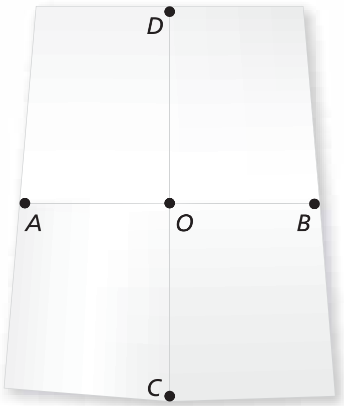
# 3.4 Proofs with Perpendicular Lines

**Essential Question** What conjectures can you make about perpendicular lines?

## EXPLORATION 1 Writing Conjectures

**Work with a partner.** Fold a piece of paper in half twice. Label points on the two creases, as shown.

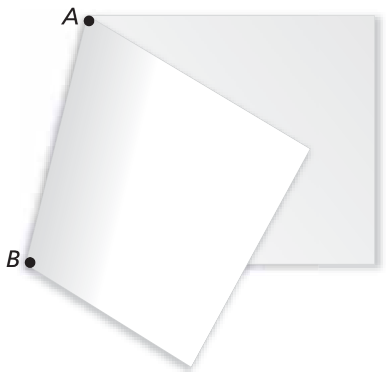



**a.** Write a conjecture about $\overline{AB}$ and $\overline{CD}$. Justify your conjecture.

**b.** Write a conjecture about $\overline{AO}$ and $\overline{OB}$. Justify your conjecture.

## EXPLORATION 2 Exploring a Segment Bisector

**Work with a partner.** Fold and crease a piece of paper, as shown. Label the ends of the crease as $A$ and $B$.




**a.** Fold the paper again so that point $A$ coincides with point $B$. Crease the paper on that fold.

**b.** Unfold the paper and examine the four angles formed by the two creases. What can you conclude about the four angles?

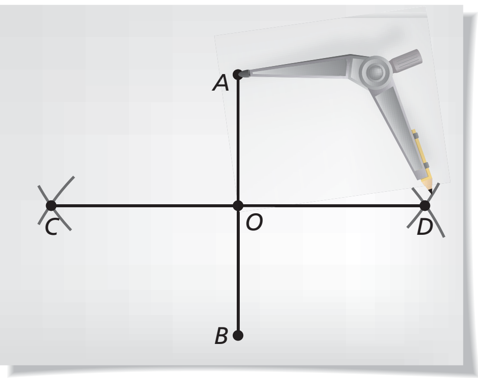
## EXPLORATION 3 Writing a Conjecture

**Work with a partner.**

**a.** Draw $\overline{AB}$, as shown.

**b.** Draw an arc with center $A$ on each side of $\overline{AB}$. Using the same compass setting, draw an arc with center $B$ on each side of $\overline{AB}$. Label the intersections of the arcs $C$ and $D$.

**c.** Draw $\overline{CD}$. Label its intersection with $\overline{AB}$ as $O$. Write a conjecture about the resulting diagram. Justify your conjecture.




**CONSTRUCTING VIABLE ARGUMENTS**

To be proficient in math, you need to make conjectures and build a logical progression of statements to explore the truth of your conjectures.

## Communicate Your Answer

**4.** What conjectures can you make about perpendicular lines?

**5.** In Exploration 3, find $AO$ and $OB$ when $AB = 4$ units.

# 3.4 Lesson

## Core Vocabulary

distance from a point to a line, *p. 148*
perpendicular bisector, *p. 149*

## What You Will Learn

- Find the distance from a point to a line.
- Construct perpendicular lines.
- Prove theorems about perpendicular lines.
- Solve real-life problems involving perpendicular lines.

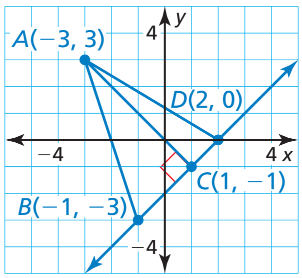
## Finding the Distance from a Point to a Line

The **distance from a point to a line** is the length of the perpendicular segment from the point to the line. This perpendicular segment is the shortest distance between the point and the line. For example, the distance between point $A$ and line $k$ is $AB$.


distance from a point to a line

### EXAMPLE 1 Finding the Distance from a Point to a Line

Find the distance from point $A$ to $\overleftrightarrow{BD}$.




**REMEMBER**

Recall that if $A(x_1, y_1)$ and $C(x_2, y_2)$ are points in a coordinate plane, then the distance between $A$ and $C$ is

$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

**SOLUTION**

Because $\overline{AC} \perp \overleftrightarrow{BD}$, the distance from point $A$ to $\overleftrightarrow{BD}$ is $AC$. Use the Distance Formula.

$$AC = \sqrt{(-3 - 1)^2 + [3 - (-1)]^2} = \sqrt{(-4)^2 + 4^2} = \sqrt{32} \approx 5.7$$

► So, the distance from point $A$ to $\overleftrightarrow{BD}$ is about 5.7 units.

## Monitoring Progress

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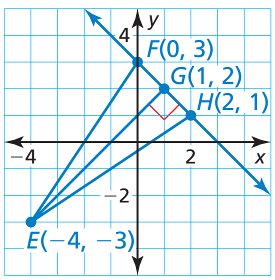
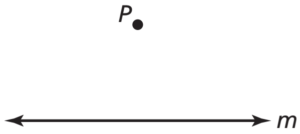
**1.** Find the distance from point $E$ to $\overleftrightarrow{FH}$.



## Constructing Perpendicular Lines

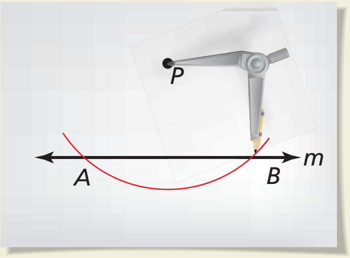
**CONSTRUCTION** **Constructing a Perpendicular Line**

Use a compass and straightedge to construct a line perpendicular to line $m$ through point $P$, which is not on line $m$.



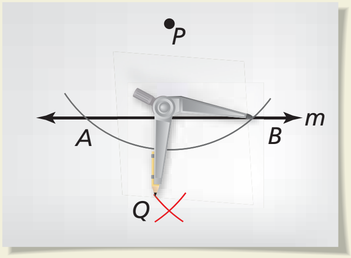

**SOLUTION**

**Step 1**



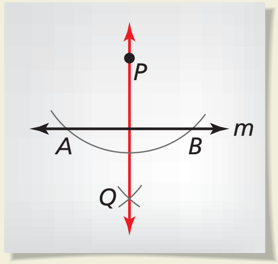

**Draw arc with center *P*** Place the compass at point $P$ and draw an arc that intersects the line twice. Label the intersections $A$ and $B$.

**Step 2**

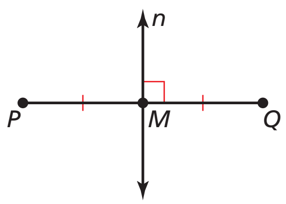



**Draw intersecting arcs** Draw an arc with center $A$. Using the same radius, draw an arc with center $B$. Label the intersection of the arcs $Q$.

**Step 3**




**Draw perpendicular line** Draw $\overleftrightarrow{PQ}$. This line is perpendicular to line $m$.




The **perpendicular bisector** of a line segment $\overline{PQ}$ is the line $n$ with the following two properties.

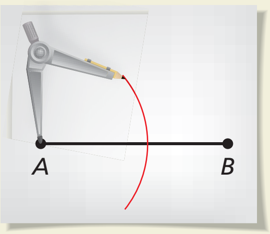
- $n \perp \overline{PQ}$
- $n$ passes through the midpoint $M$ of $\overline{PQ}$.

**CONSTRUCTION** **Constructing a Perpendicular Bisector**

Use a compass and straightedge to construct the perpendicular bisector of $\overline{AB}$.

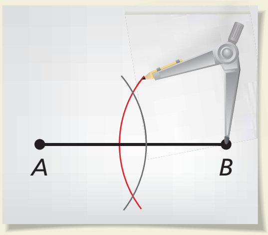

**SOLUTION**

**Step 1**



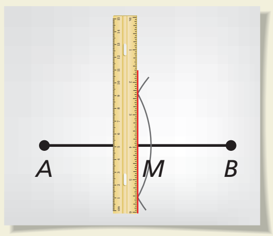

**Draw an arc** Place the compass at $A$. Use a compass setting that is greater than half the length of $\overline{AB}$. Draw an arc.

**Step 2**




**Draw a second arc** Keep the same compass setting. Place the compass at $B$. Draw an arc. It should intersect the other arc at two points.

**Step 3**




**Bisect segment** Draw a line through the two points of intersection. This line is the perpendicular bisector of $\overline{AB}$. It passes through $M$, the midpoint of $\overline{AB}$. So, $AM = MB$.

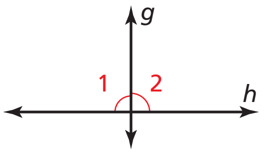
## Proving Theorems about Perpendicular Lines

### Theorems

**Theorem 3.10 Linear Pair Perpendicular Theorem**

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

If $\angle 1 \cong \angle 2$, then $g \perp h$.




*Proof* Ex. 13, p. 153

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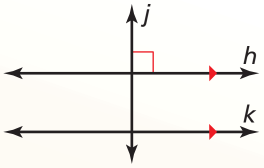
**Theorem 3.11 Perpendicular Transversal Theorem**

In a plane, if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line.

If $h \parallel k$ and $j \perp h$, then $j \perp k$.




*Proof* Example 2, p. 150; Question 2, p. 150

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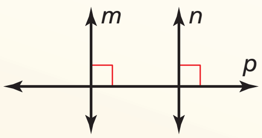
**Theorem 3.12 Lines Perpendicular to a Transversal Theorem**

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

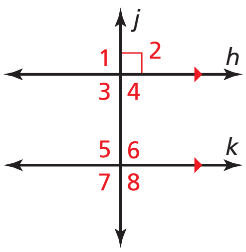
If $m \perp p$ and $n \perp p$, then $m \parallel n$.




*Proof* Ex. 14, p. 153; Ex. 47, p. 162

### EXAMPLE 2 Proving the Perpendicular Transversal Theorem

Use the diagram to prove the Perpendicular Transversal Theorem.




**SOLUTION**

**Given** $h \parallel k$, $j \perp h$

**Prove** $j \perp k$

| STATEMENTS | REASONS |
|---|---|
| **1.** $h \parallel k$, $j \perp h$ | **1.** Given |
| **2.** $m\angle 2 = 90°$ | **2.** Definition of perpendicular lines |
| **3.** $\angle 2 \cong \angle 6$ | **3.** Corresponding Angles Theorem (Theorem 3.1) |
| **4.** $m\angle 2 = m\angle 6$ | **4.** Definition of congruent angles |
| **5.** $m\angle 6 = 90°$ | **5.** Transitive Property of Equality |
| **6.** $j \perp k$ | **6.** Definition of perpendicular lines |

### Monitoring Progress

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**2.** Prove the Perpendicular Transversal Theorem using the diagram in Example 2 and the Alternate Exterior Angles Theorem (Theorem 3.3).

## Solving Real-Life Problems

### EXAMPLE 3 Proving Lines Are Parallel

The photo shows the layout of a neighborhood. Determine which lines, if any, must be parallel in the diagram. Explain your reasoning.


**SOLUTION**

Lines $p$ and $q$ are both perpendicular to $s$, so by the Lines Perpendicular to a Transversal Theorem, $p \parallel q$. Also, lines $s$ and $t$ are both perpendicular to $q$, so by the Lines Perpendicular to a Transversal Theorem, $s \parallel t$.

▶ So, from the diagram you can conclude $p \parallel q$ and $s \parallel t$.

### Monitoring Progress

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**Use the lines marked in the photo.**


**3.** Is $b \parallel a$? Explain your reasoning.

**4.** Is $b \perp c$? Explain your reasoning.

# 3.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The perpendicular bisector of a segment is the line that passes through the ________ of the segment at a ________ angle.

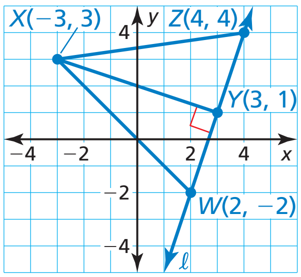
2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Find the distance from point $X$ to line $\overleftrightarrow{WZ}$.

Find $XZ$.

Find the length of $\overline{XY}$.

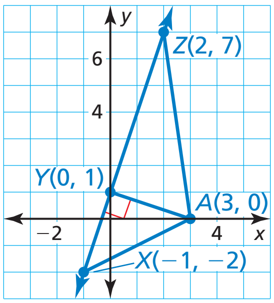
Find the distance from line $\ell$ to point $X$.



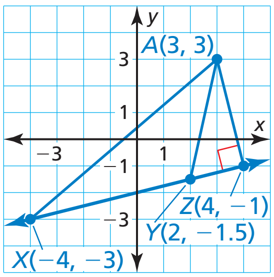

## Monitoring Progress and Modeling with Mathematics

**In Exercises 3 and 4, find the distance from point $A$ to $\overleftrightarrow{XZ}$.** *(See Example 1.)*

3.

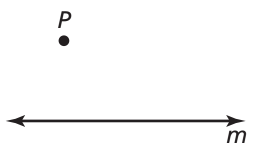



4.

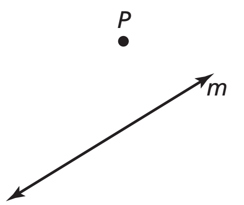



**CONSTRUCTION** **In Exercises 5–8, trace line $m$ and point $P$. Then use a compass and straightedge to construct a line perpendicular to line $m$ through point $P$.**

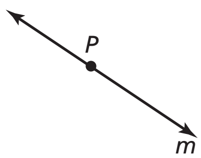
5.



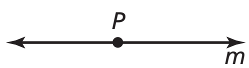

6.




7.

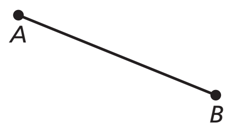



8.

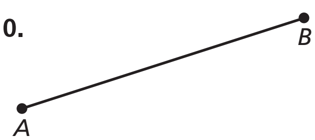



**CONSTRUCTION** **In Exercises 9 and 10, trace $\overline{AB}$. Then use a compass and straightedge to construct the perpendicular bisector of $\overline{AB}$.**

9.

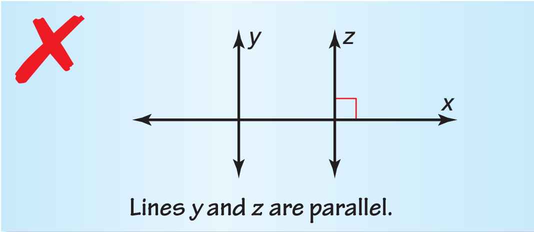



10.

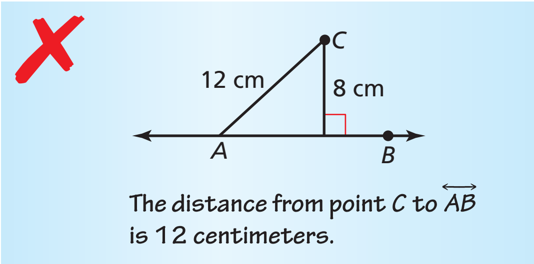


**ERROR ANALYSIS** **In Exercises 11 and 12, describe and correct the error in the statement about the diagram.**

**11.**




**12.**




**PROVING A THEOREM** **In Exercises 13 and 14, prove the theorem.** *(See Example 2.)*

**13.** Linear Pair Perpendicular Theorem (Thm. 3.10)

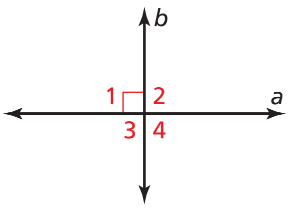
**14.** Lines Perpendicular to a Transversal Theorem (Thm. 3.12)

**PROOF** **In Exercises 15 and 16, use the diagram to write a proof of the statement.**

**15.** If two intersecting lines are perpendicular, then they intersect to form four right angles.

**Given** $a \perp b$

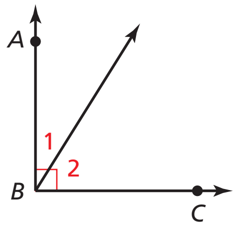
**Prove** $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are right angles.




**16.** If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

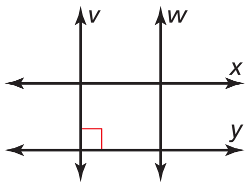
**Given** $\overrightarrow{BA} \perp \overrightarrow{BC}$

**Prove** $\angle 1$ and $\angle 2$ are complementary.

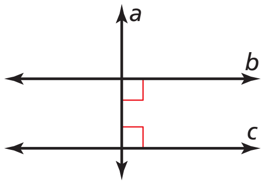



**In Exercises 17–22, determine which lines, if any, must be parallel. Explain your reasoning.** *(See Example 3.)*

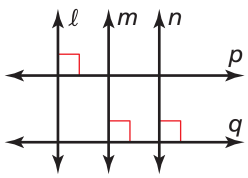
**17.**



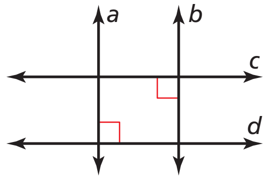

**18.**



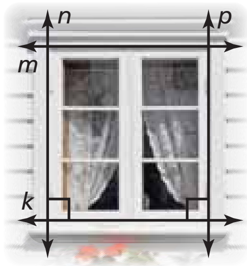

**19.**



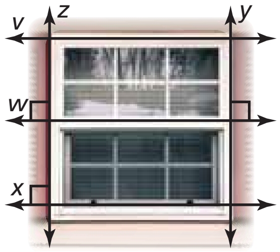

**20.**



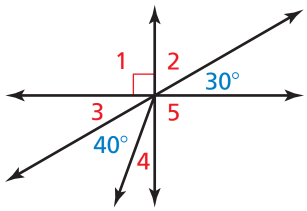

**21.**




**22.**

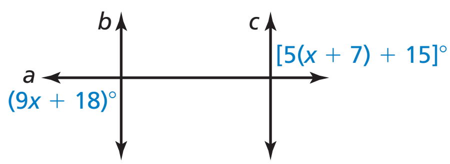



**23.** **USING STRUCTURE** Find all the unknown angle measures in the diagram. Justify your answer for each angle measure.

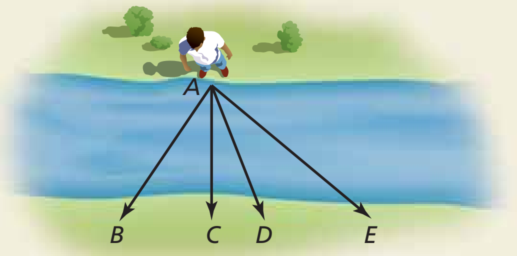



**24.** **MAKING AN ARGUMENT** Your friend claims that because you can find the distance from a point to a line, you should be able to find the distance between any two lines. Is your friend correct? Explain your reasoning.

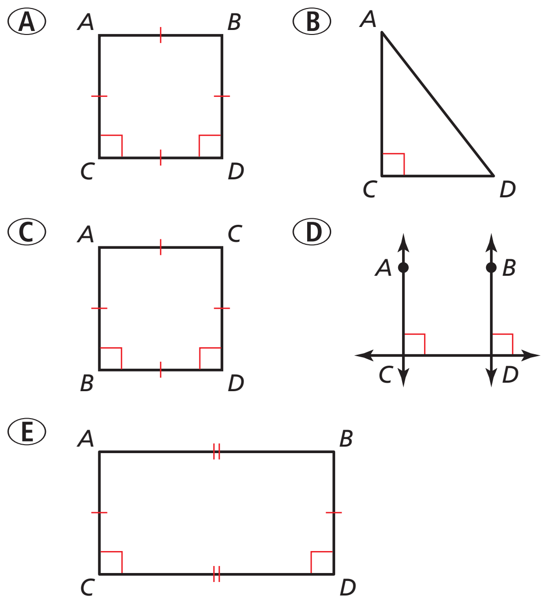
**25.** **MATHEMATICAL CONNECTIONS** Find the value of $x$ when $a \perp b$ and $b \parallel c$.



**26. HOW DO YOU SEE IT?** You are trying to cross a stream from point $A$. Which point should you jump to in order to jump the shortest distance? Explain your reasoning.



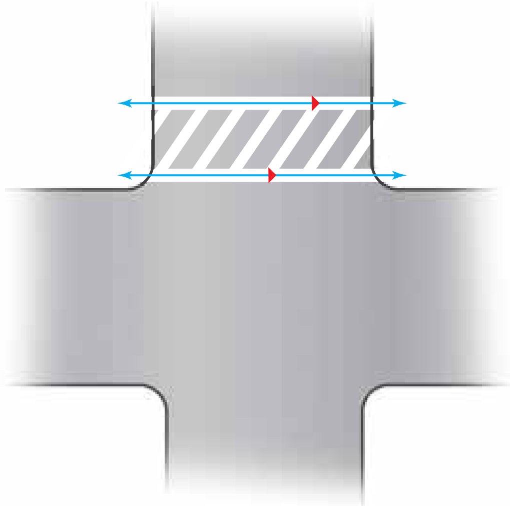

**27. ATTENDING TO PRECISION** In which of the following diagrams is $\overline{AC} \parallel \overline{BD}$ and $\overline{AC} \perp \overline{CD}$? Select all that apply.




**28. THOUGHT PROVOKING** The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, how many right angles are formed by two perpendicular lines? Justify your answer.

**29. CONSTRUCTION** Construct a square of side length $AB$.


**30. ANALYZING RELATIONSHIPS** The painted line segments that form the path of a crosswalk are usually perpendicular to the crosswalk. Sketch what the segments in the photo would look like if they were perpendicular to the crosswalk. Which type of line segment requires less paint? Explain your reasoning.



**31. ABSTRACT REASONING** Two lines, $a$ and $b$, are perpendicular to line $c$. Line $d$ is parallel to line $c$. The distance between lines $a$ and $b$ is $x$ meters. The distance between lines $c$ and $d$ is $y$ meters. What shape is formed by the intersections of the four lines?

**32. MATHEMATICAL CONNECTIONS** Find the distance between the lines with the equations $y = \frac{3}{2}x + 4$ and $-3x + 2y = -1$.

**33. WRITING** Describe how you would find the distance from a point to a plane. Can you find the distance from a line to a plane? Explain your reasoning.

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Simplify the ratio.** *(Skills Review Handbook)*

**34.** $\dfrac{6-(-4)}{8-3}$ **35.** $\dfrac{3-5}{4-1}$ **36.** $\dfrac{8-(-3)}{7-(-2)}$ **37.** $\dfrac{13-4}{2-(-1)}$

**Identify the slope and the $y$-intercept of the line.** *(Skills Review Handbook)*

**38.** $y = 3x + 9$ **39.** $y = -\frac{1}{2}x + 7$ **40.** $y = \frac{1}{6}x - 8$ **41.** $y = -8x - 6$