Proposition logic and argument

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Where are my glasses?

I know the following statements are true.

1. If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.
2. If my glasses are on the kitchen table, then I saw them at breakfast.
3. I did not see my glasses at breakfast.
4. I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.
5. If I was reading the newspaper in the living room then my glasses are on the coffee table.

More likely scenario for you: where are bugs in my CS1 program?

you might find yourself reasoning through your code: if a>0 here, then this line is executed, which makes b=0, ….

Algebraic for logic

Recall that you know

1+10=10+1, 45+23=23+45, … and in general, a+b=b+a,

2 * (3+10)=2*3+2*10, … in general a(b+c)=ab+ac

….

- Similarly, definitions of formal logic were developed to capture natural or intuitive logic used by people
- Benefits:
  - allow us to see structures (forms) of arguments more clearly
  - avoid fallacy (or logic errors)

Outline

- Review: (propositional) logic
- Logic Equivalence
- Arguments
  - Rule of inferences
  - Fallacy

Statements: simple and compound

- A **statement (proposition)** is a statement that is true or false, but not both.
- **Compound statement** can be formed from **simple statement** as follows:
  - Given statement *p*,q,
    - "~*p*" ("not *p*", "It is not the case that *p*") is called **negation of *p*.**
    - "$p \wedge q$" ("*p* and *q*") is **conjunction of *p* and *q*.**
    - "$p \vee q$" ("*p* or *q*") is **disjunction of *p* and *q***
    - "p⊕q" (p exclusive or q)
    - "p→q" (if p then q) is conditional
    - "p↔q" (p if and only if q) is biconditional

English to Symbols

Write following sentences symbolically, letting
h = “It is hot” and *s* = “It is sunny.”

a. It is not hot but it is sunny.

b. It is neither hot nor sunny.

Truth Table definition of negation

<u>Negation of a statement</u> *is a statement that exactly expresses the original statement to be false.*

• Definition

If p is a statement variable, the **negation** of p is "not p" or "It is not the case that p" and is denoted $\sim p$. It has opposite truth value from p: if p is true, $\sim p$ is false; if p is false, $\sim p$ is true.

summarized in a *truth table:*

Truth Table for ~*p*

p	$\sim p$
T	F
F	T

Definition of conjunction (and)

• Definition

If p and q are statement variables, the **conjunction** of p and q is "p and q," denoted $p \wedge q$. It is true when, and only when, both p and q are true. If either p or q is false, or if both are false, $p \wedge q$ is false.

summarized in a truth table.

Truth Table for $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Definition of Disjunction (or)

• Definition

If p and q are statement variables, the **disjunction** of p and q is "p or q," denoted $p \vee q$. It is true when either p is true, or q is true, or both p and q are true; it is false only when both p and q are false.

Here is the truth table for disjunction:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Truth Table for $p \vee q$

Exclusive Or

Consider statement

$$(p \vee q) \wedge \sim(p \wedge q).$$

This means "*p* or *q, and not both p and q*", i.e., **exclusive or.**

This is often abbreviated as

$$p \oplus q \text{ or } p \text{ XOR } q.$$

Evaluating the Truth of More General Compound Statements

Now that truth values have been assigned to $\sim p$, $p \wedge q$, and $p \vee q$, consider the question of assigning truth values to more complicated expressions such as $\sim p \vee q$, $(p \vee q) \wedge \sim(p \wedge q)$, and $(p \wedge q) \vee r$. Such expressions are called *statement forms* (or *propositional forms*).

• Definition

A **statement form** (or **propositional form**) is an expression made up of statement variables (such as p, q, and r) and logical connectives (such as $\sim$, $\wedge$, and $\vee$) that becomes a statement when actual statements are substituted for the component statement variables. The **truth table** for a given statement form displays the truth values that correspond to all possible combinations of truth values for its component statement variables.

Conditional Statements

Let *p* and *q* be statements. A sentence of the form "If *p* then *q*" is denoted symbolically by "$p \rightarrow q$"; *p* is called the *hypothesis* and *q* is called the *conclusion, e.g.,*

$$\text{If } \underbrace{4{,}686 \text{ is divisible by } 6}_{\text{hypothesis}}, \text{ then } \underbrace{4{,}686 \text{ is divisible by } 3}_{\text{conclusion}}$$

• Definition

If p and q are statement variables, the **conditional** of q by p is "If p then q" or "p implies q" and is denoted $p \rightarrow q$. It is false when p is true and q is false; otherwise it is true. We call p the **hypothesis** (or **antecedent**) of the conditional and q the **conclusion** (or **consequent**).

Meaning of Conditional Statements

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table for $p \leftrightarrow q$

- Based on its everyday, intuitive meaning.

Manager: *"If you show up for work Monday morning, then you will get the job"*

When will you be able to say that the manager lies?
Only if you show up on Monday morning, but you did not get the job.

- When "if" part (hypothesis) is false, the whole conditional statement is true, regardless of whether conclusion is true or false.
  - In this case, we say the conditional statement is **vacuously true** or **true by default**.

thus the statement is vacuously true if you do not show up for work Monday morning.

A Conditional Statement with a False Hypothesis

Consider the statement:

If 0 = 1 then 1 = 2.

As strange as it may seem, since the hypothesis of this statement is false, the statement as a whole is true.

Only If

"*p* only if *q*" means that ***p* can take place *only* if *q* takes place also**.

- i.e., if *q* does not take place, then *p* cannot take place.
- or, if *p* occurs, then *q* must also occur.

• Definition

It *p* and *q* are statements,

p **only if** *q* means "if not *q* then not *p*,"

or, equivalently,

"*if p* then *q*."

Interpreting Only If

• Definition

It p and q are statements,

p **only if** q means "if not q then not p,"

or, equivalently,

"*if* p then q."

Rewrite following statement in if-then forms:

John will break the world's record for the mile run only if he runs the mile in under four minutes.

If and only If (Biconditional)

• Definition

Given statement variables p and q, the **biconditional of p and q** is "p if, and only if, q" and is denoted $p \leftrightarrow q$. It is true if both p and q have the same truth values and is false if p and q have opposite truth values. The words *if and only if* are sometimes abbreviated **iff.**

The biconditional has the following truth table:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth Table for $p \leftrightarrow q$

Necessary and Sufficient Conditions

• Definition

If r and s are statements:

r is a **sufficient condition** for s	means	"if r then s."
r is a **necessary condition** for s	means	"if not r then not s."

- "*r* is a sufficient condition for *s*" means that the occurrence of *r* is *sufficient* to guarantee the occurrence of *s*.

- "*r* is a necessary condition for *s*" means that if *r* does not occur, then *s* cannot occur either. The occurrence of *r* is necessary to obtain the occurrence of *s*.

r is a necessary condition for s	also means	"if s then r."

Necessary and Sufficient Conditions

Consider statement

If John is eligible to vote, then he is at least 18 years old.

The truth of the condition “John is eligible to vote” is *sufficient* to ensure the truth of the condition “John is at least 18 years old.”

In addition, the condition “John is at least 18 years old” is *necessary* for the condition “John is eligible to vote” to be true.

If John were younger than 18, then he would not be eligible to vote.

Only If and the Biconditional

According to the separate definitions of *if* and *only if*, saying "p if, and only if, q" should mean the same as saying both "p if q" and "p only if q."

The following annotated truth table shows that this is the case:

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

$p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$ always have the same truth values, so they are logically equivalent

Truth Table Showing that $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

Exercise: If and Only If

Rewrite the following statement as a conjunction of two if-then statements:

This computer program is correct if, and only if, it produces correct answers for all possible sets of input data.

Precedence of logical operators

The full hierarchy of operations for the five logical operators is:

Order of Operations for Logical Operators

1. $\sim$ Evaluate negations first.
2. $\wedge, \vee$ Evaluate $\wedge$ and $\vee$ second. When both are present, parentheses may be needed.
3. $\rightarrow, \leftrightarrow$ Evaluate $\rightarrow$ and $\leftrightarrow$ third. When both are present, parentheses may be needed.

Practice: $p \vee \sim q \wedge r \rightarrow p \wedge r$

$p \vee \sim q \rightarrow \sim p$

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Logical Equivalence

Statements

(1) 6 is greater than 2

(2) 2 is less than 6

are saying same thing,
because of definition of phrases ***greater than*** and ***less than***.

Logical Equivalence

Statements

(1) Dogs bark and cats meow

and

(2) Cats meow and dogs bark

also say same thing (either both are true, or both be false)

- Not because of definition of the words.
- It has to do with logical form of the statements.
- Any two statements whose **logical forms** are related in same way as (1) and (2) would mean the say thing.

Logical Equivalence

Compare truth tables for logic forms of two statements:

1. statement variables *p* and *q* are substituted for component statements "Dogs bark" and "Cats meow," respectively.
2. truth table shows that for each combination of truth values for *p* and *q*, $p \wedge q$ is true when, and only when, $q \wedge p$ is true.

In such a case, statement forms are called ***logically equivalent***, and we say that (1) and (2) are *logically equivalent statements*.

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

$p \wedge q$ and $q \wedge p$ always have the same truth values, so they are logically equivalent

Logical Equivalence

• Definition

Two *statement forms* are called **logically equivalent** if, and only if, they have identical truth values for each possible substitution of statements for their statement variables. The logical equivalence of statement forms P and Q is denoted by writing $P \equiv Q$.

Two *statements* are called **logically equivalent** if, and only if, they have logically equivalent forms when identical component statement variables are used to replace identical component statements.

Testing Logical Equivalence

Testing Whether Two Statement Forms *P* and *Q* Are Logically Equivalent

1. Construct a truth table with one column for *P* and another column for *Q*.

2. Check each combination of truth values of the statement variables to see whether the truth value of *P* is the same as the truth value of *Q*.

a. If **in every row** the truth value of *P* is the same as the truth value of *Q*, then *P* and *Q* are logically equivalent.

b. If **in some row** *P* has a different truth value from *Q*, then *P* and *Q* are not logically equivalent.

Prove non-Logical Equivalence

- Use a truth table to *find rows for which their truth values differ*, or

- *Find concrete statements* for each of the two forms, one of which is true and the other of which is false.

Showing Nonequivalence

Statement forms

$\sim(p \wedge q)$ and

$\sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$		$\sim p \wedge \sim q$
T	T	F	F	T	F		F
T	F	F	T	F	T	$\neq$	F
F	T	T	F	F	T	$\neq$	F
F	F	T	T	F	T		T

$\sim(p \wedge q)$ and $\sim p \wedge \sim q$ have different truth values in rows 2 and 3, so they are not logically equivalent

Showing Nonequivalence

$\sim(p \wedge q)$ and $\sim p \wedge \sim q$

Let p be statement "$0 < 1$" and
Let q be statement "$1 < 0$."

Then

$$\sim(p \wedge q) \quad \text{is} \quad \text{"It is not the case that both } 0 < 1 \text{ and } 1 < 0,\text{"}$$

which is true.

$$\sim p \wedge \sim q \quad \text{is} \quad \text{"} 0 \not< 1 \quad \text{and} \quad 1 \not< 0,\text{"}$$

which is false.

De Morgan's Laws

The negation of an *and* statement is logically equivalent to the *or* statement in which each component is negated.

The negation of an *or* statement is logically equivalent to the *and* statement in which each component is negated.

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

$$\sim(p \vee q) \equiv \sim p \wedge \sim q.$$

named after Augustus De Morgan, who was first to state them in formal mathematical terms.

Applying De Morgan’s Laws

Write negations for following statements:

John is 6 feet tall and he weighs at least 200 pounds.

The bus was late or Tom’s watch was slow.

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Tautologies and Contradictions

• Definition

A **tautology** is a statement form that is always true regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a tautology is a **tautological statement.**

A **contradication** is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a contradication is a **contradictory statement.**

the truth of a tautological statement (and the falsity of a contradictory statement) are due to logical structure of the statements themselves, and are independent of the meanings of the statements.

Logical Equivalence Involving Tautologies and Contradictions

If **t** is a tautology and **c** is a contradiction, show that $p \wedge \mathbf{t} \equiv p$ and $p \wedge \mathbf{c} \equiv \mathbf{c}$.

Solution:

p	$\mathbf{t}$	$p \wedge \mathbf{t}$	p	$\mathbf{c}$	$p \wedge \mathbf{c}$
T	T	T	T	F	F
F	T	F	F	F	F

same truth values, so $p \wedge \mathbf{t} \equiv p$

same truth values, so $p \wedge \mathbf{c} \equiv \mathbf{c}$

Summary of Logical Equivalences

Theorem 2.1.1 Logical Equivalences

Given any statement variables p, q, and r, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold.

1. *Commutative laws:*	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. *Associative laws:*	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. *Distributive laws:*	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. *Identity laws:*	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
5. *Negation laws:*	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
6. *Double negative law:*	$\sim(\sim p) \equiv p$	
7. *Idempotent laws:*	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. *Universal bound laws:*	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
9. *De Morgan's laws:*	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
10. *Absorption laws:*	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. *Negations of* **t** *and* **c**:	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Simplifying Statement Forms

Use Theorem 2.1.1 to verify the logical equivalence

$$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv p.$$

Exercise: Truth Table for $p \vee \sim q \rightarrow \sim p$

Construct a truth table for the statement form $p \vee \sim q \rightarrow \sim p$.

Solution:

By order of operations:

$p \vee \sim q \rightarrow \sim p$ is equivalent to

$(p \vee (\sim q)) \rightarrow (\sim p)$

Exercise: Division into Cases

Show that statement forms

$$p \vee q \rightarrow r$$

and

$$(p \rightarrow r) \wedge (q \rightarrow r).$$

are logically equivalent.

(Hint: draw truth table, and explain).

Solution

p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$	$p \vee q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

$p \vee q \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ always have the same truth values, so they are logically equivalent

$\sim p \vee q$ and $p \to q$

Rewrite the following statement in if-then form.

Either you get to work on time or you are fired.

Negate Conditional Statement

By definition, $p \to q$ is false if, and only if,

its hypothesis, p, is true and its conclusion, q, is false. It follows that

> The negation of "if p then q" is logically equivalent to "p and not q."

symbolically,

$$\sim(p \to q) \equiv p \wedge \sim q$$

Negate If-Then Statements

If my car is in the repair shop, then I cannot get to class.

Negation:

If Sara lives in Athens, then she lives in Greece.

Negation:

Contrapositive of a Conditional Statement

• Definition

The **contrapositive** of a conditional statement of the form "If p then q" is

$$\text{If } \sim q \text{ then } \sim p.$$

Symbolically,

$$\text{The contrapositive of } p \to q \text{ is } \sim q \to \sim p.$$

A conditional statement is logically equivalent to its contrapositive.

Writing the Contrapositive

If Howard can swim across the lake, then Howard can swim to the island.

Contrapositive:

If today is Easter, then tomorrow is Monday.

Contrapositive:

The Converse and Inverse of a Conditional Statement

• Definition

Suppose a conditional statement of the form "If p then q" is given.

1. The **converse** is "If q then p."
2. The **inverse** is "If $\sim p$ then $\sim q$."

Symbolically,

$$\text{The converse of } p \to q \text{ is } q \to p,$$

and

$$\text{The inverse of } p \to q \text{ is } \sim p \to \sim q.$$

Writing the Converse and the Inverse

If Howard can swim across the lake, then Howard can swim to the island.

Converse:

Inverse:

If today is Easter, then tomorrow is Monday.

Converse:

Inverse:

Logically equivalent or not?

conditional statement
$p \rightarrow q$

converse:
$q \rightarrow p$

inverse:
$\sim p \rightarrow \sim q$

contrapositive:
$\sim q \rightarrow \sim p$

Outline

- Review: (propositional) logic
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  - Rule of inferences
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Valid and Invalid Arguments

- In mathematics and logic an argument is not a dispute.

- **Argument:** is a sequence of statements ending in a conclusion.

  If Socrates is a man, then Socrates is mortal.
  Socrates is a man.
  ∴ Socrates is mortal.

- In logic, we focus on whether an argument is **valid** or not — i.e., whether the conclusion follows *necessarily* from the preceding statements.

Are they valid arguments?

- **Consider the following three arguments**

If Socrates is a man, then Socrates is mortal.
Socrates is a man.
∴ Socrates is mortal.

If Scoopy is a man, then Scoopy is mortal.
Scoopy is a mortal.
∴ Scoopy is a man.

If you see Tom, then you see Jerry.
You see Tom.
∴ You see Jerry.

- Observation: arguments (1) and (3) follows same logic!

Valid and Invalid Arguments

For example, the argument

If Socrates is a man, then Socrates is mortal.
Socrates is a man.
$\therefore$ Socrates is mortal.

has the abstract form

If p then q
p
$\therefore q$

An argument is called *valid* if, and only if, whenever all the premises are true (are satisfied), the conclusion is also true.

Valid and Invalid Arguments

• **Definition**

An **argument** is a sequence of statements, and an **argument form** is a sequence of statement forms. All statements in an argument and all statement forms in an argument form, except for the final one, are called **premises** (or **assumptions** or **hypotheses**). The final statement or statement form is called the **conclusion.** The symbol ∴, which is read "therefore," is normally placed just before the conclusion.

To say that an *argument form* is **valid** means that no matter what particular statements are substituted for the statement variables in its premises, if the resulting premises are all true, then the conclusion is also true. To say that an *argument* is **valid** means that its form is valid.

the truth of the premises.

Testing validity of argument form

1. Identify premises and conclusion of argument form.

2. Construct a truth table showing truth values of all premises and conclusion, under all possible truth values for variables.

3. A row of the truth table in which all the premises are true is called a **critical row**.

- If there is a critical row in which conclusion is false, then it is possible for an argument of the given form to have true premises and a false conclusion, and so the argument form is invalid.
- If conclusion in *every* critical row is true, then argument form is valid.

Example 1 – Determining Validity or Invalidity

$$p \rightarrow q \vee \sim r$$
$$q \rightarrow p \wedge r$$
$$\therefore \; p \rightarrow r$$

						premises		conclusion
p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \rightarrow q \vee \sim r$	$q \rightarrow p \wedge r$	$p \rightarrow r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

This row shows that an argument of this form can have true premises and a false conclusion. Hence this form of argument is invalid.

S

there is one situation (row 4) where the premises are true and the conclusion is false.

Modus Ponens

- An argument form consisting of two premises and a conclusion is called a **syllogism.**
- The first and second premises are called the **major premise** and **minor premise**, respectively.
- Most famous form of syllogism in logic is called **modus ponens** with following form:

If p then q.
p
$\therefore$ q

Modus Ponens

If p then q.
p
$\therefore$ q

It is instructive to prove that modus ponens is a valid form of argument, to confirm agreement between formal definition of validity and intuitive concept.

To do so, we construct a truth table for premises and conclusion.

		premises		conclusion	
p	q	$p \to q$	p	q	
T	T	T	T	T	← critical row
T	F	F	T		
F	T	T	F		
F	F	T	F		

Modus Tollens

consider another valid argument form called **modus tollens**. with following form:

If p then q.
$\sim q$
$\therefore \sim p$

Exercise:

Use modus ponens or modus tollens to fill in the blanks of the following arguments so that they become valid inferences.

a. If there are more pigeons than there are pigeonholes, then at least two pigeons roost in the same hole.
There are more pigeons than there are pigeonholes.
∴____.

b. If 870,232 is divisible by 6, then it is divisible by 3.
870,232 is not divisible by 3.
∴____.

Valid Argument Forms

- Commonly used valid argument forms (also called **rule of inference)**

Modus Ponens	$p \to q$ p • q	**Elimination**	**a.** $p \lor q$ $\sim q$ • p **b.** $p \lor q$ $\sim p$ • q
Modus Tollens	$p \to q$ $\sim q$ • $\sim p$	**Transitivity**	$p \to q$ $q \to r$ • $p \to r$
Generalization	**a.** p • $p \lor q$ **b.** q • $p \lor q$	**Proof by Division into Cases**	$p \lor q$ $p \to r$ $q \to r$ • r
Specialization	**a.** $p \land q$ • p **b.** $p \land q$ • q		
Conjunction	p q • $p \land q$	**Contradiction Rule**	$\sim p \to c$ • p

Rule of Generalization

a. p

$\therefore p \vee q$

b. q

$\therefore p \vee q$

- Used for making generalizations.
  - in a), if p is true, then, more generally, "p or q" is true for *any* other statement q.
- e.g. to count upperclassmen in a class. You find out Anton is junior, and reason:

Anton is a junior.

∴ (more generally) Anton is a junior or Anton is a senior.

Knowing that upperclassman means junior or senior, Anton is added into your list.

Rule of Specialization

following argument forms are valid:

a. $p \wedge q$

$\therefore\ p$

b. $p \wedge q$

$\therefore\ q$

- Used for specializing, discard extra info, to concentrate on particular property of interest.
- e.g., You are looking for someone knows graph algorithms.
- You discover that Ana knows both numerical analysis and graph algorithms, and reason:

Ana knows numerical analysis and Ana knows graph algorithms.

∴ (in particular) Ana knows graph algorithms.

Rule of Elimination

a. $p \vee q$

$\sim q$

$\therefore\ p$

b. $p \vee q$

$\sim p$

$\therefore\ q$

Idea: You have only two possibilities, and you can rule one out, then the other must be the case.

Elimination example

Example: suppose you know that for a particular number *x*,

$$x - 3 = 0 \quad \text{or} \quad x + 2 = 0.$$

If you also know that *x* is not negative, then $x \neq -2$, so

$$x + 2 \neq 0.$$

By elimination, you can then conclude that

x-3=0

Rule of Transitivity

$$p \to q$$
$$q \to r$$
$$\therefore p \to r$$

- Many arguments contain chains of if-then statements.
  - From the fact that one statement implies a second and the second implies a third, you can conclude that the first statement implies the third.

Example – Transitivity

Here is an example:

If 18,486 is divisible by 18, then 18,486 is divisible by 9.

If 18,486 is divisible by 9, then the sum of the digits of 18,486 is divisible by 9.

- If 18,486 is divisible by 18, then the sum of the digits of 18,486 is divisible by 9.

Proof by Division into Cases

following argument form is valid:

$$p \vee q$$

$$p \rightarrow r$$

$$q \rightarrow r$$

$$\therefore\ r$$

- It often happens that you know one thing or another is true. If you can show that in either case a certain conclusion follows, then this conclusion must also be true.

Example: Proof by Division into Cases

- For instance, suppose you know that *x* is a particular nonzero real number. The trichotomy property of the real numbers says that any number is positive, negative, or zero. Thus (by elimination) you know that *x* is positive or *x* is negative.

You can deduce that $x^2 > 0$ by arguing as follows:

x is positive or *x* is negative.

If *x* is positive, then $x^2 > 0$.

If *x* is negative, then $x^2 > 0$.

- $x^2 > 0$.

Contradiction Rule

If you can show that the supposition that statement p is false leads logically to a contradiction, then you can conclude that p is true.

$$\sim p \rightarrow c, \text{ where } c \text{ is a contradiction}$$
$$\therefore \quad p$$

- contradiction rule is logical heart of method of **proof by contradiction.**

- A slight variation also provides the basis for solving many logical puzzles by eliminating contradictory answers: *If an assumption leads to a contradiction, then that assumption must be false.*

Where are my glasses?

I know the following statements are true.

a. If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.

b. If my glasses are on the kitchen table, then I saw them at breakfast.

c. I did not see my glasses at breakfast.

d. I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.

e. If I was reading the newspaper in the living room then my glasses are on the coffee table.

Outline

- Review: (propositional) logic
- Logic Equivalence
- Arguments
  - Rule of inferences
  - Fallacy

Fallacies

- A **fallacy** is an error in reasoning that results in an invalid argument.
- Three common fallacies are
  - **using ambiguous premises,** and treating them as if they were unambiguous,
  - **circular reasoning** (assuming what is to be proved without having derived it from the premises),
  - **jumping to a conclusion** (without adequate grounds).

- two other fallacies (**seemingly** valid argument)
  - *converse error*
  - *inverse error*

Fallacies

For an argument to be valid, every argument of the same form whose premises are all true must have a true conclusion. It follows that for an argument to be invalid means that there is an argument of that form whose premises are all true and whose conclusion is false.

Converse Error

Show that following argument is invalid:

If Zeke is a cheater, then Zeke sits in the back row.
Zeke sits in the back row.
$\therefore$ Zeke is a cheater.

- The fallacy underlying this argument form is called **converse error,** because conclusion of the argument would follow from the premises if premise $p \to q$ were replaced by its converse.

- Such a replacement is not allowed, because a conditional statement is not logically equivalent to its converse.
- also known as ***fallacy of affirming the consequent*.**

Inverse Error

Consider following argument:

If interest rates are going up, then stock market prices will go down.

Interest rates are not going up.

∴*Stock market prices will not go down.*

the form of above argument is:

$$p \rightarrow q$$
$$\sim p$$
$$\therefore \sim q$$

- Such fallacy is called **inverse error: as it confuse** premise $p \rightarrow q$ *with* its inverse. But $p \rightarrow q$ *is not equivalent to* its inverse
- also known as the *fallacy of denying the antecedent*.

Argument being valid vs Conclusion being true

Is following argument valid? Is the conclusion true?

If John Lennon was a rock star, then John Lennon had red hair.

John Lennon was a rock star.

∴ John Lennon had red hair.

Is following argument valid? Is the conclusion true?

If New York is a big city, then New York has tall buildings.

New York has tall buildings.

∴New York is a big city.

Sound Argument

Argument is valid does not mean the conclusion is true.

as some premises might be false.

If John Lennon was a rock star, then John Lennon had red hair.

John Lennon was a rock star.

∴ John Lennon had red hair.

Valid not unsound!

An argument is called sound if and only if, it is valid and all its premises are true; otherwise, an argument is called unsound.

Valid Argument Forms

Modus Ponens	$p \to q$ p • q	**Elimination**	**a.** $p \lor q$ $\sim q$ • p **b.** $p \lor q$ $\sim p$ • q
Modus Tollens	$p \to q$ $\sim q$ • $\sim p$	**Transitivity**	$p \to q$ $q \to r$ • $p \to r$
Generalization	**a.** p • $p \lor q$ **b.** q • $p \lor q$	**Proof by Division into Cases**	$p \lor q$ $p \to r$ $q \to r$ • r
Specialization	**a.** $p \land q$ • p **b.** $p \land q$ • q		
Conjunction	p q • $p \land q$	**Contradiction Rule**	$\sim p \to c$ • p