Properties of Triangles

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5.1

Name That Triangle!

Classifying Triangles on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:

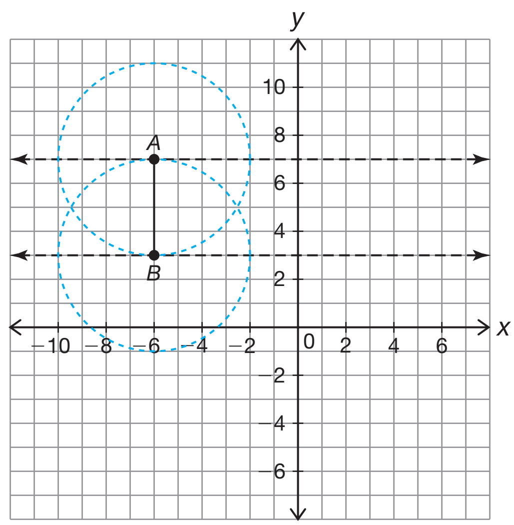
- Determine the coordinates of a third vertex of a triangle, given the coordinates of two vertices and a description of the triangle.
- Classify a triangle given the locations of its vertices on a coordinate plane.

Because you may soon be behind the steering wheel of a car, it is important to know the meaning of the many signs you will come across on the road. One of the most basic is the yield sign. This sign indicates that a driver must prepare to stop to give a driver on an adjacent road the right of way. The first yield sign was installed in the United States in 1950 in Tulsa, Oklahoma, and was designed by a police officer of the town. Originally, it was shaped like a keystone, but over time, it was changed. Today, it is an equilateral triangle and is used just about everywhere in the world. Although some countries may use different colors or wording (some countries call it a "give way" sign), the signs are all the same in size and shape.

Why do you think road signs tend to be different, but basic, shapes, such as rectangles, triangles, and circles? Would it matter if a stop sign was an irregular heptagon? Does the shape of a sign make it any easier or harder to recognize?

PROBLEM 1 Location, Location, Location!

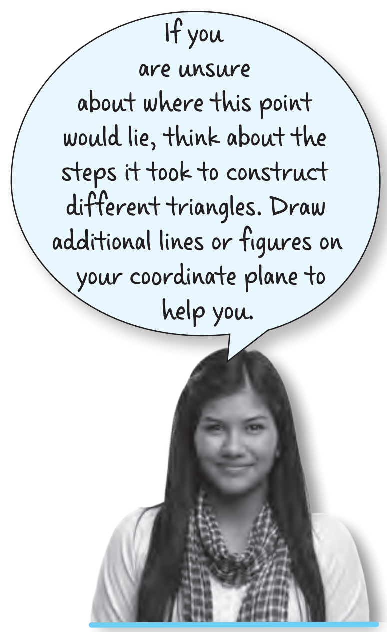
1. The graph shows line segment *AB* with endpoints at *A* (−6, 7) and *B* (−6, 3). Line segment *AB* is a radius for congruent circles *A* and *B*.



2. Using $\overline{AB}$ as one side of a triangle, determine a location for point *C* on circle *A* or on circle *B* such that triangle *ABC* is:

   a. a right triangle.

   b. an acute triangle.



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c. an obtuse triangle.

3. Using $\overline{AB}$ as one side of a triangle, determine the location for point *C* on circle *A* or on circle *B* such that triangle *ABC* is:

a. an equilateral triangle.

b. an isosceles triangle.

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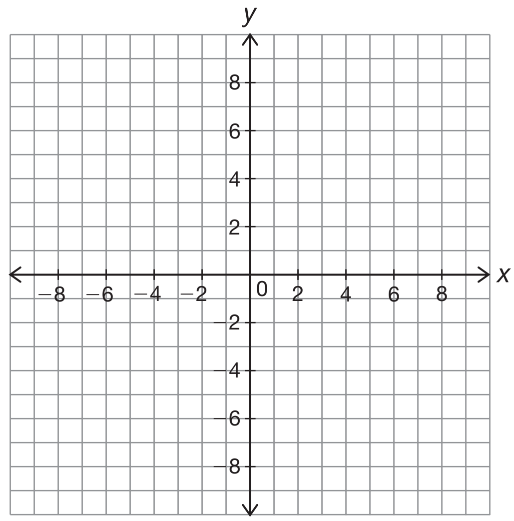
c. a scalene triangle.

PROBLEM 2 What's Your Name Again?

1. Graph triangle *ABC* using points *A* (0, −4), *B* (0, −9), and *C* (−2, −5).



2. Classify triangle *ABC*.

   a. Determine if triangle *ABC* is scalene, isosceles, or equilateral. Explain your reasoning.

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b. Explain why triangle *ABC* is a right triangle.

c. Zach does not like using the slope formula. Instead, he decides to use the Pythagorean Theorem to determine if triangle *ABC* is a right triangle because he already determined the lengths of the sides. His work is shown.

Zach

$$a^2 + b^2 = c^2$$

$$(\sqrt{5})^2 + (\sqrt{20})^2 = 5^2$$

$$5 + 20 = 25$$

$$25 = 25$$

He determines that triangle *ABC* must be a right triangle because the sides satisfy the Pythagorean Theorem. Is Zach's reasoning correct? Explain why or why not.

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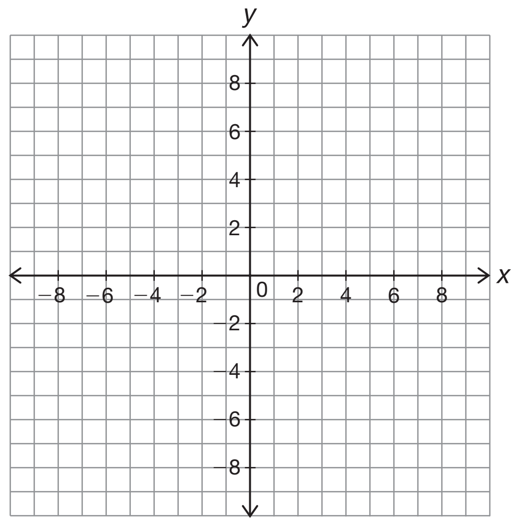
3. Graph triangle ABC using points A $(-2, 4)$, B $(8, 4)$, and C $(6, -2)$.



4. Classify triangle ABC.

   a. Determine if triangle ABC is a scalene, an isosceles, or an equilateral triangle. Explain your reasoning.

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b. Determine if triangle *ABC* is a right triangle. Explain your reasoning. If it is not a right triangle, use a protractor to determine what type of triangle it is.

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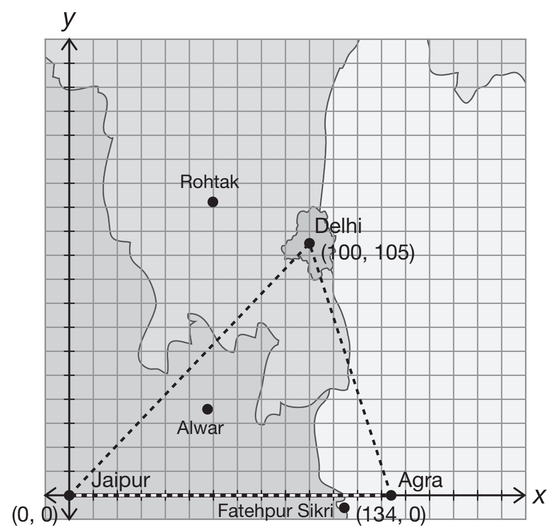
PROBLEM 3 India's Golden Triangle

1. India's Golden Triangle is a very popular tourist destination. The vertices of the triangle are the three historical cities of Delhi, Agra (Taj Mahal), and Jaipur.

   The locations of these three cities can be represented on the coordinate plane as shown.



   Classify India's Golden Triangle.

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Be prepared to share your solutions and methods.

5.2

Inside Out

Triangle Sum, Exterior Angle, and Exterior Angle Inequality Theorems

LEARNING GOALS

In this lesson, you will:

- Prove the Triangle Sum Theorem.
- Explore the relationship between the interior angle measures and the side lengths of a triangle.
- Identify the remote interior angles of a triangle.
- Identify the exterior angle of a triangle.
- Explore the relationship between the exterior angle measure and two remote interior angles of a triangle.
- Prove the Exterior Angle Theorem.
- Prove the Exterior Angle Inequality Theorem.

KEY TERMS

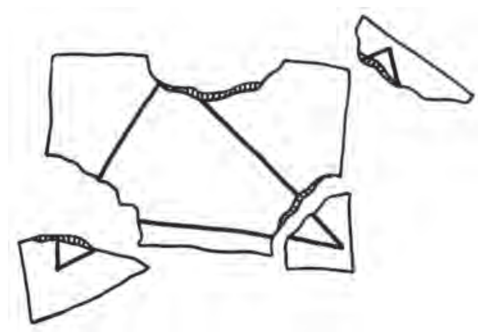
- Triangle Sum Theorem
- remote interior angles of a triangle
- Exterior Angle Theorem
- Exterior Angle Inequality Theorem

Easter Island is one of the remotest islands on planet Earth. It is located in the southern Pacific Ocean approximately 2300 miles west of the coast of Chile. It was discovered by a Dutch captain in 1722 on Easter Day. When discovered, this island had few inhabitants other than 877 giant statues, which had been carved out of rock from the top edge of a wall of the island's volcano. Each statue weighs several tons, and some are more than 30 feet tall.

Several questions remain unanswered and are considered mysteries. Who built these statues? Did the statues serve a purpose? How were the statues transported on the island?

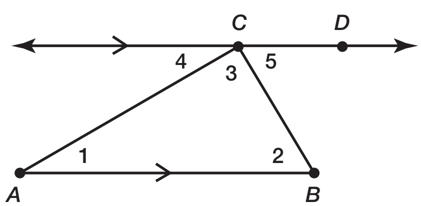
PROBLEM 1 Triangle Interior Angle Sums

1. Draw any triangle on a piece of paper. Tear off the triangle's three angles. Arrange the angles so that they are adjacent angles. What do you notice about the sum of these three angles?



The **Triangle Sum Theorem** states: "the sum of the measures of the interior angles of a triangle is 180°."

2. Prove the Triangle Sum Theorem using the diagram shown.



Given: Triangle ABC with $\overline{AB} \parallel \overline{CD}$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Think about the Angle Addition Postulate, alternate interior angles, and other theorems you know.

PROBLEM 2 Analyzing Triangles

1. Consider the side lengths and angle measures of an acute triangle.

   a. Draw an acute scalene triangle. Measure each interior angle and label the angle measures in your diagram.

   b. Measure the length of each side of the triangle. Label the side lengths in your diagram.

   c. Which interior angle is opposite the longest side of the triangle?

   d. Which interior angle lies opposite the shortest side of the triangle?

2. Consider the side lengths and angle measures of an obtuse triangle.

   a. Draw an obtuse scalene triangle. Measure each interior angle and label the angle measures in your diagram.

   b. Measure the length of each side of the triangle. Label the side lengths in your diagram.

   c. Which interior angle lies opposite the longest side of the triangle?

   d. Which interior angle lies opposite the shortest side of the triangle?

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3. Consider the side lengths and angle measures of a right triangle.

   a. Draw a right scalene triangle. Measure each interior angle and label the angle measures in your diagram.

   b. Measure each side length of the triangle. Label the side lengths in your diagram.

   c. Which interior angle lies opposite the longest side of the triangle?

   d. Which interior angle lies opposite the shortest side of the triangle?

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4. The measures of the three interior angles of a triangle are 57°, 62°, and 61°. Describe the location of each side with respect to the measures of the opposite interior angles without drawing or measuring any part of the triangle.

   a. longest side of the triangle

   b. shortest side of the triangle

5. One angle of a triangle decreases in measure, but the sides of the angle remain the same length. Describe what happens to the side opposite the angle.

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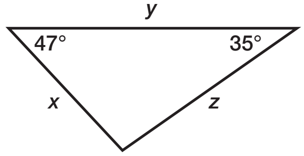
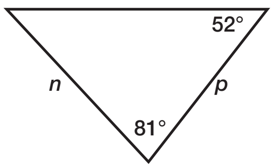
6. An angle of a triangle increases in measure, but the sides of the angle remain the same length. Describe what happens to the side opposite the angle.

7. List the sides from shortest to longest for each diagram.

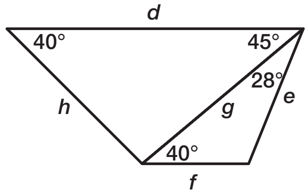
a.



b.

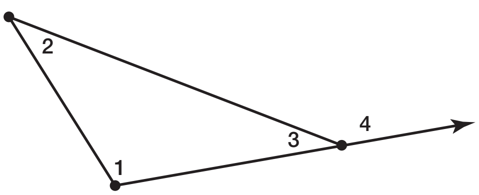


c.



PROBLEM 3 Exterior Angles

Use the diagram shown to answer Questions 1 through 12.



1. Name the interior angles of the triangle.

2. Name the exterior angles of the triangle.

3. What did you need to know to answer Questions 1 and 2?

4. What does $m\angle 1 + m\angle 2 + m\angle 3$ equal? Explain your reasoning.

5. What does $m\angle 3 + m\angle 4$ equal? Explain your reasoning.

6. Why does $m\angle 1 + m\angle 2 = m\angle 4$? Explain your reasoning.

7. Consider the sentence "The buried treasure is located on a remote island." What does the word *remote* mean?

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8. The exterior angle of a triangle is $\angle 4$, and $\angle 1$ and $\angle 2$ are interior angles of the same triangle. Why would $\angle 1$ and $\angle 2$ be referred to as "remote" interior angles with respect to the exterior angle?

The **remote interior angles of a triangle** are the two angles that are non-adjacent to the specified exterior angle.

9. Write a sentence explaining $m\angle 4 = m\angle 1 + m\angle 2$ using the words *sum, remote interior angles of a triangle*, and *exterior angle of a triangle*.

10. Is the sentence in Question 9 considered a postulate or a theorem? Explain your reasoning.

11. The diagram was drawn as an obtuse triangle with one exterior angle. If the triangle had been drawn as an acute triangle, would this have changed the relationship between the measure of the exterior angle and the sum of the measures of the two remote interior angles? Explain your reasoning.

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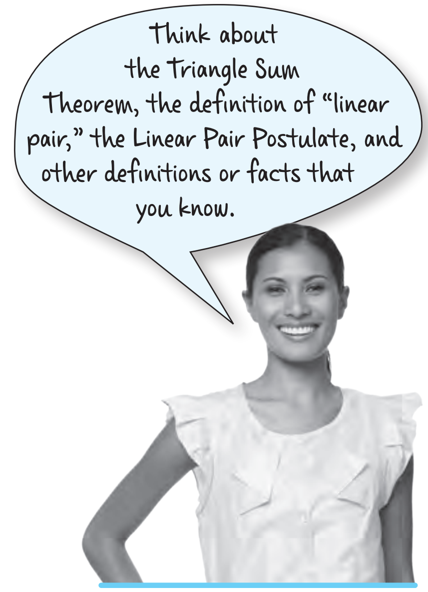
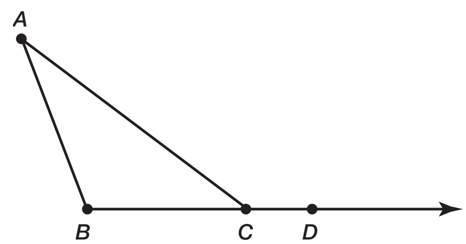
12. If the triangle had been drawn as a right triangle, would this have changed the relationship between the measure of the exterior angle and the sum of the measures of the two remote interior angles? Explain your reasoning.

The **Exterior Angle Theorem** states: "the measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle."



13. Prove the Exterior Angle Theorem using the diagram shown.



Given: Triangle ABC with exterior $\angle ACD$

Prove: $m\angle A + m\angle B = m\angle ACD$

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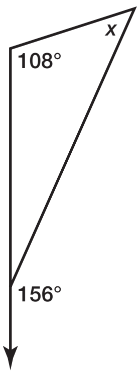
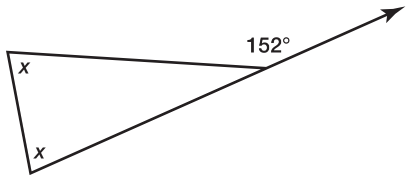
14. Solve for x in each diagram.

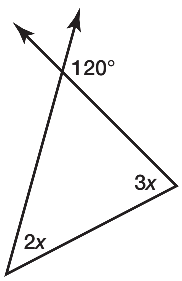
a.



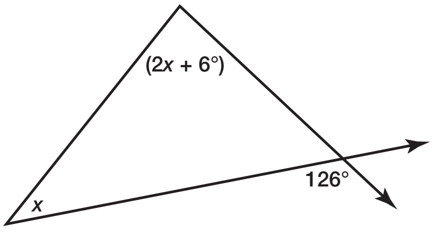
b.



c.



d.



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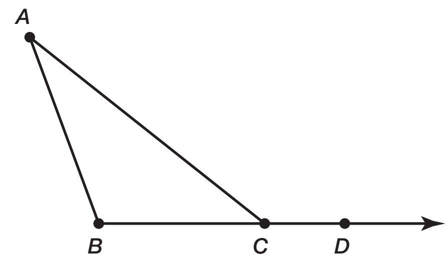
The **Exterior Angle Inequality Theorem** states: "the measure of an exterior angle of a triangle is greater than the measure of either of the remote interior angles of the triangle."

15. Why is it necessary to prove two different statements to completely prove this theorem?

16. Prove both parts of the Exterior Angle Inequality Theorem using the diagram shown.



a. Part 1

Given: Triangle *ABC* with exterior $\angle ACD$

Prove: $m\angle ACD > m\angle A$

Statements	Reasons
1.	1. Given
2.	2. Triangle Sum Theorem
3.	3. Linear Pair Postulate
4.	4. Definition of linear pair
5.	5. Substitution Property using step 2 and step 4
6.	6. Subtraction Property of Equality
7.	7. Definition of an angle measure
8.	8. Inequality Property (if $a = b + c$ and $c > 0$, then $a > b$)

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b. Part 2

Given: Triangle *ABC* with exterior $\angle ACD$

Prove: $m\angle ACD > m\angle B$

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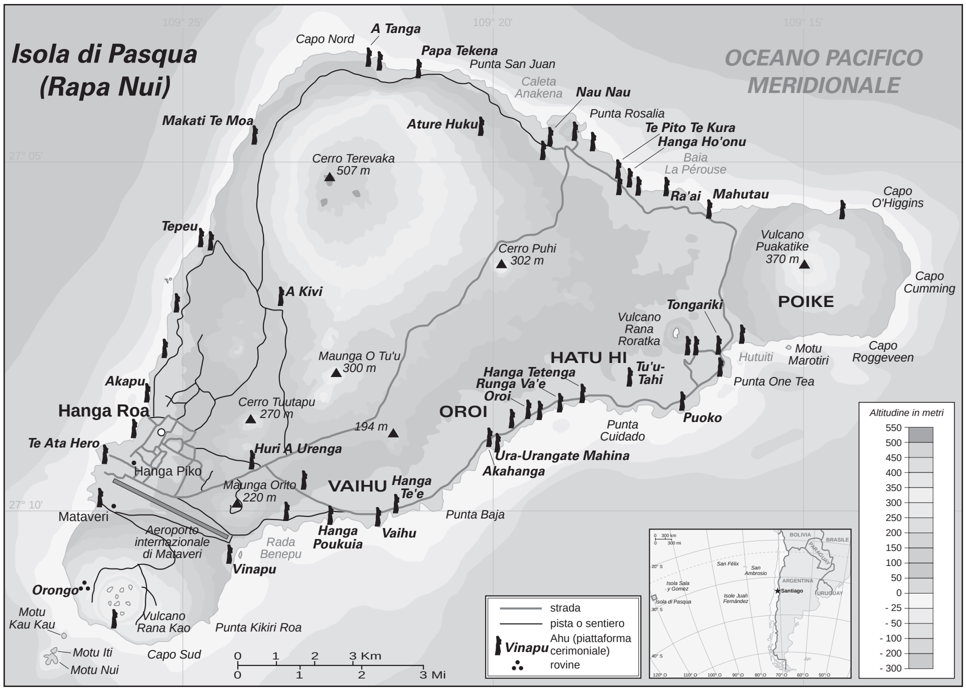
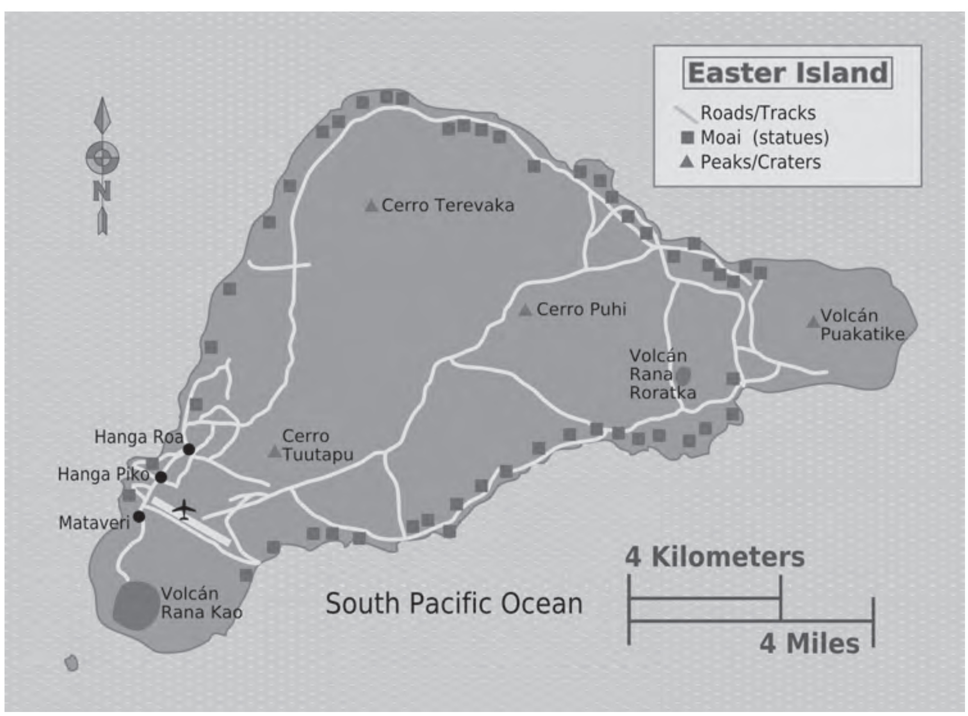
PROBLEM 4 Easter Island

Easter Island is an island in the southeastern Pacific Ocean, famous for its statues created by the early Rapa Nui people.

Two maps of Easter Island are shown.





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1. What questions could be answering using each map?

2. What geometric shape does Easter Island most closely resemble? Draw this shape on one of the maps.

3. Is it necessary to draw Easter Island on a coordinate plane to compute the length of its coastlines? Why or why not?

4. Predict which side of Easter Island appears to have the longest coastline and state your reasoning using a geometric theorem.

5. Use either map to validate your answer to Question 4.

6. Easter Island has 887 statues. How many statues are there on Easter Island per square mile?

7. Suppose we want to place statues along the entire coastline of the island, and the distance between each statue was 1 mile. Would we need to build additional statues, and if so, how many?

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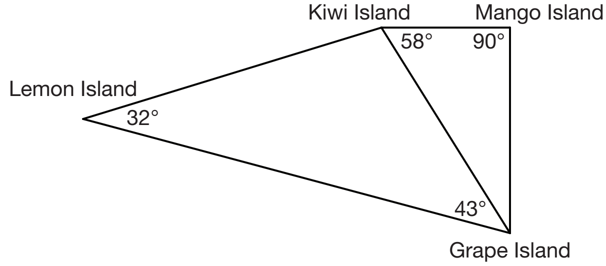
Talk the Talk

Using only the information in the diagram shown, determine which two islands are farthest apart. Use mathematics to justify your reasoning.



Be prepared to share your solutions and methods.

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5.3

Trade Routes and Pasta Anyone?

The Triangle Inequality Theorem

LEARNING GOALS

In this lesson, you will:

- Explore the relationship between the side lengths of a triangle and the measures of its interior angles.
- Prove the Triangle Inequality Theorem.

KEY TERM

- Triangle Inequality Theorem

Triangular trade best describes the Atlantic trade routes among several different destinations in Colonial times. The Triangular Trade Routes connected England, Europe, Africa, the Americas, and the West Indies. The Triangular Trade Routes included the following:

- Trade Route 1: England to Africa to the Americas
- Trade Route 2: England to Africa to the West Indies
- Trade Route 3: Europe to the West Indies to the Americas
- Trade Route 4: Americas to the West Indies to Europe

PROBLEM 1 Who Is Correct?

1. Sarah claims that any three lengths will determine three sides of a triangle. Sam does not agree. He thinks some combinations will not work. Who is correct?

2. Sam then claims that he can just look at the three lengths and know immediately if they will work. Sarah is unsure. She decides to explore this for herself.

   Help Sarah by working through the following activity.

   To begin, you will need a piece of strand pasta (like linguine). Break the pasta at two random points so the strand is divided into three pieces. Measure each of your three pieces of pasta in centimeters. Try to form a triangle from your three pieces of pasta. Try several pieces of pasta with different breaking points.

3. Collect and record your classmates' measurements.

Piece 1 (cm)	Piece 2 (cm)	Piece 3 (cm)	Forms a Triangle? (yes or no)

4. Examine the lengths of the pasta pieces that did form a triangle. Compare them with the lengths of the pasta pieces that did not form a triangle. What observations can you make?

5. Under what conditions were you able to form a triangle?

6. Under what conditions were you unable to form a triangle?

7. Based upon your observations, determine if it is possible to form a triangle using segments with the following measurements. Explain your reasoning.

   a. 2 centimeters, 5.1 centimeters, 2.4 centimeters

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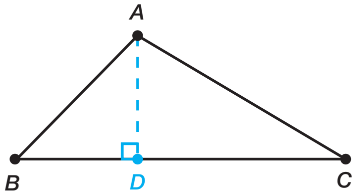
   b. 9.2 centimeters, 7 centimeters, 1.9 centimeters

The rule that Sam was using is known as the Triangle Inequality Theorem.

The **Triangle Inequality Theorem** states: "the sum of the lengths of any two sides of a triangle is greater than the length of the third side."

8. Prove the Triangle Inequality Theorem by completing each step.



Given: Triangle ABC

Prove: $AB + AC > BC$

A perpendicular line segment AD is constructed through point A to side BC.

Statements	Reasons
1. Triangle ABC	**1.** Given
2. Draw $\overline{AD} \perp \overline{BC}$	**2.** Construction
3. $\angle ADB$ is a right angle.	**3.** Definition of perpendicular.
4. $\angle ADC$ is a right angle.	**4.** Definition of perpendicular.
5. $BD^2 + AD^2 = AB^2$	**5.** Pythagorean Theorem
6. $CD^2 + AD^2 = AC^2$	**6.** Pythagorean Theorem
7. $AB^2 > BD^2$	**7.** Definition of greater than.
8. $AC^2 > DC^2$	**8.** Definition of greater than.
9. $AB > BD$	**9.**
10. $AC > DC$	**10.**
11. $AB + AC > BD + DC$	**11.**
12. $BD + DC = BC$	**12.**
13. $AB + AC > BC$	**13.**

Be prepared to share your solutions and methods.

5

5.4

Stamps Around the World

Properties of a 45°–45°–90° Triangle

LEARNING GOALS

In this lesson, you will:

- Use the Pythagorean Theorem to explore the relationship between the side lengths of a triangle and the measures of its interior angles.
- Prove the 45°–45°–90° Triangle Theorem.

KEY TERM

- 45°–45°–90° Triangle Theorem

The first adhesive postage stamp was issued in the United Kingdom in 1840. It is commonly known as the Penny Black which makes sense because it cost one penny and had a black background. It featured a profile of 15-year-old former Princess Victoria.

You may think that the very first stamp is quite rare and valuable. However, that isn't quite true. The total print run was 68,808,000. During this time envelopes were not normally used. The address and the stamp was affixed to the folded letter itself. Many people kept personal letters and ended up keeping the stamp too.

As of 2012, the most valuable stamp was the Treskilling Yellow stamp from Sweden. Only one known copy exists. In 2010 it sold at auction for over three million dollars!

PROBLEM 1 Stamp Collecting Isn't Always Square!

The first triangle-shaped U.S. stamps were issued on June 8, 1997. The pair of 32-cent commemorative stamps of triangular shape featured a mid-19th-century clipper ship and a U.S. mail stagecoach.

Each image shown is an enlargement of both stamps.

1. Can you use this enlargement to determine the measures of the angles of the actual stamp? Why or why not?

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2. Measure the angles of one of the commemorative stamps.

3. Measure the length of the sides of one of the commemorative stamps and describe the relationship between the length of each side and the measure of the angle located opposite each side.

The **45°–45°–90° Triangle Theorem** states: "the length of the hypotenuse in a 45°–45°–90° triangle is $\sqrt{2}$ times the length of a leg."

4. Use the Pythagorean Theorem to prove the 45°–45°–90° Triangle Theorem. Let c represent the length of the hypotenuse and let ℓ represent the length of each leg.

5. Using the 45°–45°–90° Triangle Theorem, what is the length of the longest side of the enlargement of the commemorative stamp?

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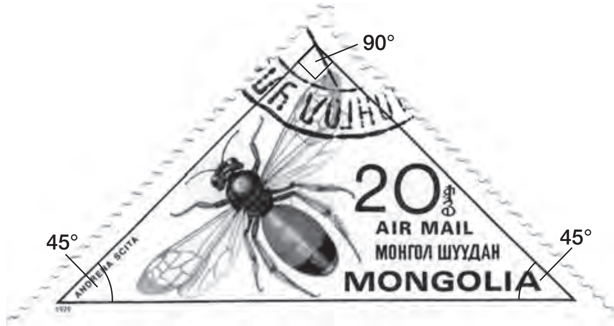
6. What additional information is needed to determine the length of the longest side of the actual commemorative stamp?

7. This stamp was issued in Mongolia.



Suppose the longest side of this stamp is 50 millimeters.

a. Use the Pythagorean Theorem to determine the approximate length of the other sides of this stamp. Round your answer to the nearest tenth of a millimeter.

b. Use the 45°–45°–90° Triangle Theorem to determine the approximate length of the other sides of this stamp. Round your answer to the nearest tenth of a millimeter.

5

8. This stamp was issued in Russia.

Suppose the longest side of this stamp is 50 millimeters. Use the 45°–45°–90° Triangle Theorem to determine the *actual* length of the shortest side of this stamp.

9. In 2007, another triangle-shaped stamp was issued in the United States. It was issued to commemorate the 400th anniversary of the Settlement of Jamestown, Virginia, by English colonists in 1607. This stamp features a painting of the three ships that carried the colonists from England to the United States. Was it a coincidence that the first fort built by the settlers was shaped like a triangle?

This is an enlargement of the Jamestown stamp.

Measure the length of the shortest side and use the 45°–45°–90° Triangle Theorem to determine the length of the longest side of the enlargement of the commemorative stamp.

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10. The first triangular stamp was issued by the Cape of Good Hope in 1853.

This is an enlargement of the Cape of Good Hope stamp.

Measure the length of the longest side and use the 45°–45°–90° Triangle Theorem to determine the length of the shortest side of the enlargement of the commemorative stamp.

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PROBLEM 2 Construction

1. Construct an isosceles right triangle with $\overline{CB}$ as a leg and $\angle C$ as the right angle.

After completing the construction, use a protractor and a ruler to confirm the following:

- $m\angle A = 45°$
- $m\angle B = 45°$
- $AC = BC$
- $AB = AC\sqrt{2}$
- $AB = BC\sqrt{2}$

2. Explain how you can use an alternate method for constructing a 45°–45°–90° triangle by constructing a square first.

Be prepared to share your solutions and methods.

5

More Stamps, Really?

5.5

Properties of a 30°–60°–90° Triangle

LEARNING GOALS

In this lesson, you will:

- Use the Pythagorean Theorem to explore the relationship between the side lengths of a triangle and the measures of its interior angles.
- Prove the 30°–60°–90° Triangle Theorem.

KEY TERM

- 30°–60°–90° Triangle Theorem

The US Postal Services doesn't have an official motto but an inscription on the James Farley Post Office in New York is well known. It reads, "Neither snow nor rain nor heat nor gloom of night stays these couriers from the swift completion of their appointed rounds."

There have been many popular characters on television who were mail carriers including Mister McFeely from the children's series *Mister Rogers' Neighborhood*, Cliff Clavin from the comedy series *Cheers*, and Newman from the comedy series *Seinfeld*.

Can you think of any other famous mail carriers? What other professions seem to inspire characters on television, the movies, or in books?

PROBLEM 1 Something Is Different About This Stamp!

This stamp was issued in Malaysia.

1. How is this stamp different from the stamps you studied in the previous lesson?

2. This Malaysian stamp is shaped like an equilateral triangle. What is the measure of each interior angle of the triangle? Explain your reasoning.

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3. Use the diagram of the stamp to draw an altitude to the base of the equilateral triangle. Describe the two triangles formed by the altitude.

4. How do you know that the two triangles formed by the altitude drawn to the base of an equilateral triangle are congruent.

5. If the length of each side of the Malaysian stamp is 50 millimeters, determine the length of the three sides in each of the two 30°–60°–90° triangles formed by the altitude drawn to the base of the equilateral triangle.

6. How does the length of the hypotenuse in each of the two 30°–60°–90° triangles relate to the length of the shortest leg?

7. How does the length of the longer leg in each of the two 30°–60°–90° triangles relate to the length of the shortest leg?

The **30°–60°–90° Triangle Theorem** states: "the length of the hypotenuse in a 30°–60°–90° triangle is two times the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg."

8. Use the Pythagorean Theorem to demonstrate the 30°–60°–90° Triangle Theorem. Let *x* represent the length of the shortest leg.

9. This stamp was issued in the Netherlands.

Suppose the length of each side of the Netherlands stamp is 40 millimeters. Use the 30°–60°–90° Triangle Theorem to determine the height of the stamp.

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10. In 1929, Uruguay issued a triangular parcel post stamp with a picture of wings, implying rapid delivery.

Suppose the height of the Uruguay stamp is 30 millimeters. Use the 30°–60°–90° Triangle Theorem to determine the length of the three sides of the stamp.

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11. A mathematical society in India designed this stamp. The pyramidal design is an equilateral triangle.

Suppose the height of the pyramidal design on the stamp is 42 millimeters. Determine the area of the pyramidal design on the stamp.

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PROBLEM 2 Construction

30°–60°–90° Triangle

Construct a 30°–60°–90° triangle by constructing an equilateral triangle and one altitude.

After completing the construction, use a protractor and a ruler to confirm that:

- one angle measure is 30°.
- one angle measure is 60°.
- one angle measure is 90°.
- the side opposite the 30° angle is one-half the length of the hypotenuse.
- the side opposite the 60° angle is one-half the hypotenuse times $\sqrt{3}$.

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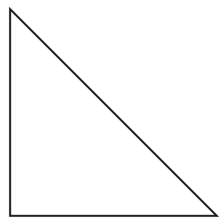
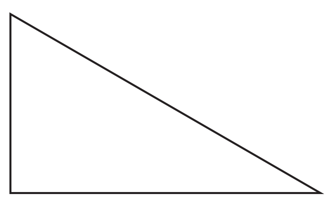
Talk the Talk

1. Label the shortest side of each triangle as x. Then label the remaining sides of each triangle in terms of x.

   a. 45°–45°–90° triangle



   b. 30°–60°–90° triangle



2. Explain how to calculate the following for a 45°–45°–90° triangle.

   a. The length of a leg given the length of the hypotenuse.

   b. The length of the hypotenuse given the length of a leg.

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3. Explain how to calculate the following for a 30°–60°–90° triangle.

   a. The length of the hypotenuse given the length of the shorter leg.

   b. The length of the hypotenuse given the length of the longer leg.

   c. The length of the shorter leg given the length of the longer leg.

   d. The length of the shorter leg given the length of the hypotenuse.

   e. The length of the longer leg given the length of the shorter leg.

   f. The length of the longer leg given the length of the hypotenuse.

Be prepared to share your solutions and methods.

5

Chapter 5 Summary

KEY TERMS

- remote interior angles of a triangle (5.2)

POSTULATES AND THEOREMS

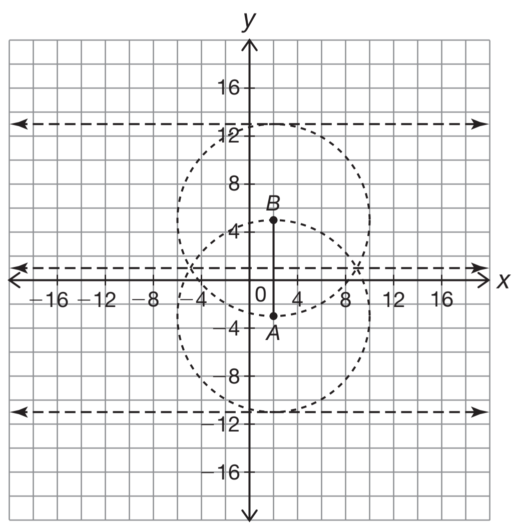
- Triangle Sum Theorem (5.2)
- Exterior Angle Theorem (5.2)
- Exterior Angle Inequality Theorem (5.2)
- Triangle Inequality Theorem (5.3)
- 45°–45°–90° Triangle Theorem (5.4)
- 30°–60°–90° Triangle Theorem (5.5)

5.1 Determining the Third Vertex of a Triangle Given Two Points

A line segment formed by two points on the coordinate plane can represent one side of a triangle. The placement of the third point depends on the type of triangle being formed.

Example

The line segment *AB* has been graphed on the coordinate plane for the points *A*(2, −3) and *B*(2, 5). Determine the location of point *C* such that triangle *ABC* is an acute triangle.



To create an acute triangle, point *C* can have an infinite number of locations as long as the location satisfies one of the following conditions:

- Point *C* could be located anywhere on circle *A* between the *y*-values of −3 and 5.
- Point *C* could be located anywhere on circle *B* between the *y*-values of −3 and 5.

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5.1 Describing a Triangle Given Three Points on a Coordinate Plane

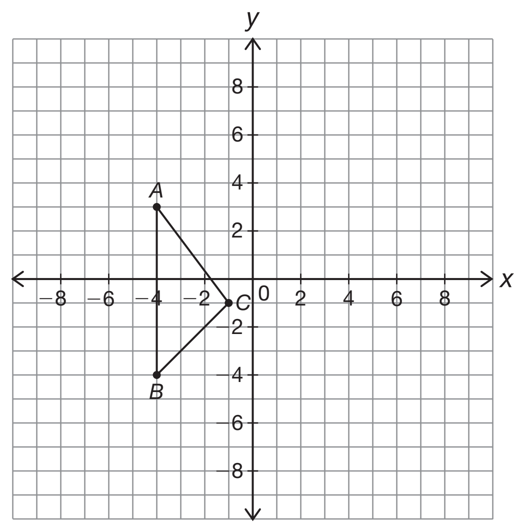
When given three points on the coordinate plane, the triangle formed can be described by the measures of its sides and angles. To determine if the triangle is scalene, isosceles, or equilateral, use the Distance Formula to determine the length of each side. To determine if the triangle is right, use the slope formula or the Pythagorean Theorem. If the triangle is not right, a protractor can be used to determine if it is obtuse or acute.

Example

Describe triangle *ABC* with points $A(-4, 3)$, $B(-4, -4)$, and $C(-1, -1)$.

$AB = 3 - (-4) = 7$

$$
\begin{aligned}
BC &= \sqrt{(-1 - (-4))^2 + (-1 - (-4))^2}\\
&= \sqrt{3^2 + 3^2}\\
&= \sqrt{9 + 9}\\
&= \sqrt{18}
\end{aligned}
$$



$$
\begin{aligned}
AC &= \sqrt{(-1 - (-4))^2 + (-1 - 3)^2}\\
&= \sqrt{3^2 + (-4)^2}\\
&= \sqrt{9 + 16}\\
&= \sqrt{25}\\
&= 5
\end{aligned}
$$

Each side is a different length, so triangle *ABC* is scalene.

$$
\begin{aligned}
a^2 + b^2 &= c^2\\
(\sqrt{18})^2 + 5^2 &= 7^2\\
18 + 25 &= 49\\
43 &\neq 49
\end{aligned}
$$

The side lengths do not satisfy the Pythagorean Theorem, so triangle *ABC* is not a right triangle. The triangle is acute because all of the angles are less than 90°.

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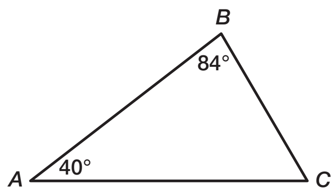
5.2 Using the Triangle Sum Theorem

The Triangle Sum Theorem states: "The sum of the measures of the interior angles of a triangle is 180°."

Example



$$m\angle A + m\angle B + m\angle C = 180°$$

$$40° + 84° + m\angle C = 180°$$

$$m\angle C = 180° - (40° + 84°)$$

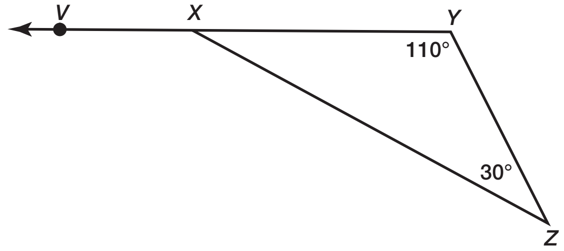
$$m\angle C = 180° - 124°$$

$$m\angle C = 56°$$

5.2 Using the Exterior Angle Theorem

The Exterior Angle Theorem states: "The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle."

Example



$$m\angle VXZ = m\angle Y + m\angle Z$$

$$m\angle VXZ = 110° + 30°$$

$$m\angle VXZ = 140°$$

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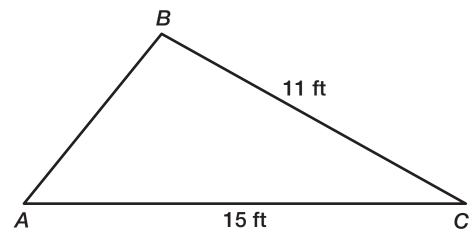
5.3 Using the Triangle Inequality Theorem

The Triangle Inequality Theorem states: "The sum of the lengths of any two sides of a triangle is greater than the length of the third side."

Example



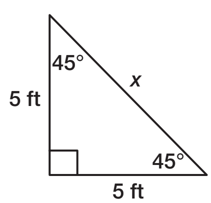
$AB < BC + AC$	$BC < AB + AC$	$AC < AB + BC$
$AB < 11 + 15$	$11 < AB + 15$	$15 < AB + 11$
$AB < 26$	$-4 < AB$	$4 < AB$

So, *AB* must be greater than 4 feet and less than 26 feet. (A length cannot be negative, so disregard the negative number.)

5.4 Using the 45°–45°–90° Triangle Theorem

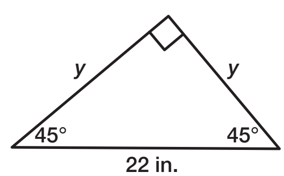
The 45°–45°–90° Triangle Theorem states: "The length of the hypotenuse in a 45°–45°–90° triangle is $\sqrt{2}$ times the length of a leg."

Examples



$x = 5\sqrt{2}$ ft

The length of the hypotenuse is $5\sqrt{2}$ feet.



$y\sqrt{2} = 22$

$$y = \frac{22}{\sqrt{2}} = \frac{22 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{22 \cdot \sqrt{2}}{2} = 11\sqrt{2} \text{ in.}$$

The length of each leg is $11\sqrt{2}$ inches.

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5.5 Using the 30°–60°–90° Triangle Theorem

The 30°–60°–90° Triangle Theorem states: "The length of the hypotenuse in a 30°–60°–90° triangle is two times the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg."

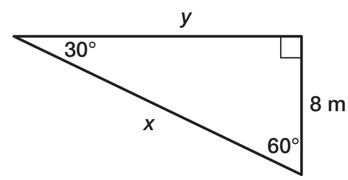
Examples

Hypotenuse:

$x = 2(8) = 16$ m

Longer leg:

$y = 8\sqrt{3}$ m



Shorter leg:

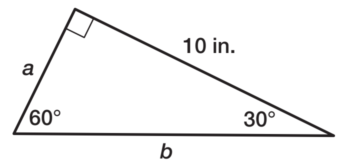
$a\sqrt{3} = 10$

$a = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$ in.

Hypotenuse:

$b = 2a = 2\left(\frac{10\sqrt{3}}{3}\right)$

$= \frac{20\sqrt{3}}{3}$ in.



Shorter leg:

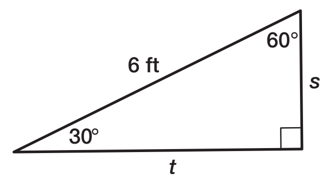
$2s = 6$

$s = 3$ ft

Longer leg:

$t = s\sqrt{3}$

$t = 3\sqrt{3}$ ft



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