



# PSAT: Math Test

## Objectives:

1. To understand the PSAT test format and scoring scheme
2. To practice the two types of PSAT questions



# Towers of Hanoi

In this ancient game, there are three pegs with a number of rings on the first peg arranged by size.





# Towers of Hanoi

The point of the game is to move all of the rings from the first peg to the last peg, but you can only place a smaller ring on top of a larger ring.





# Towers of Hanoi

You cannot place a large ring on top of a small ring. Oh, and you want to be able to do this with the least number of moves possible.



Click on the baby to play.



# Towers of Hanoi

The point here is that there are many ways to move the disks from one peg to another, but there is often a faster, more efficient way.



This is also the case with many PSAT questions.



# About the PSAT



- Originally meant “Pre-Scholastic Aptitude Test”
- Designed to measure verbal and mathematical reasoning
  - Measures ability, not knowledge (yeah, right!)
  - Gives students a chance to practice SAT-type questions and qualify for NMS



# About the PSAT

- 2 hour, 10 minute test in 5 sections
- The math portion presents questions grounded in high school math but asks them in unconventional ways.
- Test scores range from 20 – 80 on each section. Math section Averages:
  - A score of 48.9 is average for a Junior (2010)
  - A score of 44 is average for 10<sup>th</sup> (2010)
- To convert to a SAT score, add a zero



# A Junior Year Test

- PSAT is considered a Junior year test
  - You may not recognize all of the mathematics on the test!
  - No Algebra II questions (not until the SAT)





# What's on the Test?

- **Critical Reading**: two 25 minute sections (48 MC questions)
- **Mathematics**: two 25 minute sections (38 questions)
- **Writing**: one 30 minute section (39 MC questions)
- Questions are arranged in increasing difficulty in each section.



# Math Section Decomposition

Two math sections:

1. Multiple-choice: 20
2. Multiple-choice: 8  
Grid-in: 10

Tests includes number operations, algebra and functions, geometry and measurement, statistics and probability.



# Scoring

Description	Score
Correct answers (regardless of difficulty)	+1
Skipped questions	0
Incorrect multiple-choice	-1/4
Incorrect grid-in	0



## Two Facts

What does the previous scoring guide and the fact that the questions are listed in increasing difficulty tell you about test-taking strategy?

- Don't waste time on the hard questions you can't answer.
- It doesn't pay to guess unless you can eliminate at least one answer choice.



# Guessing?

What is the probability that you would choose the correct answer to a 5-choice multiple-choice question?  **$1/5$**

So, if you randomly guessed on 5 questions, you would expect to get 1 right ( **$+1$  point**) and 4 wrong ( **$-4/4 = -1$  point**).

Thus, you'd have a net gain of **0 points**.



## Guessing? Part 2

What is the probability that you would choose the correct answer to a 5-choice multiple-choice question, if you eliminated 1 choice?  **$1/4$**

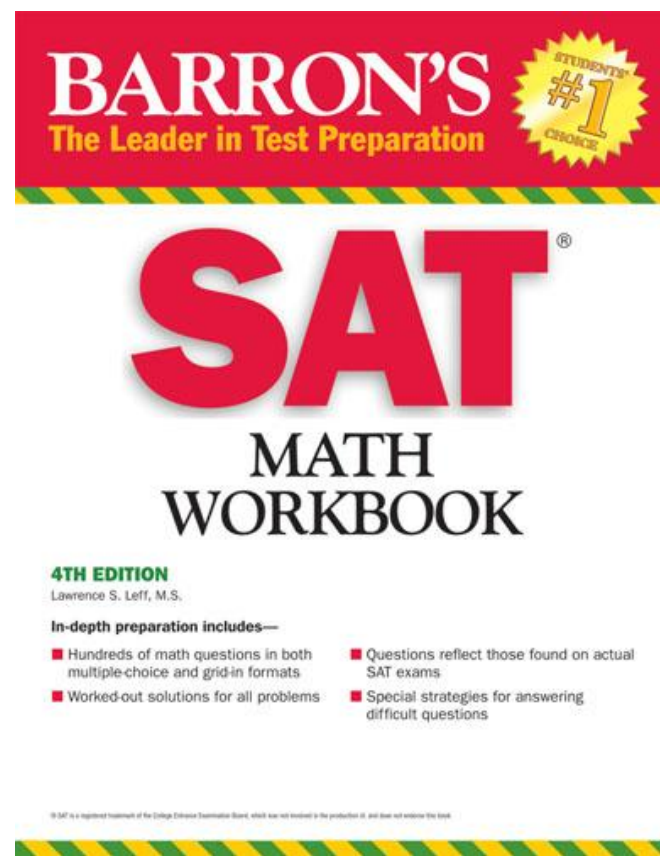
So, if you eliminated one answer choice on 4 questions, you would expect to get 1 right ( **$+1$  point**) and 3 wrong ( **$-3/4$  point**).

Thus, you'd have a net gain of  **$1/4$  point**.



# Math Strategies

Sixteen of these 17 PSAT math strategies were adapted from [Barron's Math Workbook for the New SAT](#) by Lawrence S. Leff, which I highly recommend (click the link for product info from Amazon.com). The 17th strategy comes from John Katzman of Princeton Review.





# Math Strategies: I

Use the test booklet as scratch paper

Points  $H$ ,  $J$ ,  $K$ , and  $M$  lie on the same line, in that order. If  $HK = 8$ ,  $JM = 11$ , and  $JK = 5$ , what is the ratio of  $HJ$  to  $HM$ ?

- This is an obvious one: sketch a picture and work out the problem in the booklet





## Math Strategies: 2

Find the quantity the question asks for

If  $(5 - 3)(x - 1) = 4$ , then  $x^2 =$

- Many PSAT questions will not simply ask you to solve for  $x$ . They go a step further and ask for  $x^2$  or  $2x - 3$ .
  - Notice that if you solved for  $x$ , you'd get 3, which is one of the answer choices



## Math Strategies: 3

*Plug in numbers to find a pattern*

When a positive integer  $k$  is divided by 5, the remainder is 3. What is the remainder when  $3k$  is divided by 5?

- Just find a couple of numbers that when you divide it by 5, you get a remainder of 3. Then multiply them by 3 and divide by 5. What's the remainder each time?



# Math Strategies: 4

Choose a convenient starting value when none is given

The current value of a stock is 20% less than its value when it was purchased. By what percent must the current value of the stock rise to have its original value?

- If a problem gives you no numbers to work with, just choose a convenient one.
  - Percents: Start with 100
  - Fractions: Start with the common denominator



# Math Strategies: 5

## Combine or multiply systems of equations

If  $8a - b^2 = 24$  and  $8b + b^2 = 56$ , then  $a + b =$

- Rather than give you a simple linear system of equations, PSAT will give you strange systems with exponents and fractions. Usually you can solve these with a clever substitution or by simply adding or subtracting your equations.
  - Watch what happens when you add the above equations



# Math Strategies: 6

Account for all solutions of a higher-degree equation

If  $x^2 = 4x$ , then  $x =$

- The highest power in an equation tells you how many solutions you should have.
  - So the above equation should have 2 answers, not one.



# Math Strategies: 7

*Make organized lists and look for patterns*

What is the units digit of  $3^{35}$ ?

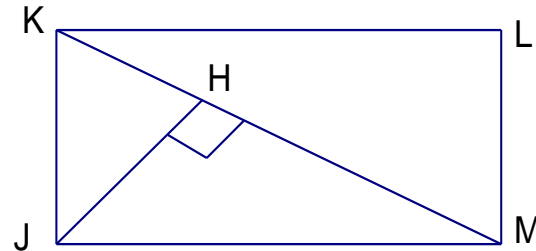
- If you try to put this into a calculator, it will give you a number that's so big, it has to convert it to scientific notation: None of the digits on the screen is the unit digit.
  - Make a list of powers of 3, starting with  $3^1$ , and watch as a pattern emerges.



# Math Strategies: 8

## Redraw figures to scale

In rectangle  $JKLM$  above, if  $JK = KL$ , what is the ratio of  $JH$  to  $KM$ ?



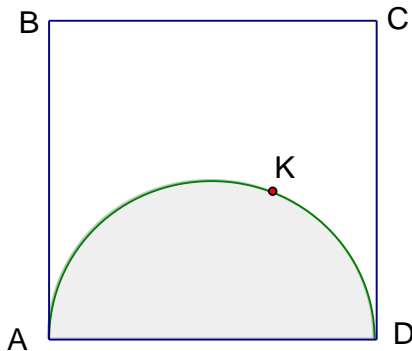
Note: Figure is not drawn to scale.

- Pictures on the PSAT are not usually drawn to scale, so redraw them according to the information in the problem.



# Math Strategies: 9

Subtract areas to find the area of an [un]shaded region



In the figure shown, what is the area of the [un]shaded region if  $ABCD$  is a square and  $AKD$  is a semicircle whose radius is 4?

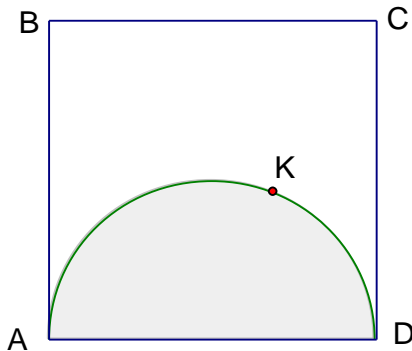
- PSAT will often have question where you have to find the area of some oddly shaped region. Just find the area of the larger figure, and then subtract the area of some other part you can easily find.





# Math Strategies: 9

Subtract areas to find the area of an [un]shaded region



In the figure shown, what is the area of the [un]shaded region if  $ABCD$  is a square and  $AKD$  is a semicircle whose radius is 4?

- Note that the actual tip should be to subtract to find the area of a shaded region. It's just the program I used to draw the figure above could not easily shade the correct region, so I just changed the problem.



# Math Strategies: IO

Test numerical answer choices in the question

If  $2^{2x-1} = 32$ , then  $x =$

- If you run into an equation you can't easily solve, just test out the answers they give you.
  - Notice that the answers are always in increasing or decreasing order.



# Math Strategies: II

## Change variable answer choices into numbers

If  $m + n$  is an odd number when  $m$  and  $n$  are positive integers, which expression always represents an even number?

- If you can't deal with the variables as given in a problem, choose numbers that make sense for the problem. Then test out each answer choice with your numbers.



# Math Strategies: 12

## Write an algebraic equation

Ticket sales receipts for a music concert totaled \$2,160. Three times as many tickets were sold for the Saturday night concert as was sold for the Sunday afternoon concert. Two times as many tickets were sold for the Friday night concert as was sold for the Sunday afternoon concert. Tickets for all three concerts sold for \$2.00 each. Find the number of tickets sold for the Saturday night concert.

- You can get by lots of PSAT questions without having to write your own algebraic equations. Sometimes, however, a problem is best solve by just defining a variable or two and writing an equation to solve.



# Math Strategies: 13

## Look at a specific case

The perimeter of a rectangle is 10 times as great as the width of the rectangle. The length of the rectangle is how many times as great as the width of the rectangle?

- This is like choosing a convenient starting value. If a question is framed in a general nature, try picking a specific case to work with. (Use inductive reasoning!)



# Math Strategies: 14

## Draw a diagram

Amy goes shopping and spends one-third of her money on a new dress. She then goes to another store and spends one-half of the money she has left on shoes. If Amy has \$56 left after these two purchases, how much money did she have when she started shopping?

- Since this question involves different fractions, draw yourself a simple diagram that breaks the money up into thirds.



# Math Strategies: 15

## Work backwards

Sara's telephone service cost \$21 per month plus \$0.25 for each local call, and long-distance calls are extra. Last month, Sara's bill was \$36.64, and it included 6.14 in long-distance charges. How many local calls did she make?

- If a question is too convoluted to work forwards, just work backwards.
  - Start with the final bill about and start subtracting off all her fees.



# Math Strategies: 16

Adopt a different point of view

A jar contains 110 marbles of which 50 are red and 60 are green. The probability of picking a red marble from the jar without looking is, therefore,  $\frac{50}{110}$ . How many red marbles must be added to the jar so that the probability of picking a red marble will be  $\frac{1}{2}$ ?





# Math Strategies: 17

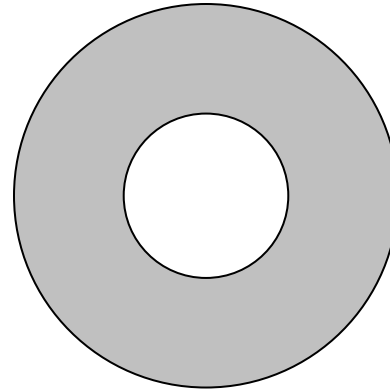
## *The Joe Bloggs Technique*

Joe Bloggs is the average test taker. When answering the easy questions, if he comes by an answer easily, it is more than likely right. By the time he gets to the last question, which is supposedly the hardest, he knows that if he is able to get the answer easily, that answer is probably wrong. So he marks out the “easy” answer and chooses something else. For more information on this crazy technique, read the “Joe Bloggs Eats Doughnuts” worksheet.



# Math Strategies: 17

## The Joe Bloggs Technique

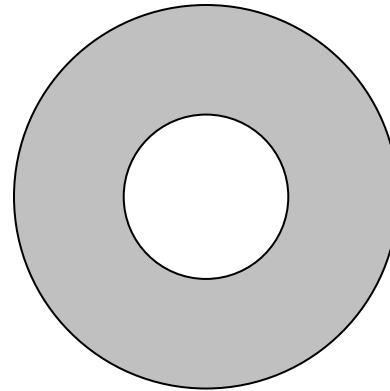


In the figure above, what is the greatest number of nonoverlapping regions into which the shaded region can be divided with exactly two straight lines?



# Math Strategies: 17

## *The Joe Bloggs Technique*



Since it would be really easy to just draw 2 intersecting lines that divide this into 4 regions, that answer must be wrong because it's too easy, and this is the last question on the test!



# Assignment

- PSAT sample questions and math strategies