## 5 $4-2$ <br> Proportions and Similar Figures

Lesson Preview


| d Check Skills You'll Need | (For help, go to the Skills Handbook and Lesson 4-1.) |  |
| :---: | :---: | :---: |
| Simplify each ratio. <br> 1. $\frac{36}{42} \frac{6}{7}$ | 2. $\frac{81}{108} \frac{3}{4}$ | 3. $26 \frac{1}{52}$ |
| Solve each proportion. |  |  |
| 4. $\frac{x}{12}=\frac{7}{30} 2 \frac{4}{5}$ | 5. $\frac{y}{12}=\frac{8}{45} 2^{2} \frac{2}{15}$ | 6. $\frac{w}{15}=\frac{12}{27} 6 \frac{2}{3}$ |
| 7. $\frac{9}{a}=\frac{81}{10} 1 \frac{1}{9}$ | 8. $\frac{25}{75}=\frac{z}{30} 10$ | 9. $\frac{n}{9}=\frac{n+1}{24} \frac{3}{5}$ |

Check Skills You'll Need
2. $\frac{81}{108} \frac{3}{4}$
9. $\frac{n}{9}=\frac{n+1}{24} \frac{3}{5}$

New Vocabulary • similar figures • scale drawing • scale
OBJECTIVE


Real-World Connection
The triangles in the quilt are the same shape, so they are similar.

## Investigation: Proportions in Triangles

The figure below shows $\triangle A C B$ and $\triangle D C E$.


1. Measure $A B, C A, C B, C D, C E$, and $D E$ using a metric ruler. See above.
2. Find each ratio.
a. $\frac{D E}{A B} 0.46$
b. $\frac{C E}{C B} 0.46$
c. $\frac{C D}{C A} 0.46$
3. Tell whether each statement is true.
a. $\frac{D E}{A B}=\frac{C E}{C B}$ true
b. $\frac{C D}{C A}=\frac{C E}{C B}$ true
c. $\frac{A B}{D E}=\frac{C A}{C D}$ true
4. Using the lengths you have measured, write two ratios that equal $\frac{C B}{C E} . \frac{C A}{C D}, \frac{A B}{D E}$

Lesson 4-2 Proportions and Similar Figures

## Ongoing Assessment and Intervention

Before the Lesson
Diagnose prerequisite skills using:

- Check Skills You'll Need

During the Lesson Monitor progress using:

- Check Understanding
- Additional Examples
- Standardized Test Prep


## After the Lesson

 Assess knowledge using:- Lesson Quiz
- Computer Test Generator CD
- Chapter Checkpoint 1 (p. 195)


## 4-2

## 1. Plan

## Lesson Preview

Check Skills You'll Need
Simplifying Fractions
Skills Handbook: p. 724
Example 2, Exercises 23-34
Ratio and Proportion
Lesson 4-1: Example 4
Exercises 14-29
Extra Practice, p. 705

## Lesson Resources

## Teaching Resources

Practice, Reteaching, Enrichment Checkpoint Quiz 1

## Reaching All Students

Practice Workbook 4-2 Spanish Practice Workbook 4-2
Reading and Math Literacy 4B Spanish Reading \& Literacy 4B
Spanish Checkpoint Quiz 1 Basic Algebra Planning Guide 4-2

## Presentation Assistant Plus!

Transparencies

- Check Skills You'll Need 4-2
- Additional Examples 4-2
- Student Edition Answers 4-2
- Lesson Quiz 4-2

PH Presentation Pro CD 4-2

## ASSEESSMMENT SYSTEM

Checkpoint Quiz 1
Computer Test Generator CD
Technology
Resource Pro ${ }^{\circledR}$ CD-ROM
Computer Test Generator CD
Prentice Hall Presentation Pro CD
www.PHSchool.com
Student Site

- Teacher Web Code: aek-5500
- Self-grading Lesson Quiz

Teacher Center

- Lesson Planner
- Resources

Plus (1)

## 2. Teach

## Math Background

Not only the corresponding sides, but also the medians, altitudes, and other corresponding lengths of similar figures are all in the same proportion. This relationship can be used to measure indirectly the height of a flagpole or the distance across a body of water.

## 1 Teaching Notes

## Investigation (Optional)

## Visual Learners

Draw a scalene triangle for the overhead projector. Project the image onto the chalkboard. Trace the triangle on the board. Move the projector closer to the board so that the image gets smaller. Trace the triangle again. Ask students to compare the triangles. same shape, different sizes

## ExaMPLE English Learners

Some students may not understand the term corresponding. Have a student look up the definition in a dictionary and read it aloud to the class. In figures, corresponding parts are in matching places. For example, $\overline{A B}$ corresponds to $\overline{F G}$ because they are between pairs of congruent angles in similar triangles.

## ${ }^{-}$Additional Examples

(1) In the figure below, $\triangle A B C \sim \triangle D E F$. Find $A B .12 \mathrm{~mm}$


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In the diagram below, $\triangle A B C$ and $\triangle F G H$ are similar. Similar figures have the same shape but not necessarily the same size. The symbol $\sim$ means "is similar to".

In similar triangles, corresponding angles are congruent and corresponding sides are in proportion. The order of the letters indicates the corresponding angles. If $\triangle A B C \sim \triangle F G H$, then the following is true.
ongruent angles have equal measures. The symbol $\cong$ means "is congruent to."

$\angle A \cong \angle F$
$\angle B \cong \angle G$
$\angle C \cong \angle H$

$\frac{A B}{F G}=\frac{A C}{F H}=\frac{B C}{G H}$
$\frac{8}{12}=\frac{10}{15}=\frac{12}{18}$

## 1 ExADIPLE Finding the Length of a Side

Geometry In the figure below, $\triangle A B C \sim \triangle D F E$. Find $D E$.


Write a proportion comparing the lengths of
Relate $\frac{A B}{D F}=\frac{A C}{D E} \quad \begin{aligned} & \text { Write a proportion co } \\ & \text { corresponding sides. }\end{aligned}$
Define Let $x=D E$.
Write $\quad \frac{15}{10}=\frac{21}{x} \quad$ Substitute 15 for $A B, 10$ for $D F, 21$ for $A C$, and $x$ for $D E$.
$15 x=10(21) \quad$ Write cross products.
$\frac{15 x}{15}=\frac{210}{15} \quad$ Divide each side by 15.
$x=14 \quad$ Simplify.
$D E$ is 14 cm .
Check Understanding
In the figure below, $\triangle F G H \sim \triangle K L M$. Find $L M .15 \mathrm{~cm}$


## Reaching All Students

Below Level Suggest that before students write a proportion involving similar figures, they point to and say to themselves the names of the corresponding sides.

Advanced Learners Challenge students to write as many proportions as possible that can be used to solve the problem in Example 1.

## English Learners

 See note on page 190 Inclusion See note on page 191.You can use proportions to find the dimensions of objects that are difficult to measure directly

## 2 <br> ExANPLE Applying Similarity

Indirect Measurement A tree casts a shadow 7.5 ft long. A woman 5 ft tall casts a shadow 3 ft long. The triangle shown for the tree and its shadow is similar to the triangle shown for the woman and her shadow. How tall is the tree?

Corresponding sides of
$\frac{3}{7.5}=\frac{5}{x}$ similar figures are in proportion.
$3 x=7.5 \cdot 5$
Write cross products.
$3 x=37.5 \quad$ Simplify.
$x=12.5$
Divide each side by 3 .


The tree is 12.5 ft tall

Check Understanding
A tree casts a $26-\mathrm{ft}$ shadow. A boy standing nearby casts a $12-\mathrm{ft}$ shadow. His height is 4.5 ft . How tall is the tree? 9.75 ft
b. A house casts a 56 -ft shadow. A girl standing nearby casts a $7.2-\mathrm{ft}$ shadow. Her height is 5.4 ft . What is the height of the house? 42 ft


Check Undersłanding
A scale drawing is an enlarged or reduced drawing that is similar to an actual object or place. Floor plans, blueprints, and maps are all examples of scale drawings. The ratio of a distance in the drawing to the corresponding actual distance is the scale of the drawing.

## 3 Exajuple Finding Distances on Maps

The scale of the map at the left is 1 inch: 10 miles. Approximately how far is it from Valkaria to Wabasso?

Map distance $=1.75$ in .

## Measure the map distance.

| $\underset{\text { actual }}{\operatorname{map}} \rightarrow \quad \frac{1}{10}$ | $=\frac{1.75}{d} \underset{\text { map }}{\leftarrow}$ actual |  | Write a proportion. |
| ---: | :--- | ---: | :--- |
| $1 \cdot d$ | $=10 \cdot 1.75$ |  | Write cross products. |
| $d$ | $=17.5$ |  | Simplify. |

Wabasso is about 17.5 mi from Valkaria.
. On the map above, measure the map distance from Grant to Gifford. Find the actual distance. about 21 mi
b. Critical Thinking If another map showed the distance from Valkaria to Wabasso but had a scale of 1 inch: 5 miles, what would the map distance be between the two locations? 3.5 in.

## Teaching Notes

Some students may wonder why the two triangles are similar. One way to prove triangles are similar is the Angle-Angle Similarity Theorem. If you can prove two pairs of corresponding angles of two triangles are congruent, then the two triangles are similar. The angles formed by the sun's rays and the standing objects are the same. Each object is standing at a right angle with the ground.

## (3) Example <br> Inclusion

Some students may have difficulty measuring because of disabilities. Let students work in pairs.

## E <br> Additional Examples

(2) A flagpole casts a shadow 102 feet long. A man 6 ft tall casts a shadow 17 feet long. How tall is the flagpole? 36 ft
(3) The scale of a map is 1 inch: 10 miles. The map distance from Valkaria to Gifford is 2.25 inches. How far is the actual distance? 22.5 mi

## Closure

Ask: How can you use proportions to find a distance that is difficult to measure? You can use the shadow and height of a figure that you can measure and the shadow of the figure you cannot measure to write a proportion Then you can solve the proportion

## 3. Practice

## Assignment Guide

1 objective
(A) (B) Core 1-8, 21, 23, 29-33
C Extension 36

## 2 objective

(A) (B) Core 9-20, 22, 24-28
C Extension 34-35
Standardized Test Prep 37-40
Mixed Review 41-48

## Visual Learners

Exercises 1-8 Suggest to students that they trace the figures onto their papers. They can then use colored pencils or markers to color code the corresponding parts.


EXERCISES
Practice and Problem Solving
A Practice by Example
The figures in each pair are similar. Identify the corresponding sides and angles.

Example 1
(page 190)

1. $\overline{A B} \cong \overline{P Q}, \overline{B C} \cong \overline{Q R}$, $C A \cong R P ; \angle A \cong \angle P$, $\angle B \cong \angle Q, \angle C \cong \angle R$
2. $\overline{D E} \cong \overline{H J}, \overline{E F} \cong \overline{J K}$, $F D \cong K H ; \angle D \cong \angle H$, $\angle E \cong \angle J, \angle F \cong \angle K$
3. 


$\triangle A B C \sim \triangle P Q R$


1-2. See left.


The figures in each pair are similar. Find the missing length.

4.

$13.3 \overline{3} \mathrm{~cm}$

5. 80 in .

7. 20.25 cm



Example 2
(page 191)
The child in the figure is $\mathbf{3} \mathbf{f t}$ tall.

9. How tall is the tree? 4.8 ft
10. The cat casts an 18 -in. shadow. How tall is the cat? 12 in.

Example 3 The scale of a map is $\mathbf{1} \mathbf{~ i n}$.: $\mathbf{1 7 . 5} \mathbf{~ m i}$. Find the actual distance corresponding to each (page 191) map distance.
11. 5 in. 87.5 mi
12. 8.3 in. 145.25 mi
13. 18.6 in. 325.5 mi
14. 20 in

350 mi


B Apply Your Skills

23a. Answers may vary. Sample: GK and RQ are not corresponding sides.


30c. Yes, the ratio of the sides is equal to the ratio of perimeters in similar figures.
d. $2 \mathrm{~m}^{2}, 18 \mathrm{~m}^{2}$
15. a. Use a ruler and the map at the left. Find the distance from each town to the others. See margin.
b. A student lives halfway between Lincoln and San Paulo and takes the shortest route to school in Duncanville. How far does the student travel each day to school? 26 mi
16. The actual distance between two towns is 28 km . Suppose you measure the distance on your map and find that it is 3.5 cm . What is the scale of your map? $1 \mathrm{~cm}: 8 \mathrm{~km}$
Using each of the following scales, find the dimensions in a blueprint of an 8 ft -by- $\mathbf{1 2} \mathrm{ft}$ room.
17. $1 \mathrm{in} .: 2 \mathrm{ft}$ 4 in. by 6 in.
18. $1 \mathrm{in} .: 3 \mathrm{ft}$ $2 \frac{2}{3}$ in. by 4 in. $\quad 2$ in. by 3 in. $\quad 3.2$ in. by 4.8 in
21. Two rectangles are similar. The first is 4 in . wide and 15 in . long. The second is 9 in . wide. Find the length of the second rectangle. 33.75 in.
22. Architecture A blueprint scale is $1 \mathrm{in} .: 9 \mathrm{ft}$. On the plan, the room measures 2.5 in . by 3 in . What are the actual dimensions of the room? 22.5 ft by 27 ft
23. Error Analysis The two figures are similar. Robert uses the proportion $\frac{G H}{P Q}=\frac{G K}{R Q}$ to find $R Q$. a. What is Robert's error?
b. What proportion should he have used? $\frac{G H}{P Q}=\frac{H L}{R Q}$


Architecture A 2-in. length in the scale drawing represents an actual length of $\mathbf{2 4} \mathbf{f t}$.
24. What is the scale of the drawing? $1 \mathrm{in} .: 12 \mathrm{ft}$
25. What are the actual dimensions of the kitchen? 9 ft by 12 ft
26. Find the actual width of the doorways that lead into the kitchen and the dining room. 3 ft
27. Find the actual area of the dining room. $216 \mathrm{ft}^{2}$
28. Can a table 7 ft long and 4 ft wide fit into the narrower section of the dining room? Explain your answer. yes; because it is 6 ft wide and 9 ft long
29. Two rectangles are similar. One is 5 cm by 12 cm . The longer side of the second rectangle is 8 cm greater than twice its shorter side. Find its length and width. 48 cm long by 20 cm wide
30. Geometry Rectangle $A B C D$ is similar to rectangle $K L M N$.
a. What is the width $w$ of rectangle $K L M N ? 6 \mathrm{~m}$
b. What is the perimeter of each rectangle? $6 \mathrm{~m}, 18 \mathrm{~m}$
c. Is the ratio of the perimeters of the rectangles (small: large) equal to the ratio of corresponding sides? Explain.
d. What is the area of each rectangle?
e. Critical Thinking Find the ratio of the areas (small: large).
Explain how the ratio of the areas is related to the ratio of the corresponding sides. $\qquad$
 Answers may vary. Sample: The area ratio is the square of the side ratio. 31. Open-Ended Give some examples of similar figures found in everyday life. See margin.

Error Prevention
Exercises 9, 10 Students may have difficulty writing the proportion correctly. Have students write labels on each term as they write the proportion. For example:
$\frac{54 \mathrm{in} \text {. (child's shadow) }}{18 \mathrm{ft}}=\frac{3 \mathrm{ft} \text { (child's height) }}{x(2)}$ 18 ft (tree's shadow) $=\frac{3 \text { (tree's height) }}{}$
By seeing the matching words, students can determine if they wrote corresponding measurements in the same ratio.

Exercise 16 Let students know that a map scale is an instance of a unit rate.

## Careers

Exercise 22 Architects draw buildings, houses, and landscapes to scale. These scale drawings must be exact so that the builders know exactly what size the building materials must be to fit together properly.

Exercise 23 Remind students that only corresponding sides of similar figures should be used to write proportions.
pages 192-195 Exercises
15a. Lincoln to
San Paulo = 16 mi
Lincoln to
Duncanville $=26 \mathrm{mi}$
San Paulo to
Duncanville $=18 \mathrm{mi}$

## 4. Assess

## Lesson Quiz 4-2

1. In the figure below, $\triangle A B C \sim \triangle D E F$. Find $D F$. about 19.7 cm

2. A boy who is 5.5 feet tall casts a shadow that is 8.25 feet long. The tree next to him casts a shadow that is 18 feet long. How tall is the tree? 12 ft
3. The scale on a map is 1 in : 20 mi . What is the actual distance between two towns that are 3.5 inches apart on the map? 70 mi

## Alternative Assessment

Group students in pairs. Instruct them to use shadows and proportions to find the height of objects they cannot measure, such as a building, a tree, or a flagpole.

## Standardized Test Prep

## Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 231
- Test-Taking Strategies, p. 226
- Test-Taking Strategies with Transparencies


## Connection to Geometry

## Exercise 37 Sometimes it is

 confusing to look at the figures. You can use the similarity statement to determine if a proportion is correct. All A's will correspond with all X's since they are the first letter written for each triangle in the similarity statement. This is also true for all $B^{\prime}$ s and $Y^{\prime}$ 's, and for all C's and Z's.
## Problem Solving Hint

For Exercise 33,
drawing a diagram can help you understand the problem.32. a. Writing Are the two cubes similar? Explain your answer
b. Explain how the ratio of volumes (small : large) is related to the ratio of their sides (small : large)
c. If the ratio of the sides of the two
 cubes is $3: 1$, what is the ratio of their
 volumes? a-c See margin.
33. Geometry The perimeter of a triangle with sides $a, b$, and $c$ is 24 cm . Side $a$ is 2 cm longer than side $b$. The ratio of the lengths of sides $b$ and $c$ is $3: 5$.
What are the lengths of the three sides of the triangle? $a=8, b=6, c=10$
34. The state of Alabama is about 335 mi long and 210 mi wide. What scale would you use to draw a map of Alabama on an $8 \frac{1}{2}$ in.-by- 11 in . paper to make the map as large as possible? about 1 in . : 24.7 mi
35. Astronomy You can block out the moon by holding a coin up at a distance from your eye that is 110 times the diameter of the coin. Using similar figures, $\frac{\text { coin diameter }}{\text { moon diameter }}=\frac{\text { coin distance }}{\text { moon distance }}$. The moon is roughly 3640 kilometers in diameter. How far away is it? $400,400 \mathrm{~km}$
36. Geometry In the figure at the right, $\triangle A B C \sim \triangle A D E$.
a. Substitute values from the diagram into the following proportion. $\frac{A D}{A B}=\frac{D E}{B C} \frac{8}{8+x}=\frac{5}{7}$
(Hint: $A B=A D+D B$.)
b. Solve the proportion for $x$. 3.2
c. Find the length of $A B .11 .2$ in.
d. What is the area of $\triangle A B C$ ? 39.2 in. ${ }^{2}$


## Standardized Test Prep

Multiple Choice
37. In the figure at the right, $\triangle A B C \sim \triangle X Y Z$. Which proportion is incorrect? B
A. $\frac{A B}{A C}=\frac{X Y}{X Z}$
B. $\frac{A B}{B C}=\frac{X Y}{X Z}$
C. $\frac{B C}{A C}=\frac{Y Z}{X Z}$
D. $\frac{A C}{X Z}=\frac{B C}{Y Z}$


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pages 192-195 Exercises
31. Answers may vary. Sample: doll house to regular house, model car to real car

32a. Yes; the sides are proportional.
b. 1:8; 1:2; answers may vary. Sample: The
volume ratio is the cube of the side ratio.
c. $27: 1$
38. A map of Kentucky is drawn with a scale of $1 \mathrm{~cm}: 11 \mathrm{~km}$. The map distance between Louisville and Bowling Green is 14.5 cm . Which is the best estimate of the actual distance?
F. 1.3 km
G. 14 km
H. 100 km
I. 160 km

Short Response

Extended Response

Take it to the NET
Online lesson quiz at www.PHSchool.com Web Code: aea-0402
39. You can paint a 6 ft -by- 5 ft rectangular wall using 0.5 gallon of paint. How many gallons of paint will you need to cover a 10 ft -by- 12 ft wall? Show your work. See margin.
40. Leonardo da Vinci's famous painting the Mona Lisa measures 77.5 cm by 55 cm . See margin.
a. Explain how you know that a $16 \mathrm{~cm}-\mathrm{by}-12 \mathrm{~cm}$ reproduction postcard is NOT similar to the original painting.
b. What dimensions would make a postcard similar to the original painting? Approximate to the nearest tenth. Show your work or explain how you found your answer.

## Chapter Checkpoint 1

To check understanding of Lessons 4-1 to 4-2:

Checkpoint Quiz 1 (p. 195)

## Teaching Resources

Checkpoint Quiz 1 (also in Prentice Hall Assessment System)

Reaching All Students Reading and Math Literacy 4B
Spanish versions available

Mixed Review

Lesson 4-1 Solve each proportion.
41. $\frac{x}{2}=\frac{9}{4} 4.5$
42. $\frac{5}{n}=\frac{3}{10} 16 \frac{2}{3}$
43. $\frac{-8}{m}=\frac{7}{20}-22 \frac{6}{7}$
44. $\frac{12}{30}=\frac{16}{v} 40$

Lesson 3-3 Solve.
45. $5 b<-20$
b $<-4$
46. $\frac{4}{7} x \geq 4 x \geq 7$
47. $-3 m>12$
$m<-4$
48. $-\frac{2}{3} h<1$ $h>-\frac{3}{2}$

## Checkpoint Quiz 1

Lessons 4-1 through 4-2

TifXII Instant self-check quiz online and on CD-ROM

1. Complete the statement 2 days $=\square$ minutes. 2880
2. Write $\$ 48$ for 8 hours as a unit rate. $\$ 6.00 / \mathrm{h}$

Solve each proportion.
3. $\frac{x}{6}=\frac{7}{4} 10.5$
4. $\frac{8}{k}=-\frac{12}{30}-20$
5. $\frac{3}{5}=\frac{y+1}{9} 4.4$
6. You are riding your bicycle. It takes you 12 min to go 2.5 mi . If you continue traveling at the same rate, how long will it take you to go 7 mi ? 33.6 min

The figures in each pair are similar. Find the missing length.

9. A $3.5-\mathrm{ft}$ child casts a 60 -in. shadow. She is standing next to a telephone pole that casts a $50-\mathrm{ft}$ shadow. How tall is the telephone pole? 35 ft
10. The scale of a map is 3 in . : 20 mi . Find the actual distance if the map distance between two towns is $5.5 \mathrm{in} .36 \frac{2}{3} \mathrm{mi}$

Lesson 4-2 Proportions and Similar Figures
39. [2] smaller area: $6 \cdot 5=$ 30; $30 \mathrm{ft}^{2}$; larger area: $10 \cdot 12=120 ; 120 \mathrm{ft}^{2}$; $\frac{0.5}{30}=\frac{x}{120}$
$0.5(120)=30 x$
$60=30 x$
$2=x$
[1] incorrect calculation for one area and proportion solved correctly OR correct area calculations but proportion set up incorrectly

