6-3 Proving That a Quadrilateral Is a Parallelogram

What You'll Learn

- To determine whether a quadrilateral is a parallelogram

. . . And Why

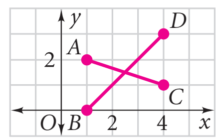
To use a parallel rule to plot a ship's course, as in Example 3

Check Skills You'll Need

GO for Help Lessons 1-8 and 3-7

Use the figure at the right.

1. Find the coordinates of the midpoints of $\overline{AC}$ and $\overline{BD}$. What is the relationship between $\overline{AC}$ and $\overline{BD}$?
2. Find the slopes of $\overline{BC}$ and $\overline{AD}$. How do they compare? **1–3. See back of book.**
3. Are $\overline{AB}$ and $\overline{DC}$ parallel? Explain.
4. What type of figure is $ABCD$? **parallelogram**



1 Is the Quadrilateral a Parallelogram?

Theorems 6-5 and 6-6 are converses of Theorems 6-1 and 6-2, respectively, from the previous lesson. They provide two ways to conclude that a quadrilateral is a parallelogram.

Key Concepts

Theorem 6-5

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

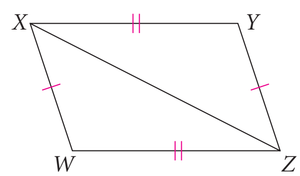
Real-World Connection

The frame remains a parallelogram as it is raised and lowered, and the backboard stays vertical.

Proof of Theorem 6-5

Given: $\overline{WX} \cong \overline{ZY}$ and $\overline{XY} \cong \overline{WZ}$

Prove: $WXYZ$ is a parallelogram.



Proof: Draw diagonal $\overline{XZ}$. Since opposite sides of $WXYZ$ are congruent, $\triangle WXZ \cong \triangle YZX$ by SSS. Using CPCTC, $\angle WXZ \cong \angle YZX$, so $\overline{WX} \parallel \overline{ZY}$. Also, $\angle WZX \cong \angle YXZ$, so $\overline{WZ} \parallel \overline{XY}$. $WXYZ$ is a parallelogram by definition.

You will complete a proof of Theorem 6-6 in Exercise 12.

Key Concepts

Theorem 6-6

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Differentiated Instruction Solutions for All Learners

Special Needs L1

Cut out the bottom of a cardboard box. Measure the opposite sides of the box to show they are congruent. Then change the angles between the adjacent edges of the box to demonstrate Theorem 6-8.

learning style: visual

Below Level L2

For Example 3, have students make a parallel ruler using straws and connectors such as brads.

learning style: tactile

6-3

1. Plan

Objectives

1 To determine whether a quadrilateral is a parallelogram

Examples

1 Finding Values for Parallelograms
2 Is the Quadrilateral a Parallelogram?
3 Real-World Connection

Professional Development

Math Background

The conditions necessary for a quadrilateral to be a parallelogram are also sufficient, as proved in this lesson. This allows using the biconditional *if and only if* to combine and catalogue the theorems and their converses in these lessons.

More Math Background: p. 304C

Lesson Planning and Resources

See p. 304E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need

For intervention, direct students to:

Finding the Midpoint
Lesson 1-8: Example 3
Extra Skills, Word Problems, Proof Practice, Ch. 1

Slope
Algebra Review, p. 165: Example 1

Checking for Parallel Lines
Lesson 3-7: Example 1
Extra Skills, Word Problems, Proof Practice, Ch. 3

2. Teach

Guided Instruction

Auditory Learners

Before students actually read the flow proof for Theorem 6-7, have them focus on the diagram of *ABCD* and suggest a Plan for Proof. Students who suggest the same basic ideas found in the proof will profit from the logical sequencing of their ideas in the proof.

Math Tip

Theorem 6-8 is the only theorem in this lesson that is not the converse of a theorem from Lesson 6-2.

1 EXAMPLE

After students find *x* and *y*, have them find the length of each segment to check their work. Ask: *How can you tell that your answers are correct?* **The diagonals bisect each other.**

Alternative Method

Encourage students to suggest other ways to prove Theorem 6-5. For example, instead of using CPCTC again, for $\angle WZX \cong \angle YXZ$ and $\overline{XY} \parallel \overline{WZ}$, use Theorem 6-8. Point out to students that two of the bonuses of learning geometry are becoming more creative mathematically and seeing many ways to prove relationships.

Theorem 6-7 is the converse of Theorem 6-3 of the previous lesson.

Key Concepts

Theorem 6-7

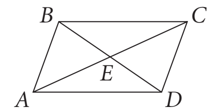
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Proof

Proof of Theorem 6-7

Given: $\overline{AC}$ and $\overline{BD}$ bisect each other at E.

Prove: $ABCD$ is a parallelogram.



$\overline{AC}$ and $\overline{BD}$ bisect each other at E. — Given

$\overline{AE} \cong \overline{CE}$, $\overline{BE} \cong \overline{DE}$ — Def. of seg. bisector

$\angle AEB \cong \angle CED$ — Vertical ∠s are ≅.

$\angle BEC \cong \angle DEA$ — Vertical ∠s are ≅.

$\triangle AEB \cong \triangle CED$ — SAS

$\triangle BEC \cong \triangle DEA$ — SAS

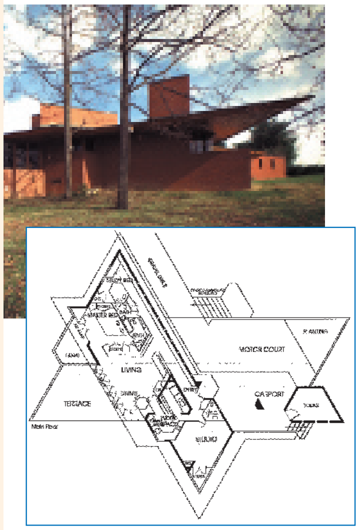
$\angle BAE \cong \angle DCE$ — CPCTC

$\angle ECB \cong \angle EAD$ — CPCTC

$\overline{AB} \parallel \overline{CD}$ — If alt. int. ∠s ≅, then lines are ∥.

$\overline{BC} \parallel \overline{AD}$ — If alt. int. ∠s ≅, then lines are ∥.

$ABCD$ is a parallelogram. — Def. of parallelogram



Real-World Connection

Frank Lloyd Wright, a famous architect, used parallelograms in designs of many houses, such as the Kraus House in Kirkwood, Missouri.

Theorem 6-8 suggests that if you keep two objects of the same length parallel, such as cross-country skis, then the quadrilateral determined by their endpoints must be a parallelogram. You will prove Theorem 6-8 in Exercise 13.

Key Concepts

Theorem 6-8

If one pair of opposite sides of a quadrilateral is both congruent and parallel, then the quadrilateral is a parallelogram.

You can use algebra and Theorems 6-7 and 6-8 to find values for which quadrilaterals are parallelograms.

Differentiated Instruction Solutions for All Learners

Advanced Learners L4

Have students find counterexamples if the word *both* is deleted from Theorems 6-5 and 6-6.

learning style: verbal

English Language Learners ELL

Have students make a table listing the five ways to prove two quadrilaterals congruent: four theorems and the definition of a parallelogram. Then discuss why *both* is such a critical term in some theorems.

learning style: visual

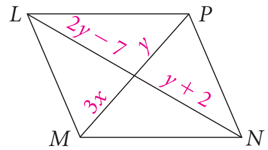
Test-Taking Tip

Read test questions carefully. Here the value of *y* is choice C, but you are asked for the value of *x*.

1 EXAMPLE Finding Values for Parallelograms

Multiple Choice For what value of x must $MLPN$ be a parallelogram?

Ⓐ 1 Ⓑ 3 Ⓒ 9 Ⓓ 27



Diagonals of a parallelogram bisect each other, so $2y - 7 = y + 2$ and $3x = y$.

$2y - 7 = y + 2$ **Write the equation with one variable.**

$y - 7 = 2$ **Collect the variables on one side.**

$y = 9$ **Solve.**

Substitute 9 for y in the second equation. Find x.

$3x = y$

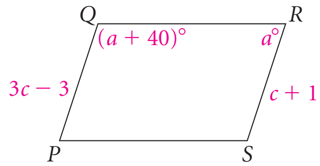
$3x = 9$ **Substitute 9 for *y*.**

$x = 3$ **Solve.**

For $x = 3$, $MLPN$ is a parallelogram. Answer B is correct.

Quick Check

1 Find the values of a and c for which $PQRS$ must be a parallelogram. **70, 2**



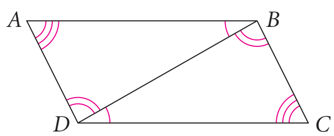
You can conclude that a quadrilateral is a parallelogram if both pairs of opposite sides are parallel. Theorems 6-5 through 6-8 provide four shortcuts to prove that a quadrilateral is a parallelogram.

2 EXAMPLE Is the Quadrilateral a Parallelogram?

Can you prove the quadrilateral is a parallelogram from what is given? Explain.

a. Given: $\angle ABD \cong \angle CDB$, $\angle BDA \cong \angle DBC$, $\angle A \cong \angle C$

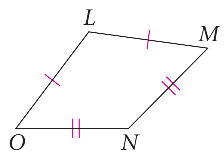
Prove: $ABCD$ is a parallelogram.



Yes, both pairs of opposite angles are congruent. $ABCD$ is a parallelogram by Theorem 6-6.

b. Given: $\overline{LM} \cong \overline{LO}$, $\overline{NM} \cong \overline{ON}$

Prove: $LMNO$ is a parallelogram.



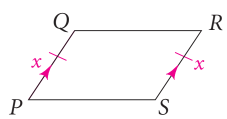
No, the given information is not enough to prove $LMNO$ is a parallelogram.

Quick Check

2 Can you prove the quadrilateral is a parallelogram? Explain. **a–b. See left.**

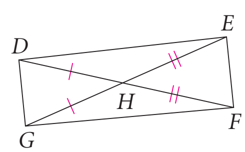
a. Given: $\overline{PQ} \cong \overline{SR}$, $\overline{PQ} \parallel \overline{SR}$

Prove: $PQRS$ is a parallelogram.

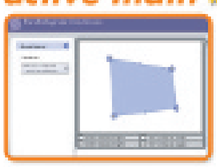


b. Given: $\overline{DH} \cong \overline{GH}$, $\overline{EH} \cong \overline{FH}$

Prove: $DEFG$ is a parallelogram.



Online active math



For: Parallelogram Activity
Use: Interactive Textbook, 6-3

2a. Yes; $\overline{PQ}$ and $\overline{SR}$ are congruent and parallel, so *PQRS* is a parallelogram.

2b. No; it is possible that the diagonals do not bisect each other, so *DEFG* would not be a parallelogram.

2 EXAMPLE Alternative Method

After students read the explanation for part a, ask: *What is another way you could prove the quadrilateral is a parallelogram?* **Two sets of alternate interior angles are congruent, so both pairs of opposite sides are parallel.**

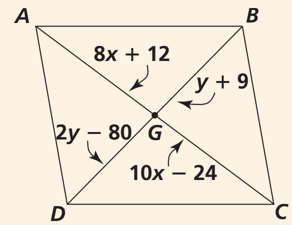
3 EXAMPLE Diversity

Many students are unfamiliar with navigation and plotting a ship's course. Ask for volunteers who can explain what a ship's compass looks like and how it is used.

PowerPoint

Additional Examples

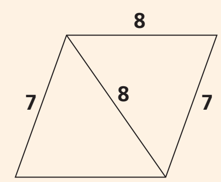
1 Find values of x and y for which $ABCD$ must be a parallelogram.



$x = 18$, $y = 89$

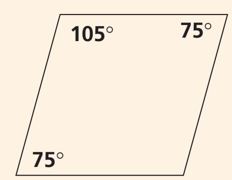
2 Determine whether the quadrilateral must be a parallelogram. Explain.

a.



No; you do not know whether both pairs of opposite sides are congruent.

b.



Yes; both pairs of opposite angles are congruent.

PowerPoint

Additional Examples

3 The captain of a fishing boat plots a course toward a school of bluefish. One side of a parallel rule connects the boat with the school of bluefish. The other side makes a 36° angle north of due east on the chart's compass. Explain how the captain knows in which direction to sail to reach the bluefish. **Because the parallel rule forms a parallelogram, the captain should sail 36° north of due east.**

Resources

- Daily Notetaking Guide 6-3 L3
- Daily Notetaking Guide 6-3—Adapted Instruction L1

Closure

Using the theorems you have learned in Chapter 6, write two different biconditionals about parallelograms. **Sample: A quadrilateral is a parallelogram if and only if both pairs of opposite angles are congruent. A quadrilateral is a parallelogram if and only if its diagonals bisect each other.**

Quick Check

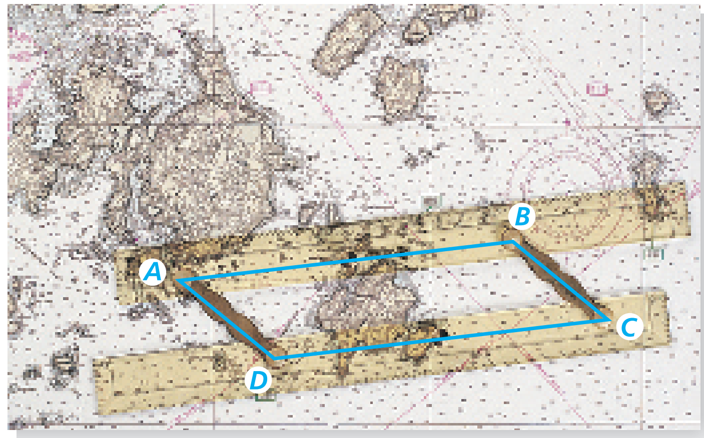
3. **Once in place, both rulers show the direction and remain ∥. Keep the second ruler in place and move the first ruler to get the compass reading.**

3 EXAMPLE — Real-World Connection

Real-World Connection

Careers A marine navigator has great responsibility for the ship, its crew, its cargo, its mission, and the surrounding natural marine environment.

Navigation A parallel rule is a navigation tool that is used to plot ship routes on charts. It is made of two rulers connected with congruent crossbars, such that $AB = DC$ and $AD = BC$. You place one ruler on the line connecting the ship's present position to its destination. Then you move the other ruler onto the chart's compass to find the direction of the route. Explain why this instrument works.



The crossbars and the sections of the rulers are congruent no matter how they are positioned. So, $ABCD$ is always a parallelogram. Since $ABCD$ is a parallelogram, the rulers are parallel. Therefore, the direction the ship should travel is the same as the direction shown on the chart's compass.

Quick Check **3** **Critical Thinking** Suppose the ruler connecting the ship's position to its destination gets in the way of reading the compass. How can you get the desired reading? **See margin.**

EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

Practice and Problem Solving

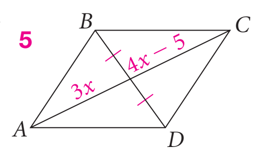
A Practice by Example

Example 1 (page 323)

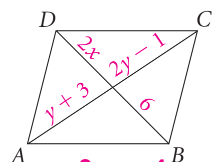
GO for Help

Algebra **Find the values of x and y for which $ABCD$ must be a parallelogram.**

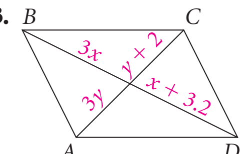
1. **5**



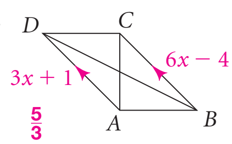
2. **$x = 3, y = 4$**



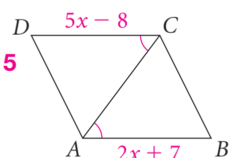
3. **$x = 1.6, y = 1$**



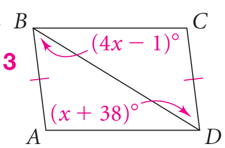
4. **$\frac{5}{3}$**



5. **5**



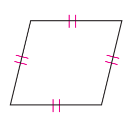
6. **13**



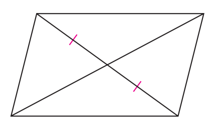
Example 2 (page 323)

Determine whether the quadrilateral must be a parallelogram. Explain. **7–9. See margin, pp. 324–325.**

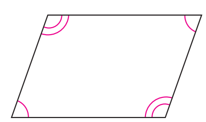
7.



8.



9.



7. **Yes; both pairs of opp. sides are ≅.**
8. **No; the quad. could be a kite.**
9. **Yes; both pairs of opp. ∠s are ≅.**

Example 3
(page 324)

10. **Fishing** Quadrilaterals are formed on on the side of this fishing tackle box by the adjustable shelves and connecting pieces. Explain why the quadrilaterals remain parallelograms no matter what position the shelves are in. **See margin.**

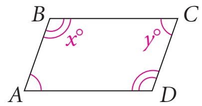
B **Apply Your Skills**

11. Combine each of Theorems 6-1, 6-2, and 6-3 with Theorems 6-5, 6-6, and 6-7, respectively, into biconditional statements. **See back of book.**

12. **Developing Proof** Complete the proof of Theorem 6-6.

Given: $\angle A \cong \angle C$ and $\angle B \cong \angle D$

Prove: $ABCD$ is a parallelogram.



Statements	Reasons
1. $x + y + x + y = 360$	1. The sum of the measures of the angles of a quadrilateral = 360.
2. $2(x + y) = 360$	a. __?__ **Distr. Prop.**
3. $x + y = 180$	b. __?__ **Div. Prop. of Eq.**
4. $\angle A$ and $\angle B$ are supplementary. $\angle A$ and $\angle D$ are supplementary.	4. Definition of supplementary
c. __?__ ∥ __?__, __?__ ∥ __?__ **See below.**	d. __?__ **If same-side int. ∠s are supp., the lines are ∥.**
6. $ABCD$ is a parallelogram.	e. __?__ **Def. of ▱**

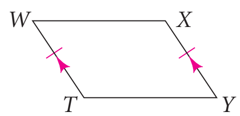
c. $\overline{AD} \parallel \overline{BC}$, $\overline{AB} \parallel \overline{DC}$

Proof **13.** Prove Theorem 6-8.

Given: $\overline{TW} \parallel \overline{YX}$ and $\overline{TW} \cong \overline{YX}$

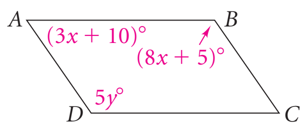
Prove: $TWXY$ is a parallelogram.

(*Hint*: Draw one or both diagonals. Find congruent triangles. Use CPCTC.)
See back of book.



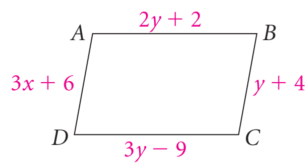
Algebra **Find the values of the variables for which *ABCD* must be a parallelogram.**

14. GPS



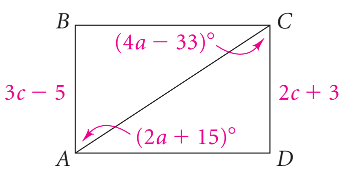
x = 15, y = 25

15.



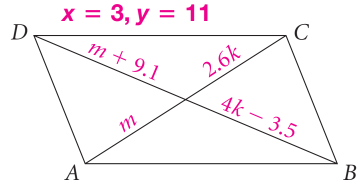
x = 3, y = 11

16.



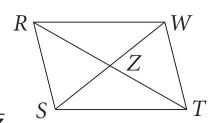
c = 8, a = 24

17.



k = 9, m = 23.4

18. **Multiple Choice** From which given information can you conclude that $RSTW$ is a parallelogram? **D**



Ⓐ $\overline{RS} \parallel \overline{WT}$, $\overline{RS} \cong \overline{ST}$
Ⓑ $\overline{RS} \parallel \overline{WT}$, $\overline{ST} \cong \overline{RW}$
Ⓒ $\overline{RS} \cong \overline{ST}$, $\overline{RW} \cong \overline{WT}$
Ⓓ $\overline{RZ} \cong \overline{TZ}$, $\overline{SZ} \cong \overline{WZ}$

19. **Open-Ended** Sketch two noncongruent parallelograms $ABCD$ and $EFGH$ such that $\overline{AC} \cong \overline{EG}$ and $\overline{BD} \cong \overline{FH}$. **See margin.**

GO Online Homework Help
Visit: PHSchool.com
Web Code: aue-0603

3. Practice

Assignment Guide

1 A B	1-25
C Challenge	26-28
Test Prep	29-32
Mixed Review	33-39

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 3, 7, 16, 22, 27.

Exercises 1–6 Have students identify the theorems they use to establish that the quadrilateral is a parallelogram.

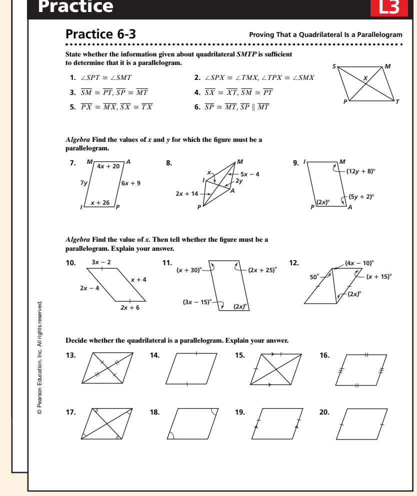
Error Prevention!

Exercise 7 Remind students that parallelograms with more precise names are still parallelograms. Ask: *Are both pairs of opposite sides congruent?* **yes** *What figure has this description?* **parallelogram**

Exercise 18 Have students copy the figure and write the givens for each answer choice to see if the given information is sufficient.

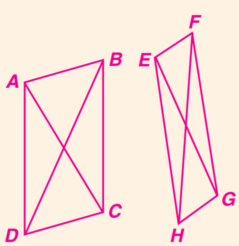
Differentiated Instruction Resources

GPS Guided Problem Solving	L3
Enrichment	L4
Reteaching	L2
Adapted Practice	L1
Practice	L3



10. **It remains a ▱ because the shelves and connecting pieces remain ∥.**

19. **Answers may vary. Sample:**



4. Assess & Reteach

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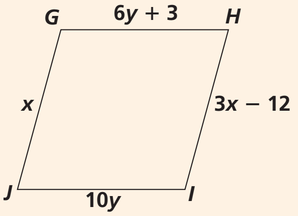
Lesson Quiz

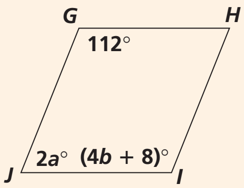
Find the values of the variables for which *GHIJ* must be a parallelogram.

1.



$x = 6, y = 0.75$

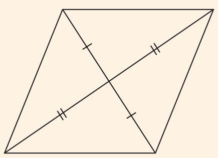
2.



$a = 34, b = 26$

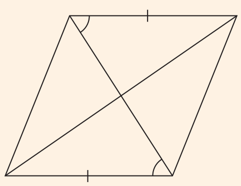
Determine whether the quadrilateral must be a parallelogram. Explain.

3.



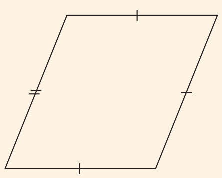
Yes; the diagonals bisect each other.

4.



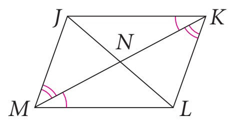
Yes; one pair of opposite sides is both congruent and parallel.

5.

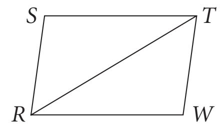


No; both pairs of opposite sides are not congruent.

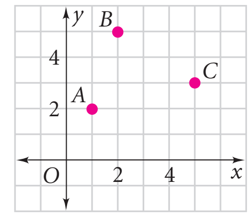
Proof **20. Given:** $\angle JKN \cong \angle LMN$
$\angle LKN \cong \angle JMN$
Prove: $JKLM$ is a parallelogram.



21. Given: $\triangle TRS \cong \triangle RTW$
Prove: $RSTW$ is a parallelogram.
20–21. See back of book.



Coordinate Geometry **Given points A, B, and C in the coordinate plane as shown, find the fourth point described below.**



22. point D so that $ABCD$ is a parallelogram **(4, 0)**

23. point E so that $ABEC$ is a parallelogram **(6, 6)**

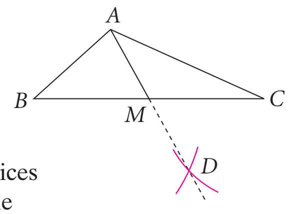
24. point F so that $AFBC$ is a parallelogram **(−2, 4)**

25. Writing Summarize the ways to show that a quadrilateral is a parallelogram. **See margin.**

Challenge

26. Probability If two opposite angles of a quadrilateral measure 120 and the measures of the other angles are multiples of 10, what is the probability that the quadrilateral is a parallelogram? $\frac{1}{6}$

Proof **27.** In the figure at the right, point D is constructed by drawing two arcs. One has center C and radius AB. The other has center B and radius AC. Prove that $\overline{AM}$ is a median of $\triangle ABC$. **See margin.**



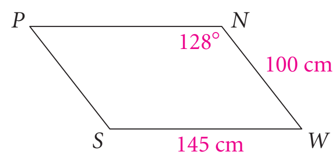
28. Coordinate Geometry The diagonals of quadrilateral $EFGH$ intersect at $D(-1, 4)$. Two vertices of $EFGH$ are $E(2, 7)$ and $F(-3, 5)$. What must be the coordinates of G and H to ensure that $EFGH$ is a parallelogram?
$G(-4, 1)$, $H(1, 3)$

Test Prep

Multiple Choice

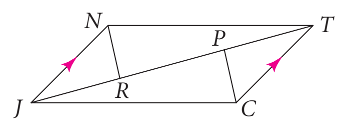
29. In ▱$PNWS$, what is $m\angle W$? **C**
A. 128 **B.** 90 **C.** 52 **D.** 26

30. In ▱$PNWS$, what is $m\angle S$? **F**
F. 128 **G.** 90 **H.** 52 **J.** 26



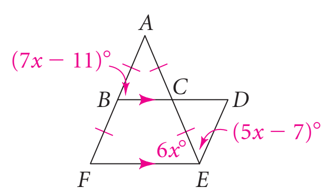
Short Response

31. Given: $\triangle NRJ \cong \triangle CPT$, $\overline{JN} \parallel \overline{CT}$
Prove: $JNTC$ is a parallelogram.
See margin, p. 327.



Extended Response

32. a. Write an equation and solve for x.
b. Is $\overline{AF} \parallel \overline{DE}$? Explain.
c. Is $BDEF$ a parallelogram? Explain.
See back of book.



25. You can show a quad. is a ▱ if both pairs of Opp. sides are ∥ or ≅, if both pairs of opp. ∠s are ≅, if diagonals bisect each other, if all consecutive ∠s are suppl., or if one pair of opp. sides are both ∥ and ≅.

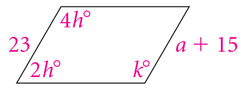
27. Answers may vary. Sample:
1. $\overline{AB} \cong \overline{CD}$, $\overline{AC} \cong \overline{BD}$ (Given)
2. $ACDB$ is a ▱. (If opp. sides are ≅, then it is a ▱.)
3. M is the midpoint of $\overline{BC}$. (The diagonals of a ▱ bisect each other.)
4. $\overline{AM}$ is a median. (Def. of a median)

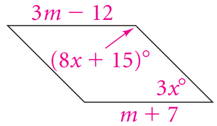
31. [2] Statements Reasons
1. $\triangle NRJ \cong \triangle CPT$ (Given)
2. $\overline{NJ} \cong \overline{CT}$ (CPCTC)
3. $\overline{NJ} \parallel \overline{TC}$ (Given)
4. $JNTC$ is a ▱. (If opp. sides of a quad. are both ∥ and

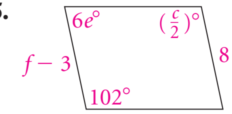
Mixed Review

GO for Help

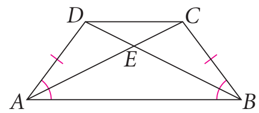
Lesson 6-2 Algebra **Find the value of each variable in each parallelogram.** **33–35. See margin.**

33.


34.


35.


Lesson 4-7 **36.** Explain how you can use overlapping congruent triangles to prove $\overline{AC} \cong \overline{BD}$. **See margin.**



Lesson 2-2 **Write the two conditional statements that make up each biconditional.**

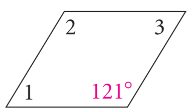
37. The diagonals of a quadrilateral bisect each other if and only if the quadrilateral is a parallelogram. **See margin.**

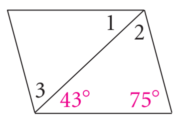
38. Two lines are parallel if and only if the two lines and a transversal form corresponding angles that are congruent. **See margin.**

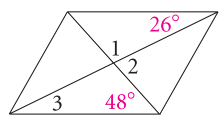
39. Two nonvertical lines are perpendicular if and only if the product of their slopes is -1. **See margin.**

Checkpoint Quiz 1 — Lessons 6-1 through 6-3

Find the measures of the numbered angles for each parallelogram. **1–3. See left.**

1.


2.


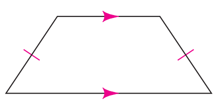
3.


1. $m\angle 1 = 59$, $m\angle 2 = 121$, $m\angle 3 = 59$

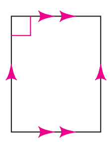
2. $m\angle 1 = 43$, $m\angle 2 = 62$, $m\angle 3 = 62$

3. $m\angle 1 = 106$, $m\angle 2 = 74$, $m\angle 3 = 26$

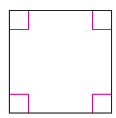
Classify each quadrilateral in as many ways as possible.

4.


trapezoid, isosc. trapezoid

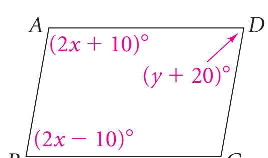
5.


rectangle, ▱

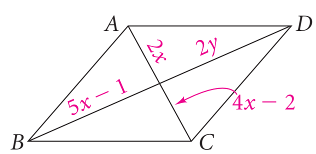
6.


rectangle, ▱

Algebra **Find the values of the variables for which *ABCD* is a parallelogram.**

7.


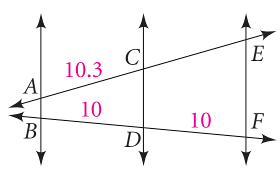
$x = 45$, $y = 60$

8.


$x = 1$, $y = 2$

9. In the figure at the right, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$. Find AE. **20.6**

10. What is the most precise name for a quadrilateral with vertices (3, 5), (−1, 4), (3, −5), and (7, 4)? **kite**



Alternative Assessment

Give pairs of students a set of two straws of unequal lengths with which to construct the diagonals of a parallelogram. Have them use the theorems in this lesson to explain why the construction works.

Test Prep

Resources
For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 361
- Test-Taking Strategies, p. 356
- Test-Taking Strategies with Transparencies

Checkpoint Quiz

Use this Checkpoint Quiz to check students' understanding of the skills and concepts of Lessons 6-1 through 6-3.

Resources
Grab & Go

- Checkpoint Quiz 1

≅, then the quad. is a ▱.)
[1] proof missing steps

33. $a = 8$, $h = 30$, $k = 120$

34. $m = 9.5$, $x = 15$

35. $e = 13$, $f = 11$, $c = 204$

36. It is given that $\overline{AD} \cong \overline{BC}$ and $\angle DAB \cong \angle CBA$. By the Reflexive Prop. of ≅ $\overline{AB} \cong \overline{AB}$, thus $\triangle DAB \cong \triangle CBA$ by SAS, so $\overline{AC} \cong \overline{BD}$ by CPCTC.

37. If a quad. is a ▱, then the diagonals bisect each other; if the diagonals of a quad. bisect each other, then it is a ▱.

38. If two lines and a transversal form ≅ corr. ∠s, then the two lines are ∥; if two lines are ∥, then a transversal forms ≅ corr. ∠s.

39. If the prod. of the slopes of two nonvertical lines is −1, then they are ⊥; if two nonvertical lines are ⊥, then the prod. of their slopes is −1.