1.67 \times 10^{-27} kg}}$$
$$= 1.38 \times 10^4\ m/s$$

⇒ So even with only 1.0 V, the velocity is still large because the mass of the proton is very small.

CHARGED PARTICLES IN AN ELECTRIC FIELD 2

CONTENT – MOVING CHARGED PARTICLES

Previously we looked at what happens to a charged particle when it is placed in a uniform electric field. However, the question that comes up after is what happens to the charged particle if it moves across the parallel plates. The acceleration due to the electric field is constant and depends on the electric field and the properties of the charged particle. Since it is constant, we can use 2D kinematics that we learned in *module 1* (i.e. projectile motion). Suppose we have an electron placed just outside the parallel plates, the position in two dimensions is (x,y). The initial velocity of the electron is v_i. Recall that the displacement of a moving object for a constant acceleration and given the initial velocity is



If we decompose the displacement in two dimensions we get

$$x = v_x t + \frac{1}{2}a_x t^2 = v_x t$$
$$y = v_y t + \frac{1}{2}a_y t^2 = \frac{1}{2}a_y t^2$$

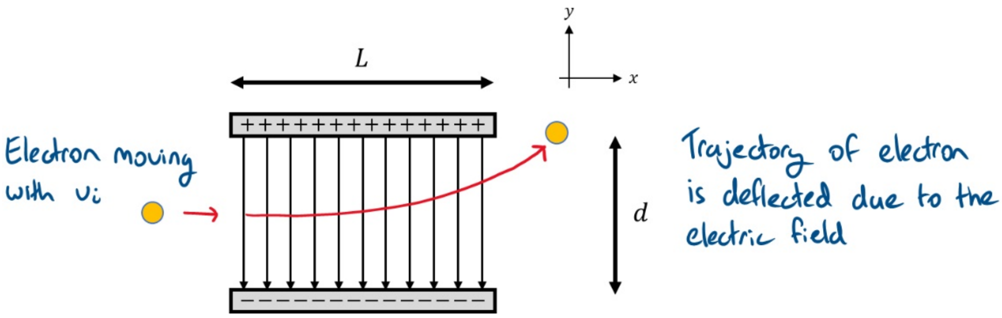
We arrive at the expression above by taking acceleration in the x-direction as zero ($a_x = 0$) and the velocity in the y direction also zero ($v_y = 0$). For the electron moving through the parallel plates, the initial velocity is $v_x = v_i$ and the acceleration from the electric field is $a_y = qE/m$. Thus the position of the electron moving through the parallel plates at a given time is

$$x = v_i t$$
$$y = \frac{qE}{2m}t^2$$

If we rearrange the equation for x to make t the subject ($t = x/v_i$) we can substitute this term to the equation for y. The displacement of the electron in the y direction is then

$$y = \frac{qE}{2m}\left(\frac{x}{v_i}\right)^2 = \left(\frac{qE}{2mv_i^2}\right)x^2$$

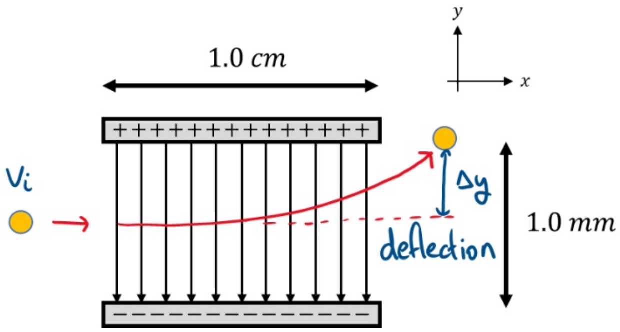
The terms in the parenthesis are constant, and the trajectory of the electron is parabolic. The sharpness of the parabola depends on the electric field and the mass and velocity of the particle. An illustration of the electron's trajectory is shown below.



The electron is deflected in the y-direction because of the electric field, and it deflects towards the positively charged plate because of the negative charge on the electron. If the particle is a proton instead, then the particle will be deflected towards the negative plate.

Worked Example

An electron moves through a uniform electric field produced by two parallel conducting plates separated by 1.0 mm and 1.0 cm in length shown in the diagram to the right (not to scale). The initial velocity of the electron is 1.5 x 10^6 m/s and the potential difference between the plates is 9.0 V. Find (a) the acceleration due to the electric field, (b) the time taken for the electron to pass through the field and (c) the deflection of the electron in the y-direction.



a)

⇒ We first need to determine the electric field produced by the conducting plates

$$E = \frac{V}{d} = \frac{1.0\ V}{1.0 \times 10^{-3}\ m} = 1000\ V/m$$

⇒ Given the electric field, we can calculate the acceleration of the particle in the y-direction

$$a_y = \frac{qE}{m} = \frac{1.6 \times 10^{-19}\ C \times 1000\ V/m}{9.11 \times 10^{-31}\ kg}$$
$$= 1.76 \times 10^{14}\ m/s^2$$

b)

⇒ The total displacement in the x-direction is 1.0 cm, then the total time for the electron to move through the plates is is

$$x = \frac{v_i}{t} \rightarrow t = \frac{x}{v_i}$$
$$t = \frac{1.0 \times 10^{-2}\ m}{1.5 \times 10^6\ m/s} = 6.67 \times 10^{-9}\ s$$

c)

⇒ For the deflection in y, we use the kinematic displacement equation

$$y = \frac{1}{2} a_y t^2$$
$$= \frac{1}{2} \times 1.76 \times 10^{14}\ m/s^2\ \times (6.67 \times 10^{-9}\ s)^2$$
$$= 3.9 \times 10^{-3}\ m$$

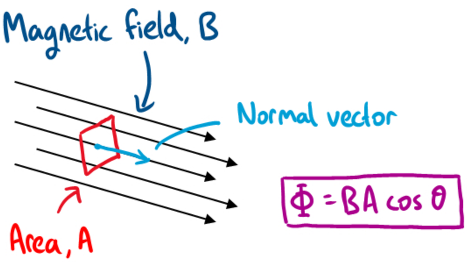
⇒ Therefore, the electron is deflected in the y-direction by 3.9 mm as a result of the electric field

Question – Internet research

Explain how the phenomenon of charged particles moving in an electric field is used in cathode ray tubes (CRT) and subsequently how is it exploited in old CRT televisions.

ELECTROMAGNETIC INDUCTION

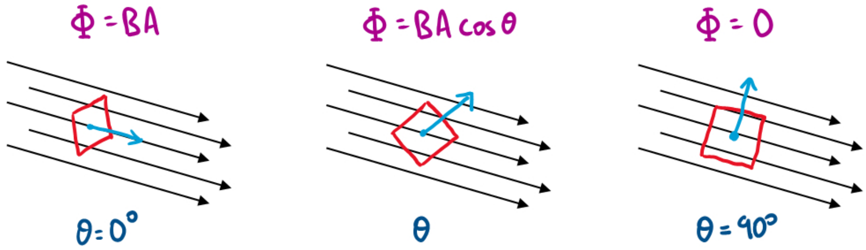
CONTENT – MAGNETIC FLUX



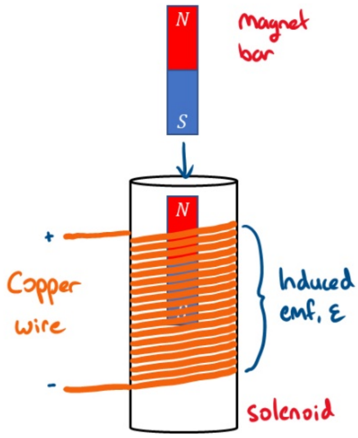
Magnetic fields, written with the symbol *B*, are measured in units of Telsa (*T*). To describe magnetic fields, we introduce a quantity called the magnetic flux. The magnetic flux quantifies the total magnetic field passing through a certain surface area. Following the diagram on the right, the total magnetic field coming through the small rectangular area is

$$\Phi = B_{\parallel}A = BA\cos\theta \qquad (1)$$

where Φ is the magnetic flux having units of Weber (Wb). The angle in the cosine is the angle between the normal vector of the area and the magnetic field line. If the area is perpendicular to the direction of the magnetic field, $\theta = 0°$, then the cosine term becomes one, which means that the magnetic flux is at a maximum. When the area is tilted by an angle other than zero then the magnetic flux through the area will be less than the maximum. As a final case, if the angle is 90º, then the magnetic flux through the area is zero.



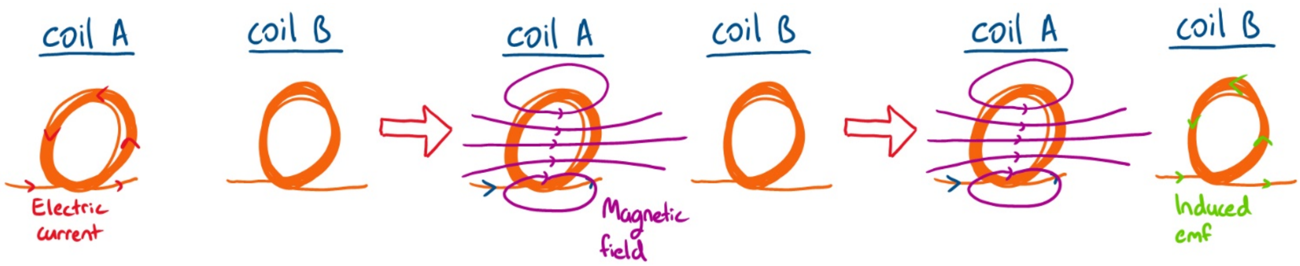
CONTENT – ELECTROMAGNETIC INDUCTION



Suppose we have a solenoid with a copper wire wrapped around *N* times. If we move a magnetic bar through the solenoid, the magnetic flux will change along the way. This change in magnetic flux induces an electromotive force (emf) ε to the solenoid. The electromotive force is the potential difference and measured in units of Voltage (*V*). This induction of emf to the solenoid is known as Faraday's law of induction. The direction of the emf will be opposite to the magnetic flux and is known as Lenz's law. Combining Faraday's and Lenz's laws the induced emf due to changing magnetic flux is

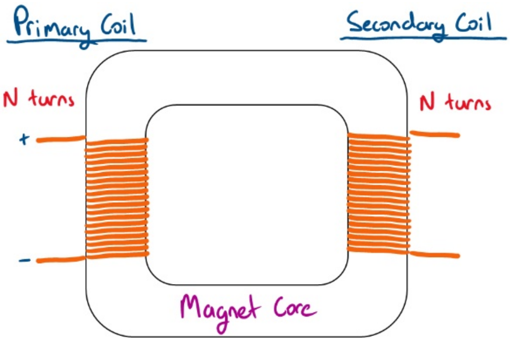
$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} \qquad (2)$$

Suppose now we have two copper coils positioned side-by-side as shown below (basically two solenoids). When a current passes through coil one, a magnetic field is produced around the coil. If coil B is close to coil A, then the magnetic field produced by coil one will move the electrons in coil B. Following the induction laws, coil two will produce an emf due to the change in magnetic flux produced by coil A. Thus, energy is transferred from one coil to another propagated through space.



CONTENT – TRANSFORMERS

The induction laws are used in devices such as transformers. Transformers transfer electricity between two or more circuits using electromagnetic induction (i.e. not connected by wires). Consider the magnet core below with copper wires wrapped around on each side with *N* turns.



The induced emf on the secondary coil depends on the ratio of the number of turns of the primary, N_p, to the secondary, N_s, coil. If the emf in the primary and secondary is V_p and V_s respectively, the relationship between two coils is written as

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} \tag{3}$$

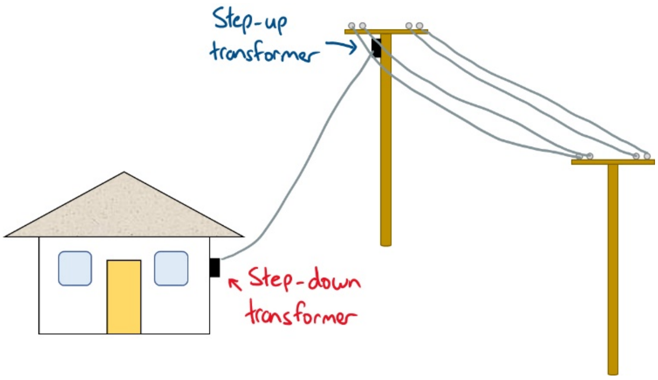
If the number of turns in the primary and secondary coils are the same, then the emf induced in the secondary coil is the same. This holds true for an ideal transformer where there is no loss of current due to resistance in the copper and no magnetic flux leakage. Following the law of conservation of energy, the power in the primary coil must be equal to the power out of the secondary coil (power going in must be equal to power going out). Since power from electric circuit can be calculated by multiplying voltage by the current ($P = VI$), the power through the coils is

$$V_p I_p = V_s I_s \tag{4}$$

Thus, if the emf in the secondary is smaller than the primary, then the current in the secondary is greater than the primary and vice-versa.

We have defined ideal transformers where there is no energy loss in transmission. However, real transformers are far from ideal, and the efficiency of the transmission is less than 100%. One reason for a loss of energy during transmission is an *incomplete flux linkage*. This happens when the magnetic flux produced by the first coil is not fully linked to the second coil reducing the emf induced. Another issue is the generation of *resistive heat* from *eddy currents*. Eddy currents can be reduced by using a better material or laminated core.

REAL WORLD APPLICATION – POWER LINES



Transformers are used in the distribution of electricity through high-voltage power lines. The strength of electric current drops over long distance and the signal need to be boosted again for the signal to continue. From equation (3), if the number of turns in the primary coil is smaller than the secondary coil then induced emf will be greater than emf in the primary. This is an example of a *step-up* transformer where the voltage is boosted. If the number of turns in the primary is greater than the secondary then the opposite will occur, i.e. reduces the induced emf. This is an example of a *step-down* transformer. Step-up transformers are used to boost signals for long-range power. In NSW the voltage in power lines can be as high as 66,000 V. The power outlet in houses output 240 V. Thus a step-down transformer is required to reduce the voltage down to the appropriate value.

MAXWELL AND CLASSICAL THEORY

CONTENT

In the 1800s, scientists were fascinated with two seemingly separate fields: electricity and magnetism. Preliminary research had been done relating magnetism to light, including the extraordinary work of Mary Somerville, who published a paper on the magnetisation of violet rays. But it was still largely not understood that these fields were related.

In 1865, Maxwell published his work combining electricity and magnetism in a new theory called electromagnetism. This work was summarised by four equations which combined and summarised all the current research of the time in both electricity and magnetism. Maxwell was able to mathematically connect magnetism and electricity by combining them in equations. For example, if we look at the following equation (don't worry about what the ∇ or $\times$ mean, we will go through each term separately):

$$\nabla \times \vec{B} = \mu_0 (\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t})$$
$$curl\ of\ magnetic\ field = static\ electric\ current + changing\ electric\ field$$

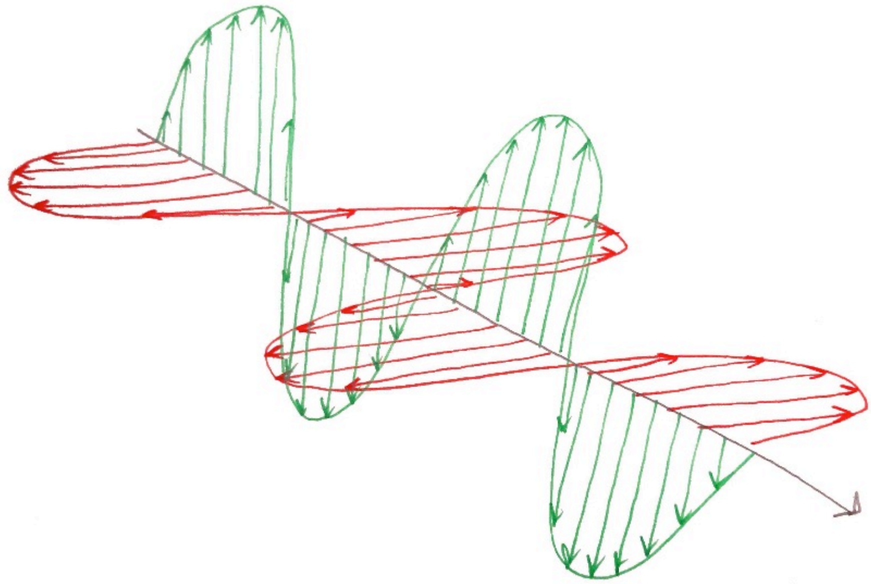
The term on the left, $\nabla \times \vec{B}$, refers to how the magnetic field curls around, much like when we use the right-hand rule to curl around a solenoid to determine the direction of the magnetic field. The first term on the right, $\mu_0 \vec{J}$, is related to the electric current, while, the other term on the right-hand side, $\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$, is related to how the electric field changes with time. Altogether, this equation means an electric current and/or a changing electric field produces a magnetic field.

While we don't need to know how to use this equation, because there are terms for both magnetic field and electric field, we can clearly see it relates the two. This equation summarises Maxwell's contribution to unifying electricity and magnetism.

From this equation, we can also predict the existence of electromagnetic waves. The two constants included are ϵ_0, the electric permittivity, and μ_0, magnetic permeability of free space. We have already seen these constants in Module 4: Electricity and Magnetism. They can be combined to determine the speed of light in a vacuum using the equation below:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

The combination of electricity and magnetism in this equation predicts not only that the speed of light is a constant, but that light is an electromagnetic wave. That is, it is an electric wave and a magnetic wave travelling together.



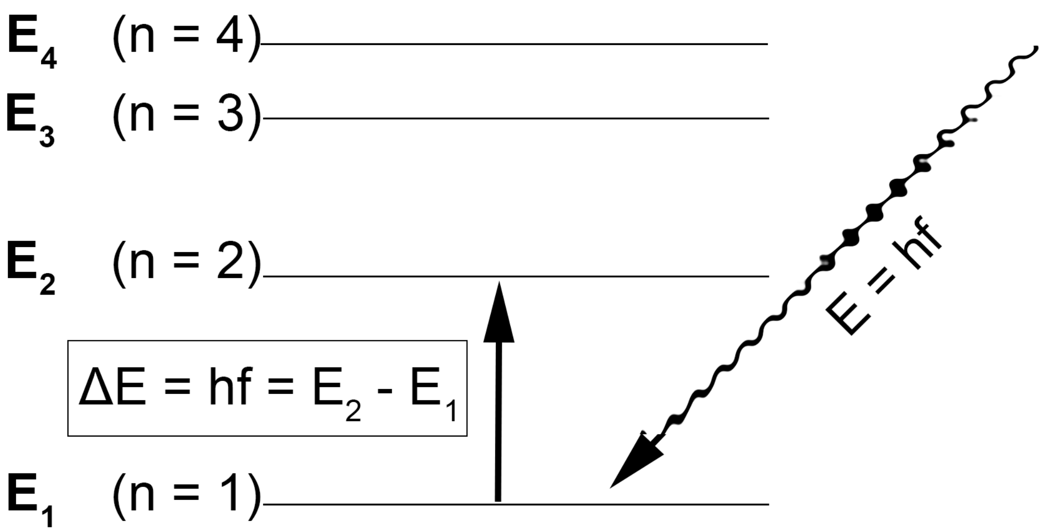
The prediction of electromagnetic waves comes directly from the definition of the speed of light in a vacuum. Thus, not only did Maxwell's equations predict light was part of the electromagnetic spectrum, they also predicted that the velocity of these waves, c, and set that velocity as a constant.

INVESTIGATION: SPECTROSCOPY

INTRODUCTION

The electrons orbiting around an atom have very specific energy levels they are allowed to be in. These electrons can absorb the energy of a photon to jump up an energy level, but this photon energy must be exactly the same as the energy difference between the two energy levels of the electron. Likewise, if an electron drops down an energy level it releases a photon which has the exact energy of the difference between the energy levels. As a result, we can see the bright lines called emission lines when the electron emits a photon and the absorption lines when the electron absorbs a photon.

The wavelengths of these lines depend on the element or molecule that has absorbed/emitted the photon and these lines are called the spectral lines or the spectra.



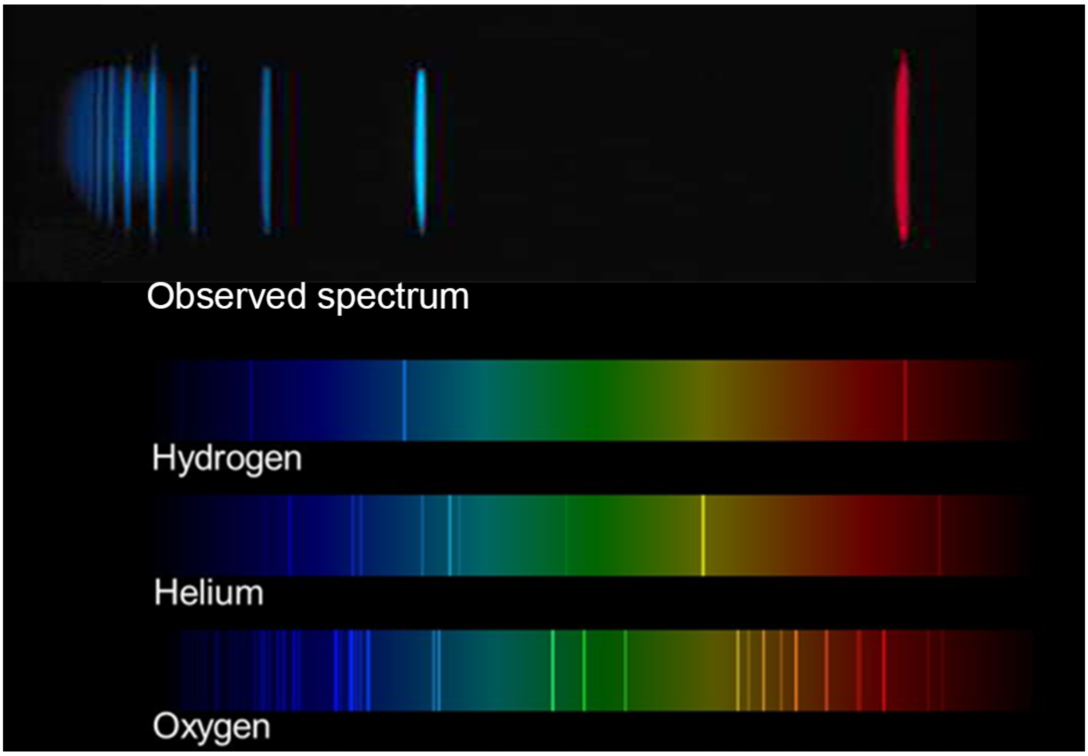
Absorption of a photon by an electron in an atom

In this experiment we will observe the spectra of several sources to see how they vary and what information we can glean from observing these features. We will also investigate the applications of spectral lines in both industry and astrophysics.

1. QUESTIONING AND PREDICTING

Let us think about the aim of this investigation:

1. What lines can we see when observing the different sources and how do these lines differ across the sources?
2. What could cause changes in the spectral lines?



How can we determine what element the observed spectrum matches?

Hypothesis

The wavelength where a spectral line is observed depends on the ______________________________

The multiple lines in a spectrum for one element are due to ____________________________________

2. Planning Investigation

This investigation has been planned for you. It is most suited to being performed by a whole class.

You will need at least one incandescent filament (a lamp filled with specific elements or molecules) and a spectrograph or spectroscope to spread the light into its component wavelengths.

1. Set up the incandescent filament so it is visible, and students can look at it with their spectroscopes.
2. Consider other sources of light to study with the spectroscope, these could include lights in the classroom or light from the sun.
3. Collect spectra of various elements to be able to compare and identify sources.

3. Conducting Investigation

For each element you observe, measure the wavelengths of the spectral lines that you see from your spectroscope. Record the wavelengths of all the spectral lines you observe.

Source	Measured Spectral Lines (wavelength nm)

Did you make any changes to the method? Did you have design problems to solve? Did you have some 'smart' ways of doing the investigation?

4. Processing and Analysis

For each of the sources you observed and your measured spectral lines, try and identify which element or elements are present in the source. To do this compare your measured wavelengths with the wavelengths of the spectral lines of known elements.

Are you able to identify all the sources?

What did you notice about the spectral lines of different sources? Where they the same? Similar?

If they were different, what could cause the differences?

Were there any difficulties identifying the sources? What were they? How could you fix them?

5. Problem Solving

All elements and molecules have very a specific set of spectral lines related to their electron orbits. Since these orbits occur at precise energies, when these electrons jump from orbit to orbit the photons they absorb or emit to do so will be of very specific frequencies. This means the differences we see in the spectral lines of the different sources is due to the different molecules present and their specific electron orbits.

As each element has a unique spectrum, we can observe the spectrum of an unknown source and compare it to known sources to identify which elements are present. You have just used this process of elemental identification to determine which elements are present in your various sources.

Discuss whether this process could be applicable in astronomy when trying to identify the elements present in stars.

6. Conclusions

Spectral lines from different sources containing different elements or molecules were ____________________ (different/the same)

Some applications of studying spectral lines are:

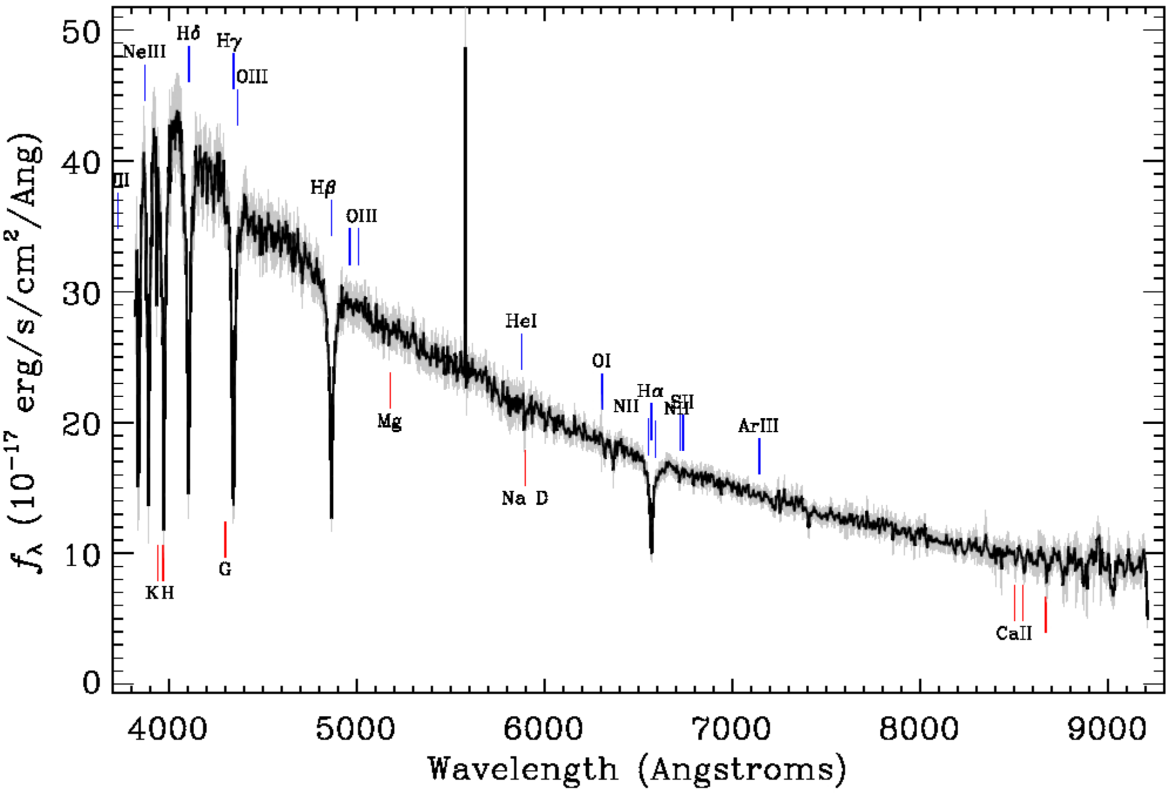
SPECTRA OF STARS

CONTENT

We have already discussed the use of spectral lines that we can observe from light sources, to identify the elements present in that source. However, we can also obtain the full spectra of astronomical objects like stars and obtain additional information including the surface temperature, rotational and translational velocity, density and chemical composition of a star.

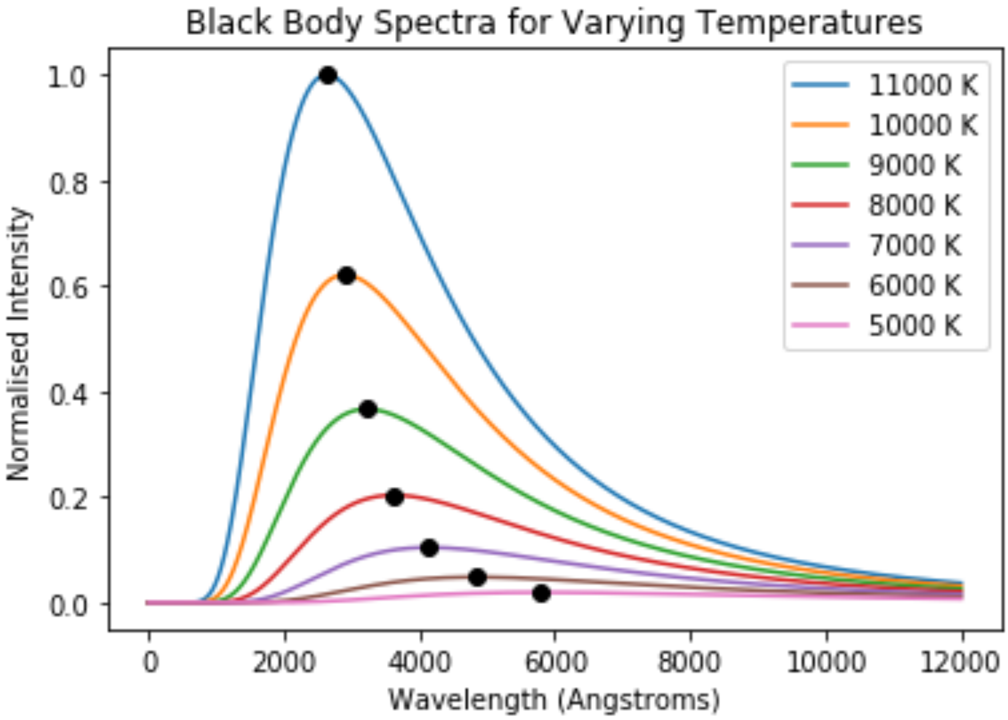
Before we extract all this information, we will first look at what the spectra of stars look like. The overall shape of these spectra follows a black body curve, which means that while they have a peak wavelength where they emit the highest intensity of light, they also emit light at other wavelengths. Here is a spectrum of a star from the Sloan Digital Sky Survey (SDSS) where we can see this overall black body envelope. We can also see various sharp peaks of absorption and emission lines due to the chemicals present in the stellar atmosphere.

We will now go through and discuss information we can gather about the star from its spectrum.



Surface Temperature

The peak wavelength of the spectra depends on the temperature of the star. This can be seen in the graph below where the black body curve is plotted for different temperatures and the maximum value for each curve is marked by the black dot. In this graph, we can see that the peak of the black body curves shifts to longer wavelengths for lower temperatures.



In order to calculate the surface temperature of the star we use Wien's Law

$$\lambda_{max} = \frac{b}{T_{surface}}$$

where λ_{max} is the peak wavelength of the black body curve in meters, $T_{surface}$ is the surface temperature of the star in Kelvin and b is Wien's Displacement Constant 2.898×10^{-3}Km. The peak temperature of stars is used to classify the spectral class of stars as O, B, A, F, G, K or M (O type stars have the hottest surface temperature and M the coolest), each of these classes can be broken down into 10 smaller classes, 1-10 for example A1 stars are hotter than A10 stars.

EXAMPLE 1

Going back to the spectrum of the star above, we can see there is a peak at around 4200Å. Calculate the surface temperature of this star.

⇒ Firstly, we will convert the wavelength from Angstroms to metres and then using Wien's Law we will calculate the surface temperature of the star. So:

$$\lambda = 4200\,Å = 4.2\times10^{-7}\,m$$

$$\lambda_{max} = \frac{b}{T}$$

$$T = \frac{b}{\lambda_{max}} = \frac{2.898\times10^{-3}}{4.2\times10^{-7}} = 6900\,K$$

Chemical Composition

When we look at the stellar spectra on the first page, we can see there are sharp drops in the brightness at very specific wavelengths. These drops are due to the absorption of very specific wavelengths of light by chemicals, called absorption lines. Each chemical element absorbs light at different wavelengths depending on the orbits and energies of the electrons in the atoms. For example, at 6563Å we see an absorption line. This is called the Hydrogen Alpha (Hα) line (it is labelled as such on the spectrum shown on page 1). Since we can calculate exactly which wavelengths we expect absorption lines to be at for each element, we can observe the spectra of stars and see which lines are present and figure out what elements are present in the star (you can check out the Spectroscopy Investigation to give this a go yourself).

We can also use these lines to determine if the star is moving (either towards or away from us), if it is rotating and also the density of the star.

Density

If the star has a low surface density (the gas of the out layer of the star is at a lower pressure), the spectral lines are sharper. The inverse is also true, if the star has a dense outer layer, meaning the gas is at a high pressure, the spectral lines are broader. While this is a similar effect to the broadening of spectral lines because of the rotational velocity (described below), they are subtly different allowing for astronomers to distinguish between the two.

Giant stars, like red supergiants, have a very low density and pressure in the outer layer. When we look at their spectra, we can see sharp spectral lines compared to white dwarfs which have a very dense outer layer. Comparing the spectra of the two allows astronomers to classify the Luminosity class of the star:

- Ia: Bright Supergiants
- Ib: Supergiants
- II: Bright Giants
- III: Giants
- IV: Sub Giants
- V: Main Sequence

- VI: Sub Dwarf
- VII: Dwarf

Our sun is a G2 V star because its surface temperature is quite cool (around 6,000K) and it is a main sequence star.

Rotational and Translational Velocity

Using spectral lines and the Doppler Effect, we can determine the different motions of the star, i.e. whether there is a translation velocity or rotational velocity. If you are unfamiliar with the Doppler Effect, you can revise it in the Doppler and Beats worksheets in Module 3.

Since chemicals absorb photons of very specific wavelengths (due to the structure of the atom or molecule) we see dramatic decreases in the intensity (or absorption lines) at those specific wavelengths. In the example above, we can see some have been identified. For example, the Hydrogen Alpha (Hα) absorption line is at 6563Å. When we put all this information together, we can identify what spectral lines are present and which elements are causing the absorption lines. We can also use the Doppler Effect to determine if the star is travelling towards or away from us (translational velocity) or is rotating on its own axis (rotational velocity). Both types of velocity have a measurable effect on the absorption lines in a spectrum.

Translational Velocity

If the star is moving towards or away from us, these absorption lines will be shifted to bluer and redder wavelengths respectively. So, we can compare the expected wavelength for a given absorption line (or the rest wavelength) with the observed wavelength and calculate if the star has any radial translational velocity. Similarly, we can calculate how quickly the star is moving. The amount of shift from the rest wavelength depends on how fast the object is moving. Using the equation from Year 11 below, we can calculate how fast the star is moving away from us

$$f' = f\frac{v_{wave} + v_{observer}}{v_{wave} - v_{source}}$$

where v_{wave} is the velocity of the wave, $v_{observer}$ is the velocity of the observer, v_{source} is the velocity of the source emitting the waves, f' is the observed frequency and f is the original frequency emitted.

It is important to note, we only see this effect if some component of the velocity of the star is directly towards or away from us. If the star is moving perpendicular to our line of sight (the imaginary line from the observer to the star), we see no effect on the absorption lines. This means we can only use the spectra to measure the line of sight velocity (called the radial velocity) rather than the total three-dimensional velocity.

Rotational Velocity

Just like with the translational velocity, if the star is rotating about its axis, the absorption lines will be shifted to redder or bluer wavelengths depending on the motion of the object emitting the light. When a star rotates, one side of the star will be moving towards the observer while the other side will be moving away. When we record the spectrum of a star, we observe one spectrum for the star. This combines the effect of the Doppler shift due to the rotation and as a result we see the same line shifted to redder and bluer wavelengths. Thus, the absorption line is actually blurred out and we see a broader emission line than we expect. Again, if the star is rotating very quickly, each side with shift the absorption line more, this means the broader the absorption line, the faster the star is rotating.

Example 2

An astronomer is studying the spectra of a star and calculated the Hα absorption line at a wavelength of 656.4nm. However, the expected wavelength for the Hα line is 656.3nm. Given the speed of light is 3×10^8m/s and assuming the observer was not moving when the spectrum was taken, what is the radial velocity of the star?

⇒ Firstly, we will write down all the values we have and convert our wavelengths to frequency using $v = f\lambda$:

Variable	Value
$\lambda_{observed}$	656.4nm
$f_{observed}$	$4.57038 \ldots \times 10^{14}$Hz
$\lambda_{expected}$	656.3nm
$f_{expected}$	$4.57108 \ldots \times 10^{14}$Hz
$v_{observer}$	0m/s
v_{wave}	3×10^8m/s
v_{source}	?

⇒ Now, we will rearrange the Doppler Shift equation to make v_{source} the subject and then calculate:

$$f' = f\frac{V_{wave} + V_{obs}}{V_{wave} - V_{source}}$$

$$f'(V_{wave} - V_{source}) = f(V_{wave} + V_{obs})$$

$$V_{wave} - V_{source} = \frac{f}{f'}(V_{wave} + V_{obs})$$

$$\Rightarrow V_{source} = V_{wave} - \left(\frac{f}{f'}(V_{wave} + V_{obs})\right)$$

$$V_{source} = 3\times10^8 - \left(\frac{4.57108}{4.57038}\times10^{14}(3\times10^8)\right)$$

$$= 3\times10^8 - (1.00015\ldots \times 3\times10^8)$$

$$= 3\times10^8 - 3.000457\ldots\times10^8$$

$$= -45.710\ldots\times10^3$$

$$\Rightarrow V_{source} = -45.71\ \text{kms}^{-1}$$

NEWTON VS HUYGENS

CONTENT

In the seventeenth century there was growing interest in the true nature of light; namely, whether it was a particle or a wave. Many experiments were done by different physicists in order to provide enough evidence for either case. Notably, Isaac Newton supported a particle model or corpuscular model while Christian Huygens argued a wave model. Both models had supporting evidence but were unable to completely explain the behaviour of light. At this time, it was already known that light travelled in straight lines, could reflect off surfaces and refract in mediums. In this worksheet we outline the experimental evidence for Newton's model and Huygens' model and then combine all the evidence to provide a conclusion.

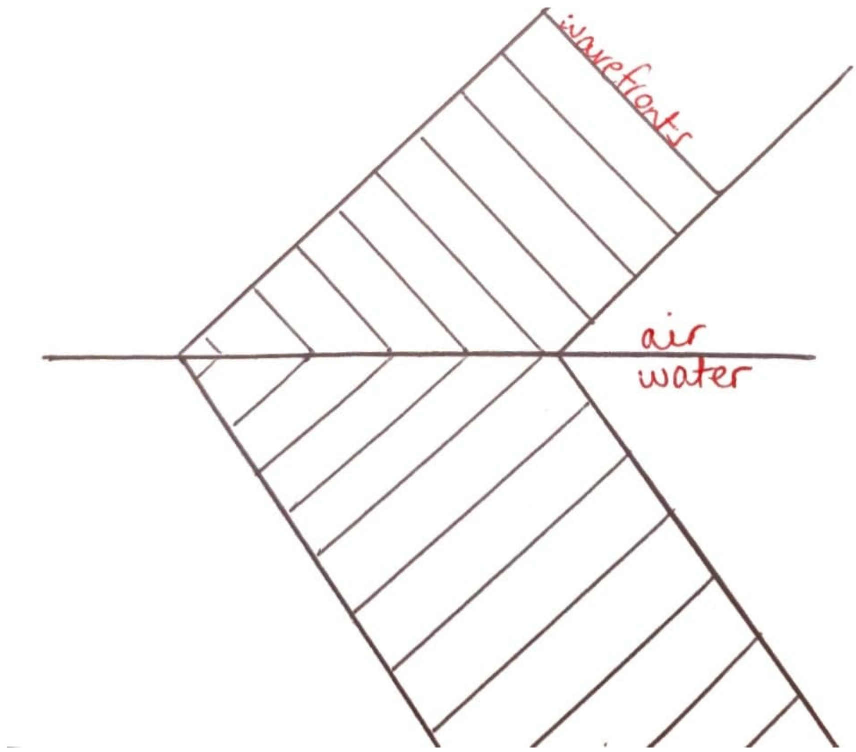
NEWTON'S MODEL

Newton supported the theory that light was made of tiny, rigid and massless particles known as corpuscles. These corpuscles were emitted in a constant stream from sources and travelled in straight lines away from the source. This can easily explain the observation that light travels in straight lines, it also does not require a medium for the particles to travel through which, as we will see in Huygens' model, avoids an added complication. Since these corpuscles are rigid massless particles, Newton was able to explain the reflection of light using the perfectly elastic collisions of the particles with a surface. This was also able to explain Snell's Law regarding the angles involved in reflections.

While Newton was able to explain the reflection of light and how it travels in straight lines using corpuscular theory, he had to introduce many complications to this theory to explain the refraction of light in a medium. Newton required a force of attraction between the corpuscles and the particles within the water that changes the direction of the particles. However, for this to be true, it requires light to travel faster in water than in air. On top of this, when Young performed his double slit experiment in 1801, Newton's model of light failed to explain these results at all. While the corpuscular theory of light was widely accepted from the 17th century (in part due to Newton's prestige over Huygens'), this result was the final nail in the coffin that led to a wave model being preferred.

Huygens' Model

Opposed to Newton was Huygens who proposed that light was a wave not a particle. However, at the time, it was believed waves required a medium in which to travel. Thus, Huygens also proposed an aether, an all pervasive, undetectable medium which allowed the light waves to travel through space. This caused a lot of doubt about Huygens' model and it was not readily accepted. Despite this, Huygens' model was able to explain reflection, refraction and, most notably, diffraction. Huygens proposed that sources emit spherical wavelets which travel outwards producing a wavefront. Waves allow the light to 'bend' around surfaces or diffract. This could explain the Fresnel Bright Spot, a bright dot observed in the middle of the shadow of a solid object. A wave model could easily explain how light is reflected off surfaces as this was already well known. But it also could simply explain refraction, compared to Newton's corpuscle model, and required that light travels slower in water, a fact we now know. We can see in the diagram below how the wavefront moves as it passes through the surface of water.



Conclusion

Both Huygens' and Newton's models have merit and are able to explain some properties of light, however neither is complete. With the addition of a quantum approach to light, our current model combines both a particle model and a wave model. However, the nature of the particles and the waves are slightly altered to the original models. We now know light as a wave is an electromagnetic wave, both an electric wave and a magnetic wave propagating together without the need for a medium. Similarly, we know photons are massless particles, however to explain diffraction we no longer need a complex force of attraction between particles.

SPECIAL RELATIVITY

CONTENT

We already know of Einstein's two postulates: 1) the speed of light in a vacuum is an absolute constant; 2) all inertial frames of reference are equivalent. From the second postulate, we can conclude that the laws of physics must also be the same in each inertial frame. In order for these two postulates to be true then other properties that we consider to be constant in Newtonian physics, such as mass, momentum, and time must not be constant when measured from different frames of reference. These concepts will be familiar to you from the old syllabus, however their description has changed somewhat in the new syllabus.

Under the old syllabus, and also in some older physics textbooks, it is said that the mass of an object increases with its speed:

$$m_v = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In this equation, m_0 is the rest mass of the object, which is the mass of the object when the measurer is at rest relative to the mass. This is an intrinsic property of an object that does not depend on speed. This has very important implications for momentum, although the old syllabus did not include it. Using the definition of momentum as mass times velocity, we get

$$p_v = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

That is, the momentum of an object is greater than expected from Newtonian physics $p = m_0 v$, by the so-called Lorentz factor. If we were to calculate momentum using our traditional equation we find that between inertial frames the conservation of momentum is not always met, defying the second postulate. When velocities are small, i.e. $v \ll c$, this is a fine approximation but as velocities approach the speed of light this approximation no longer holds. This is also why it becomes harder and harder to accelerate an object as its speed approaches the speed of light.

In the new syllabus, and more generally in modern physics, we no longer use the first equation and we don't say that the mass of an object increases with speed but rather focus on the increase in momentum. In this more modern description the term 'mass' always refers to the rest mass. Hence, in the new syllabus (and in most textbooks), mass is given the symbol *m*, without the subscript and momentum is described as,

$$p_v = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where m is the rest mass, v is the velocity and c is the speed of light in a vacuum, roughly 2.998×10^8m/s. So the momentum of an object is greater than expected from Newtonian physics, and increases steeply towards infinity as the speed approaches c. For this reason, it becomes progressively harder to accelerate an object as it gets faster, and it is impossible to accelerate it up to the speed of light. However, when we calculate the total relativistic momentum in different inertial frames the conservation of momentum is not violated, as is the case when we calculate the total momentum in different inertial frames using the Newtonian equation.

N.B. This entire discussion does not apply to a particle with zero mass. According to Special Relativity, such a particle always travels at c and photons are the best-known example.

EXAMPLE 1

Calculate the relativistic momentum for a muon travelling at 0.99 times the speed of light and with a rest mass of 1.9×10^{-28}kg. Assume the speed of light is 3×10^{8}m/s.

⇒ Firstly, we write down all the variables we have:

Variable	Value
m	1.9×10^{-28}kg
v	$0.99c$
p	?

We will use the relativistic equation for momentum:

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

⇒ Now, to calculate the value of the relativistic momentum:

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1.9 \times 10^{-28} \times 0.99 \times 3 \times 10^{8}}{\sqrt{1 - \left(\frac{0.99c}{c}\right)^2}}$$

$$= \frac{5.643 \times 10^{-20}}{\sqrt{1 - 1.0199}}$$

$$= 4.0 \times 10^{-19} \text{kgms}^{-1}$$

EXAMPLE 2

Using the same values for velocity from Example 1, calculate the classical momentum of the muon and compare it with the calculated relativistic momentum in Example 1.

⇒ So, firstly, let us write down our variables and the equation we will use:

Variable	Value
m	?
p	4.0×10^{-19}kgm/s
v	$0.99c$

We will use the classical equation for momentum:

$$p = mv$$

⇒ Now, to calculate the mass:

$$p = mv$$

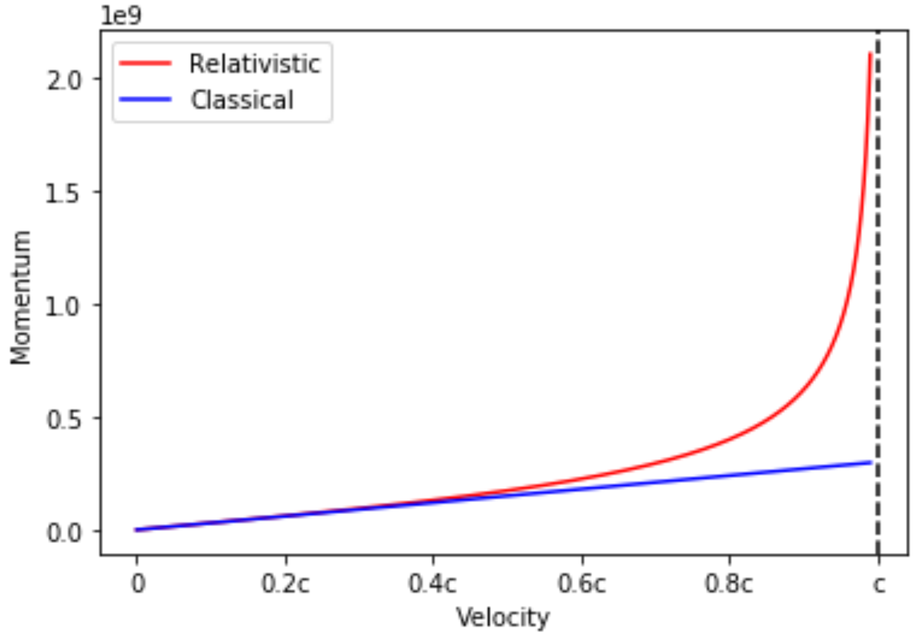
$$= 1.9 \times 10^{-28} \times 0.99 \times 3 \times 10^{8}$$

$$= 5.6 \times 10^{-20}$$

⇒ For the same mass, the classical momentum is less is than the relativistic momentum

EXAMPLE 3

Consider the graph below of the momentum for a particle with unit mass. Using both classical momentum and relativistic momentum justify the use of classical momentum for small velocities and relativistic momentum for velocities approaching the speed of light.



⇒ To tackle this question, we will consider it in two parts: smaller velocities below ~$0.5c$ and larger velocities above ~$0.5c$

⇒ Firstly, the smaller velocities. From the graph, we can see both classical and relativistic momentum are almost identical until around $0.5c$. This proves that while classical momentum doesn't account for relativity, it is a reasonable approximation for low velocities where the differences between the classical momentum and relativistic momentum are small.

⇒ However, for the second case (larger velocities above $0.5c$), we can see these two cases produce significantly different values for momentum, suggesting that the approximation of classical is no longer applicable at large velocities. As we approach the speed of light, c, it is only the relativistic momentum that correctly accounts for the maximum velocity imposed on a particle by special relativity. This is seen by the dramatic rise of relativistic momentum as we approach the asymptote at $x = c$. Classical momentum continues straight and would continue beyond the speed of light despite it being a maximum value; this is not a physical situation and thus only relativistic momentum is correct for large velocities and can account for velocities approaching c

ENERGY MASS EQUIVALENCE, E=MC²

CONTENT

One of the best-known results from Einstein's Special Relativity Theory is the equation that relates energy and mass:

$$E = mc^2$$

where E is the energy, m is mass and c is the speed of light in a vacuum, 3.00×10^8ms^{-1}. This result has many applications and is able to explain where energy and mass go in certain reactions without violating the law of conservation of energy. Since the speed of light is a **very** large number, and in this equation it is being squared to make an even larger number, $E = mc^2$ shows how a very small amount of mass can be converted to produce a huge amount of energy. This had many implications, from explaining the energy of the sun, to where the energy in the combustion of conventional fuels and particle-anti-particle annihilations came from.

For many years, scientists had tried to explain where the energy of the sun came from. Calculations suggested it could only produce the energy it is currently producing for a period of at most a few million years. However, scientists had already aged rocks on the Earth to be over 4 billion years old. The energy mass equivalence explains where the energy from the sun comes from and how it is able to sustain itself for many billions of years.

Inside the sun, a process of nuclear fusion is occurring, a process that fuses, or combines, multiple atoms together to form a new product. The sun is fusing types of Hydrogen into Helium through several complicated reactions. However, the end product of this reaction has less mass than the beginning products. Using $E = mc^2$ we now know this missing mass is converted into energy.

Just like the nuclear fusion process in the sun, when a particle and its corresponding anti-particle collide, they annihilate each other and their mass is turned into energy via $E = mc^2$. If we know the mass of the two particles before the collision and there are no particles left after, we know all the mass has been converted to energy.

EXAMPLE

The mass of an electron is about 9.11×10^{-31}kg and a positron (its anti-particle counterpart) has the same mass. If an electron and positron collide and no particle is left after, how much energy is released by the conversion of mass to energy in the collision?

⇒ Firstly, we add the mass of the electron and the positron since the total mass converted to energy is the combined mass of the two. So, our m is 1.822×10^{-30}kg

⇒ Now to sub our values into $E = mc^2$ and calculate the energy released by the conversion of mass:

$$E = mc^2$$
$$= 1.822 \times 10^{-30} \times (3 \times 10^8)^2$$
$$= 1.64 \times 10^{-13} \text{ J (3 sig fig)}$$

⇒ Therefore, when all the mass of an electron and positron annihilation is converted to energy, 1.64×10^{-13}J are released.

EMISSION AND ABSORPTION SPECTRA

BLACK BODY RADIATION

Black-body radiation is the name given to the electromagnetic radiation emitted by an idealised opaque and non-reflective object, which is in thermal equilibrium with its surroundings. Put simply, a black body does not **reflect** any of the light which hits it, and all of the light which hits it is **absorbed**. However, despite its name a black body is still able to **emit** radiation. Approximating stars as black bodies is generally a reasonable approximation. Even though stars do reflect light the amount of reflected light is many orders of magnitude lower than that emitted due to black body radiation. A comparison between the observed spectrum of our sun, and a theoretical black body of the same temperature is shown in Figure 1. The overall shape is a good approximation; however, the solar spectrum has significantly more visible and infrared light, and less UV light.

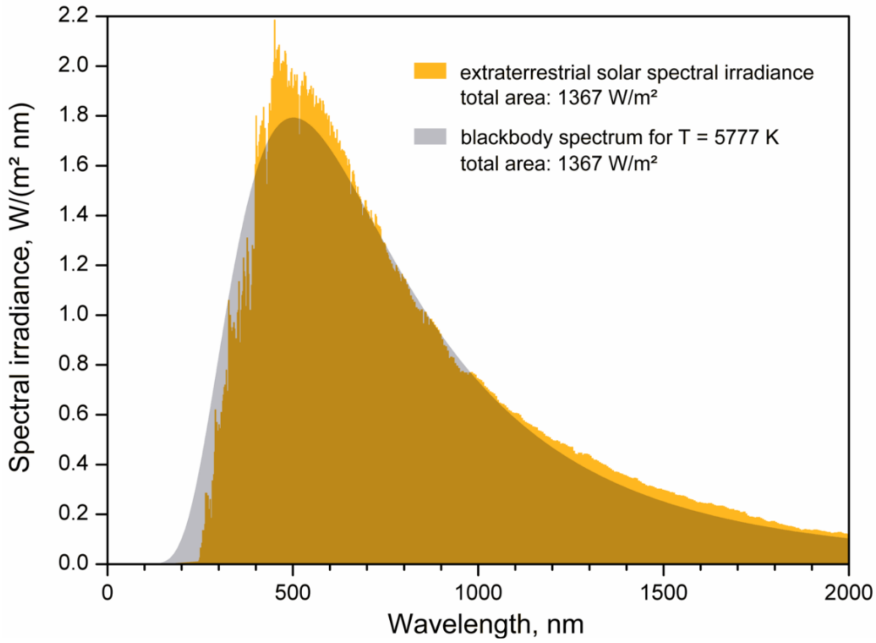


Figure 1. Comparison of observed solar spectrum and theoretical black body radiation
Image Credit: User:Sch/Wikimedia Commons/CC BY-SA 3.0

The wavelengths of the black body radiation are determined solely by the temperature of the material and not the composition of the material. Hotter temperatures shift the distribution to shorter wavelengths (bluer in colour) and colder temperatures produce longer wavelengths (redder in colour). Examples of these black body emissions for a range of temperatures are shown in Figure 2.

Another requirement for the black body radiation description to be valid is that the material must have a sufficiently large number of particles. Recall that there are discrete energy levels electrons can occupy in individual atoms. As we join atoms together the number of possible energy levels increases, and in the limit of a very large number of particles the number of energy levels grows to a point where it is no longer discrete as in a single atom but these possible energy levels become essentially continuous. This is why we get a seemingly continuous distribution in blackbody radiation. This is contrasted with the spectra we observe for gases, where the atoms are far enough apart that they maintain their discrete electron energy level structure. The emitted radiation from such a diffuse material appears as discrete 'spectral lines', which correspond directly to the discrete energy levels (this is covered in more detail in the resource 'Quantised Energy Levels').

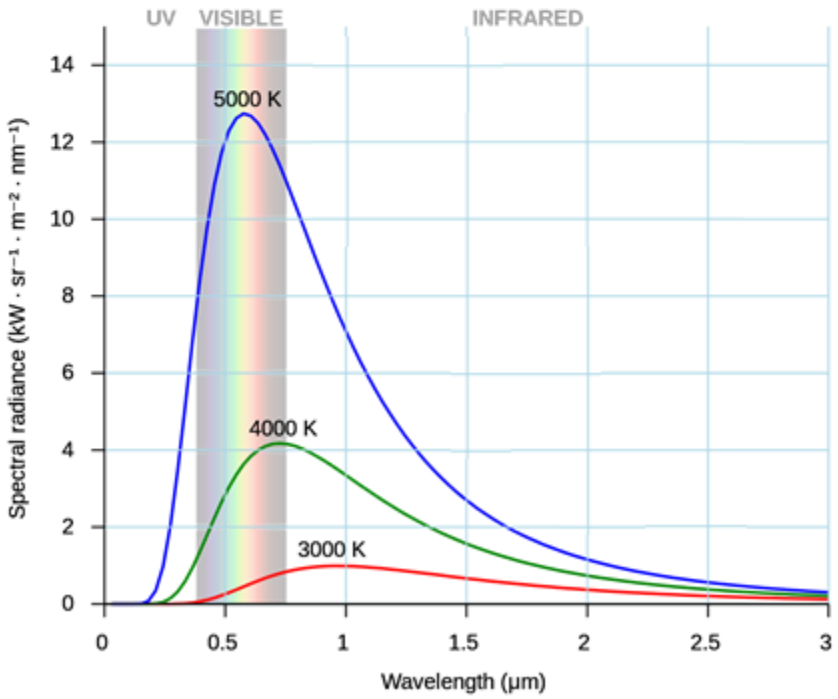


Figure 2. Black body radiation distributions for three different temperatures

Absorption and Emission

Although the bulk of a star is dense enough to produce continuous black body radiation, the outer layers are diffuse enough that the atoms in these outer layers maintain their discrete electron energy level structure. The electrons in these atoms can absorb specific wavelengths of light which correspond to these energy levels. This process causes absorption lines to appear superimposed on the continuous black body spectrum.

The inverse of absorption spectra are emission spectra. These spectra are observed when a diffuse region of gas (e.g. the outer layers of massive stars, or a galactic nebula) is excited by light from a star and they selectively re-emit this energy at the characteristic wavelengths of that gas. These emission spectra are found by observing the gaseous object from a direction which is out of line with the background source that is exciting the gas. These processes are depicted diagrammatically in Figure 3. Emission spectra can be used to determine the composition of gas clouds between the stars and the Earth.

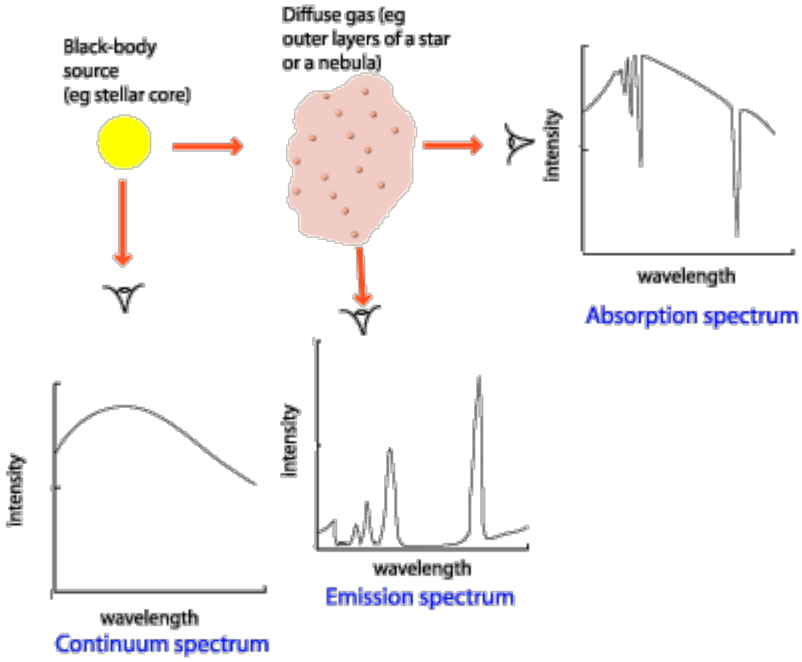


Figure 3. Creation of emission and absorption spectra
Image Credit: http://www.atnf.csiro.au/outreach/education/senior/astrophysics/

A real stellar spectrum is shown below in Figure 4. The most apparent or 'strongest' absorption lines correspond to the visible wavelengths of the Hydrogen atom known as the Balmer series.

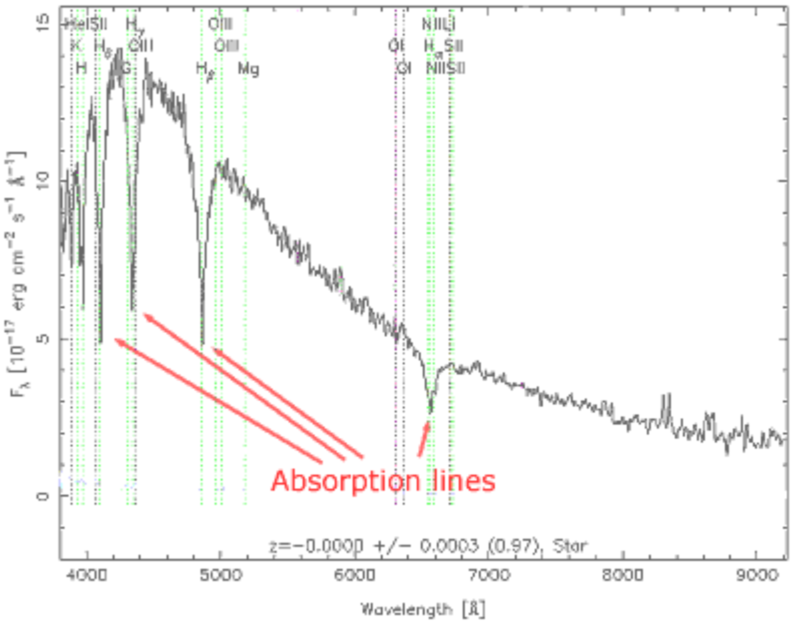


Figure 4. Real spectrum of a typical 'main sequence' star
Image Credit: The Sloan Digital Sky Survey

We can use these stellar spectra to classify stars by their temperature. By temperature we actually mean the 'effective temperature' of the star which corresponds to the temperature of the outer surface of the star that we actually observe. The inner core of the star is much hotter than the surface. The 'effective temperature' is determined by comparing the peak of the spectrum with the peak of the theoretical black body spectrum. Most stars can be classified into the following spectral classes, listed from hottest to coldest are O, B, A, F, G, K, M. The typical spectral features of these star are summarised in the table below.

Spectral Class	**Effective Temperature (K)**	**Colour**	**H Balmer Features**	**Other Features**	**M/M_{Sun}**	**L/L_{Sun}**
O	28,000 - 50,000	Blue	weak	ionised He^+ lines, strong UV continuum	20 - 60	90,000 - 800,000
B	10,000 - 28,000	Blue-white	medium	neutral He lines	3 - 18	95 - 52,000
A	7,500 - 10,000	White	strong	strong H lines, ionised metal lines	2.0 - 3.0	8 -55
F	6,000 - 7,500	White-yellow	medium	weak ionised Ca^+	1.1 - 1.6	2.0 - 6.5
G	4,900 - 6,000	Yellow	weak	ionised Ca^+, metal lines	0.85 - 1.1	0.66 - 1.5
K	3,500 - 4,900	Orange	very weak	Ca^+, Fe, strong molecules, CH, CN	0.65 - 0.85	0.10 - 0.42
M	2,000 - 3,500	Red	very weak	molecular lines, eg TiO, neutral metals	0.08 - 0.05	0.001 - 0.08

Table 1. Spectral features of stellar categories
Credit: Table reproduced from http://www.atnf.csiro.au/outreach//education/senior/astrophysics)

There are also rarer classes of stars that are distinguished by their relative abundance of carbon. For example R stars have the same temperature as K stars but have high abundances of carbon, and N stars are carbon-rich stars with the same temperature as M stars. Wolf-Rayet stars (classified as WN and WC), have the same temperature as O stars, but show strong broad emission lines of nitrogen or carbon. It is also important to mention that the light from distant stars often passes through diffuse gas clouds in between the star and earth. This can further complicate the interpretation of these absorption spectra. Aside from determining the chemical composition and temperature of stars, we can also use absorption spectra to infer the translational and rotational speed of a star. This is achieved by using the Doppler Effect and looking for the shift between the spectral lines observed in the stars and those from stationary sources here on earth.

HERTZSPRUNG RUSSELL DIAGRAM

CONTENT

The **Hertzsprung–Russell diagram**, or more simply **HR diagram,** was first created in 1910 by Ejnar Hertzsprung and Henry Norris Russell. It is a now famous scatter plot of stars which describes the relationship between the absolute magnitude or luminosity of stars and their stellar classifications or effective temperatures. More simply, each star is plotted on a graph with the star's brightness on the vertical axis and its temperature or colour on the horizontal axis. While there are several versions of the HR diagram, they all share the same general layout, with more luminous stars towards the top of the diagram, and fainter stars towards the bottom. Stars with higher surface temperature sit toward the left side of the diagram, and those with lower surface temperature sit to the right side of the diagram. Since stars are essentially hot black bodies, their colour is directly related to their surface temperature, with blue stars being the hottest and red stars being the coolest. The concept of blackbody radiation is covered in more detail in a previous resource. A descriptive version of the HR diagram is shown in Figure 1.

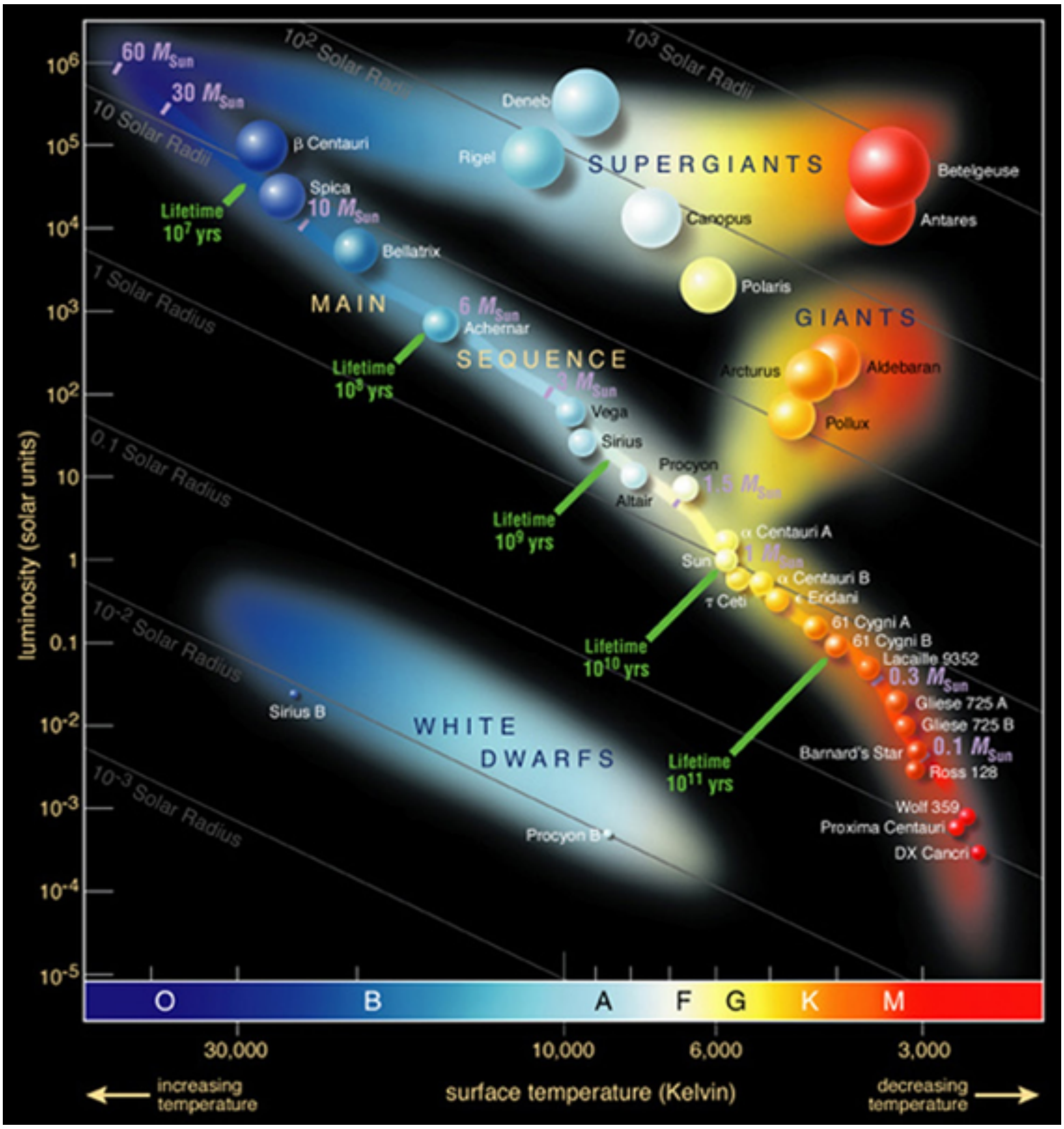


Figure 1: The Hertzsprung-Russell diagram
Image Credit: European Southern Observatory/**CC BY 4.0**

The life cycle of stars can also be tracked through the HR diagram. The basic stellar evolution of both high and low mass stars is covered in Figure 2. All stars begin as a star forming nebula of gas, and when enough mass has accumulated hydrogen fusion begins. The remains of the nebula that has not coalesced into a star is blown away by the energy generated by fusion. Once the fusion process has begun, the protostar is "born". All stars, regardless of their mass, begin their life on the "main sequence" (this grouping of stars can be seen in Figure 1). Very low mass stars about 1/10 the mass of our sun, which can be found in the bottom right corner of the HR diagram, are only massive enough to fuse hydrogen into helium. Once their hydrogen supply is close to running out they move to the left of the HR diagram and become blue dwarfs, and once their fuel is consumed entirely they will slowly cool, eventually becoming white and then black dwarfs. There are no blue dwarfs yet as the lifetime of these small stars is very long, and there has not been enough time to exhaust their fuel given the limited age of the universe. All white dwarfs that currently exist are the remnants of mid-sized stars such as the sun.

Mid-size stars, such as our sun, begin fusing hydrogen on the "main-sequence" but are massive enough to fuse their helium through the CNO cycle (covered in more details in the 'Origin of the Elements' resource). Once the helium burning stage begins, these stars grow significantly in radius and move vertically up the HR diagram as they get much brighter. The outer surface temperature also decreases and the stars become redder in colour. This is due to the larger size of the star, as the surface is now further from the core where most of the energy and heat is being created. These stars are now off the "main-sequence" and in the "giant" area. This cooler surface temperature gives these stars the commonly used name "red giant". Once the helium is exhausted the star collapses, creating a planetary nebula and ultimately a white dwarf. White dwarfs are located in the lower left of the HR diagram. These stars are small in radius and low luminosity as much of their mass is lost to the planetary nebula. However, they are still quite hot/blue when they are first created. They slowly move in a diagonal right-down direction as they slowly cool and become dimmer, eventually becoming so cool and dim that we can no longer observe them (black dwarfs).

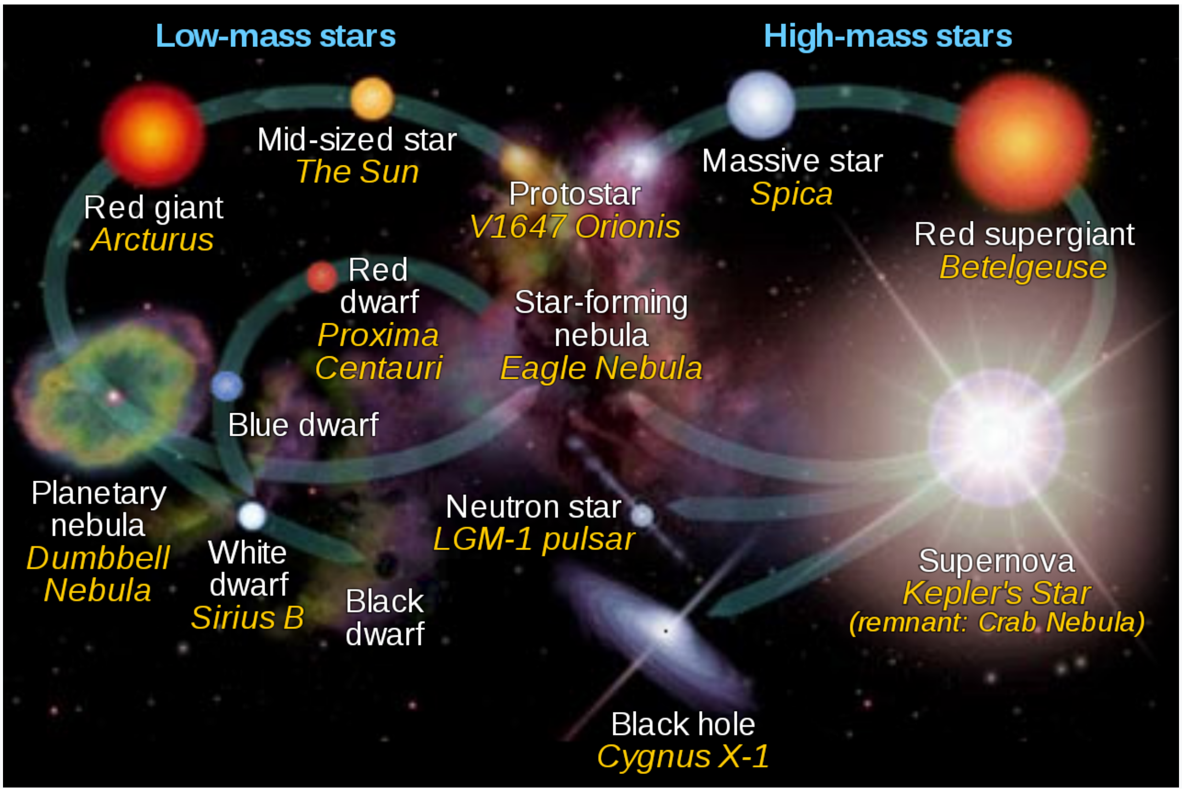


Figure 2: Life cycles of low and high mass stars
Image Credit: NASA Goddard Space Flight Center/**CC BY-SA 4.0**

High mass stars (>10 x mass of the sun) also begin on the "main sequence" as they consume their hydrogen. These stars quickly initiate helium fusion after they have exhausted their hydrogen, and are so massive that they will be able to continue fusing heavier elements after the helium fuel is exhausted. These stars will even fuse heavier elements up to iron until they develop an iron core. While these stars fuse these heavier elements they move off the main sequence towards the right. However, the most massive of these stars often don't make it as far right as less massive supergiants. This is because they are so energetic and massive that their outer layers are completely blown away before the star goes supernova. A supernova occurs once the fuel begins to run out and the gravitational pressure on the iron core becomes so large that the matter reaches a threshold where the repulsive force between the electron clouds that keeps atoms separate forces a collapse into the atoms' nuclei to form what is essentially one huge atomic nucleus made up of neutrons. We call this a neutron star. If the mass of the star is large enough then in a similar way the neutrons can also collapse and become a black hole, although this exact process is not well understood. These final states of the supergiant stars do not appear on the HR diagram.

The HR diagram can also be used by scientists to roughly measure the distances from Earth to a particular star cluster. This is achieved by comparing the apparent magnitudes of the unknown cluster stars to the absolute magnitudes of stars with known distances. The observed cluster is then shifted in the vertical direction on the HR diagram until the main sequence of the cluster overlaps with the main sequence of the reference stars of known distances. This technique is known as main sequence fitting, and by comparing the difference in luminosity between the unknown cluster and the reference stars, the distance to the unknown cluster can be determined.

ORIGINS OF THE ELEMENTS

CONTENT

The most famous equation in all of physics is, probably, Einstein's description of the equivalence of energy and mass. This relationship tells us that energy can be converted into mass and vice versa.

$$E = mc^2$$

The first thing that we notice from this equation is that due to the large value of the speed of light (c), it takes a lot of energy to create a small amount of mass. For example, the amount of energy equivalent in a cup (250mL) of water is 2.24×10^9 MJ. This is enough energy to power around 41,000 homes for 1 whole year! Let's take a look at how we calculated these numbers.

EXAMPLE 1

To demonstrate how to use this relationship we will calculate how much energy is equivalent to 250 mL of water.
First we need to know the mass of a cup of water. Given the density of water is 1000 kg/m^3, we can calculate the mass with the following equation:

$$\begin{aligned} mass &= density(kg/m^3) \times volume(m^3) \\ &= 1000 \times 0.00025 \\ &= 0.25\ kg \end{aligned}$$

Now that we have the mass of the water, we can calculate the equivalent amount of energy:

$$\begin{aligned} E &= mc^2 \\ &= 0.25\ \mathrm{kg} * (2.998 \times 10^8\ \mathrm{m/s})^2 \\ &= 2.247 \times 10^{16}\mathrm{J} \\ &= 2.25 \times 10^{10}\ \mathrm{MJ} \end{aligned}$$

This gives us the energy in Joules. However, we mostly measure electrical power consumption in units of kWh. We can use the conversion factor of 1MJ = 0.2778 kWh, thus

$$\begin{aligned} E_{kWh} &= 2.25 \times 10^{10} \times 0.2778 \\ &= 6.25 \times 10^9\ \mathrm{kWh} \end{aligned}$$

The average Australian household consumes 40 kWh per day, meaning that the total number of houses that could be powered for a year would be:

$$Houses = \frac{6.25 \times 10^8}{40 \times 365} = 428{,}116$$

Nuclear fusion reactions

It is the conversion of mass to energy that is responsible for the enormous energy released by stars. Nuclear reactions known as fusion reactions are responsible for this release of energy. There are two major fusion reactions that take place in stars.

In each example below the mass of the helium nucleus produced is slightly less than the mass of the four hydrogen nuclei consumed to produce it. Slightly less than 1% of the mass involved in the reaction is converted to energy.

Proton-proton fusion

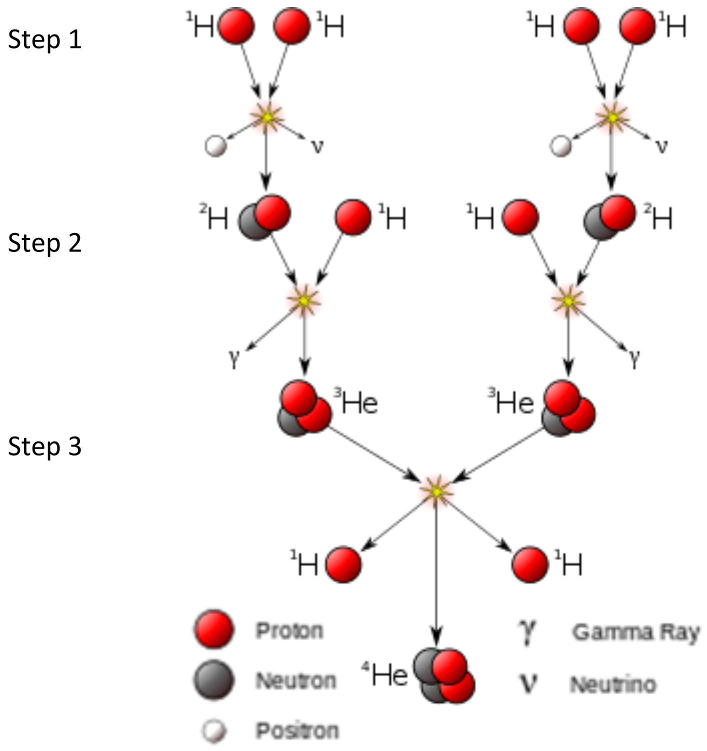


Figure 1. Schematic diagram of the proton-proton cycles
Image credit: User: Borb/Wikimedia Commons/CC BY-SA 3.0

$$4H \rightarrow He_4$$

Step 1: Two hydrogen nuclei, each a single proton, fuse together to create deuterium. This fusion process results in the emission of a positron (e+) and a neutrino.

Step 2: The newly-formed deuterium nucleus fuses with another hydrogen nucleus present inside the star. Together they form a ^{3}He nucleus, emitting a single photon.

Step 3: Two ^{3}He nuclei formed through the two steps above now fuse together, resulting in a stable ^{4}He nucleus and the release of two hydrogen nuclei back into the star.

This entire fusion cycle releases around 26.2 MeV of energy each time. This proton-proton cycle is the dominant source of energy in a main sequence star like our Sun.

CARBON-NITROGEN-OXYGEN FUSION (CNO)

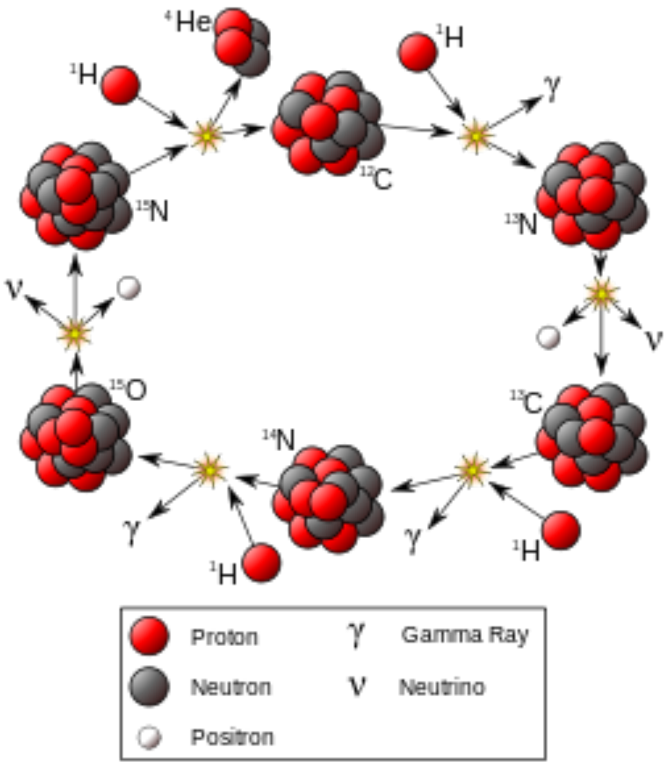


Figure 2. Shemcatic diagram of the CNO cycle
Image credit: User:Borb/Wikimedia Commons/**CC BY-SA 3.0**

Another way that stars can convert hydrogen into helium is through the carbon-nitrogen-oxygen (CNO) cycle. The CNO cycle is a catalytic process which involves multiple steps where heavier nuclei are involved in the reaction; however these nuclei are not used up in the process. Since the carbon nucleus is reconstructed at the end of the cycle, the overall reaction can be represented simply as $4H \rightarrow He_4$, as before. There are many slightly different versions of the CNO cycle, which are divided into two main categories of cold and hot CNO cycles, with the former being limited by the rate of initial proton captures, and the latter being limited by the rate of beta decays of the intermediate nuclei. An example of a cold CNO cycle is given below. This is the most common CNO cycle found in typical conditions within stars.

Step 1: A carbon-12 (^{12}C) nucleus fuses with a proton to form a nitrogen-13 (^{13}N) nucleus, emitting a photon in the process.

Step 2: The nitrogen nucleus releases a positron (e+) and a neutrino to become a carbon-13 (^{13}C) nucleus.

Step 3: The new carbon nucleus fuses with a proton to form nitrogen-14 (^{14}N), releasing a photon.

Step 4: The nitrogen nucleus fuses with a proton to form oxygen-15 (^{15}O), releasing a photon.

Step 5: The oxygen nucleus releases a positron and a neutrino to become a nitrogen-15 (^{15}N) nucleus.

Step 6: The nitrogen-15 (^{15}N) nucleus fuses with one final proton to produce a helium-4 (^{4}He) nucleus and a carbon-12 (^{12}C) nucleus that can be used once again for step 1, beginning the cycle over again.

Each complete CNO cycle produces about 25 MeV of energy. This is the main source of energy in higher-mass post-main sequence stars. This is because the repulsive electromagnetic force of the six-proton carbon nucleus prevents fusion without an incredibly high temperature and pressure to overcome it. Thus, it can only occur in the very centre of low mass stars, but is widespread in more massive stars. In the most massive stars even heavier elements, up to iron and nickel can form. Elements heavier than these typically don't form as iron and nickel are very stable and the star loses energy when fusing even heavier elements together. The heaviest elements tend to form a dense core in the centre of the star, with the lighter elements forming the outer layers of the star.

NUCLEAR MODEL OF THE ATOM

CONTENT

In the late 1800's and early 1900's there was a fierce debate about the composition of matter. While there were many ideas at the time, two models dominated the discourse. These were Thomson's 'plum-pudding' model, and Rutherford's 'atomic' model. The electron was discovered by J. J. Thomson in 1897 through his famous cathode ray experiment, and although this experiment wasn't conclusive, it suggested that the electron was a very light particle, about 2000 times lighter than the hydrogen atom. This result was conclusively determined by Millikan's oil drop experiment, where the exact charge of the electron was confirmed. This idea about a small and mobile electron was incorporated into both the Thomson model and the Rutherford model. Descriptive depictions of these models are shown in Figure 1.

Both models depicted the electrons in the same way (shown in blue), as small localised and mobile sources of negative charge. It was known that matter contains some amount of positive charge which balances out the negative charge of the electrons. It was also known that unlike the negative charge of electrons which are highly mobile, the positive charge contained within matter is locked in place. The key difference between the two models is the distribution of this positive charge. In Thomson's model the positive charge is spread out over a large volume, which makes up the bulk of the volume of matter. This can explain why the positive matter is not mobile as it is not condensed into small particles. Rutherford presumed that the positive matter should take on a much more similar form to the electrons. He thought that the positive charge should be localised as a discrete concentration of particles. Although much larger than the electrons, they still represented most of the mass of matter. This did suggest, however, that most of the space within matter is made up of nothing more than empty space. This is probably the most counter-intuitive part of Rutherford's model.

Definitive evidence for Rutherford's model came from the Geiger-Marsden experiment. A schematic diagram of this experiment is shown in Figure 1. An alpha particle source emits energetic particles in all directions. A collimated beam of alpha particles is created by encapsulating an alpha source in a box with a small hole in it. This beam is directed towards a very thin film of gold foil. Gold was chosen due to the fact that it is the most malleable metal and can be formed into incredibly thin sheets. The sheets had to be very thin as alpha particles have very poor penetration power and if the material was too thick then the alpha particles would be absorbed by the material. The expected results were quite different for the two models. Thomson's model predicted that due to the spread out nature of the positive charges the repulsive forces from the positive charge would be small and the deflection of the alpha particles would be minor. However, due to the high concentrations of positive matter in Rutherford's model, when an alpha particle approached one of these high concentrations of charge the predicted forces would be immense and the degree of deflection would be dramatic. The results were clear and clearly supported the 'atomic' model of the atom. After this discovery it was quickly surmised that the place of elements in the periodic table or 'atomic number' corresponded to the nuclear charge of the atom, and that the smallest atom (hydrogen) contained just one of these particles, which came to be named protons.

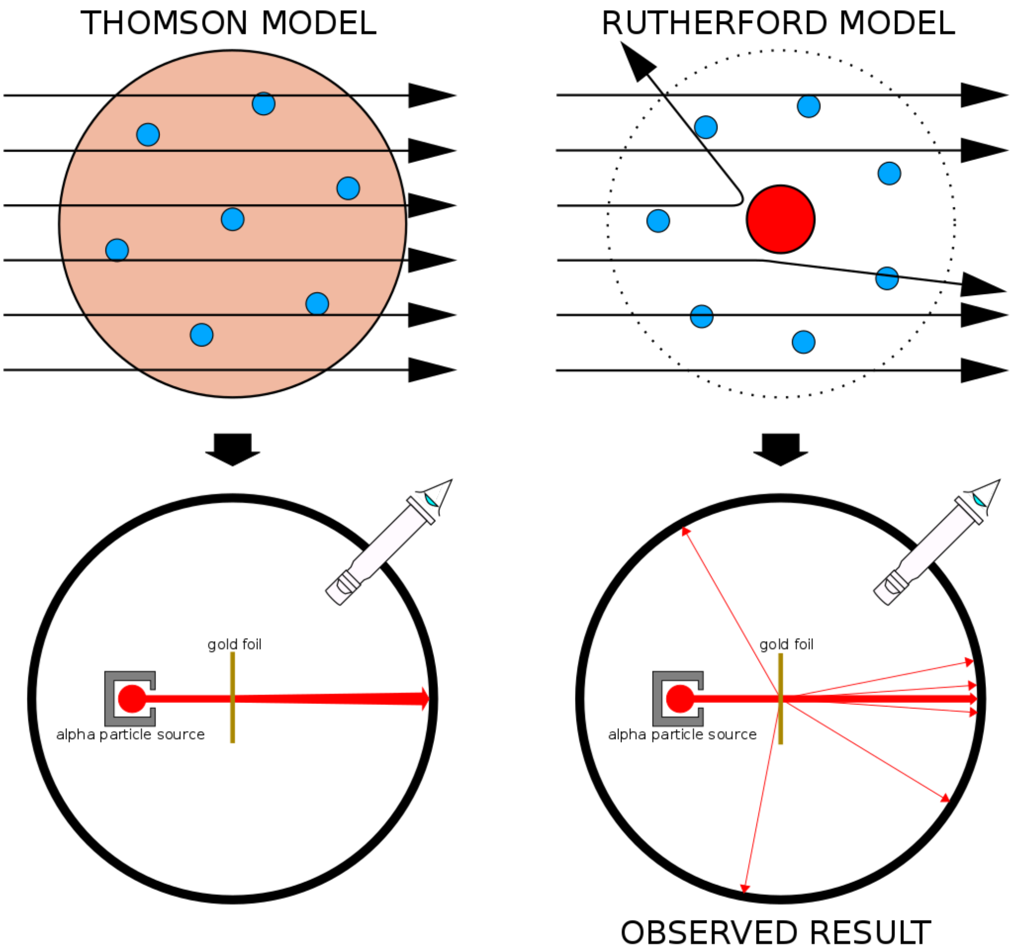


Figure 1. Comparison of Thomson and Rutherford models of the atom
Image credit: User:Kurzon/ Wikimedia Commons/CC BY-SA 3.0

After the validation of the atomic model by Rutherford, a more detailed version of Rutherford's model, called the Rutherford-Bohr model, or just the Bohr model was developed by Rutherford and Bohr. A schematic diagram of this model is shown in Figure 2. The core feature of this model, which differs to the earlier Rutherford model, is the postulated fact that the electrons are able to occupy certain stable orbits without emitting radiation (in conflict to classical electromagnetism). Electrons cannot occupy orbits between these stable orbits and the stationary orbits are located at distances which correspond to an electron angular momentum that is a multiple of the reduced Planck's constant ($n\hbar$). The final major point of the Bohr model is that electrons can move to different stable energy levels by the absorption and emission of electromagnetic radiation with a frequency that corresponds to the energy difference between the energy levels. This model was able to explain the atomic spectra of Hydrogen and similar atoms. This was a major improvement on previous models and suggested that there was some validity to the concept of 'allowed electron energy levels' in atomic structure. Despite these improvements, the Bohr model was not able to predict atomic spectra for the majority of atoms; it could not explain why some spectral lines were much more intense than others, it could not explain the Zeeman effect (where spectral lines are split in the presence of a magnetic fields, and most importantly it could not explain any reasoning as to why these energy levels were allowed and others were forbidden. It would take the introduction of quantum mechanics to answer these questions.

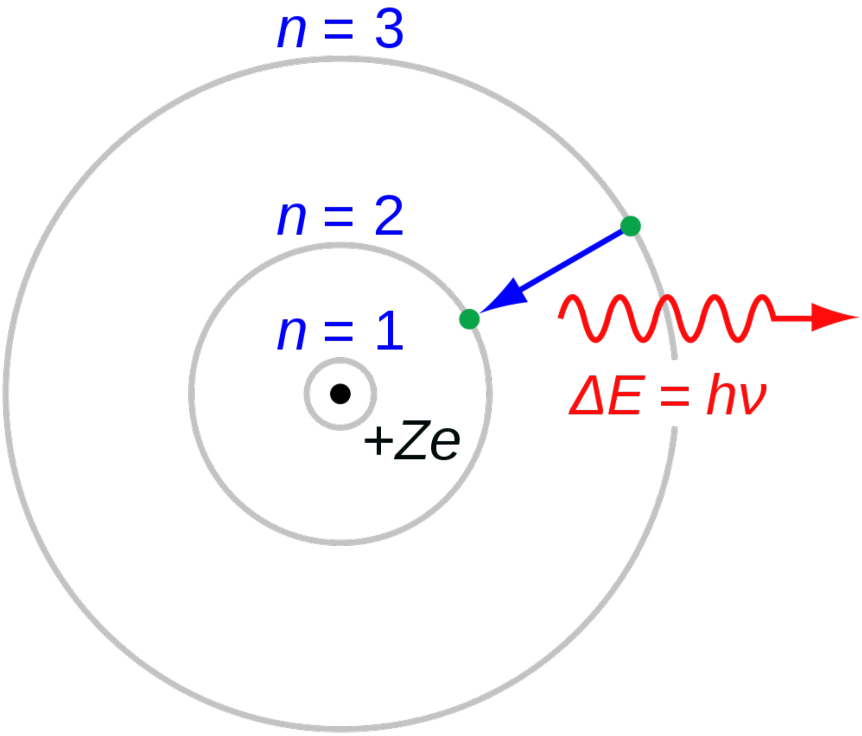


Figure 2: The Bohr model of the atom
Image credit: User:JabberWok/Wikimedia Commons/ CC BY-SA 3.0

The discovery of nuclear isotopes (that is, atoms that have the same atomic number and chemical nature, but a different atomic weight) was the first major piece of evidence that suggested the existence of the neutron. It was found that the mass of these isotopes could always be expressed as a whole number of proton masses, suggesting the existence of a particle that had the same mass as a proton but was neutral in terms of electric charge. Due to the fact that the neutron has no electric charge and the difficulty in creating free neutrons separate from the atomic nucleus, it would remain difficult to prove its existence.

An unknown and highly penetrating form of radiation was observed to be emitted when alpha particles struck specific light elements, such as beryllium. Initially, this radiation was thought to be gamma rays, but it was more highly penetrating than any gamma rays previously observed. It was also found that this unknown radiation caused the ejection of protons from paraffin wax at very high energies of 5 MeV. In order for gamma rays to be responsible for this interaction they must have an unbelievably high energy. James Chadwick was one of the physicists who did not believe the gamma ray explanation and designed the experiment shown in Figure 3 in order to study the reaction. By carefully measuring the range of the ejected protons and the interaction of the unknown radiation with a number of gases, Chadwick was able to determine that the unknown radiation could be interpreted as a neutral particle with about the same mass as the proton. Chadwick's discovery of the neutron provided the final piece in the atomic model puzzle and, in 1935, won him the Nobel Prize on Physics.

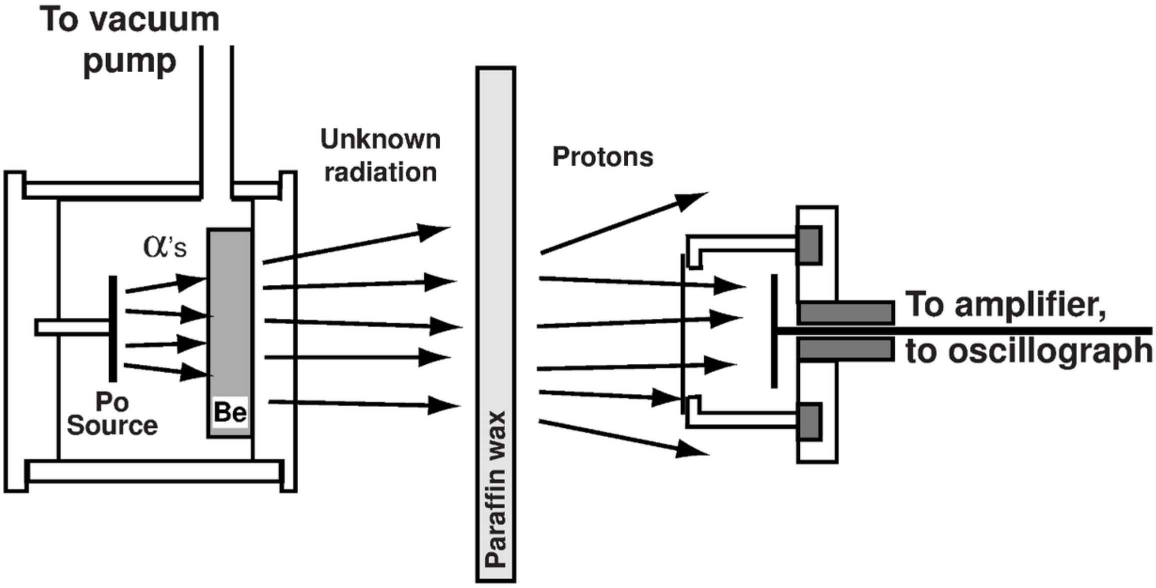


Figure 3: Schematic diagram of the experiment Chadwick used to discover the neutron
Image credit: User:Bdushaw/Wikimedia commons/CC BY-SA 4.0

ORIGINS OF THE UNIVERSE

CONTENT

Current scientific theory holds that at some point in history, there was no universe. In fact, time and space themselves didn't even exist before the explosion of radiation known as the 'Big Bang'. Suddenly, nearly 14 billion years ago, energy burst into being from a singularity and ultimately created everything that exists in our universe today, from atoms to stars to galaxies. But how did a bunch of light become physical matter?

Scientists believe that the universe was born at an incredibly high temperature, somewhere around 10^{32} K. With this much heat energy around, physical matter like particles and atoms couldn't exist, and the four fundamental forces of the universe we're familiar with - gravity, electromagnetism, and the strong and weak nuclear forces - were unified into a single superforce. We can trace a somewhat speculative timeline of the early universe as it expanded and cooled.

TIMELINE

- *Inflationary phase* (t = 10^{-43} to 10^{-35} s): The universe undergoes exponential expansion and cools down rapidly to ~10^{27} K. Gravity separates itself out as an independent force.

- *Age of leptons* (t = 10^{-35} to 10^{-6} s): The strong nuclear force separates, followed by the weak nuclear force and electromagnetism. Fundamental particles like quarks, leptons and photons come into existence.

- *Age of nucleons* (t = 10^{-6} to 225 s): Composite particles made up of two or more quarks known as hadrons come into being. Thanks to the high temperature, photons combine to form matter/anti-matter pairs, which then annihilate each other to create more photons in a thermal equilibrium. These processes are detailed in the equations following.

$$\gamma + \gamma \Leftrightarrow e^- + e^+$$

$$\gamma + \gamma \Leftrightarrow p + \bar{p}$$

$$\gamma + \gamma \Leftrightarrow n + \bar{n}$$

As this process continues, the universe cools down to 10^{11} K, too cold for continued proton and neutron anti-matter pairs. There is slightly more matter than antimatter overall.

- *Age of nucleosynthesis* (t = 225 s to 1,000 years): Protons and neutrons, collectively known as nucleons, begin to combine with each other to form nuclei which then react with each other to form bigger nuclei. By the end of this stage about 25% of the universe is helium, and the remainder mainly hydrogen.

- *Age of ions* (t = 1,000 to 3,000 years): The temperature of the universe is still high enough to ionize any atoms formed, so it's made mostly of photons, light nuclei and other charged particles.

• *Age of atoms* (t = 3,000 to 300,000 years): The temperature drops to 10^5 K, allowing nucleons and electrons to form stable atoms of hydrogen and helium. Following the production of stable atoms, photons no longer interact frequently with normal matter in the thermal equilibrium described above. This electromagnetic radiation spreads out to fill the universe. Today, this ancient radiation can still be detected as a faint, omnipresent glow known as the cosmic microwave background, with a temperature equivalent to about 3 K.

• *Age of stars and galaxies* (t = 300,000 years to present): Matter aggregates into clumps. Extremely dense clumps allow hydrogen atoms to undergo nuclear fusion, and create the first stars.

HUBBLE AND THE EXPANDING UNIVERSE

Up until the early 1900s most scientists assumed that the universe was overall static. This means that while it was thought possible that stars and galaxies could move relative to each other, it was thought that the space in which all objects exist in does not expand or contract in any way. However, since we know that gravity is an attractive force between all matter, why has the matter in the universe not coalesced into one gigantic mass? This question was pondered by many; in fact even Newton raised this as a serious issue with his theory of gravitation.

In the 1920s, the American astronomer Edwin Hubble conducted an investigation into the observations that other astronomers had made of the red-shift of distant 'nebulae' (now known to be galaxies) a known distance away – especially studies by astrophysicist Henrietta Leavitt and her observation of variable stars in the Small and Large Magellanic Clouds. It was observed that characteristic spectral lines of stars (covered in detail in the resource 'Stellar Spectra') were red-shifted for distant galaxies when compared with the wavelengths of these same spectral lines when measured on earth. It was initially assumed that these shifts were due to the Doppler Effect. This was a known process where the relative motion between a source and an observer causes a shift in the wavelengths measured. This process is depicted in Figure 1. However, we now know that this interpretation is not accurate.

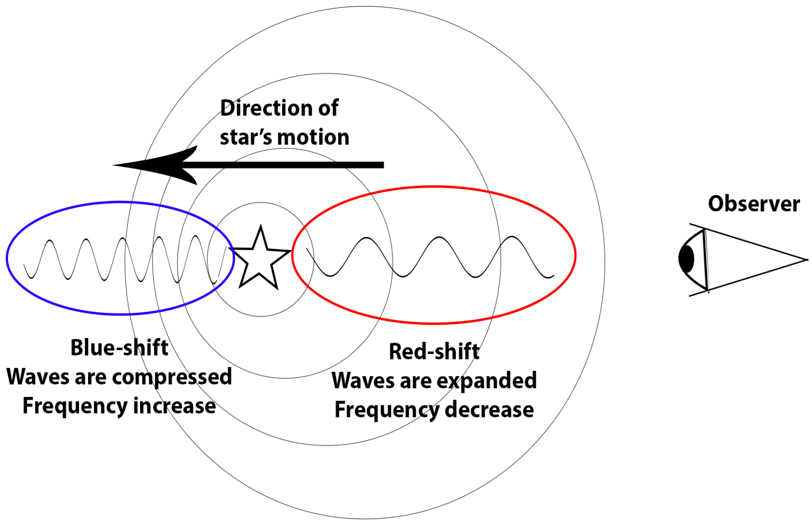


Figure 1. The initial interpretation of Hubble's Data was Red/blue shift from stars motion

Two remarkable results were found from analysing Hubble's data. First, it was found that the speed with which a galaxy is moving away from us is proportional to its distance, with the most distant galaxies moving the fastest and the closest galaxies moving the slowest. This relationship is called Hubble's Law. The second major discovery is that the light from *all* of the galaxies is 'redshifted' indicating that all galaxies are moving away from us. We have no reason to think that we are located at the centre of the universe, or that the region of space that we are located in is special in any way. This is a consequence of what is known as the cosmological principle, which states that at a large scale the universe is homogeneous and isotropic. This leads us to conclude that from any galaxy it appears that all other galaxies are moving away or that from any point in the universe it appears that the universe is expanding in all directions. Hubble's law supports the theory of a Big Bang, as it indicates that all of the matter in the universe was far more concentrated in the past than it is now.

Our understanding of the universe has improved with the introduction of the theory of general relativity. According to general relativity, the redshift in the observed wavelength of light is not caused by a Doppler shift as distant galaxies expand into a previously empty space. Rather, the observed redshift comes from the expansion of space itself and everything in intergalactic space, including the wavelengths of light traveling to us from distant sources. This can be a difficult concept to grasp at first. Consider the example shown in Figure 2, this shows a section of the 'universe'. Note that the relative position (grid-coordinates) of the galaxies remains the same, while the distance between the galaxies has expanded, and the wavelength of the light between the galaxies has also expanded. This is a direct effect of the fact that it is the space on which the grid, galaxies, and light are placed which is expanding. The picture below is a little misleading as it still shows a 'finite' universe which seems to expand into 'empty' space on the page. This is where this simple illustration breaks down, as our best current evidence suggests that the universe is infinite. This means that there is nothing outside the universe, it has no edges, and it is not expanding into anything. This is a hard concept to capture in an illustration!

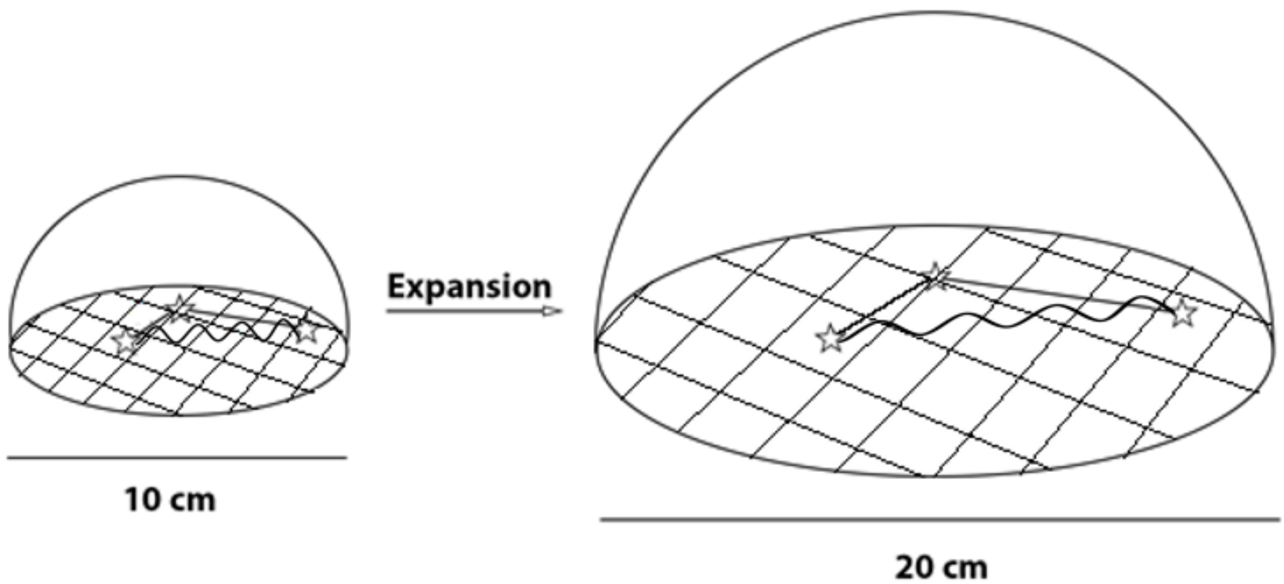


Figure 2. The general relativity view of the expansion of the universe

QUANTISED ENERGY LEVELS

CONTENT

The Balmer series refers to a series of visible spectral lines that are observed when hydrogen gas is excited. These lines are shown in Figure 1 (upper). The four most prominent lines occur at wavelengths of 410 nm, 434 nm, 486 nm, and 656 nm. The combination of these spectral lines is responsible for the familiar pinkish/purplish glow that we associate with hydrogen discharges and gaseous nebulae, shown in Figure 1 (lower).

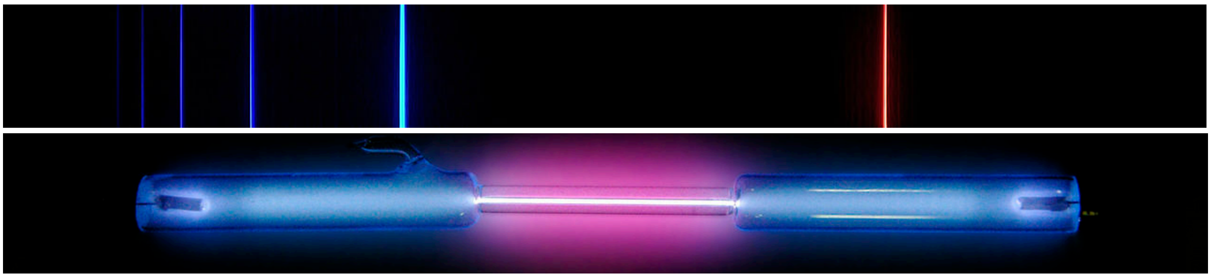


Figure1. The six lines of the Balmer series (upper), Hydrogen discharge (lower)
Image Credit: User: Jan Homann/Wikimedia Commons/CC BY-SA 3.0 & User: Alchemist-hp/(www.pse-mendelejew.de)/FAL

These spectral lines were observed long before an empirical equation was discovered by Johann Balmer in 1885 which could describe the wavelengths of the lines. However, it was not known at the time why this equation matched the observed wavelengths. Balmer's equation for wavelengths is given as

$$\lambda = B\left(\frac{n^2}{n^2 - m^2}\right),$$

where λ is the predicted wavelength, B is a constant with the value of 364.50682 nm, *m* is equal to 2, and *n* is an integer such that $n > m$.

We now know that this equation was able to describe the wavelengths of the Balmer series because these wavelengths correspond to electron transitions between higher energy levels and the n=2 energy level. As electrons transition from higher energy levels to lower energy levels, conservation of energy is maintained by the emission of a photon of energy equal to the difference between the final and initial energy levels of the electron. Such a transition is depicted in Figure 2, for the H_α line. We also know the difference between successive energy levels decreases as *n* increases. In addition, in the limit as n goes to infinity, the wavelength approaches the value of the constant B. This limit corresponds to the amount of energy required to ionise the hydrogen atom and eject the electron. This is also referred to as the electron binding energy.

As previously stated, the law of conservation of energy dictates that the frequency of the emitted photon be equal to the energy difference between the transitioning electron energy levels in the atom. This relation is quantified by the Einstein-Planck equation (sometimes referred to, in the context of electron transitions, as Bohr's frequency condition), given as

$$E = hf,$$

where E represents the difference in electron energy levels (this is also equal to the photon energy), *h* is Planck's constant (6.626 ×10^{-34} J.s), and *f* is the frequency of the emitted photon. We can also use the wave equation,

$$c = f\lambda,$$

where *c* is the speed of light, *f* is the frequency of the light, and λ is the wavelength of the light, in order to relate the energy of the electron transitions directly to the wavelength of the emitted photon

$$E = \frac{hc}{\lambda}.$$

Other hydrogen spectral series were also discovered, for instance the Lyman series describes transitions that end on the *n* = 1 energy level. As a result, the photons emitted from these transitions are higher in energy than the Balmer series, and the Lyman series is in the ultraviolet region of the electromagnetic spectrum. Another example is the Paschen series; this series describes transitions to the *n* = 3 level, so the emitted photons are lower in energy than the Balmer series and are categorised as infrared.

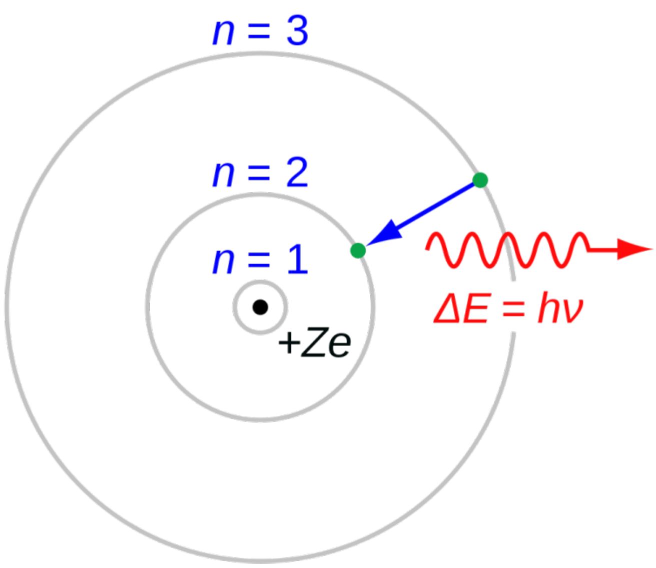


Figure 2: The Bohr model of the atom showing the H_α line
Image credit: User: JabberWok/Wikimedia Commons/ CC BY-SA 3.0

The Balmer equation was generalised by Rydberg in order to describe the wavelengths of all of the spectral series of hydrogen, including for instance the Paschen and Lyman series. The Rydberg equation is described as:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right),$$

where λ is the wavelength of electromagnetic radiation emitted in vacuum, R_H is the Rydberg constant for hydrogen with a value of 1.097×10^7m^{-1}, and n_f and n_i correspond to the final and initial energy levels of the transition.

Example 1:

Determine the wavelength of the 2nd line in the Lyman series (electron transition from n = 3 to n = 1), known as Lyman β

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{8}{9}\right)$$

$$\lambda = 1.0255 \times 10^{-7}\,\mathrm{m}$$

$$\lambda = 103\ \mathrm{nm}$$

DE BROGLIE AND SCHRÖDINGER

CONTENT

Louis de Broglie was a French physicist who postulated that electrons could exhibit wave behaviour. This was based on the explanation Einstein gave for the photoelectric effect. Recall from the previous module 'The Nature of Light' that the photoelectric effect was an experimental result involving the interaction between light and metallic surfaces. These results could not be explained by the wave model of light but were explained successfully by Einstein using a particle model of light. He described the energy and momentum of the individual particles of light (photons) as:

$$E = hf$$
$$p = \frac{h}{\lambda}$$

In 1924, as a part of his PhD thesis, De Broglie postulated that this relationship between momentum and wavelength could apply to electrons as well. By rearranging the momentum equation, and substituting in the classical equation for momentum ($p = mv$), he was able to predict the wavelength of electrons as:

$$\lambda = \frac{h}{mv}$$

These waves are referred to as 'matter waves' or 'De Broglie waves', and this relationship is now known to hold true for all types of matter, not only electrons. This was verified experimentally for the first time in 1927 with the Davisson–Germer experiment. After this verification De Broglie won the Nobel prize in physics for his prediction. The Davisson–Germer experiment was based on the principle of diffraction. Diffraction is the general principle that occurs when waves (light or other) encounter an object or aperture (covered in the previous module 'The Nature of Light'). It was previously determined by Bragg that the crystal lattices of certain materials can cause diffraction patterns when X-rays pass through them. The Davisson–Germer experiment was simple in principle: an electron beam was fired at a nickel target and a detector was moved about in angle to search for reflected electrons, as shown in Figure 1.

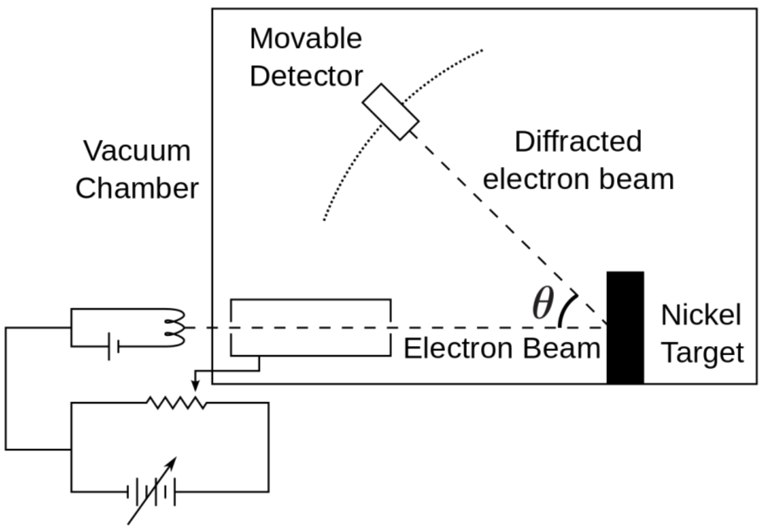


Figure 1. Experimental setup of the Davisson–Germer experiment
Image Credit: User:Roshan/Wikimedia Commons/ CC BY-SA 3.0

This experiment was not initially designed to look for diffraction patterns but for looking at how electrons are reflected from the surface of the material. One day when the nickel target was accidentally exposed to air, an oxide layer developed on its surface. In order to remove this oxide layer the nickel target was heated in a high temperature oven. While this removed the oxide surface, it also had the unintended effect of turning the polycrystalline structure of the nickel into much larger areas of consistent crystal structure. These consistent crystal areas were now large enough to cover the whole width of the electron beam. When the experiment was repeated with the new target, a diffraction pattern with unexpected peaks was observed, similar to those observed with x-rays by Bragg. The results were published by Davisson and Germer but they did not understand what to make of them. It was Max Born who saw the diffraction patterns and realised that they were in fact the first experimental confirmation of the De Broglie Hypothesis.

The idea of matter waves was extended by Erwin Schrödinger through a series of three papers published in 1926. In these papers he presented what is now known as the Schrödinger equation, which describes what we now know as 'wave-functions'. The wave function is a mathematical description of the state of a quantum system (e.g. electrons bound to atoms). It also describes the probabilities for the outcomes of making measurements of that system. These concepts formed the basis of quantum physics and have led to Schrödinger being referred to as the 'father of quantum physics'. Schrödinger also provided solutions to the Schrödinger equation that corresponded to the energy levels for hydrogen or hydrogen like atoms (atoms with only one electron). This model of the atom was able to correctly predict the spectral lines associated with these energy levels. This new quantum model was also able to account for other things that the previous model (Bohr model) of the atom could not, such as the various intensities of spectral lines, and the Zeeman Effect. We still use the Schrodinger equation and the concept of wave functions to describe atoms, and we now refer to the states of the electrons in atoms as atomic orbitals. An example of the atomic orbitals found in the hydrogen atom is shown in Figure 2. Note that unlike the electron orbits of earlier atomic models these orbitals take on many different shapes, not only spherical. Also, rather than having an electron as a discrete particle orbiting a nucleus, the atomic orbitals are more akin to a cloud which surrounds the nucleus. The probability of finding the electron (if we attempt to measure it) is greatest where the cloud is most dense.

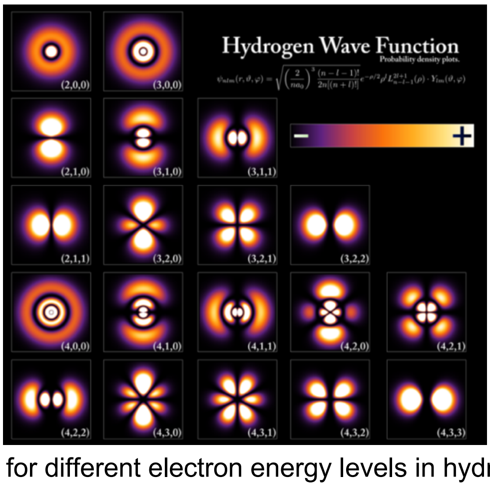


Figure 2. Atomic orbitals for different electron energy levels in hydrogen. The probability of finding the electron is given by the colour shown in the key (upper right).

RADIOACTIVE DECAY

CONTENT

Soon after the initial discovery of radioactivity, researchers found that applying an electric field would cause the radiation to split into three different types, as shown in Figure 1. This result indicated that alpha particles are positively charged, beta particles are negatively charged, and gamma rays have no electric charge.

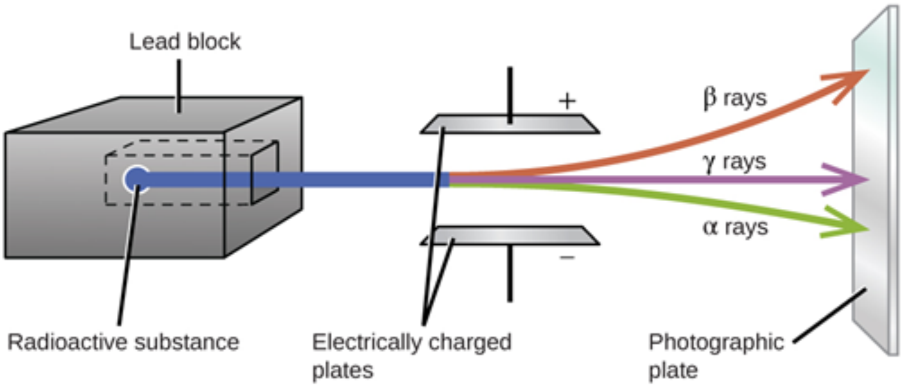


Figure 1. The discovery of three different types of radiation

Image Credit: http://letslearnnepal.com/class-12/physics/modern-physics/radioactivity

These types of radiation were named by their penetrating power, with alpha radiation being the least penetrating, being shielded by a few cm of air or just a sheet of paper. Beta radiation was found to be somewhat more penetrating, requiring a few mm of metal, such as aluminium in order to be blocked. Gamma rays are by far the most penetrating requiring several centimetres of lead in order to be absorbed. All of these are referred to as 'ionising radiation'. This means that the energy of this radiation is sufficient to ionise atoms and break chemical bonds. This is biologically important, as this is the type of radiation that can damage our DNA and cells, causing injury or even death. Examples of non-ionising radiation found in our everyday lives are microwaves and radio waves. While alpha radiation is the least penetrating it is actually the most ionising of these three forms of radiation. Although this may be counter-intuitive at first, it is the fact that the alpha particles so readily ionise the material they encounter, and transfer their energy into that material, that leads to them penetrating less material and dumping all of their energy in a small volume.

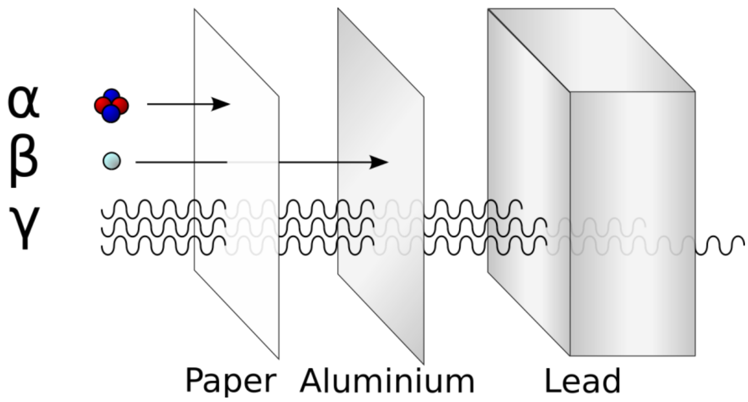


Figure 2. Penetrating power of alpha, beta, and gamma radiation

User: Stannered derivative work: Ehamberg/Wikimedia Commons/ CC BY 2.5

The composition of alpha, beta and gamma radiation is also different. An alpha particle is actually a helium-4 nucleus, which itself consists of two protons and two neutrons. Alpha decay typically occurs only in heavy nuclei. With the exception of beryllium-8 which spontaneously decays into two alpha particles, there are no elements below tellurium (atomic number 52) that undergo alpha decay. However, all elements heavier than lead (atomic number 82) are unstable, with the majority decaying through alpha radiation. Alpha decay has the largest effect on the decaying nucleus reducing the atomic mass number by 4 and atomic number by 2. A common isotope which is regularly used by students to observe an alpha decay is americium-241. Americium-241 has a relatively long half-life of 432.2 years, and the decay process is summarised in the equation below:

$$^{241}_{95}Am \rightarrow \ ^{237}_{93}Np + \ ^{4}_{2}\propto + \ \gamma \ 59 \text{ keV}$$

As you can see, this decay process also results in the emission of a gamma ray with an energy of 59 keV. Gamma rays always occur with another decay process. This is because the gamma rays are not actually emitted during the initial alpha or beta decay, but rather from the resultant nucleus. In the americium-241 example above the neptunium nucleus is actually created in an excited state, and this excited state rapidly decays (with a very short half-life) into a stable ground state with the emission of a gamma photon. This is very similar to how electrons that are bound to atoms can decay from excited states to lower energy states with the emission of a photon.

A beta particle is an electron that has been ejected from a nucleus. The electron is created in the nucleus of the atom by a neutron decaying into a proton and an electron. The proton remains in the nucleus and the electron is emitted. Beta decay typically occurs in nuclei that have a large ratio of neutrons to protons. Since beta decay will reduce the number of neutrons by one and increase the number of protons by one, the ratio of neutrons to protons is made even by this reaction. A common example of beta decay is caesium-137, and the reaction is summarised in the equation below:

$$^{137}_{55}Cs \rightarrow \ ^{137}_{56}Ba + \ ^{0}_{-1}\beta$$

For alpha, beta and gamma radiation it is impossible to predict when any specific nucleus will decay. That is why we describe these reactions as 'probabilistic', and we are only able to predict the average fraction of nuclei that will decay over a certain period of time. This introduces us to the characteristic time scale which we call 'half-life' ($t_{1/2}$). This is the time it takes for half of the nuclei in a sample to decay. If we consider the number of nuclei remaining after a period of many half-lives the amount of remaining nuclei follows an exponential decay, with half the nuclei remaining after one 'half-life', and half of that half (a quarter) remaining after another 'half-life', and so on. This is depicted diagrammatically in Figure 3. This relationship is described mathematically by:

$$N_t = N_0 e^{-\lambda t}$$

where N_t is the number of particles at time *t*, N_0 is the number of particles present at *t=0*, $\lambda = \frac{\ln(2)}{t_{1/2}}$ is called the decay constant, and $t_{1/2}$ is the half-life.

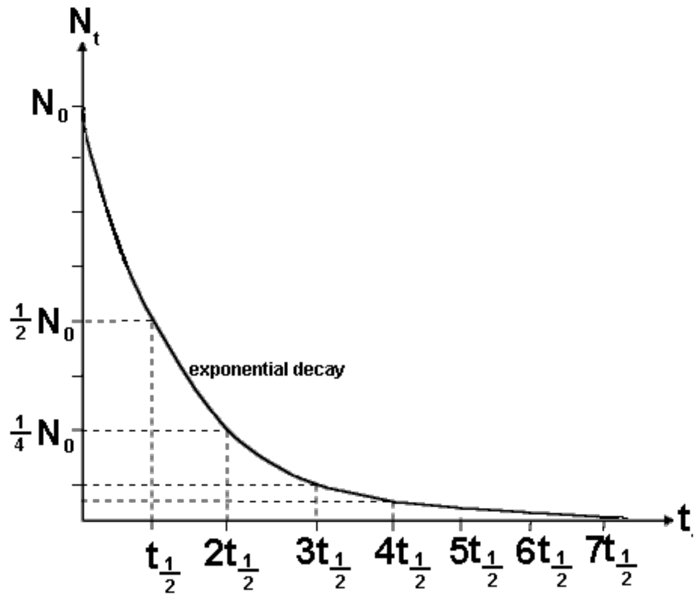


Figure 3. Number of nuclei versus number of half-life periods
Image Credit: https://www.cyberphysics.co.uk/topics/radioact/Radiodecay.htm

Example 1: In a student lab it is common to use americium-241 to demonstrate alpha decay. Say that a student's source contains 0.1g of americium-241. How much americium will be left after 1000 years? The half-life of americium-241 is 432.2 years.

$$N_t = N_0 e^{-\lambda t}$$

$$N_t = 0.1 e^{(-\frac{\ln(2)}{432.2})1000}$$

$$N_t = 0.020\ g$$

Example 2: Iodine-131 is a very dangerous substance with a short half-life (8.3 days), it is produced in significant quantities in nuclear reactors and weapons. Let's say a lake is polluted with iodine-131, how many days do we have to wait for 99.9% of the iodine to decay and for the water to be safe to drink?

$$N_t = N_0 e^{-\lambda t}$$

$$0.1 = 100 e^{-\lambda t}$$

$$1 \times 10^{-3} = e^{-\lambda t}$$

$$\lambda t = -\ln(1 \times 10^{-3})$$

$$t = \frac{-\ln(1 \times 10^{-3})}{\lambda}$$

$$t = \frac{-\ln(1 \times 10^{-3})}{(\frac{\ln(2)}{t_{1/2}})}$$

$$t = \frac{-\ln(1 \times 10^{-3})}{(\frac{\ln(2)}{8.3})}$$

$$t = 82.7\ days$$

$$t = 83\ days$$

FISSION AND FUSION

CONTENT

FISSION:

Nuclear fission can be described as either a nuclear reaction or a radioactive decay process. What defines this process is the splitting of the atom's nucleus into smaller, lighter nuclei. In the case of the fission of heavy elements, this process typically emits individual neutrons and gamma rays, and releases a very large amount of energy. It is the difference in mass between the original fuel nucleus and the resulting lighter nuclei that results in either the release or absorption of energy. Specifically, for fission to produce net energy, the resulting elements must have less mass than that of the original element. This difference in mass arises due to the difference in atomic "binding energy" between the atomic nuclei before and after the reaction (an example for this is given in the resource 'Nuclear Energy Calculations'). The energy contained in a nuclear fuel (e.g. ^{235}U, ^{239}Pu) is millions of times the amount of energy contained in a similar mass of chemical fuel. This makes nuclear fission an incredibly dense source of energy. It is nuclear fission which produces the energy of nuclear power plants and also drives the explosive force of nuclear weapons.

After a fission event, the resulting nuclei are no longer the same element as the original atom. Fission is, therefore, a type of nuclear transmutation. This process usually results in two nuclei of comparable sizes, but in around 0.2% of fission events three fragments are created. This special case is known as ternary fission. The fact that the products of fission reactions are not the same every time is a key difference between this and other types of radioactive decay, such as alpha and beta decay which give the same result in every decay reaction. These fission products are generally more radioactive than the original fuel elements and remain dangerously radioactive for long periods of time. This creates a significant issue for the safe disposal and storage of this nuclear waste. An additional issue is the diversion of nuclear fuel for use in weapons. These are the biggest roadblocks to more widespread use of nuclear energy.

The neutrons that are emitted during a fission event are emitted at a very high velocity, typically 7% the speed of light. A material is described as *fissionable* if it undergoes a fission reaction after absorbing one of these fast neutrons. It is preferable for reactor safety and efficiency that the fuel does not depend on these fast, neutron-induced fission reactions. It is preferable that the fuel is able to absorb much slower "thermal" neutrons to undergo a fission reaction. Such fuels are known as *fissile* materials. Some of the most commonly used fissile fuels are ^{233}U, ^{235}U and ^{239}Pu. An example of a fissionable element that is not fissile is ^{238}U, which is the most abundant form of uranium. It will undergo induced fission when impacted by a fast neutron. However, the average energy of the neutrons produced through the fission of ^{238}U is too low to induce further fission. For this reason, a chain reaction is not possible with ^{238}U.

A chain reaction of fission events can take place so long as the neutrons emitted from a fission event create at least one more fission event. Another way of thinking about this is also as the rate of generation of neutrons within the fuel versus the loss of neutrons from the fuel. As long as the rate of generation is greater a chain reaction will take place. A reactor which allows for a sustained nuclear chain reaction is commonly referred to as a critical assembly. In the case of a bomb, where the device is mainly just nuclear fuel, it is called a critical mass. An example of a sustained fission reaction is shown in Figure 1. Critical fission reactors are the most common type of nuclear reactor.

The fundamental physics of the fission chain reaction in a nuclear reactor is similar to a nuclear weapon. However, the two types of devices are very different in design. A nuclear weapon must release all its energy at once, while a reactor is designed to generate a steady supply of useful power. Put simply the main differences are the degree of criticality and the use of fast versus slow neutrons. In a reactor the ratio of neutrons produced in the fuel to those lost is only slightly greater than 1. Also, the time between the production of a neutron to its use in a fission event is on the order of tens of seconds. This results in a rate of power change of about 10% per second. This rate of change is slow enough that it can be controlled by reactor mechanisms. In a weapon, the time between the production of a neutron and its use in a fission event is much quicker, typically on the order of milliseconds. This results in a rate of power change of around 1,000,000% per second. This satisfies the purpose of the weapon, which is to create as much energy as fast as possible. Such a reaction would obviously not be able to be controlled in a reactor. Due to these and other design factors, it not possible for a nuclear reactor to explode with the same destructive power as a nuclear weapon.

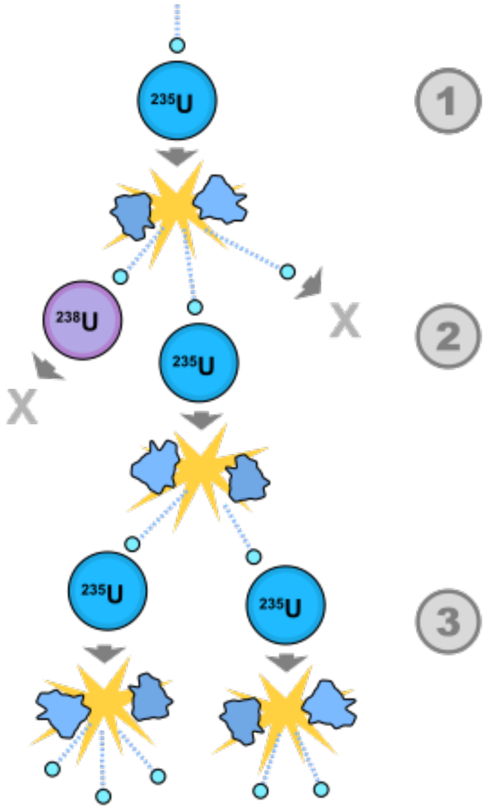


Figure 1: Diagram of a fission chain reaction

FUSION:

Nuclear fusion is a reaction in which two or more atomic nuclei are combined to form a number of different atomic nuclei and subatomic particles. As with fission, it is the difference in mass between the reactants and products which determines the amount of energy that is either released or absorbed. A fusion reaction that produces a nucleus **lighter** than iron will result in a net release of energy. The opposite is true for nuclear fission, where elements **heavier** than iron produce a net release of energy.

Hydrogen to helium fusion is the primary source of energy in all stars. Mid-size stars will also fuse helium into heavier elements such as oxygen, carbon, and nitrogen. Supermassive stars will even fuse elements all the way up to iron. Fusion to heavier elements does not occur in significant amounts as these processes take energy from the star, and when the fusion energy begins to wane the gravitational force collapses the star and if the star is massive enough a supernova results. The supernova is so energetic that elements heavier than iron are created in significant amounts and this is the main way that these heavier elements are created in the universe.

Nuclear fusion has the potential to provide us within an abundant source of energy without the long-lived nuclear waste issues that arise with nuclear fission. This is because the products of fusion reactions we might use for power generation are either stable or have comparatively short half-lives.

Unlike nuclear fission, where we were able to create controlled nuclear fission in a relatively short period of time following our initial discovery of the phenomenon, we have been unable to create controlled nuclear fusion with a net output of energy despite many decades of research. The main reason we have been unable to create controlled nuclear fusion is that it requires temperatures of millions of degrees Celsius. When materials reach these temperatures (or even at much lower temperatures) the electrons separate from the positive nucleus to form free electrons and ions. A material such as this is generally considered the fourth state of matter and is called plasma. Since these ions and electrons are charged particles, combinations of electric and magnetic fields are used to control this material. This is challenging due to the extremely high energy of the plasma particles. A high degree of confinement of this plasma is necessary as any contact with materials at normal temperatures will rapidly cool the plasma and stop any fusion from occurring. There are many different approaches to confining these fusion plasmas, and the most successful to date is a device known as the Tokomak. This is a doughnut-shaped device which confines a plasma in a toroidal magnetic field.

NUCLEAR ENERGY CALCULATIONS

CONTENT

The laws of conservation of mass and energy are two fundamental premises in physics which state that both mass and energy cannot be created or destroyed and are therefore conserved. In most cases it serves us well to consider separately that the total mass and total energy is conserved, as this is true in all mechanical and chemical processes. However, when it comes to nuclear processes a significant change in the amount of mass *and* energy in the system is observed. These reactions highlight the fact that the laws of conservation of mass and energy are not correct in all circumstances. It is more accurate for us to state that it is the total amount of mass *and* energy that is conserved, and that mass and energy can be transformed into one another using Einstein's most famous equation expressing the equivalence of mass and energy, $E = mc^2$.

BINDING ENERGY

Binding energy is the term given to the energy that holds together the individual nucleons (protons and neutrons) within the nucleus of an atom. This is quite a large amount of energy, which is a result of how close the protons in the nucleus are to each other. Recall that particles which have like charges will generate a repulsion force between them and this force increases with the inverse square of the distance between them. The source of the binding energy is a small fraction of the mass of the nucleons; this mass is converted to binding energy according to $E = mc^2$.

Example 1:
Let's take a look at the binding energy of the helium nucleus. The most common form of helium is known as ^{4}He which contains two protons and two neutrons. In order to calculate the magnitude of the binding energy in a single atom of helium we need to compare the mass of 2 individual protons and 2 individual neutrons, with the mass of a helium nucleus.

Looking up a few values we find that

Mass of proton = $1.6726219 \times 10^{-27}$ kg

Mass of neutron = 1.674929×10^{-27} kg

Mass of helium nucleus = $6.6464807 \times 10^{-27}$ kg

So, the difference in mass between the individual nucleons and the helium atom is:

$$(2 \times mass\ of\ proton + 2 \times mass\ of\ neutron) - (mass\ of\ helium\ nucleus)$$

$$(6.69510 \times 10^{-27}) - (6.64466 \times 10^{-27})$$

$$0.050441 \times 10^{-27}\ \text{kg}$$

We then use the mass energy equivalence to convert to energy:

$$E = mc^2$$

$$E = 0.050441 \times 10^{-27} \times (3 \times 10^8)^2$$

$$E = 4.5334 \times 10^{-12}\ \mathrm{J}$$

This seems like a very small amount of energy, and it is, but remember that this is for only one atom, and there are *a lot* of atoms in the amounts of things that we are more familiar with. To put this in perspective there are about 1.5 x 10^{23} atoms in 1 gram of helium, which gives us a binding energy of 680MJ per gram of helium, and this is quite a lot of energy!

The binding energy is relevant to fusion and fission reactions as it is a requirement **that the products of the reaction be more stable than the reactants** for either of these reactions to release a net gain of energy.

Energy Released In Nuclear Reactions

Example 2:
Let us calculate the energy released in the fusion reaction between deuterium and tritium. This is the most promising reaction for a potential fusion power source. The reaction is summarised below.

$$^2_1D + ^3_1T \rightarrow ^4_2He + ^1_0n$$

Here, a tritium nucleus fuses with a deuterium nucleus to produce a helium nucleus and a free neutron. By comparing the difference in the masses of the products and reactants, we can use E = mc^2 to calculate the energy released in this reaction. We can find from the literature the mass for the elements of the reaction

Mass of Deuterium: 3.34 x 10^{-27} kg
Mass of Tritium: 5.01 x 10^{-27} kg

Mass of Helium: 6.65 x 10^{-27} kg
Mass of Neutron: 1.67 x 10^{-27} kg

Calculating the energy released in joules

$$Energy\ Released = (Mass\ of\ reactants - Mass\ of\ products) \times speed\ of\ light^2$$

$$= \left((mass_T + mass_D) - (mass_{He} + mass_n)\right) \times speed\ of\ light^2$$

$$= \left(\left((3.34 + 5.01) - (6.65 + 1.67)\right) \times 10^{-27}\right) \times 3.00 \times 10^8$$

$$= 2.8 \times 10^{-12}\ \mathrm{J}$$

Nuclear reactions are typically described in units of MeV. In order to convert to MeV we must first divide by the charge of an electron, to convert from J to eV. Then divide by 10^6 to convert eV to MeV.

$$Energy\ Released = 17.5\ MeV$$

This is consistent with the documented value of 17.59 MeV. This same method can be used to determine the energy released in any nuclear transmutation, including naturally occurring radioactive decay (alpha, beta, and gamma) and fission.

BINDING ENERGY PER NUCLEON

Let us now consider the changes in binding energy in the deuterium-tritium reaction. We won't go through calculating the binding energy for all of these nuclei as we did in the last example, but we can easily look up the binding energies in order to compare them.

Values:

Binding energy for Deuterium, 2224 keV

Binding energy of Tritium, 8,481 keV

Binding energy of ^{4}He, 28300 keV

We can clearly see that the binding energy for the helium nucleus is much greater than the deuterium and tritium nuclei, which indicates that the helium nucleus is more tightly bound and hence is a more stable nucleus. For this reason, it is a more energetically stable configuration for the nucleons (proton and neutrons) to exist in than as a deuterium and tritium nucleus.

However, there is one more caveat in understanding the relationship between binding energy and stability. The number that truly represents stability is not the total binding energy of the nucleus, but the binding energy per nucleon. This number accurately represents stability as it represents how easy it is to pull the individual nucleons apart.

For example, consider a nucleus that has 100 nucleons and a total binding energy of 200,000 keV, this would be a binding energy per nucleon of 2,000 keV. Now consider a nucleus with just 10 nucleons and a total binding energy of 100,000 keV, this would be a binding energy per nucleon of 10,000 keV. Despite the fact that the larger nucleus has more total binding energy, it is the smaller nucleus that is actually more stable as it has 5 times more binding energy per nucleon and hence requires 5 times more energy to separate an individual nucleon.

If we were to calculate the binding energy per nucleon for our previous deuterium-tritium example, we can see that it is still greater for helium than deuterium and tritium, hence the reaction can occur and release energy.

Binding energy per nucleon of Deuterium, 1112keV

Binding energy per nucleon of Tritium, 2827keV

Binding energy per nucleon of ^{4}He, 7075keV

This brings us to the final figure which helps us understand binding energy. This graph plots the binding energy per nucleon for a range of different nuclei. On the left-hand side of the graph we can see that as the sizes of the nuclei increase, the binding energy per nucleon also increases. Hence, in this region it is possible for fusion reactions to occur and release energy. This is because the larger nuclei are more stable than the smaller reactants. However, on the right-hand side of the graph the opposite is true; these large nuclei are actually more stable and release energy when they split apart to make smaller nuclei (fission). This is the opposite process to fusion and is called fission. The most stable nucleus shown on this graph is ^{56}Fe.

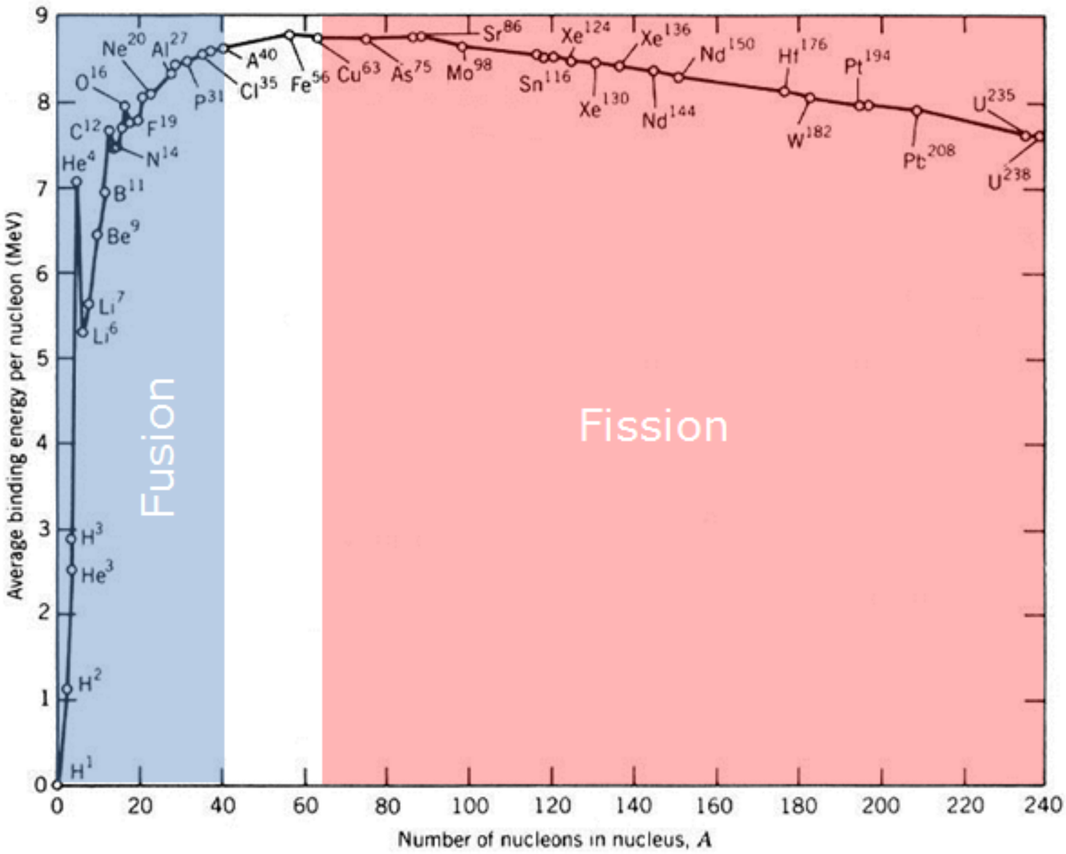


Figure 1. Binding energy per nucleon for a variety of atomic nuclei
Image Credit: http://www.earth-site.co.uk/Education/binding-energy/