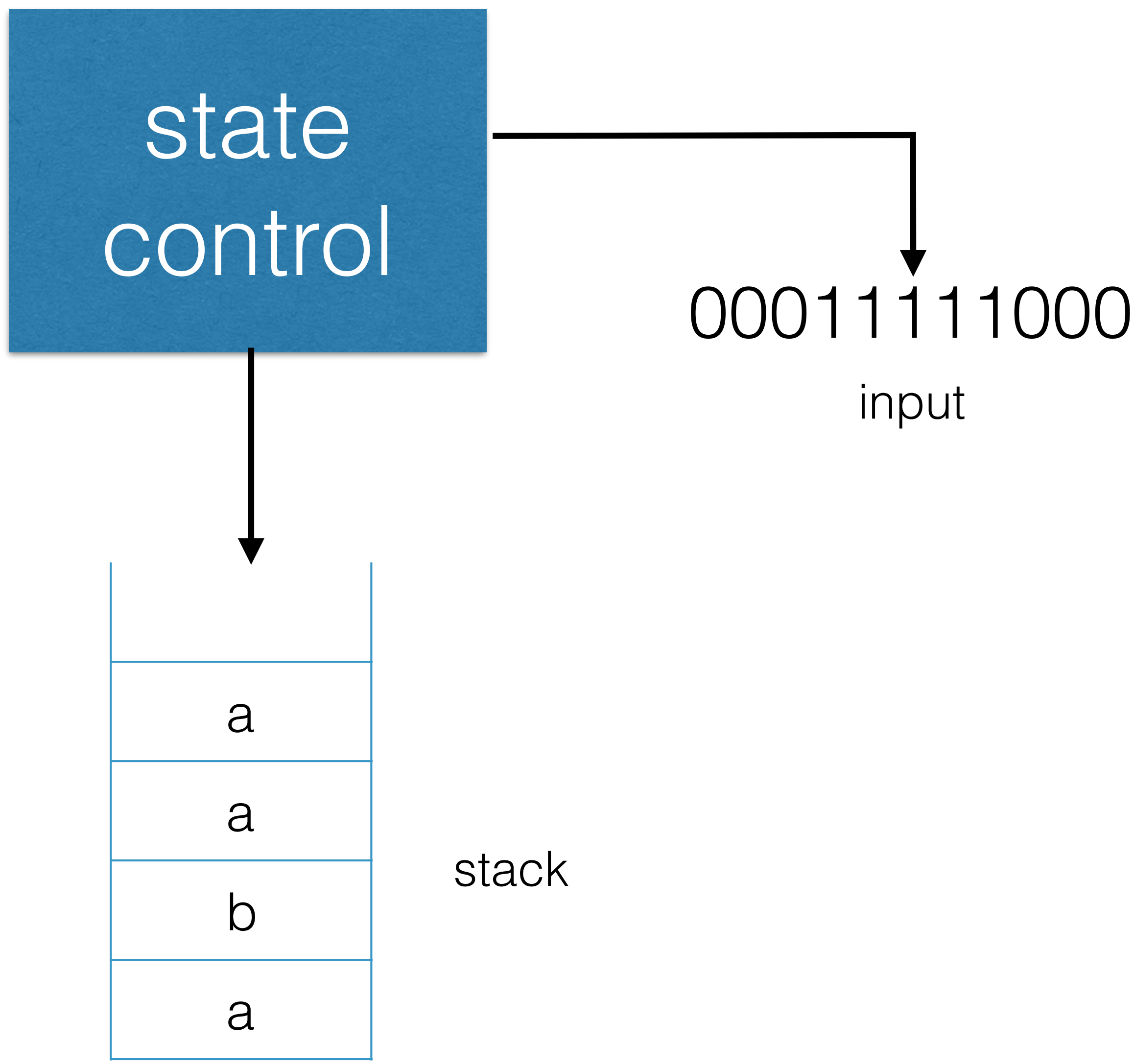
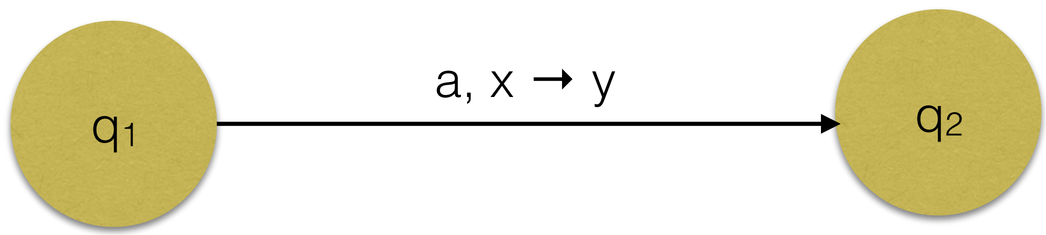
Pushdown Automata (PDA)



Pushdown Automata (PDA)

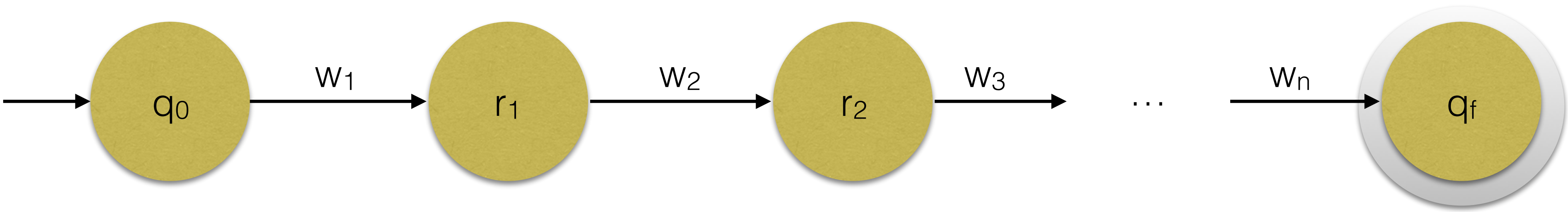
If the input symbol is a **and**
the top stack symbol is x **then**
q_1 to q_2, pop x, push y, advance read head



If a = ε do not advance read head
If x = ε do not read from stack
If y = ε do not write to stack

When does a PDA accept a string?

input: $w_1w_2 \ldots w_n$

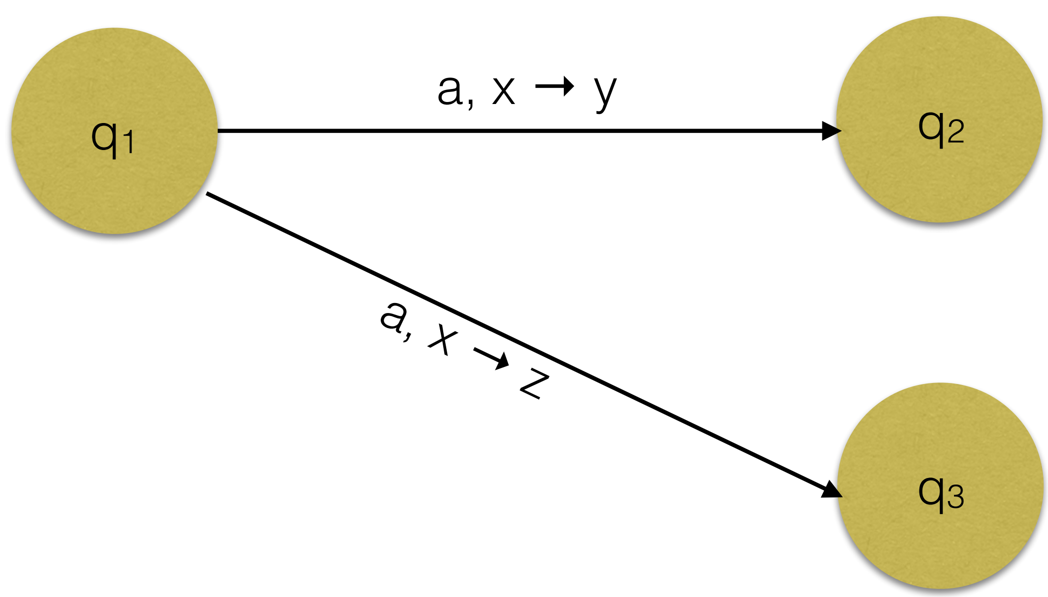


accept if any branch accepts

Pushdown Automata (PDA)

$(Q, \Sigma, \Gamma, \delta, q_0, F)$

$\delta: Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \rightarrow \mathscr{P}(Q \times \Gamma_\varepsilon)$



Theorems

Not every nondeterministic PDA has an equivalent deterministic PDA

A language is context-free iff some PDA recognizes it

CFL

vs.

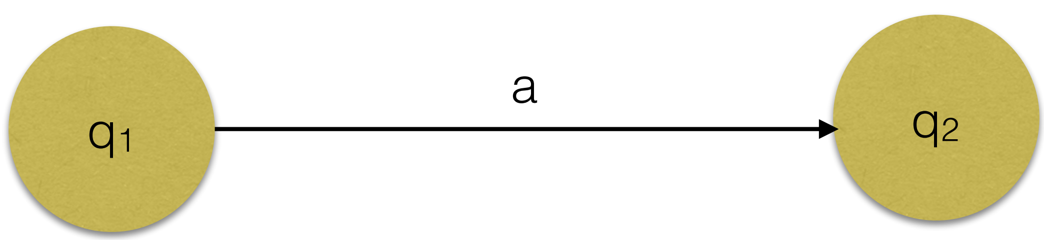
Regular Languages

CFL vs. Regular Languages

~~PDA to NFA~~

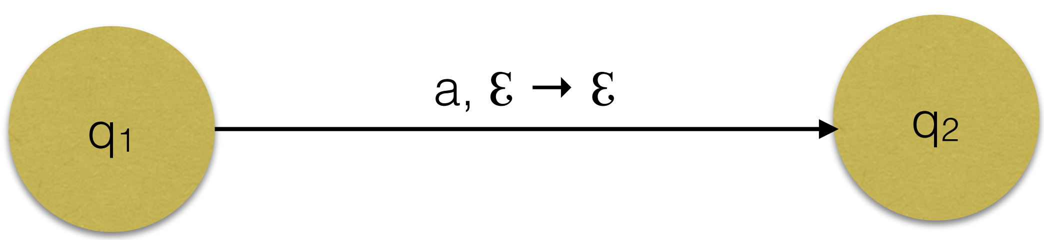
CFL vs. Regular Languages

NFA to PDA



CFL vs. Regular Languages

NFA to PDA



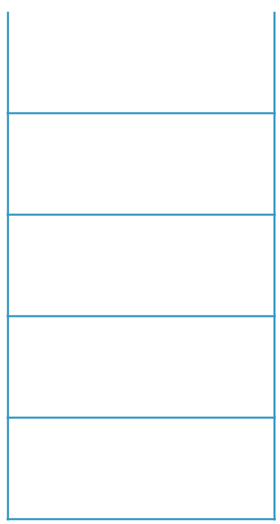
PDA Design
Examples

PDA Design

$$\{0^n1^n \mid n \geq 0\}$$

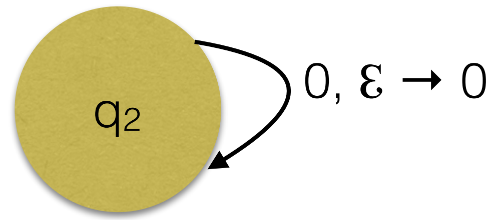
input: 000111

$\{0^n 1^n \mid n \geq 0\}$

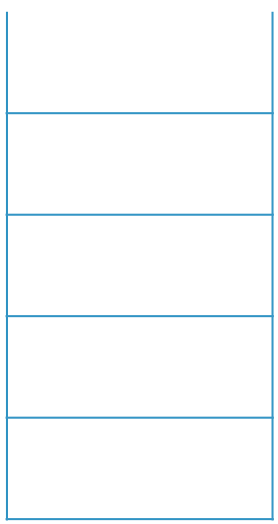


stack

input: 000111



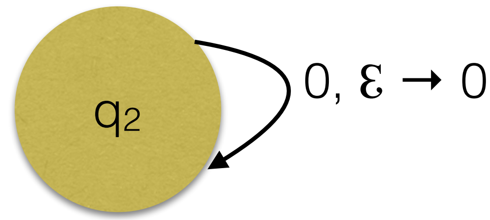
$$\{0^n1^n \mid n \geq 0\}$$

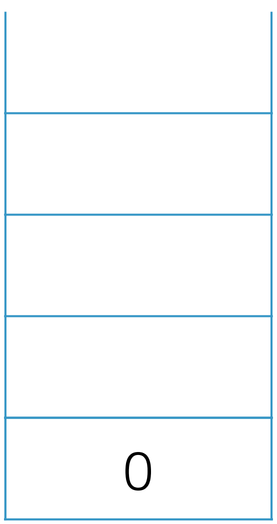


stack

input: **0**00111

$\{0^n 1^n \mid n \geq 0\}$

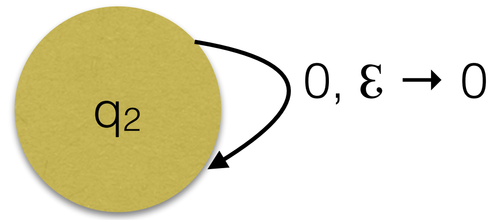


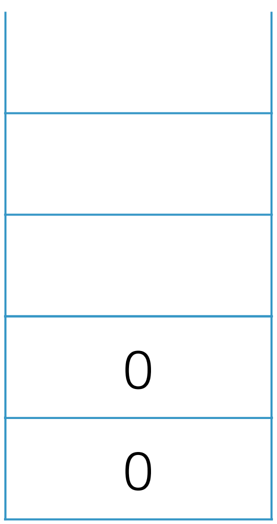


stack

input: 0**0**0111

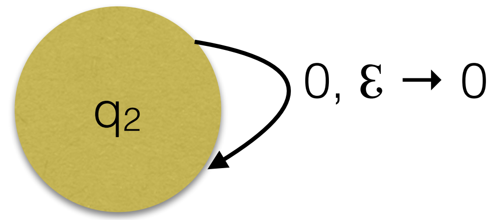
$\{0^n 1^n \mid n \geq 0\}$



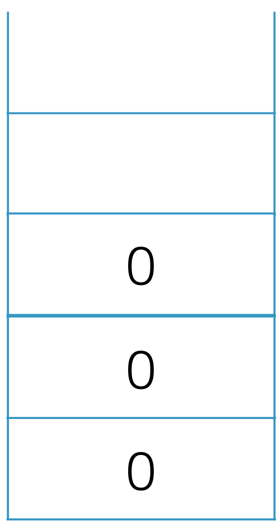


stack

input: 00**0**111



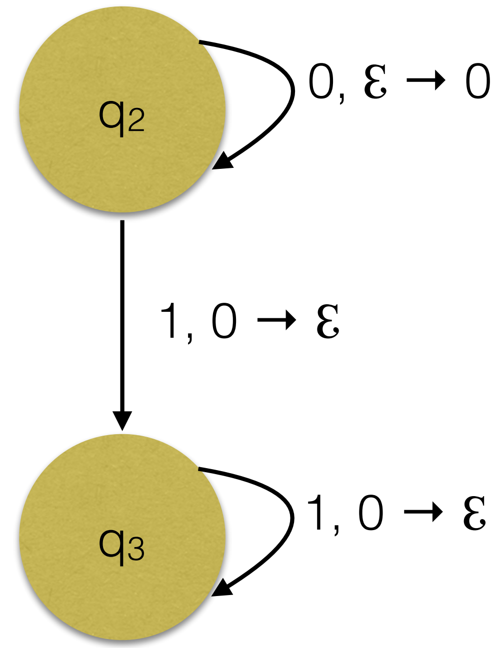
$$\{0^n 1^n \mid n \geq 0\}$$

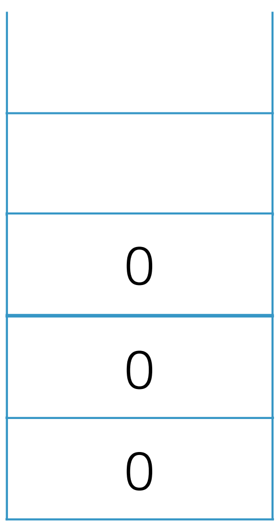


stack

input: 000111

$$\{0^n 1^n \mid n \geq 0\}$$

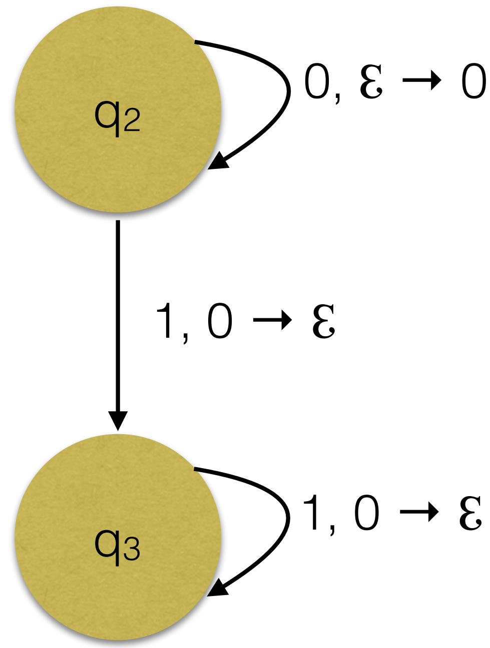


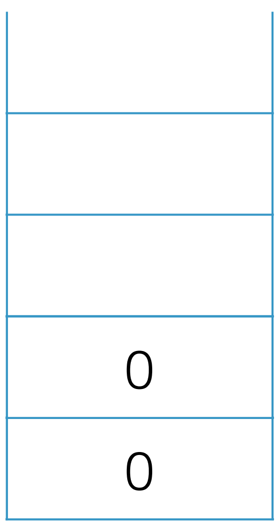


stack

input: 000111

$$\{0^n 1^n \mid n \geq 0\}$$

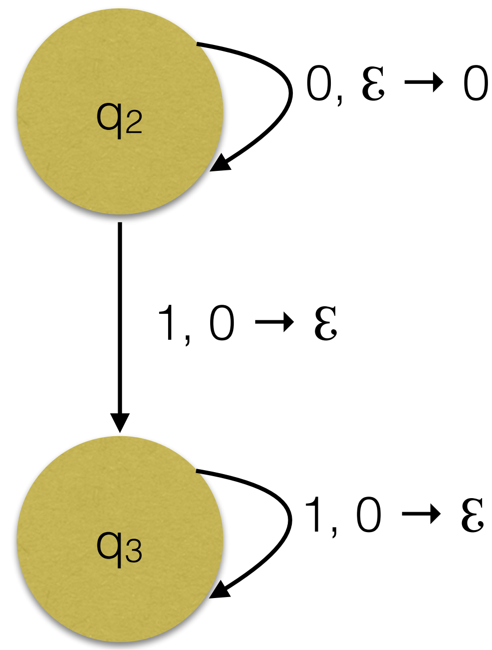


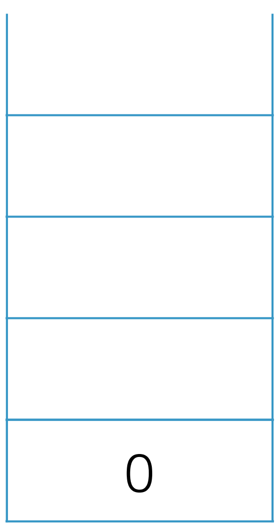


stack

input: 000111

$\{0^n 1^n \mid n \geq 0\}$

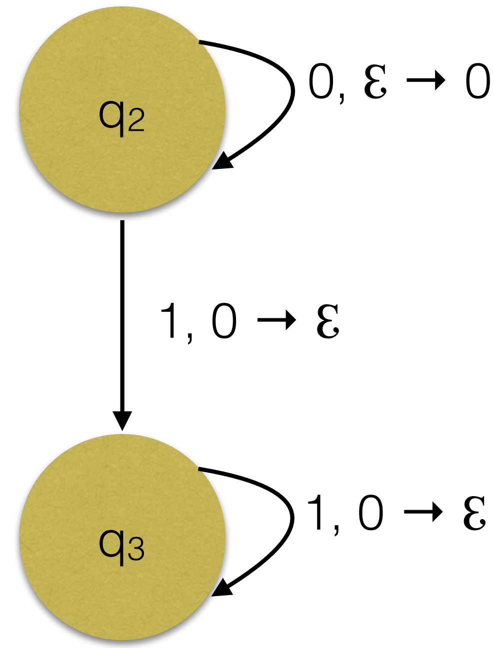


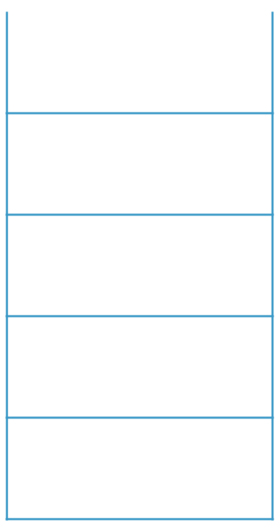


stack

input: 00011**1**

$$\{0^n 1^n \mid n \geq 0\}$$

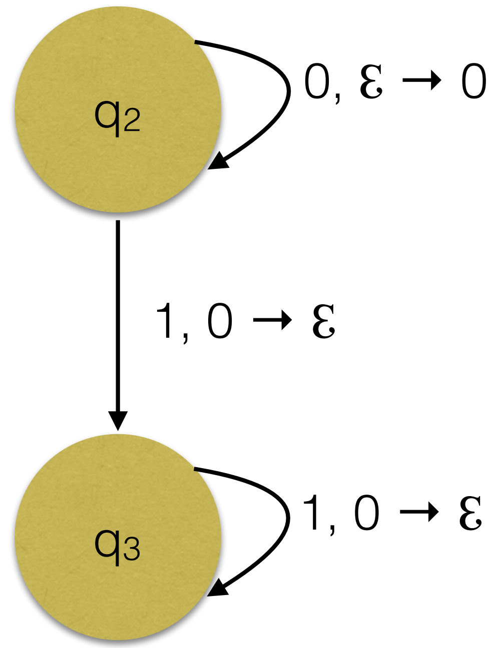


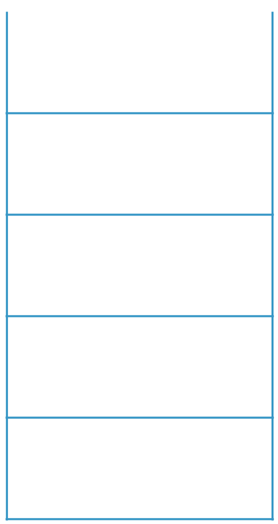


stack

input: 000111_

$$\{0^n 1^n \mid n \geq 0\}$$

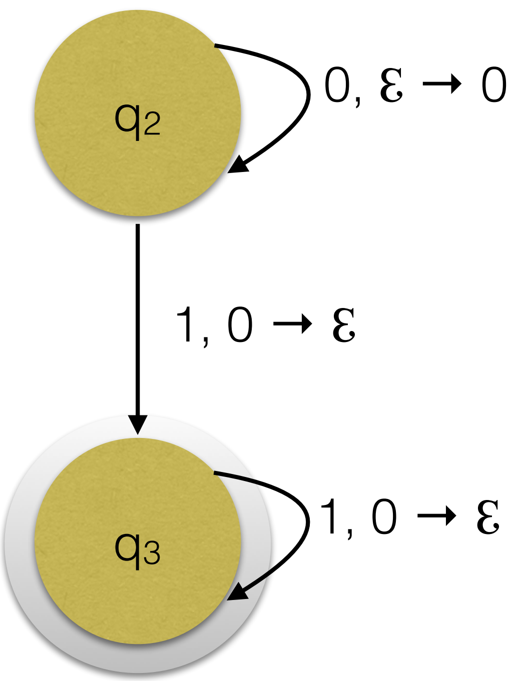




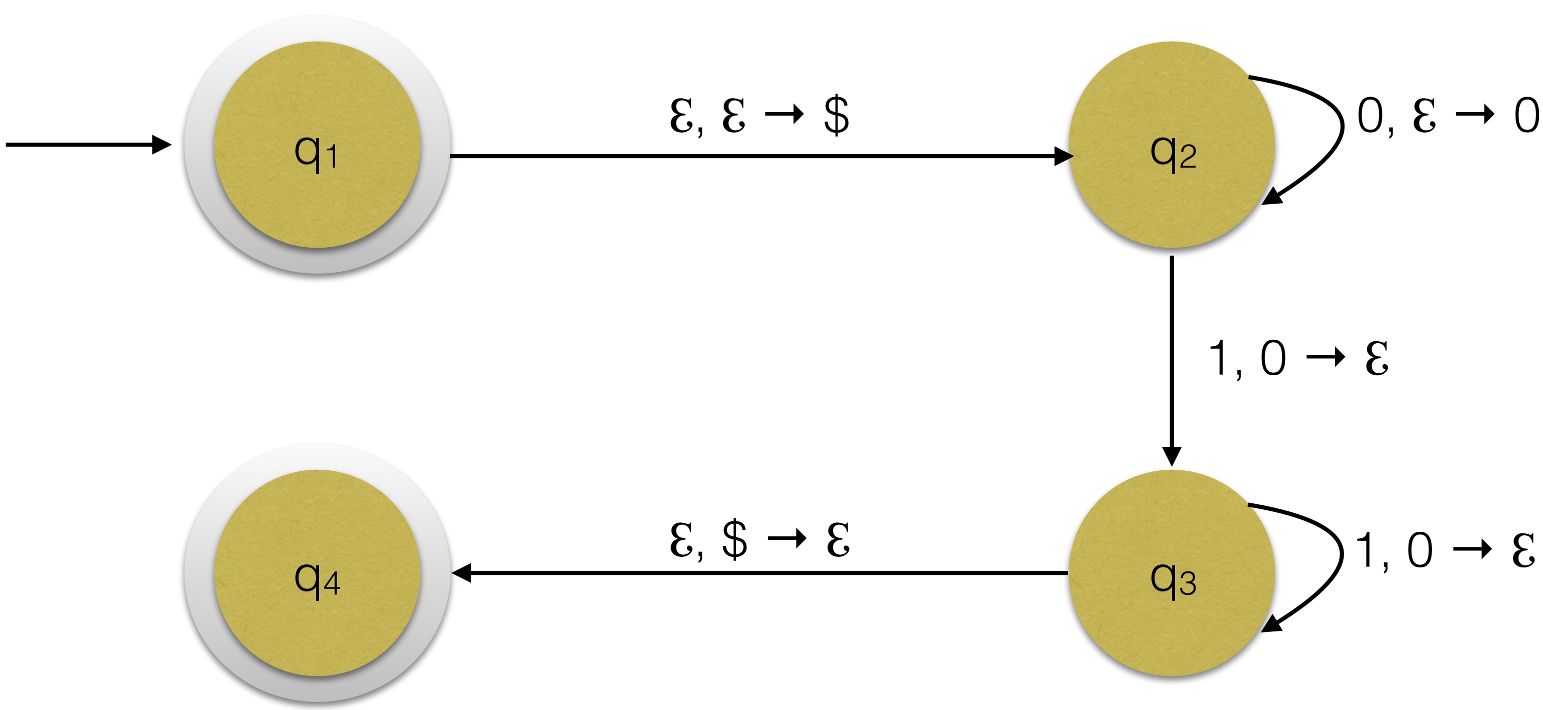
stack

Does this work?

$\{0^n 1^n \mid n \geq 0\}$



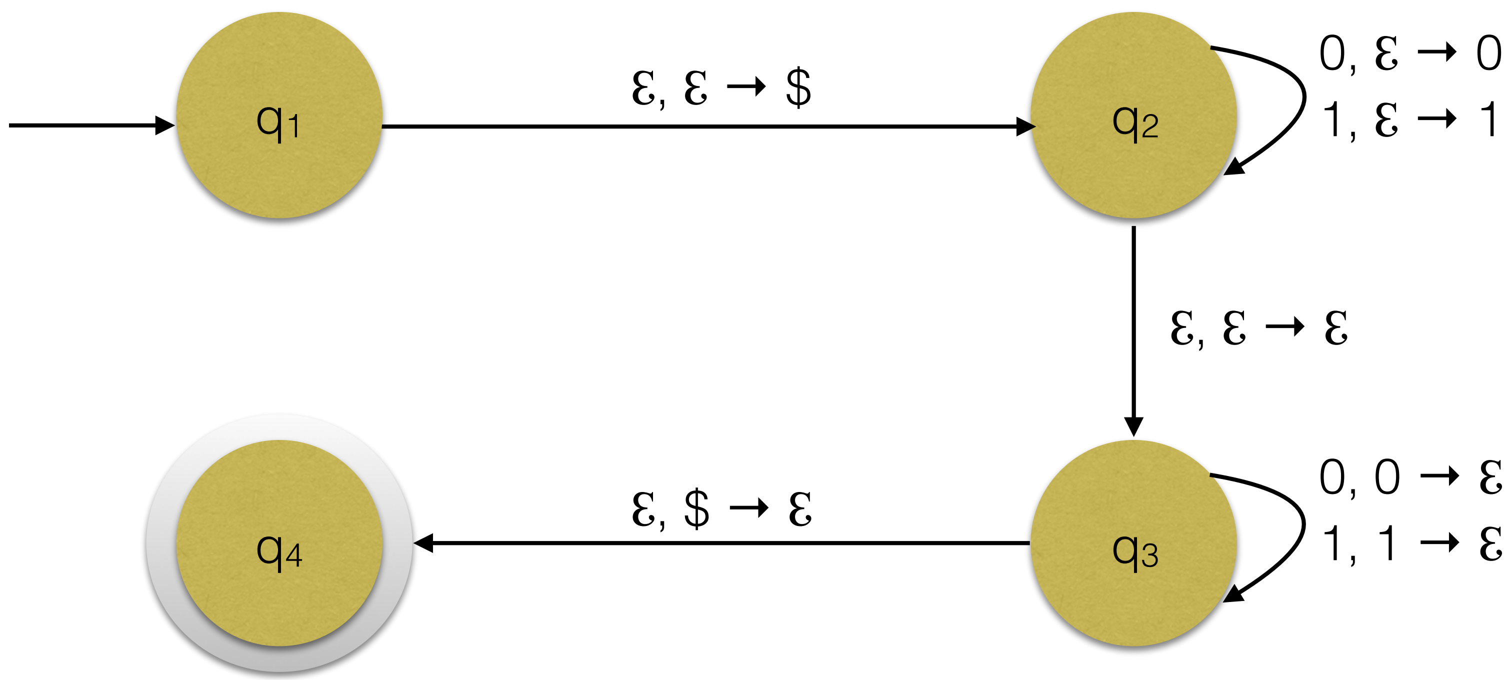
$\{0^n 1^n \mid n \geq 0\}$



PDA Design

$\{ww^R \mid w \in \{0, 1\}^*\}$

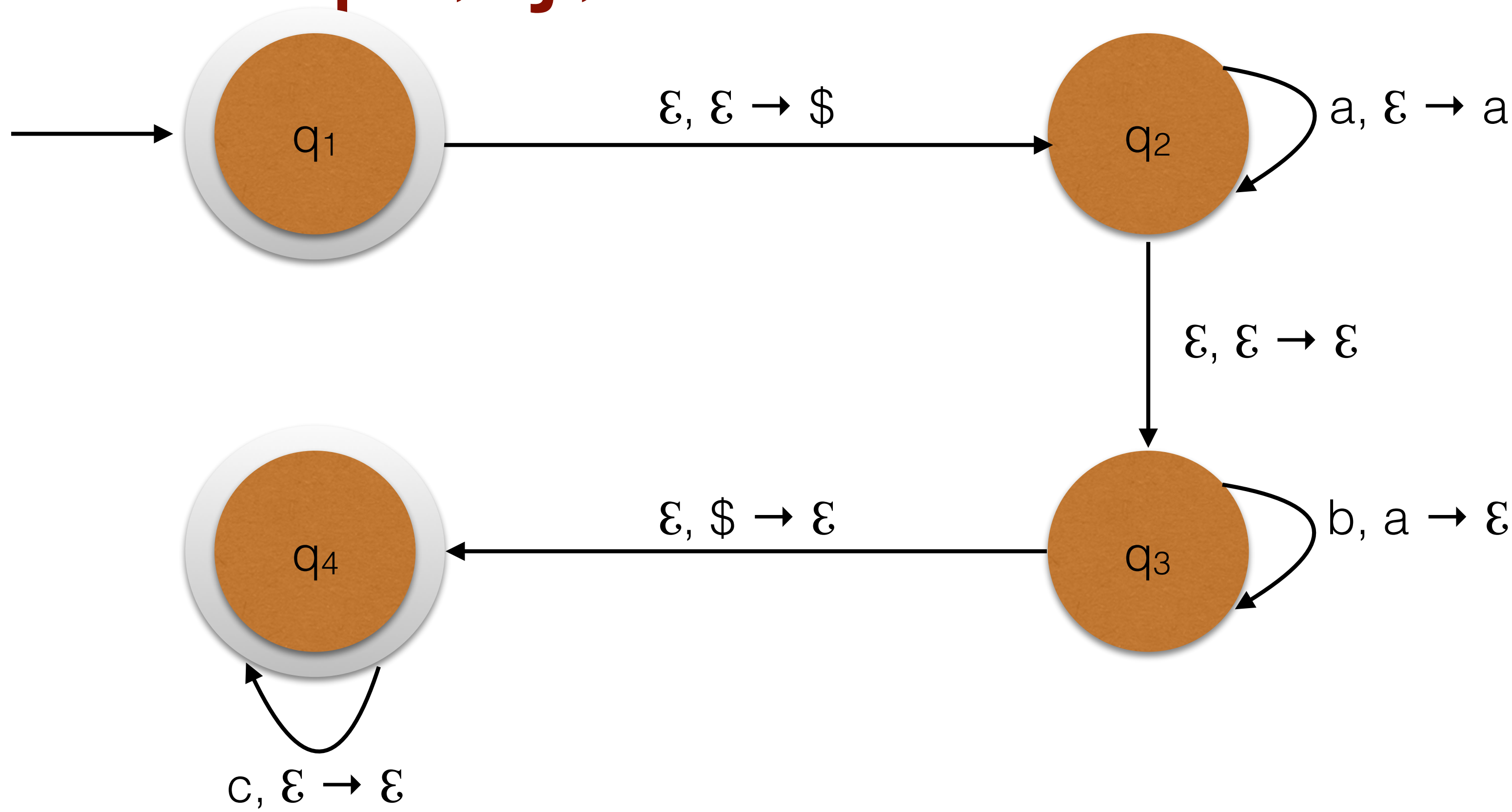
$\{ww^R \mid w \in \{0, 1\}^*\}$



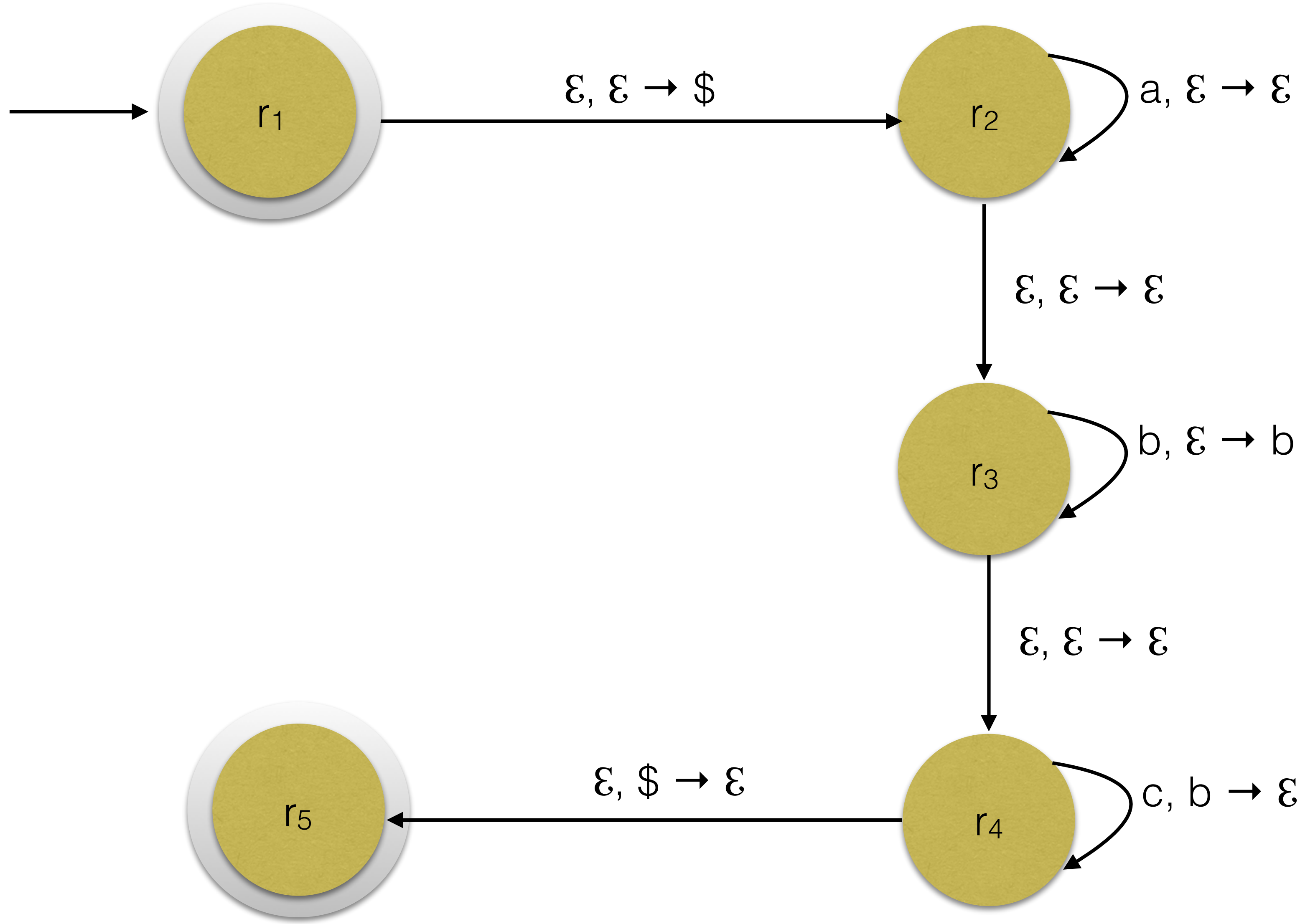
PDA Design

$\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j \text{ or } j = k\}$

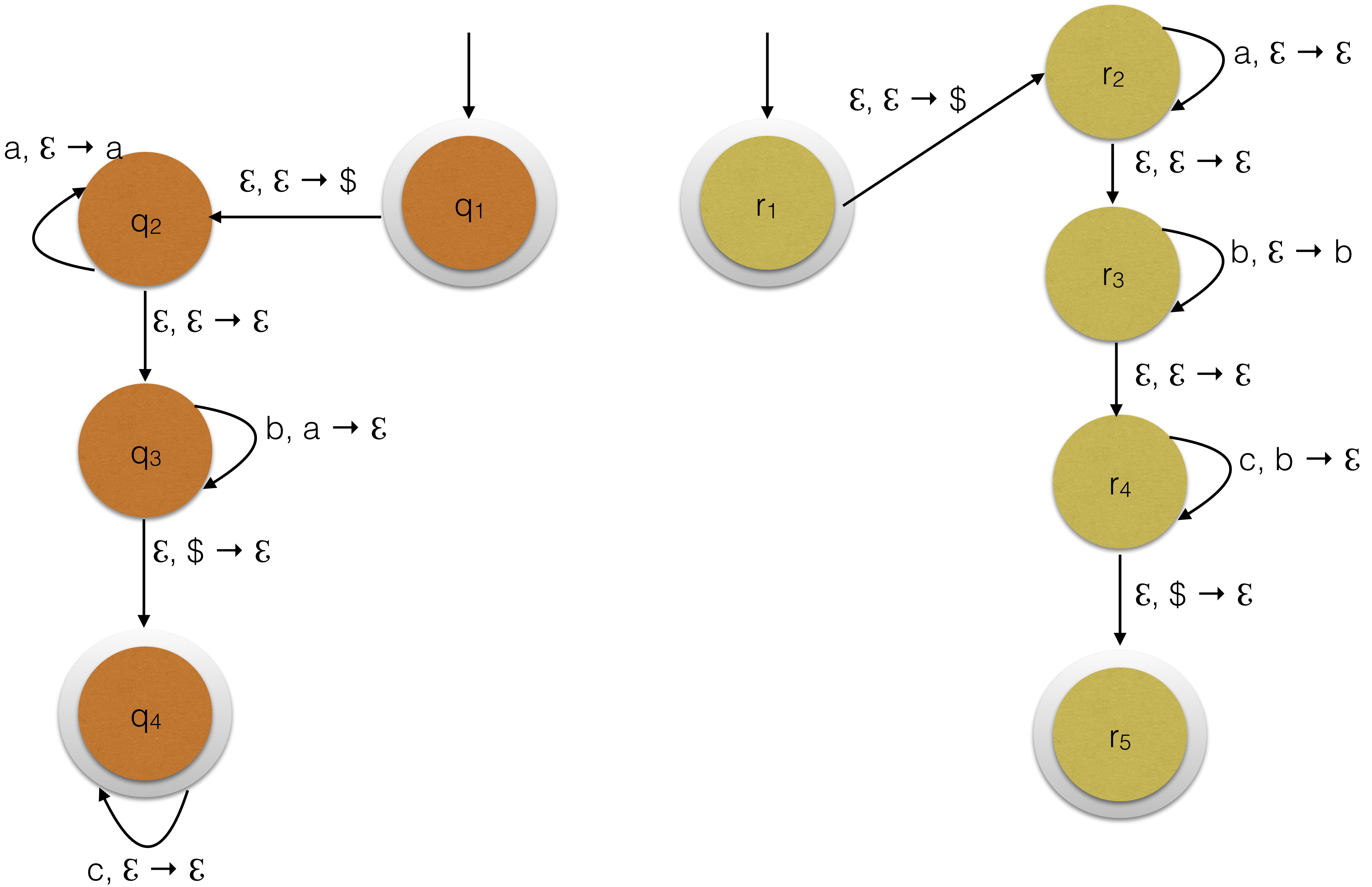
$\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j\}$



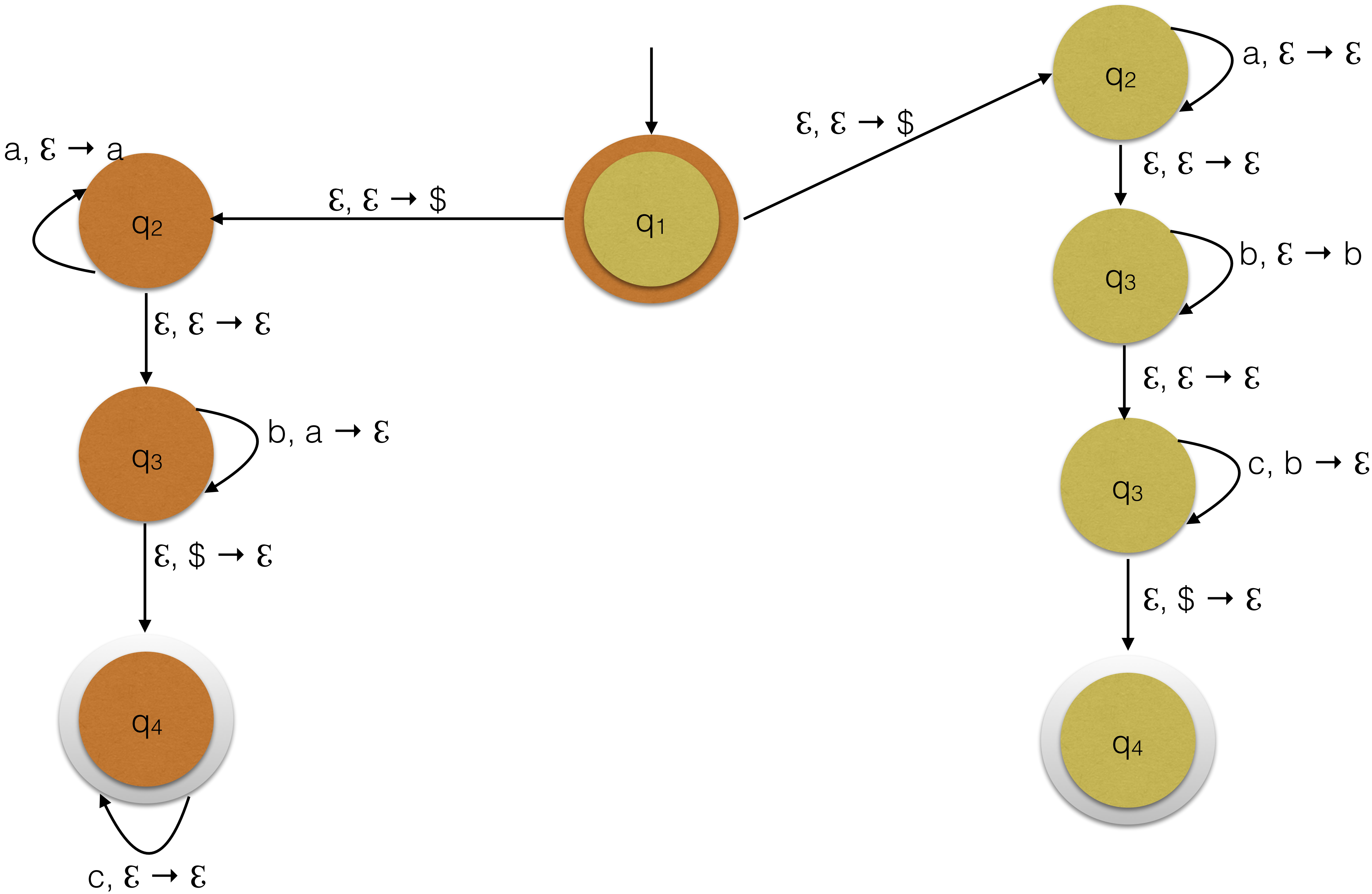
$\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } j = k\}$



$\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i=j \text{ or } j = k\}$



$\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i=j \text{ or } j = k\}$



Pumping Lemma:
Regular Languages

If A is a regular language,
then there is a pumping length p st
if $s \in A$ with $|s| \geq p$ then we can write $s = xyz$
so that

- $\forall i \geq 0 \; xy^iz \in A$
- $|y| > 0$
- $|xy| \leq p$

To prove $\{0^n1^n \mid n \geq 0\}$ is *not* regular using the Pumping Lemma

1. Suppose $\{0^n1^n \mid n \geq 0\}$ is regular
2. Call its pumping length p
3. Find string $s \in A$ with $|s| \geq p$. Let $s = 0^p1^p$
4. The pumping lemma says that for *some* split $0^p1^p = xyz$ *all* the following conditions hold
   - $\forall i \geq 0\ xy^iz \in A$
   - $|y| > 0$
   - $|xy| \leq p$

⇒ y is a non-empty string of 0s

To prove A is *not* regular using the Pumping Lemma

1. Suppose A is regular
2. Call its pumping length p
3. Find string $s \in A$ with $|s| \geq p$
4. The pumping lemma says that for *some* split $s = xyz$ *all* the following conditions hold
    - $\forall i \geq 0 \; xy^iz \in A$
    - $|y| > 0$
    - $|xy| \leq p$