# Proportions and Similar Figures

Section 2-8

# Goals

**Goal**

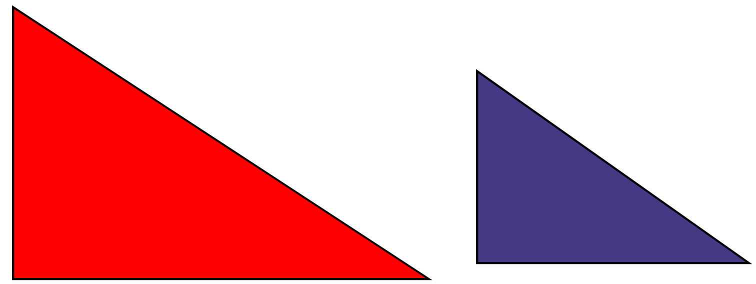
- To find missing lengths in similar figures.
- To use similar figures when measuring indirectly.

# Vocabulary

- Similar Figures
- Scale Drawing
- Scale
- Scale model

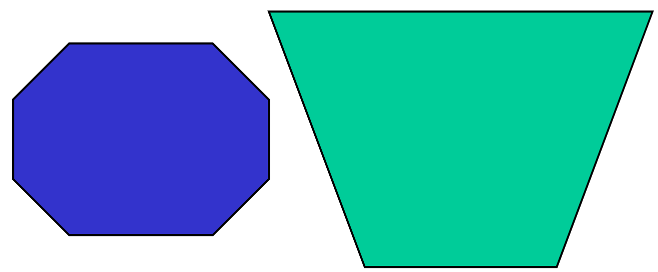
# What is Similarity?

**Not Similar**



**Similar**

**Similar**



**Not Similar**

# Similar Figures

Figures that have the same shape but not necessarily the same size are **similar figures**. But what does "same shape mean"? Are the two heads similar?

# Similar Figures

Similar figures can be thought of as **enlargements** or **reductions** with no irregular distortions.

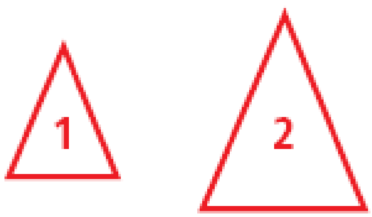
- So two figures are similar if one can be enlarged or reduced so that it is congruent (means the figures have the same dimensions and shape, symbol ≅) to the original.

# Similar Triangles

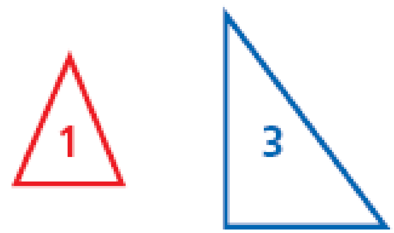
- When triangles have the same shape but may be different in size, they are called ***similar triangles***.

- We express similarity using the symbol, ~.
  **(i.e. $\Delta ABC \sim \Delta PRS$)**

# Example - Similar Triangles




$\triangle 1$ is similar to $\triangle 2 (\triangle 1 \sim \triangle 2)$.




$\triangle 1$ is not similar to $\triangle 3 (\triangle 1 \nsim \triangle 3)$.

# Similar Figures

*Similar* figures have exactly the same shape but not necessarily the same size.

*Corresponding sides* of two figures are in the same relative position, and *corresponding angles* are in the same relative position. Two figures are similar if and only if the lengths of corresponding sides are proportional and all pairs of corresponding angles have equal measures.

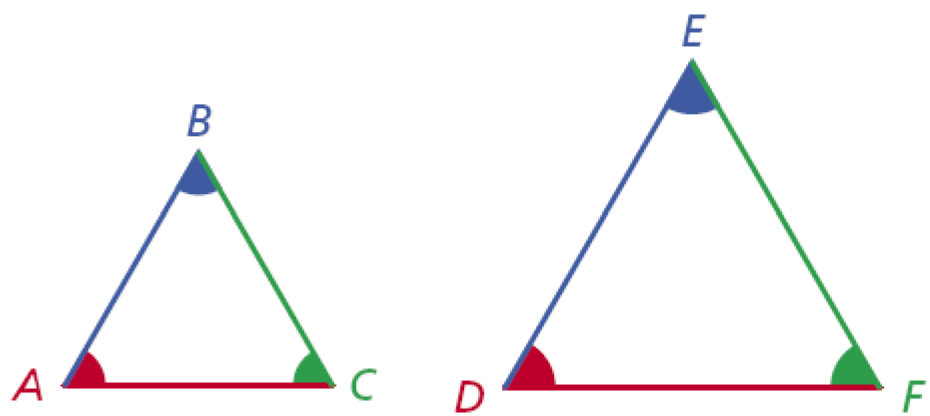
# Similar Figures

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$m\angle A = m\angle D$$

$$m\angle B = m\angle E$$

$$m\angle C = m\angle F$$




When stating that two figures are similar, use the symbol ~. For the triangles above, you can write $\Delta ABC \sim \Delta DEF$. Make sure corresponding vertices are in the same order. It would be incorrect to write $\Delta ABC \sim \Delta EFD$.

You can use proportions to find missing lengths in similar figures.

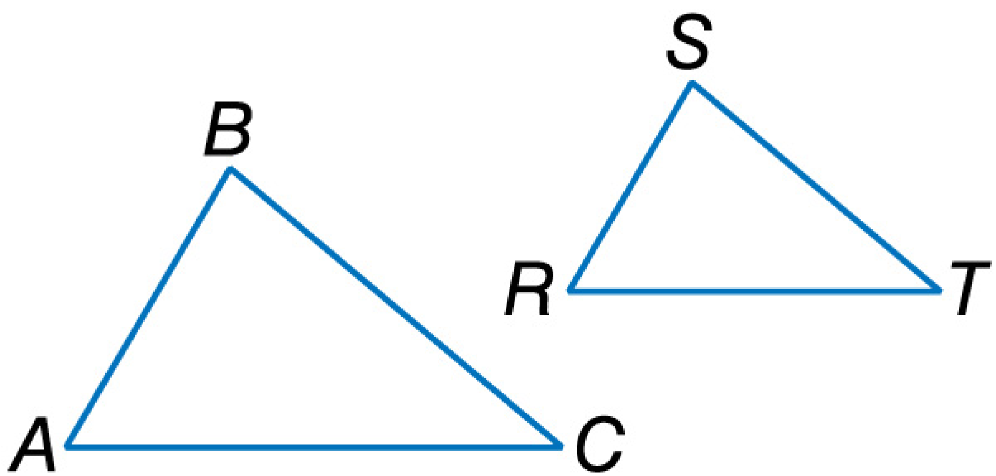
## Reading Math

- $\overline{AB}$ means segment *AB*. *AB* means the length of $\overline{AB}$.
- <*A* means angle *A*.

m<*A* the measure of angle *A*.

# Example 1

- **If $\Delta ABC \sim \Delta RST$, list all pairs of congruent angles and write a proportion that relates the corresponding sides.**

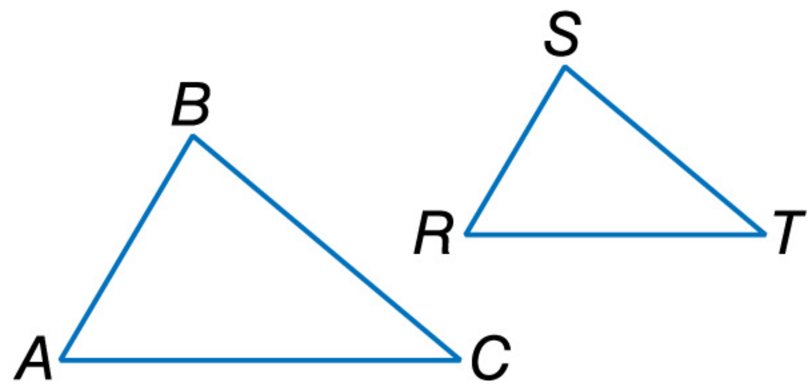


# Example 1

- Use the similarity statement.

*ABC* *RST*

**Answer:**




<A ~ <R    <B ~ <S    <C ~ <T

Proportion: $\frac{AB}{RS} = \frac{BC}{ST} = \frac{AC}{RT}$

# Your Turn:

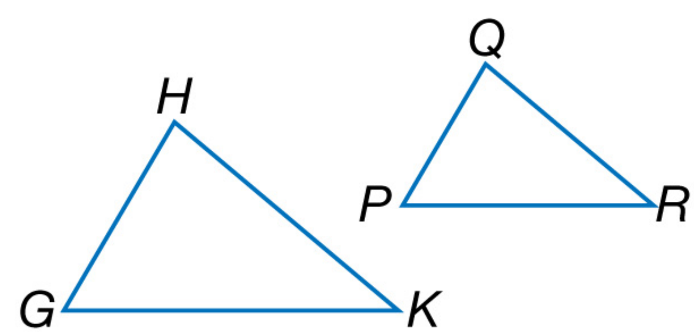
- **If $\Delta GHK \sim \Delta PQR$, determine which of the following similarity statements is not true.**

**A.** $HK \sim QR$

**B.** $\frac{GH}{PQ} = \frac{GK}{PR}$

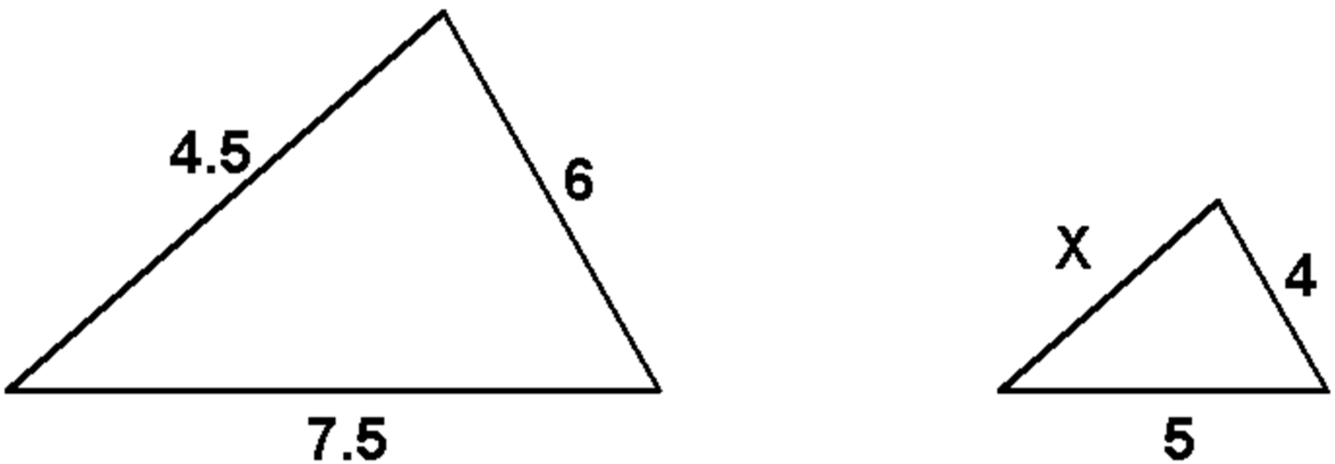
**C.** $\angle K \sim \angle R$

**D.** $\angle H \sim \angle P$



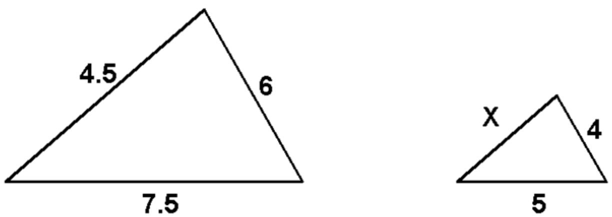
# Example: Finding the length of a Side of Similar Triangles

The two triangles below are similar, determine the length of side *x.*




$$\frac{7.5}{5} = \frac{4.5}{x}$$

# Example: Continued




$$\frac{7.5}{5} = \frac{4.5}{x}$$

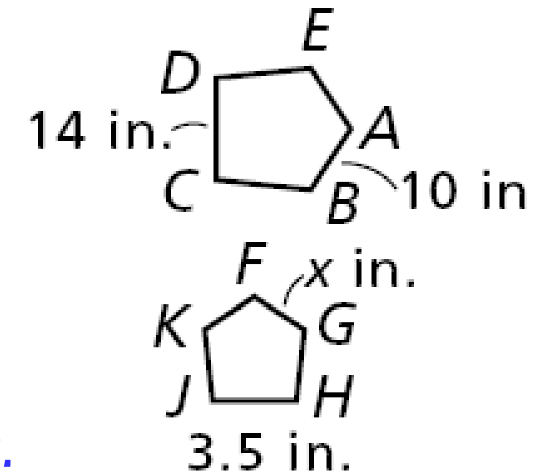
$$5(4.5) = 7.5x$$

$$22.5 = 7.5x$$

$$3 = x$$

# Example: Finding the length of a Side of Similar Figures

Find the value of *x* the diagram.




*ABCDE* ~ *FGHJK*

$$\frac{14}{3.5} = \frac{10}{x}$$

$$\frac{CD}{HJ} = \frac{AB}{FG}$$

$$14x = 35$$

*Use cross products.*

$$\frac{14x}{14} = \frac{35}{14}$$

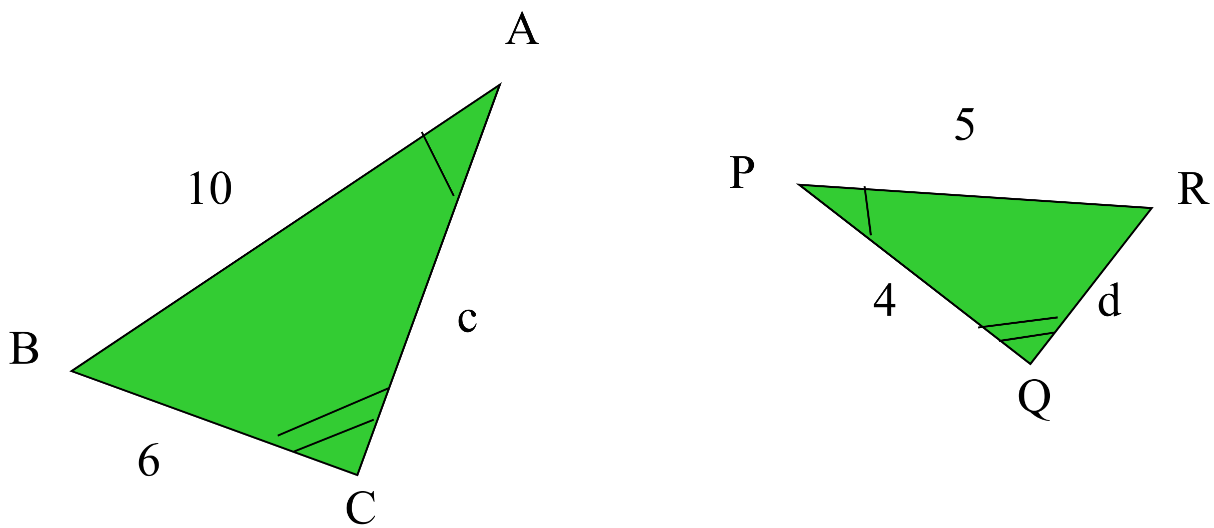
*Since x is multiplied by 14, divide both sides by 14 to undo the multiplication.*

$$x = 2.5$$

The length of $\overline{FG}$ is 2.5 in.

# Your Turn:

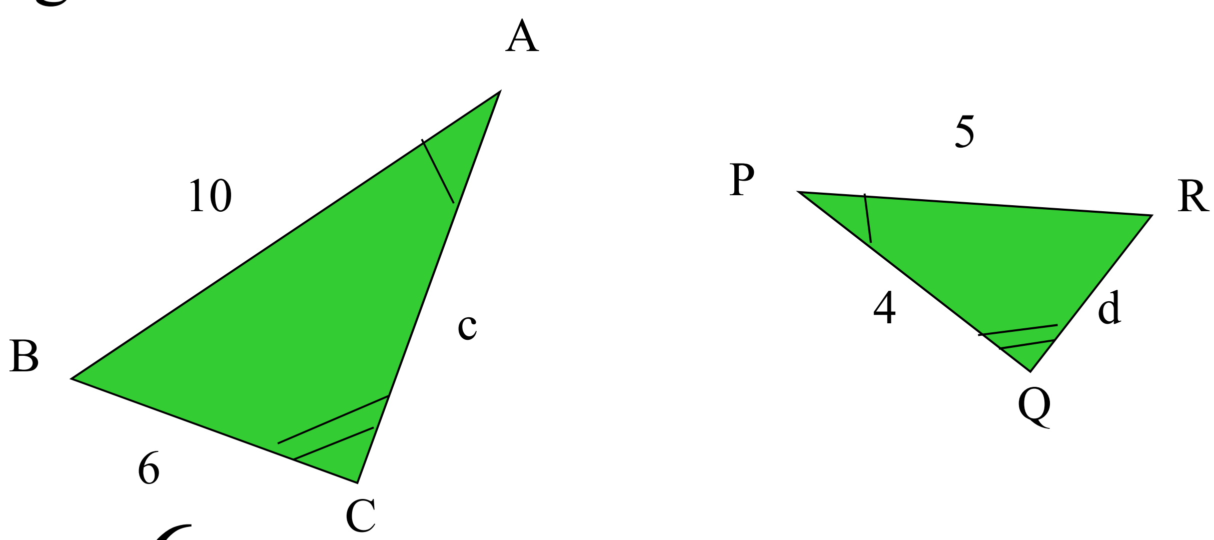
In the figure, the two triangles are similar. What is the length of c?




$$\frac{10}{5} = \frac{c}{4} \qquad 40 = 5c \qquad 8 = c$$

# Your Turn:

In the figure, the two triangles are similar. What is the length of d?




$$\frac{10}{5} = \frac{6}{d} \quad 30 = 10d \quad 3 = d$$

# Indirect Measurement

- You can use similar triangles and proportions to find lengths that you cannot directly measure in the real world.
- This is called *indirect measurement*.
- If two objects form right angles with the ground, you can apply indirect measurement using their shadows.

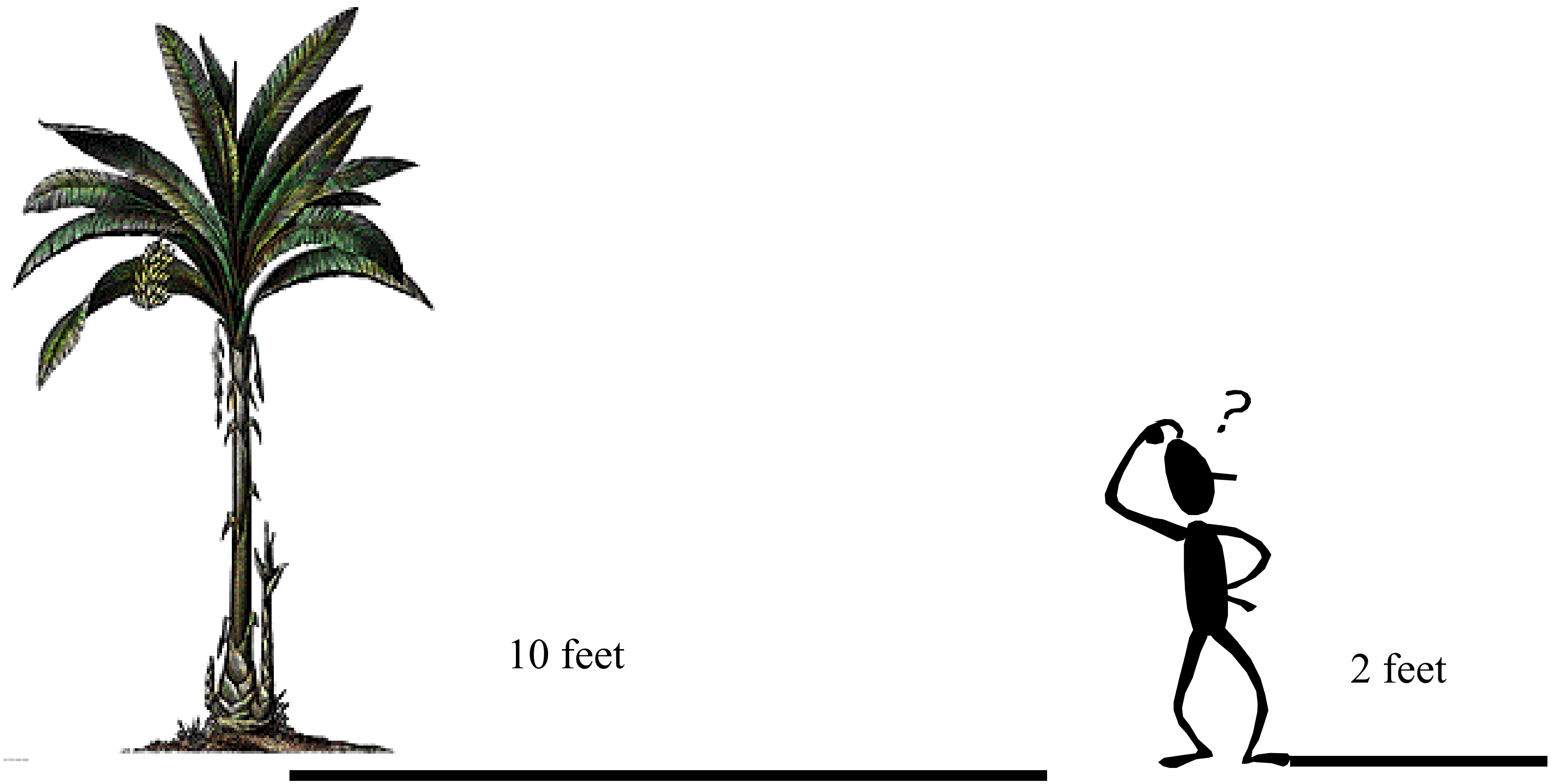
# Similarity is used to answer real life questions.

- Suppose that you wanted to find the height of this tree.
- Unfortunately all that you have is a tape measure, and you are too short to reach the top of the tree.

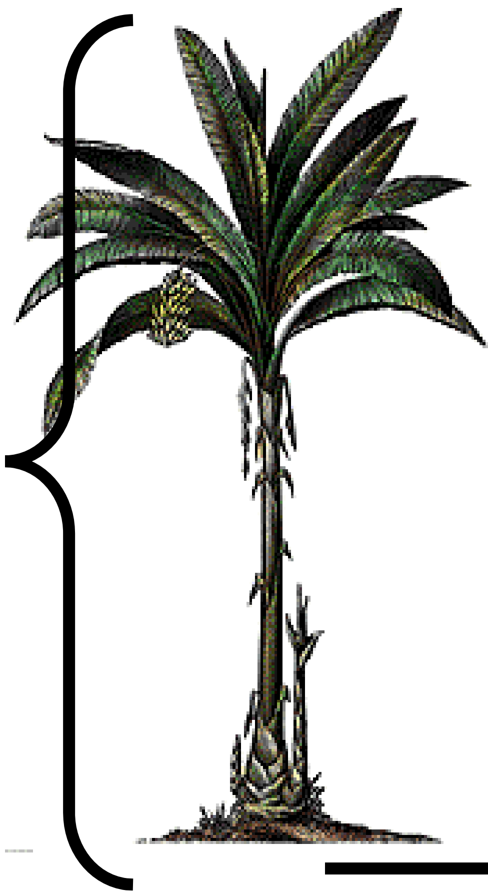
You can measure the length of the tree’s shadow.

10 feet

# Then, measure the length of your shadow.



If you know how tall you are, then you can determine how tall the tree is.



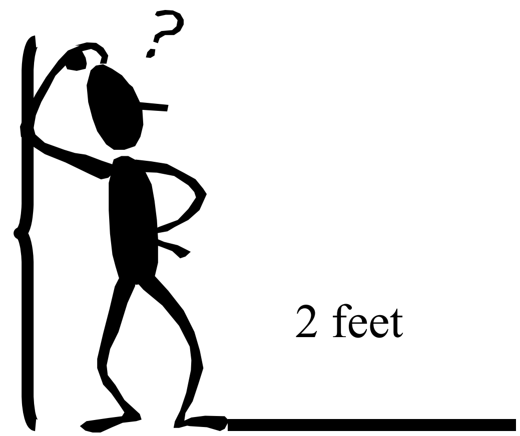
$$\frac{h}{10}=\frac{6}{2}$$

$$2h=60$$

$$h=30$$

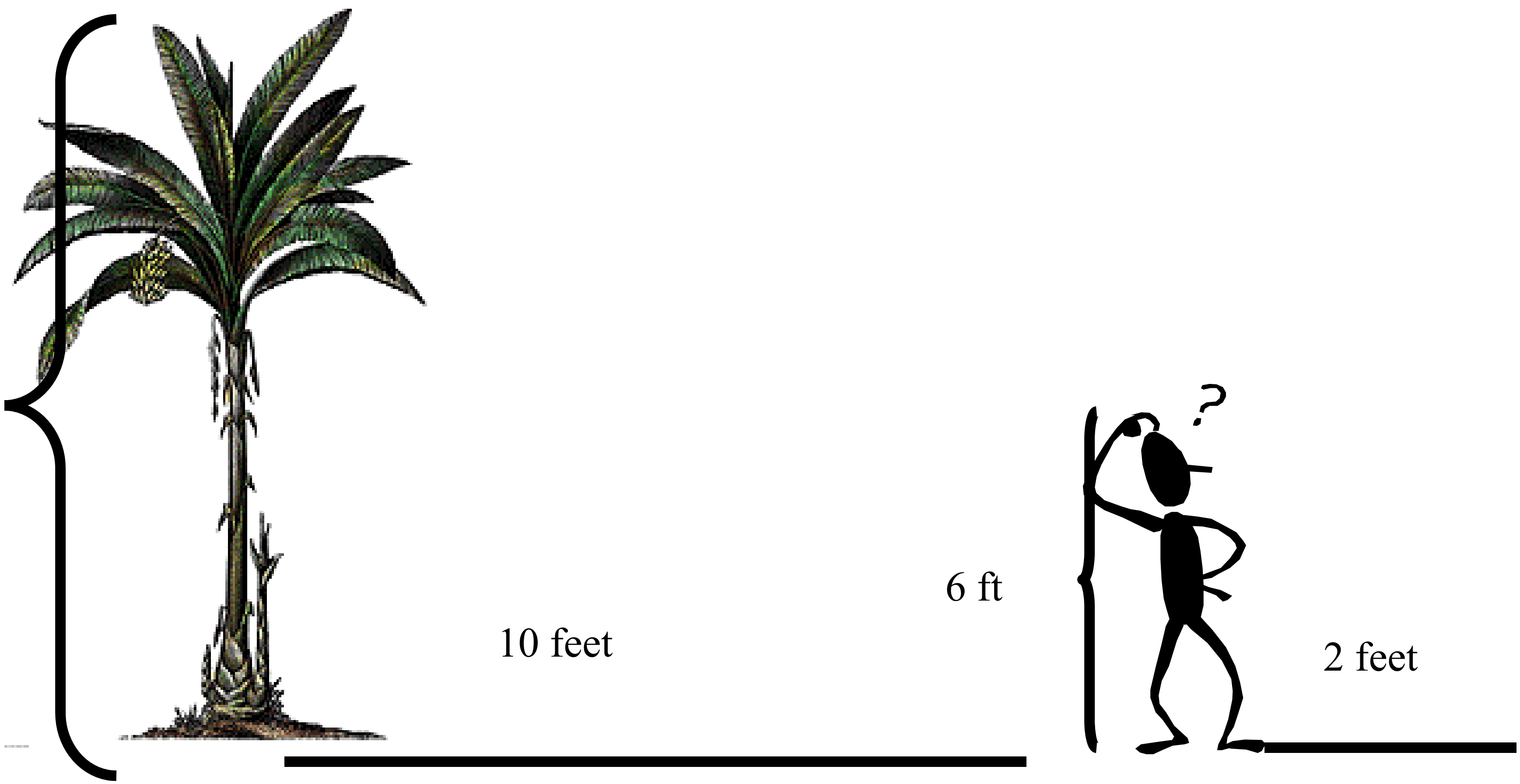
6 ft

10 feet



2 feet

The tree is 30 ft tall.
Boy, that's a tall tree!



# Example: Indirect Measurement

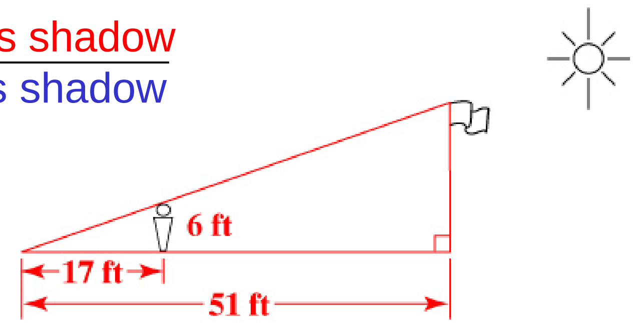
When a 6-ft student casts a 17-ft shadow, a flagpole casts a shadow that is 51 ft long. Find the height of the flagpole.

Set up a proportion for the similar triangles.

**Words**

$$\frac{\text{flagpole's height}}{\text{student's height}} = \frac{\text{length of flagpole's shadow}}{\text{length of student's shadow}}$$

Let $h$ = the flagpole's height.




**Proportion**

$$\frac{h}{6} = \frac{51}{17}$$

$17h = 6 \cdot 51$ ← Write the cross products.

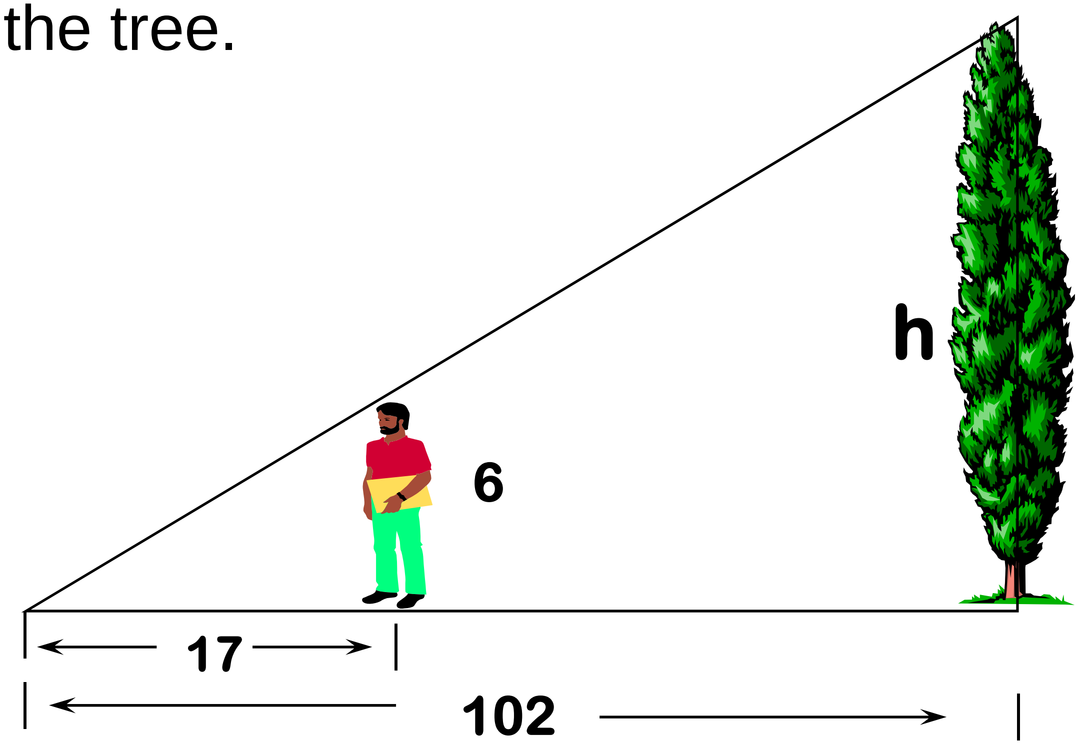
$\frac{17h}{17} = \frac{6 \cdot 51}{17}$ ← Divide each side by 17.

$h = 18$ ← Simplify.

The height of the flagpole is 18 ft.

# Your Turn:

When a 6-ft student casts a 17-ft shadow, a tree casts a shadow that is 102 ft long. Find the height of the tree.

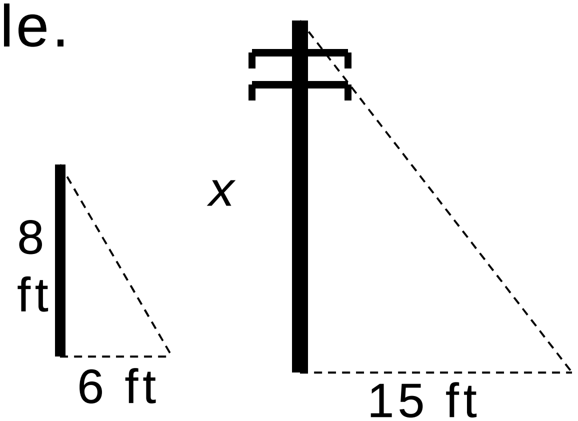



$$\frac{17}{102} = \frac{6}{h}$$

$$36 = h$$

# Your Turn:

**1.** Use the similar triangles to find the height of the telephone pole.




**20 feet**

**2.** On a sunny afternoon, a goalpost casts a 75 ft shadow. A 6.5 ft football player next to the goal post has a shadow 19.5 ft long. How tall is the goalpost?

**25 feet**

# Definition

- Proportions are used to create *scale drawings* and *scale models.*
- *Scale* - a ratio between two sets of measurements, such as 1 in.:5 mi.
- *Scale Drawing or Scale Model* - uses a scale to represent an object as smaller or larger than the actual object.
  - A map is an example of a scale drawing.

# Example: Scale Drawing

A contractor has a blueprint for a house drawn to the scale 1 in.:3 ft.

A wall on the blueprint is 6.5 inches long. How long is the actual wall?

$$\frac{\text{blueprint}}{\text{actual}} \rightarrow \frac{1\text{in.}}{3\text{ft.}}$$

*Write the scale as a fraction.*

$$\frac{1\text{ in.}}{3\text{ ft.}} = \frac{6.5}{x}$$

*Let x be the actual length.*

*Use cross products to solve.*

$$x \bullet 1 = 3(6.5)$$

$$x = 19.5$$

The actual length is 19.5 feet.

# Example: Scale Drawing

A contractor has a blueprint for a house drawn to the scale 1 in.:3 ft.

A wall in the house is 12 feet long. How long is the wall on the blueprint?

$$\frac{\text{blueprint}}{\text{actual}} \rightarrow \frac{1\text{in.}}{3\text{ft.}}$$

*Write the scale as a fraction.*

$$\frac{1}{3} = \frac{x}{12}$$

*Let x be the blueprint length.*

*Use cross products to solve.*

$$x \cdot 3 = 1(12)$$

$$x = 4$$

The blueprint length is 4 inches.

**Reading Math**

A scale written without units, such as 32:1, means that 32 units of any measure corresponds to 1 unit of that same measure.

# Your Turn:

The actual distance between North Chicago and Waukegan is 4 mi. What is the distance between these two locations on the map?


$$\frac{\text{map}}{\text{actual}} \rightarrow \frac{1}{18}$$

*Write the scale as a fraction.*

$$\frac{x}{4} = \frac{1}{18}$$

*Let x be the map distance.*

$$18x = 4$$

$$x \approx 0.2$$

*Use cross products to solve.*

The distance on the map is about 0.2 in.

# Your Turn:

A scale model of a human heart is 16 ft long. The scale is 32:1 How many inches long is the actual heart that the model represents?

$$\frac{\text{model}}{\text{actual}} \rightarrow \frac{32}{1}$$

*Write the scale as a fraction.*

$$\frac{32}{1} = \frac{16}{x}$$

*Let x be the actual distance.*

*Use cross products to solve.*

$$32x = 16$$

$$x = 0.5$$

The actual heart is 0.5 feet or 6 inches.

# *Joke Time*

- What kind of coffee was served on the Titanic?
- Sanka.

- And what kind of lettuce was served on the Titanic?
- Iceberg.

- Why do gorillas have big nostrils?
- Because they have big fingers.