

PSI AP Physics I

Rotational Motion Chapter Questions

1. What property of real matter leads to the need to analyze rotational motion?
2. What is the axis of rotation? Does the axis of rotation of a rotating tire on a car touch the rubber in the tire?
3. Explain why the radian is a more physically natural unit than the degree when working rotation problems.
4. A small bug is on a spinning record near the center of the record. His friend is on the outside edge of the record. As the record rotates, compare the linear displacement (arc length, s) to the angular displacement of the two bugs.
5. For a rotating disc, what are the two types of linear acceleration? How do their magnitudes depend on how far an object on the disc is from the center of the disc?
6. You are stepping on a merry-go-round with two rings of fiberglass horses – an inner ring and an outer ring. You get motion sick very easily. Should you choose a horse in the inner or outer ring to ride? Why?
7. What assumption about angular acceleration is made in deriving the angular kinematics equations? What is a good way to write the rotational kinematics equations if you know the linear kinematics equations? Is it necessary to know what causes the object to move if you want to use the rotational kinematics equations?
8. What is the rotational analog to Force? A mass subjected to a constant force will move with a constant acceleration. If that same force is applied on a wrench gripping a nut, it will cause an angular acceleration of the nut. How can the angular acceleration of the nut be changed while still applying the same force?
9. When you open a door, why do you push as far away from the door hinges (axis of rotation) and as perpendicular to the surface as you can? When you want to keep a door open, you place a door stop between the bottom of the door and floor. Where do you place the door stop (in terms of its distance from the door hinges) and why?
10. Torque is expressed in Newton meters. Energy is expressed in Joules. These units are mathematically equivalent. So, why does torque never use Joules as a unit?
11. If you have a flat tire, and you're using a wrench to loosen the nuts that hold the tire rim to the axle, is it advantageous to have a longer wrench or a shorter one? Why?
12. Explain the importance of locating the fulcrum when you are using a metal bar to lift a heavy rock. Should the fulcrum be closer to the heavy rock or to your hands where you are pushing down on the bar?
13. There are two equal mass objects with the same radius; one is a solid cylinder, and the other is a hollow cylinder. Explain, without using an equation, which one has a greater moment of inertia and why.
14. A solid cylinder and a hollow cylinder of equal mass and radius are at rest at the top of an inclined plane. They are released simultaneously, and roll down the plane without slipping. Without using an equation, explain which object reaches the bottom of the incline first, and why.
15. When an object undergoes rotational and translational motion, what does the phrase "rotate without slipping mean?" What relationship between ω and v can be used in this case?
16. An ice skater is spinning very fast with her arms tucked into her side. She wants to slow her rate of rotation. Without digging her skates into the ice (whereby the increased friction between her skates and the ice would apply an external torque to her), how can she change her rotation rate?

Chapter Problems

I. Axis of Rotation and Angular Properties

Classwork

1. How many radians are subtended by a 0.10 m arc of a circle of radius 0.40 m?
2. How many degrees are subtended by a 0.10 m arc of a circle of radius of 0.40 m?
3. A ball rotates 2π rad in 4.0 s. What is its angular velocity?
4. A toy car rolls in a circular path of radius 0.25 m and the car wheels rotate an arc length of 0.75 m in 8.0 s. What is the angular velocity of the wheels?
5. A mouse is running around in a circle. It starts from rest and accelerates to $\omega = 12$ rad/s in 0.90 s. What is the mouse's angular acceleration?
6. A bicycle tire moving at $\omega = 3.5$ rad/s accelerates at a constant rate to 4.2 rad/s in 3.0 s. What is its angular acceleration?
7. A bicycle with tires of radius equal to 0.30 m is moving at a constant angular velocity of 2.5 rad/s. What is the linear velocity of the bicycle?
8. A motorbike has tires of radius 0.25 m. If the motorbike is traveling at 4.0 m/s, what is the angular velocity of the tires?
9. An elementary school student pushes, with a constant force, a merry-go-round of radius 4.0 m from rest to an angular velocity of 0.80 rad/s in 1.5 s.
 - a) What is the angular acceleration, α ?
 - b) What is the merry-go-round's tangential acceleration at $r = 4.0$ m?
 - c) What is the merry-go-round's radial (centripetal) acceleration at $r = 4.0$ m?
 - d) What is the merry-go-round's total linear acceleration at $r = 4.0$ m?
 - e) What is the frequency of rotation (f) of the merry-go-round at $t = 1.5$ s?

Homework

10. A bicycle tire of radius 0.33 m moves forward, without slipping, a distance of 0.27 m. How many radians does the wheel rotate?
11. A bicycle tire of radius 0.33 m moves forward, without slipping, a distance of 0.27 m. How many degrees does the wheel rotate?
12. A flat disc rotates 1.5π rad in 12 s. What is its angular velocity?
13. An electric train rides a circular track of radius 0.39 m and its wheels rotate an arc length (linear distance) of 0.96 m in 4.0 s. What is the angular velocity of the wheels?

14. A boy pulls a wagon in a circle. The wagon starts from rest and accelerates to an angular velocity of 0.25 rad/s in 1.2 s . What is the wagon's angular acceleration?
15. A car tire rotating with an angular velocity of 42 rad/s accelerates at a constant rate to 51 rad/s in 3.0 s . What is its angular acceleration?
16. A bicycle has wheels with a radius of 0.25 m . The wheels are moving at a constant angular velocity of 3.1 rad/s . What is the linear velocity of the bicycle?
17. A motorbike has tires of radius 0.25 m . If the motorbike is traveling at 1.7 m/s , what is the angular velocity of the tires (assume the tires are rotating without slipping)?
18. A cyclist accelerates uniformly from rest to a linear velocity of 1.0 m/s in 0.75 s . The bicycle tires have a radius of 0.38 m .
 - a) What is the angular velocity of the each bicycle tire at $t = 0.75 \text{ s}$?
 - b) What is the angular acceleration of each tire?
 - c) What is the tangential acceleration of each tire at $r = 0.38 \text{ m}$?
 - d) What is the centripetal (radial) acceleration of each tire at $r = 0.38 \text{ m}$?
 - e) What is the total linear acceleration of each tire at $t = 0.75 \text{ s}$?
 - f) What is the rotational frequency (f) of each tire?
19. Unlike record players which rotate at a constant angular velocity, the CD-ROM driver in a CD player rotates the CD at different angular velocities so that the optical head maintains a constant tangential velocity relative to the CD. Assume the CD has a diameter of 0.120 m and that the CD needs to move at a tangential velocity of 1.26 m/s relative to the optical head so the data (music) can be read correctly.
 - a) What is the angular velocity when the CD-ROM is reading data at the outer edge of the CD? Give the answer in rpm and rad/s.
 - b) At the inner edge of the CD, the CD-ROM rotates at 500 rpm . How far from the center of the CD is the inner edge ($v_{\text{tangential}} = 1.26 \text{ m/s}$)?
 - c) What is the centripetal (radial) acceleration at the outer edge of the CD?
 - d) What is the rotational frequency (f) of the CD when it is reading data at its outer edge?

II. Rotational Kinematics

Classwork

20. A tricycle wheel of radius 0.11 m is at rest and is then accelerated at a rate of 2.3 rad/s^2 for a period of 8.6 s . What is the wheel's final angular velocity?
21. A tricycle wheel of radius 0.11 m is at rest and is then accelerated at a rate of 2.3 rad/s^2 for a period of 8.6 s . What is the wheel's final linear velocity?
22. A potter's wheel is rotating with an angular velocity of $\omega = 3.2 \text{ rad/s}$. The potter applies a constant force, accelerating the wheel at 0.21 rad/s^2 . What is the wheel's angular velocity after 6.4 s ?

23. A potter's wheel is rotating with an angular velocity of $\omega = 3.2 \text{ rad/s}$. The potter applies a constant force, accelerating the wheel at 0.21 rad/s^2 . What is the wheel's angular displacement after 6.4 s?
24. A bicycle wheel with a radius of 0.38 m accelerates at a constant rate of 4.8 rad/s^2 for 9.2 s from rest. How many revolutions did it make during that time?
25. A bicycle wheel with a radius of 0.38 m accelerates at a constant rate of 4.8 rad/s^2 for 9.2 s from rest. What was its linear displacement during that time?
26. A Frisbee of radius 0.15 m is accelerating at a constant rate from 7.1 revolutions per second to 9.3 revolutions per second in 6.0 s. What is its angular acceleration?
27. A Frisbee of radius 0.15 m is accelerating at a constant rate from 7.1 revolutions per second to 9.3 revolutions per second in 6.0 s. What is its angular displacement during that time?
28. A record is rotating at 33 revolutions per minute. It accelerates uniformly to 78 revolutions per minute with an angular acceleration of 2.0 rad/s^2 . Through what angular displacement does the record move during this period?
29. What is the angular acceleration of a record that slows uniformly from an angular speed of 45 revolutions per minute to 33 revolutions per minute in 3.1 s?

Homework

30. A tricycle wheel of radius 0.13 m is at rest and is then accelerated uniformly to a final angular velocity of 4.4 rad/s after 3.4 s. What was the wheel's angular acceleration?
31. A tricycle wheel of radius 0.13 m is at rest and is then accelerated uniformly to a final angular velocity of 4.4 rad/s after 3.4 s. What was the wheel's tangential acceleration at its rim?
32. A Micro-Hydro turbine generator is accelerating uniformly from an angular velocity of 610 rpm to its operating angular velocity of 837 rpm. The radius of the turbine generator is 0.62 m and its rotational acceleration is 5.9 rad/s^2 . What is the turbine's angular displacement (in radians) after 3.2 s?
33. A Micro-Hydro turbine generator rotor is accelerating uniformly from an angular velocity of 610 rpm to its operating angular velocity of 837 rpm. The radius of the rotor is 0.62 m and its rotational acceleration is 5.9 rad/s^2 . What is the rotor's angular velocity (in rad/s) after 3.2 s?
34. A bicycle wheel with a radius of 0.42 m accelerates uniformly for 6.8 s from an initial angular velocity of 5.5 rad/s to a final angular velocity of 6.7 rad/s . What was its angular acceleration?
35. A bicycle wheel with a radius of 0.42 m accelerates uniformly for 6.8 s from an initial angular velocity of 5.5 rad/s to a final angular velocity of 6.7 rad/s . What was its angular displacement during that time? What was its linear displacement?
36. An aluminum pie plate of radius 0.12 m spins and accelerates at a constant rate from 13 rad/s to 29 rad/s in 5.4 s. What was its angular acceleration?
37. An aluminum pie plate of radius 0.12 m spins and accelerates at a constant rate from 13 rad/s to 29 rad/s in 5.4 s. What was its angular displacement during that time?
38. A record turntable rotating at 78.0 rpm is switched off and slows down uniformly to a stop in 30.0 s. What was its angular acceleration? How many revolutions did it make while slowing down?
39. A wheel starts from rest and accelerates with a constant $\alpha = 3.0 \text{ rad/s}^2$. During a 4.0 s interval it rotates through an angular displacement of 120 radians. How long had the wheel been in motion at the start of that interval?

III. Rotational Dynamics

Classwork

40. A lug wrench is being used to loosen a lug nut on a Chevrolet's wheel rim, so that a flat can be changed. A force of 250.0 N is applied perpendicularly to the end of the wrench, which is 0.540 m from the lug nut. Calculate the torque experienced by the lug nut due to the wrench.
41. A novice tire changer is applying a force of 250.0 N to a lug wrench which is secured to a lug nut 0.540 m away. The lug nut requires 135 N-m of torque to loosen. The novice is applying the force at an angle of 30° to the length of the wrench. Will the lug nut rotate? Why or why not?
42. Two students are on either end of a see-saw. One student is located at 2.3 m from the center support point and has a mass of 55 kg. The other student has a mass of 75 kg. Where should that student sit, with reference to the center support point, if there is to be no rotation of the see-saw?
43. A rock of mass 170 kg needs to be lifted off the ground. One end of a metal bar is slipped under the rock, and a fulcrum is set up under the bar at a point that is 0.65 m from the rock. A worker pushes down (perpendicular) on the other end of the bar, which is 1.9 m away from the fulcrum. What force is required to move the rock?
44. You have two screwdrivers. One handle has a radius of 2.6 cm, and the other, a radius of 1.8 cm. You apply a 72 N force tangent to each handle. What is the torque applied to each screwdriver shaft?
45. What torque needs to be applied to an antique sewing machine spinning wheel of radius 0.28 m and mass 3.1 kg (model it as a hoop, with $I = MR^2$) to give it an angular acceleration of 4.8 rad/s^2 ?
46. What is the angular acceleration of a 75 g lug nut when a lug wrench applies a 135 N-m torque to it? Model the lug nut as a hollow cylinder of inner radius 0.85 cm and outer radius 1.0 cm ($I = M (r_1^2 + r_2^2)$). What is the tangential acceleration at the outer surface? Why are these numbers so high – what factor was not considered?
47. A baseball player swings a bat, accelerating it uniformly from rest to 4.2 revolutions/second in 0.25 s. Assume the bat is modeled as a uniform rod ($I = 1/3 ML^2$), and has $m = 0.91 \text{ kg}$ and is 0.86 m long. Find the torque applied by the player to the bat.
48. Two masses of 3.1 kg and 4.6 kg are attached to either end of a thin, light rod (assume massless) of length 1.8 m. Compute the moment of inertia for:
 - a) The rod is rotated about its midpoint.
 - b) The rod is rotated at a point 0.30 m from the 3.1 kg mass.
 - c) The rod is rotated about the end where the 4.6 kg mass is located.
49. A large pulley of mass 5.21 kg (its mass cannot be neglected) is rotated by a constant Tension force of 19.6 N in the counterclockwise direction. The rotation is resisted by the frictional torque of the axle on the pulley. The frictional torque is a constant 1.86 N-m in the clockwise direction. The pulley accelerates from 0 to 27.2 rad/s in 4.11 s. Find the moment of inertia of the pulley.

50. A baton twirler has a baton of length 0.42 m with masses of 0.53 kg at each end. Assume the rod itself is massless. The rod is first rotated about its midpoint. It is then rotated about one of its ends, and in both cases uniformly accelerates from 0 rad/s to 1.8 rad/s in 3.0 s.
- Find the torque exerted by the twirler on the baton when it is rotated about its middle.
 - Find the torque exerted by the twirler on the baton when it is rotated about its end.

Homework

51. What torque is applied by a person when he pushes with a force of 22 N perpendicularly to the plane of a door at a distance of 0.90 m from the hinges that hold the door to the frame?
52. The same person pushes with a force of 22 N at an angle of 65° to the plane of the door at a distance of 0.90 m from the hinges. What torque is exerted on the door?
53. A light metal rod of negligible mass is balanced on a fulcrum and is free to rotate. A mass of 0.11 kg is placed on one end of the rod at a distance of 0.091 m from the fulcrum. Where must a mass of 0.22 kg be placed on the other side of the fulcrum so that the rod does not rotate?
54. Archimedes stated "Give me a place to stand and I will move the earth." Imagine that you have found that place, and you have a fulcrum and a rod, both strong enough and long enough to withstand the strain put on them by the earth's mass of 5.97×10^{24} kg. You can exert a force of 411 N. Assume the earth's center of mass is located at a distance of 7.11×10^6 m away from the fulcrum and resting on the impossible rod. How far from the fulcrum must you push down on the rod to move the earth? Research how big the "observable universe" is and comment on Archimedes' claim.
55. You have two screwdrivers. One handle has a radius of 2.2 cm, and the other, a radius of 1.8 cm. You apply a 92 N force tangent to each handle. What is the torque applied to each screwdriver shaft?
56. What torque needs to be applied to a hula hoop of radius 0.58 m and mass 2.5 kg (model it as a hoop, with $I = MR^2$) to give it an angular acceleration of 6.8 rad/s^2 ?
57. A cricket batsman swings his bat, accelerating it uniformly from rest to 17.3 rad/s in 0.21 s. Assume the bat is modeled as a flat plate ($I = \frac{1}{3} Mh^2 + \frac{1}{2} Mw^2$), and has $m = 1.36$ kg, $h = 0.97$ m, and $w = 0.11$ m. Find the torque applied by the batsman to the bat.
58. Two masses of 4.2 kg and 5.8 kg are attached to either end of a thin, light rod (assume massless) of length 2.4 m. Compute the moment of inertia for:
- The rod is rotated about its midpoint.
 - The rod is rotated at a point 0.5 m from the 4.2 kg mass.
 - The rod is rotated about the end where the 5.8 kg mass is located.
59. A large pulley of mass 6.91 kg (its mass cannot be neglected) is rotated by a constant Tension force of 22.3 N in the counterclockwise direction. The rotation is resisted by the frictional torque of the axle on the pulley. The frictional torque is a constant 2.12 N-m in the clockwise direction. The pulley accelerates from 0 to 31.2 rad/s in 5.63 s. Find the moment of inertia of the pulley.

60. A baton twirler has a baton of length 0.36 m with masses of 0.48 kg at each end. Assume the rod itself is massless. The rod is first rotated about its midpoint. It is then rotated about one of its ends, and in both cases uniformly accelerates from 0 rad/s to 2.4 rad/s in 3.6 s.
- Find the torque exerted by the twirler on the baton when it is rotated about its middle.
 - Find the torque exerted by the twirler on the baton when it is rotated about its end.

IV. Rotational Kinetic Energy

Classwork

61. A solid cylinder ($I = \frac{1}{2} MR^2$) of mass 0.56 kg and radius 0.042 m rolls, without slipping, down an incline of height 0.67 m. What is the speed of the cylinder at the bottom of the incline? Does its speed depend on the mass and radius of the cylinder?
62. Two uniform spheres ($I = \frac{2}{5} MR^2$) roll, without slipping, down an incline of height 0.72 m. Sphere 1 has a mass of 1.1 kg and a radius of 0.18 m and Sphere 2 has a mass of 1.8 kg and a radius of 0.14 m. Which sphere gets to the bottom of the incline quicker? What is the velocity of each sphere?
63. What is the rotational kinetic energy of a 0.82 kg sphere ($I = \frac{2}{5} MR^2$), with a radius of 0.058 m, rolling with an angular velocity of 5.2 rad/s?
64. A 0.36 kg, 0.11 m radius, thin hoop ($I = MR^2$) is rotating, without slipping, while moving linearly with an angular velocity of 4.8 rad/s along a path. What is its total kinetic energy (translational plus rotational)?
65. A solid cylinder rolls down a hill without slipping. How much work does the frictional force between the hill and the cylinder do on the cylinder as it is rolling? Why?
66. How much work is required to uniformly slow a merry-go-round of mass 1850 kg and a radius of 8.30 m from a rotational rate of 1 revolution per 7.40 s to a stop? Model the merry-go-round as a solid cylinder ($I = \frac{1}{2} MR^2$). If the merry-go-round is stopped in 7.40 s, what power is exerted?

Homework

67. A thin hoop ($I = MR^2$) of mass 0.56 kg and radius 0.042 m rolls, without slipping, down an incline of height 0.67 m. What is the speed of the hoop at the bottom of the incline? Does its speed depend on the mass and radius of the hoop?
68. Two solid cylinders ($I = \frac{1}{2} MR^2$) roll, without slipping, down an incline of height 0.85 m. Sphere 1 has a mass of 1.1 kg and a radius of 0.11 m and Sphere 2 has a mass of 2.1 kg and a radius of 0.14 m. Which cylinder gets to the bottom of the incline quicker? What is the velocity of each cylinder?
69. What is the rotational kinetic energy of a 1.3 kg solid cylinder ($I = \frac{1}{2} MR^2$), with a radius of 0.043 m, rolling with an angular velocity of 4.9 rad/s?
70. A 0.42 kg, 0.09 m radius, sphere ($I = \frac{2}{5} MR^2$) is rotating, without slipping, while moving linearly with an angular velocity of 5.2 rad/s along a path. What is its total kinetic energy (translational plus rotational)?
71. A sphere slides down a hill without rotating. Does the frictional force between the hill and the sphere do work on the sphere while it is sliding? Why?

72. How much work is required to uniformly accelerate a merry-go-round of mass 1490 kg and a radius of 7.80 m from rest to a rotational rate of 1 revolution per 8.33 s? Model the merry-go-round as a solid cylinder ($I = \frac{1}{2} MR^2$). How much power is required to accelerate the merry-go-round to that rate in 8.33 s?

V. Angular Momentum

Classwork

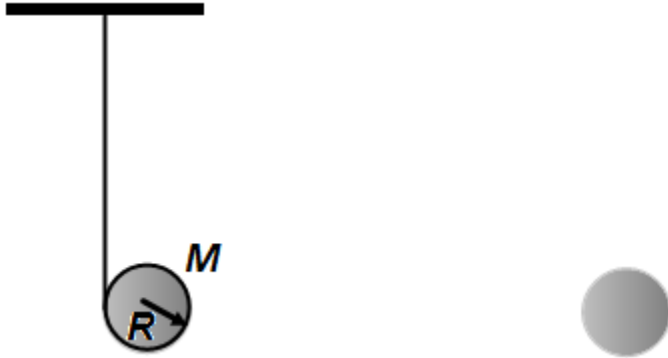
73. You spin a ball of mass 0.18 kg that is attached to a string of length 0.98 m at $\omega = 5.2$ rad/s in a circle. What is the ball's angular momentum?
74. A student is standing, with her arms outstretched, on a platform that is rotating at 1.6 rev/s. She pulls her arms in and the platform now rotates at 2.2 rev/s. What is her final moment of inertia (I) in terms of her original moment of inertia (I_0)?
75. An LP record is spinning on an old fashioned record player with an angular velocity of ω . The record changer drops an identical record on top of the spinning record. What is the new angular velocity of both records (assume the record player doesn't add additional torque to keep the records spinning at the original ω)?
76. Calculate the angular momentum of a ballet dancer who is spinning at 1.5 rev/sec. Model the dancer as a cylinder ($I = \frac{1}{2} MR^2$) with a mass of 62 kg, a height of 1.6 m and a radius of 0.16 m.
77. A student of mass 42 kg is standing at the center of a merry-go-round of radius 3.4 m and a moment of inertia of 840 kg-m² that is rotating at $\omega = 1.8$ rad/s. The student walks to the outer edge of the merry-go-round. What is the angular velocity of the merry-go-round when he reaches the edge?

Homework

78. A ball of mass 0.14 kg attached to a string of length 0.64 m is spun in a circle with $\omega = 4.9$ rad/s. What is the ball's angular momentum?
79. A platform is rotating at 2.2 rev/s and a student is standing in the middle of it with his arms at his side. He extends his arms straight out and the platform now rotates at 1.4 rev/s. What is his final moment of inertia (I) in terms of his original moment of inertia (I_0)?
80. A potter spins his wheel at 0.98 rev/s. The wheel has a mass of 4.2 kg and a radius of 0.35 m. He drops a chunk of clay of 2.9 kg directly onto the middle of the wheel. The clay is in the shape of a pancake and has a radius of 0.19 m. Assume both the wheel and the chunk of clay can be modeled as solid cylinders ($I = \frac{1}{2} MR^2$). What is the new tangential velocity of the wheel and the clay?
81. What is the angular momentum of a roller skater who is spinning at 1.5 rev/sec? Model the skater as a cylinder ($I = \frac{1}{2} MR^2$) with a mass of 81 kg, a height of 1.8 m and a radius of 0.18 m.
82. A student of mass 59 kg is standing at the edge of a merry-go-round of radius 4.2 m and a moment of inertia of 990 kg-m² that is rotating at $\omega = 2.1$ rad/s. The student walks to the middle of the merry-go-round. What is the angular velocity of the merry-go-round when he reaches the middle?

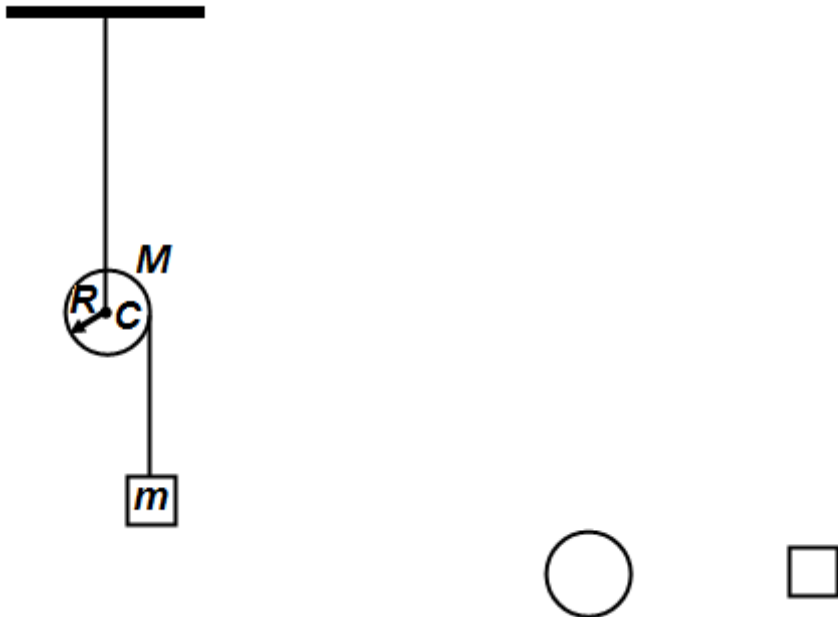
VI. General Problems

83. A very light cotton tape is wrapped around the outside surface of a uniform cylinder of mass M and radius R . The free end of the tape is attached to the ceiling. The cylinder is released from rest and as it descends it unravels from the tape without slipping. The moment of inertia of the cylinder about its center is $I = \frac{1}{2} MR^2$.



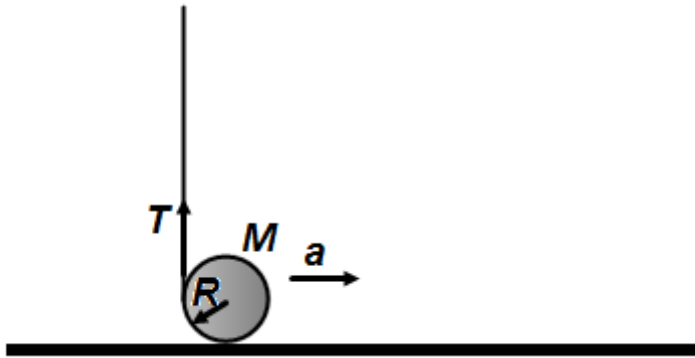
- On the circle above, show all the forces applied on the cylinder.
- Find the acceleration of the center of the cylinder when it moves down.
- Find the tension force in the tape.

84. A uniform cylinder of mass M and radius R is fixed on a frictionless axle at point C . A block of mass m is suspended from a light cord wrapped around the cylinder and released from rest at time $t = 0$. The moment of inertia of the cylinder is $I = \frac{1}{2} MR^2$.



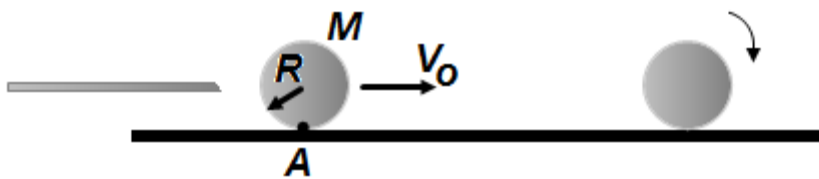
- On the circle and the square above, show all the applied forces on the cylinder and the block.
- Find the acceleration of the block as it moves down.
- Find the tension in the cord.
- Express the angular momentum of the cylinder as a function of time t .

85. A uniform cylinder of mass M and radius R is initially at rest on a rough horizontal surface. A light string is wrapped around the cylinder and is pulled straight up with a force T whose magnitude is $0.80 Mg$. As a result, the cylinder slips and accelerates horizontally. The moment of inertia of the cylinder is $I = \frac{1}{2} MR^2$ and the coefficient of kinetic friction is 0.40 .



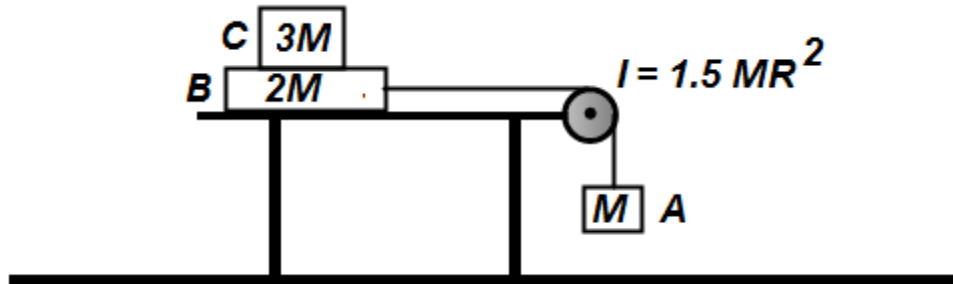
- On the circle above, show all the forces applied on the cylinder.
- In terms of g , determine the linear acceleration, a of the center of the cylinder.
- Determine the angular acceleration, α of the cylinder.
- Explain the difference in results of linear acceleration a , and $R\alpha$.

86. A billiard ball of mass M and radius R is struck by a cue stick along a horizontal line through the center of mass of the ball. The ball initially slides with a velocity v_0 . As the ball moves across the rough billiard table its motion gradually changes from pure translational through rolling with slipping to rolling without slipping. The moment of inertia of the ball is $I = \frac{2}{5} MR^2$ and the coefficient of kinetic friction is μ .



- Express the linear velocity, v of the center of mass of the ball as a function of time t while it is rolling with slipping.
- Express the angular velocity, ω of the ball as a function of time while it is rolling with slipping.
- Find the time at which the ball begins to roll without slipping.
- When the ball is struck it acquires an angular momentum about the fixed point A on the surface of the table. During the motion the angular momentum about point A remains constant despite the friction force. Explain why this occurs.

87. A block, A of mass, M is suspended from a light string that passes over a pulley and is connected to block B of mass $2M$. Block B sits on the surface of a smooth table. Block C, of mass $3M$, sits on the top of block B. The surface between block C and block B is not frictionless. When the system of three blocks is released from rest, block A accelerates downward with a constant acceleration, a , and the two blocks on the table move relative to each other. The moment of inertia of the pulley is $I = 1.5 MR^2$. Present all results in terms of M , g , and a .

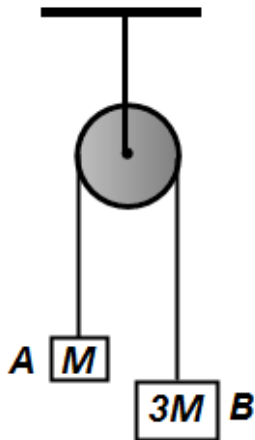


- Find the tension force in the vertical section of the string.
- Find the tension force in the horizontal section of the string.

The acceleration of block A was determined from a series of experiments: $a = 2 \text{ m/s}^2$.

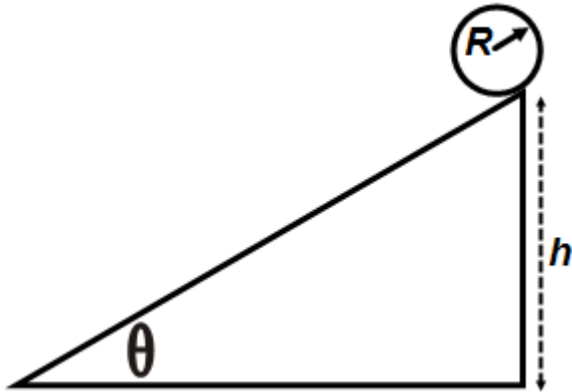
- Find the coefficient of kinetic friction between the two blocks on the table.
- Find the acceleration of block C.

88. A pulley of radius, R and moment of inertia, $I = 2 MR^2$ is mounted on an axle with negligible friction. Block A, with a mass M , and Block B, with a mass $3M$, are attached to a light string that passes over the pulley. Assuming that the string doesn't slip on the pulley, answer the following questions in terms of M , R , and fundamental constants.



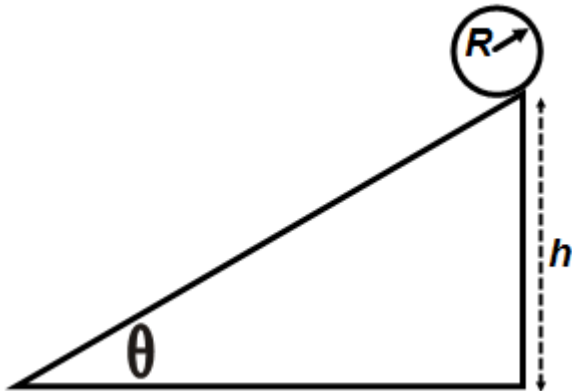
- What is the acceleration of the two blocks?
- What is the tension force in the left section of the string?
- What is the tension force in the right section of the string?
- What is the angular acceleration of the pulley?

89. A solid uniform sphere of mass, M and radius, R is placed on an inclined plane at a distance, h from the base of the incline. The inclined plane makes an angle, θ with the horizontal. The sphere is released from rest and rolls down the incline without slipping. The moment of inertia of the sphere is $I = \frac{2}{5} MR^2$.



- Determine the translational kinetic energy of the sphere when it reaches the bottom of the inclined plane.
- Determine the rotational kinetic energy of the sphere when it reaches the bottom of the inclined plane.
- Determine the linear acceleration of the sphere when it is on the inclined plane.
- Determine the friction force acting on the sphere when it is rolling down the inclined plane.
- If the solid sphere is replaced with a hollow sphere of identical mass and radius, how would that change the answer to question (b)? Explain.

90. A solid uniform cylinder of mass M and radius R is placed on an inclined plane at a distance h from the base of the incline. The inclined plane makes an angle θ with the horizontal. The cylinder is released from rest and rolls down the incline without slipping. The moment of inertia of the cylinder is $I = \frac{1}{2} MR^2$.

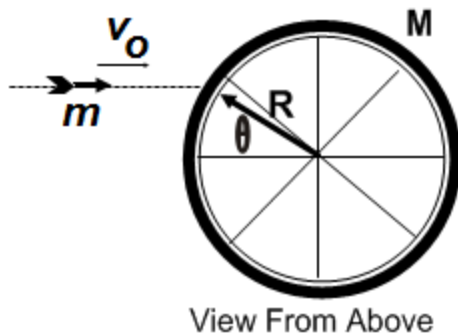


- Determine the translational kinetic energy of the cylinder when it reaches the bottom of the inclined plane.
- Determine the rotational kinetic energy of the cylinder when it reaches the bottom of the inclined plane.
- Determine the linear acceleration of the cylinder when it is on the inclined plane.
- Determine the friction force acting on the cylinder when it is rolling down the inclined plane.
- If the solid cylinder is replaced with a hollow hoop of identical mass and radius, how would that change the answer to question (b)? Explain.

91. In a physics experiment, students made a lab cart of a wooden block of mass $5m$ and four wheels each of mass, m and radius r . The moment of inertia of each wheel is $I = \frac{1}{2}mr^2$. The cart is released from rest and rolls without slipping from the top of an inclined plane of height h . After the cart reaches the bottom of the inclined plane, it collides with an elastic spring with negligible mass and a spring constant k .



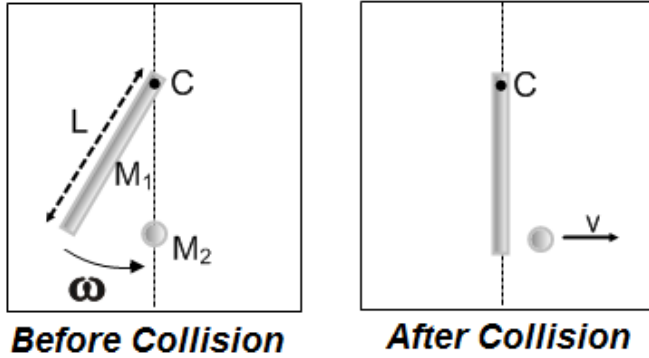
- Determine the moment of inertia of all four wheels.
 - Determine the speed of the cart at the bottom of the inclined plane.
 - Determine the maximum compression of the spring after the collision.
92. A bicycle wheel of mass M and radius R is placed on a vertical axle and is able to rotate without friction. A dart of mass, m is fired toward the wheel with an initial velocity V_0 . The dart strikes the wheel and sticks in the tire. The moment of inertia of the wheel is $I = MR^2$.



- What is the initial linear momentum of the dart?
- Is the linear momentum of the dart conserved? Explain.
- Is the angular momentum of the dart conserved? Explain.
- What is the initial angular momentum of the dart with respect to the center of the wheel?
- What is the angular velocity of the wheel after the dart strikes?
- Find the ratio of the final kinetic energy of the system, after the dart strikes, and the initial kinetic energy of the dart.

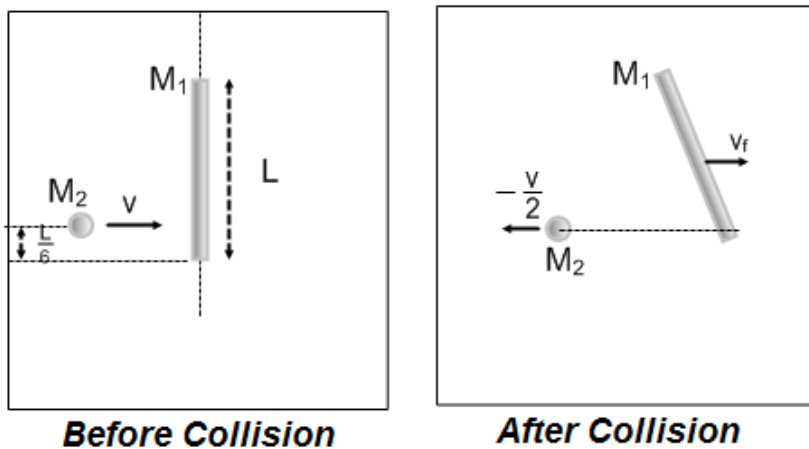
93. A uniform rod of mass, M_1 and length, L is fixed on one end, and the other is free to rotate with respect to point C on a frictionless horizontal table. The angular velocity of the rod is ω and initially stays unchanged. The rod strikes a stationary sphere of mass, M_2 with the free end. After the collision, the rod stops and the sphere moves to the right with a velocity, v . The moment of inertia of the rod with respect to point C is $I = \frac{1}{3}ML^2$. Present all answers in terms of: M_1 , M_2 , L , and ω .

Views From Above



- Determine the velocity of the sphere, v after the collision by using the conservation of angular momentum.
 - Determine the linear momentum of the rod just before the collision.
 - Determine the linear momentum of the sphere after the collision.
 - Is the linear momentum of the system of two objects conserved during the collision? Explain.
 - Is the angular momentum of the system of two objects conserved? Explain.
94. A thin, uniform rod of mass, M_1 and length, L is initially at rest on a frictionless horizontal tabletop. A small mass M_2 moves perpendicularly toward the rod with an initial velocity, v and strikes the rod at a distance of $\frac{L}{6}$ from its end. After the collision, M_2 moves in the opposite direction with a velocity, $-\frac{v}{2}$. The moment of inertia of the rod with respect to its center of mass is $I = \frac{1}{12}ML^2$.

Views From Above



- Determine the velocity of the center of mass of the rod after the collision.
- Determine the angular velocity of the rod about its center of mass after the collision.
- Determine the change in kinetic energy of the system as a result of the collision.

Answers

Chapter Questions

1. Real matter is not just a point – it has a structure. It doesn't just move in a linear fashion – it can rotate about itself.
2. The axis of rotation is a line about which the object rotates. For a tire, the axis goes through the empty space within the tire – if the tire is on a car, the axle would be the axis of rotation, so, no, the axis doesn't touch the rubber in the tire.
3. The degree was an angle of measurement chosen by humans based on culture or their system of mathematics. The radian is a property of the circle, as it relates the angle subtended by a length on the circumference of the circle (arc length) to the radius of the circle.
4. Both bugs move through the same angular displacement, but the bug near the edge of the record moves through a greater linear displacement (arc length).
5. Tangential acceleration and centripetal acceleration. Both increase as an object is further from the axis of rotation.
6. Both rings are moving with the same angular acceleration and velocity. However, the further you are away from the center of the merry-go-round, the greater the tangential and centripetal acceleration and the velocity. This is what you feel, so if you are prone to motion sickness, you want slower magnitudes of the linear quantities. You should choose the inner ring.
7. Angular acceleration is assumed to be constant. If you know the linear kinematics equations, you just replace the linear variables, x , v and a , with their angular equivalents, θ , ω and α . No – the reason why objects move is covered in Dynamics. Kinematics just deals with how objects move, not why.
8. Torque. By increasing the distance of where the force is applied to the object to be rotated. Also, the force should be applied perpendicular to the line connecting the force application with the rotated object.
9. Torque is dependent upon the distance between the force application and the object. It also depends on the angle that it makes with a line connecting the force to the object. Both are maximized by pushing at the far end of the door in a perpendicular fashion, resulting in a greater angular acceleration. The door stop is positioned as far away from the door hinges as possible to increase its torque which opposes any torque applied to/by the door to shut it.
10. Torque is a vector – it has a direction and a magnitude. Torque is maximized when the force is applied perpendicular to the line connecting it and the rotated object. Work and Energy are scalars – and they are maximized when a force is applied parallel to their motion.
11. A longer wrench will maximize the torque due to a given force (your strength), and will result in a greater angular acceleration of the wheel nuts. The goal is to provide a greater torque than what currently is holding the nuts in place – friction between the nut and the wheel rim and bolt.
12. The fulcrum should be closer to the heavy rock. The rock is providing a torque at the fulcrum point in one direction (let's say, clockwise), and you are providing a torque in the opposite direction (counter clockwise). To lift the rock, the torque that you provide has to be greater than the rock's torque. As you have a limited amount of strength (force), by having a longer section of the board on your side of the fulcrum will maximize your applied torque.
13. The moment of inertia of the hollow cylinder is greater than the sphere (assuming equal masses and radii). The moment of inertia depends on the mass distribution of the object – the more mass that is located further from the rotation axis, the greater the moment of inertia. More of the mass of the hollow cylinder is further from the rotation axis – so it has a greater moment of inertia than the sphere.
14. The hollow cylinder has a greater moment of inertia. Therefore, its rotational kinetic energy is greater than the rotational kinetic energy of the solid cylinder. Since the total energy of each rotating object is the same that leaves less translational kinetic energy for the hollow cylinder. Therefore, its velocity is less, and it will reach the bottom of the inclined plane after the solid cylinder.
15. The linear distance (translational) covered by an object in translational motion equals the arc length distance (rotational) traveled by a point on the rotating object. If this condition holds, then the equation $v = r\omega$ can be used.
16. She should throw her arms out to the side, increasing her moment of inertia (more of her mass is distributed further from her axis of rotation). Since there is no external torque applied (because she did

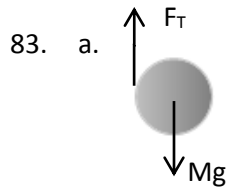
not dig her skates into the ice), her angular momentum will be conserved, $L = I_1\alpha_1 = I_2\alpha_2$. Since $I_2 > I_1$, α_2 will be less than α_1 . Thus, her angular acceleration decreases and she slows down her rate of rotation.

Chapter Problems

1. 0.25 rad
2. 14°
3. 1.6 rad
4. 0.38 rad/s
5. 13 rad/s²
6. 0.23 rad/s²
7. 0.75 m/s
8. 16 rad/s
9.
 - a. 0.53 rad/s²
 - b. 2.1 m/s²
 - c. 2.6 m/s²
 - d. 3.3 m/s²
 - e. 0.13 Hz
10. 0.82 rad
11. 47°
12. 0.39 rad
13. 0.62 rad/s
14. 0.21 rad/s²
15. 3.0 rad/s²
16. 0.78 m/s
17. 6.8 rad/s²
18.
 - a. 2.6 rad/s
 - b. 3.3 rad/s²
 - c. 1.3 m/s²
 - d. 2.6 m/s²
 - e. 2.9 m/s²
 - f. 0.41 Hz
19.
 - a. 21.0 rad/s; 201 rpm
 - b. 0.0241 m
 - c. 26.5 m/s²
 - d. 3.34 Hz
20. 20 rad/s
21. 2.2 m/s
22. 4.5 rad/s
23. 25 rad
24. 32 revolutions
25. 77 m
26. 2.3 rad/s²
27. 310 rad
28. 14 rad
29. -0.41 rad/s²
30. 1.3 rad/s²
31. 0.17 m/s²
32. 230 rad
33. 83 rad/s
34. 0.18 rad/s²
35. 42 rad; 18 m
36. 2.8 rad/s²

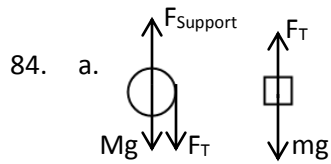
37. 120 rad
38. -0.272 rad/s^2 ; 195 revolutions
39. 8.0 s
40. 135 N-m
41. 67.5 N-m. The lug nut will not move – it requires a torque of 135 N-m before it will have an angular acceleration.
42. 1.7 m
43. 570 N
44. 1.9 N-m; 1.3 N-m
45. 0.58 N-m
46. $2.1 \times 10^7 \text{ rad/s}$; $2.1 \times 10^5 \text{ m/s}$. These numbers are way too high. But, what's missing is the friction force provided by the threads of the lug nut, which opposes the applied force and reduces the total torque. The wrench would start applying the force, the lug nut would move and the wrench would lose contact, reducing the applied torque and the angular acceleration. This is where a simple basic physics model fails in explaining a fairly simple project.
47. 21 N-m
48. a) 6.2 kg-m^2 b) 11 kg-m^2 c) 10 kg-m^2
49. 1.05 kg-m^2
50. a) 0.0028 N-m b) 0.0014 N-m
51. 20 N-m
52. 18 N-m
53. 0.046 m
54. $1.01 \times 10^{30} \text{ m}$. Archimedes might have been wrong here. The limit of the observable universe (that which we have received light from is $2.6 \times 10^{26} \text{ m}$. Archimedes would need more room (or more strength).
55. 2.0 N-m; 1.7 N-m
56. 5.7 N-m
57. 36 N-m
58. a) 14 kg-m^2 b) 22 kg-m^2 c) 24 kg-m^2
59. 3.64 N-m
60. a) 0.021 N-m b) 0.041 N-m
61. 3.0 m/s. No, it only depends on g and the height of the incline.
62. They both get to the bottom at the same time; 3.2 m/s.
63. 0.015 J
64. 0.10 J
65. Zero work. At the point of contact between the cylinder and the ground, the cylinder is moving first down then up. The friction exerted by the incline on the cylinder is parallel to its linear movement. Hence the frictional force is perpendicular to the displacement of the cylinder at the point of contact – zero work.
66. $-2.30 \times 10^4 \text{ J}$; 3.10 kW
67. 2.1 m/s; No, it is mass and radius independent.
68. Both arrive at the same time; 3.3 m/s
69. 0.014 J
70. 0.064 J
71. Yes. The frictional force is along the incline, and the displacement of the sphere is along the incline (it is sliding, not rotating). $W = Fd \cos \theta$ and $\theta = 0$.
72. $1.71 \times 10^4 \text{ J}$; 2.05 kW
73. $0.90 \text{ kgm}^2/\text{s}$
74. $I = 0.73 I_0$
75. $\omega = \omega_0/2$
76. $7.5 \text{ kgm}^2/\text{s}$
77. 1.1 rad/s
78. $0.28 \text{ kgm}^2/\text{s}$
79. $\omega = 0.64 \omega_0$
80. 1.8 m/s
81. $12 \text{ kgm}^2/\text{s}$
82. 4.3 rad/s

General Problems:



b. $\frac{2g}{3}$

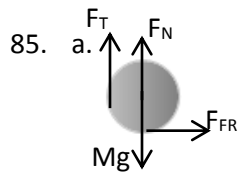
c. $\frac{mg}{3}$



b. $\frac{mg}{M/2+m}$

c. $\frac{Mmg}{M+2m}$

d. $\frac{MmgtR}{M+2m}$



b. $0.08g$

c. $\frac{1.44g}{R}$

d. Because the cylinder slips, αR is greater than a .

86. a. $v = v_0 - \mu gt$

b. $\omega = \frac{5\mu gt}{2R}$

c. $\frac{2v_0}{7\mu g}$

d. Point A, with the addition of the friction force, does not rotate, as the rotational forces cancel out.

87. a. $M(g - a)$

b. $Mg - 2.5Ma$

c. 0.03

d. 0.3 m/s^2

88. a. $\frac{g}{3}$

b. $\frac{4mg}{3}$

c. $2mg$

d. $\frac{g}{3R}$

89. a. $\frac{5Mgh}{7}$

b. $\frac{2Mgh}{7}$

c. $\frac{5g \sin \theta}{7}$

d. $\frac{2mg \sin \theta}{7}$

e. The moment of inertia, I , would increase. This would result in a lower tangential velocity, as more of the total Kinetic Energy is transformed into rotational Kinetic Energy.

90. a. $\frac{2Mgh}{3}$

b. $\frac{Mgh}{3}$

c. $\frac{2g \sin \theta}{3}$

d. $\frac{Mg \sin \theta}{3}$

e. It would lower the moment of inertia I , increasing the value of the answer to (b).

91. a. $2mr^2$

b. $\sqrt{\frac{18gh}{11}}$

c. $\sqrt{\frac{18mgh}{k}}$

92. a. mv_0

- b. No. There is an unbalanced force at the center of the wheel.
- c. Yes. Torque at the center of the wheel is equal to 0. No external torque acts on the system.
- d. $mRv_o \sin \theta$
- e. $\frac{m \sin \theta v_o}{R(m+M)}$
- f. $\frac{m \sin^2 \theta}{m+M}$

93. a. $\frac{M_1 L \omega}{3M_2}$

b. $\frac{M_1 L \omega}{2}$

c. $\frac{M_1 L \omega}{3}$

d. No. Initially, linear momentum was equal to $\frac{M_1 L \omega}{2}$. The final linear momentum is equal to $\frac{M_1 L \omega}{3}$, showing that they are not conserved. This is because there is an unbalanced force at point C.

e. Yes. The torque at point C is equal to zero. No external torque acts on the system.

94. a. $\frac{3M_2 v}{2M_1}$

b. $\frac{6M_2 v}{M_1 L}$

c. $\frac{3}{8} M_2 v^2 - \frac{21M_2^2 v^2}{8M_1}$