Shanghai American School
An International Community

Puxi High School Examinations
Semester 1, 2009 – 2010

AP Calculus (BC)

Part 1

Wednesday, December 16th, 2009
12:45 pm – 3:15 pm

Time: 45 minutes

Teacher: Mr. Surowski

Testing Site: HS Gymnasium

Student Name:___________________________________

Instructions to the Candidate

- No food or drink to be brought into examination room.
- No cell phones/iPods are allowed during the examination at any time. You will be dismissed from the testing site if you are seen with one out. Please do not talk during the examination.
- If you have a problem please raise your hand and wait quietly for a teacher.
- Please do not open the examination booklet until directed to do so.
- Please ensure that you have the correct examination in front of you.
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- Write your name clearly in the space above when directed to do so.
- At the conclusion of your examination please refrain from speaking until you are outside the exam room as there may still be other examinations still in progress.
- Students are reminded that they are not permitted to leave the examination room early.

Special Instructions:

- Graphing Calculators NOT allowed.
- Be sure to answer ALL questions.
- Part 1 has **9 pages** including the cover page and the Student Bubble Sheet.

There are 23 questions in this part (1 point each). A .25 point penalty is in force for incorrect answers.

Multiple-Choice Bubble Sheet
AP Calculus (BC)—Fall, 2009

Name: ______________________________ **Date:** ____________

Part 1:

1. Ⓐ ● Ⓒ Ⓓ Ⓔ
2. Ⓐ Ⓑ Ⓒ Ⓓ ●
3. ● Ⓑ Ⓒ Ⓓ Ⓔ
4. Ⓐ Ⓑ ● Ⓓ Ⓔ
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21. Ⓐ Ⓑ ● Ⓓ Ⓔ
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23. Ⓐ ● Ⓒ Ⓓ Ⓔ

Part 2:

1. Ⓐ Ⓑ Ⓒ Ⓓ ●
2. Ⓐ ● Ⓒ Ⓓ Ⓔ
3. ● Ⓑ Ⓒ Ⓓ Ⓔ
4. Ⓐ Ⓑ Ⓒ ● Ⓔ
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11. ● Ⓑ Ⓒ Ⓓ Ⓔ
12. ● Ⓑ Ⓒ Ⓓ Ⓔ
13. Ⓐ Ⓑ ● Ⓓ Ⓔ

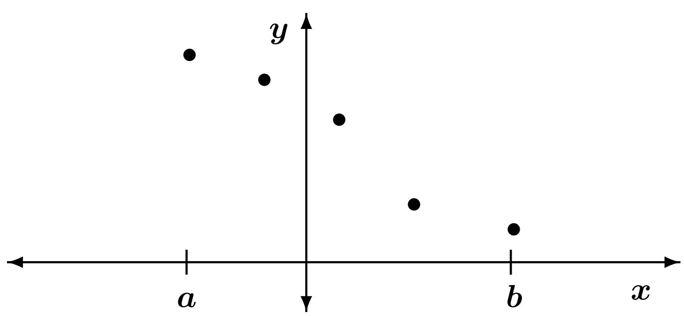
1. $\lim_{h \to 0} \dfrac{\sec(\pi + h) - \sec(\pi)}{h}$

(A) $= -1$ (B) $= 0$ (C) $= \dfrac{1}{\sqrt{2}}$ (D) is not defined (E) $= \sqrt{2}$

2. If $y = (x^3 + 1)^2$, then $\dfrac{dy}{dx} =$

(A) $(3x^2)^2$
(B) $2(x^3 + 1)$
(C) $2(3x^2 + 1)$
(D) $3x^2(x^3 + 1)$
(E) $6x^2(x^3 + 1)$

3. The points depicted below are on the graph $y = f(x)$ of a twice-differentiable function.



Consider the following statements.

I. There exists $x,\ a < x < b$ such that $f''(x) > 0$.
II. $f'(x) < 0$ for all $x,\ a < x < b$.
III. $f(x) > 0$ for all $x,\ a < x < b$.

(A) I only (B) II only (C) III only (D) I and II (E) I, II, and III

4. Which of the following limits defines the derivative of the function f at the point $x = a$:

(A) $\lim_{h \to a} \frac{f(h+a) - f(a)}{h}$

(B) $\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(\Delta x)}{\Delta x}$

(C) $\lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x}$

(D) $\lim_{x \to 0} \frac{f(x) - f(a)}{x - a}$

(E) $\lim_{x \to a} \frac{f(x+a) - f(x)}{x - a}$

5. The function f given by $f(x) = 3x^5 - 4x^3 - 3x$ is both increasing and concave up over which of the following intervals?

(A) $(-\infty, \sqrt{2/5})$ (B) $(-\sqrt{2/5}, 0)$ (C) $(-1, 1)$ (D) $(\sqrt{2/5}, \infty)$ (E) $(1, \infty)$

6. Define the function $F(x) = \int_1^{x^2} \frac{t\,dt}{1+t^2}$. Then $F'(x) =$

(A) $\frac{x^2}{1+x^4}$

(B) $\frac{1-x^4}{(1+x^4)^2}$

(C) $\frac{1-x^2}{(1+x^2)^2}$

(D) $\frac{2x^3}{1+x^4}$

(E) $\frac{2x^2}{1+x^2}$

7. If $y = \dfrac{e^x}{e^x - 1}$, then $y' =$

(A) $-\dfrac{e^x}{(e^x-1)^2}$ (B) $\dfrac{1}{e^x-1}$ (C) $-\dfrac{1}{(e^x-1)^2}$ (D) 0 (E) $\dfrac{e^x-2}{e^x-1}$

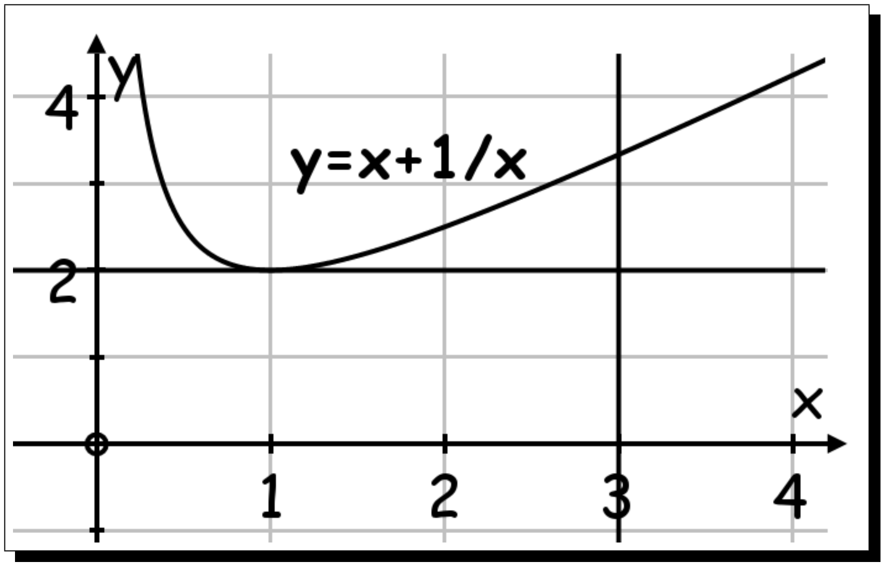
8. If $f(x) = 2xe^{-x},\ -\infty < x < \infty$, then f is concave up on

(A) $(-\infty, 0) \cup (1, \infty)$
(B) $(-\infty, 0) \cup (2, \infty)$
(C) $(-\infty, 1) \cup (2, \infty)$
(D) $(2, \infty)$
(E) $(-\infty, \infty)$

9. Let a be a positive constant. Then $2\displaystyle\int_{-a}^{a} \sqrt{a^2 - x^2}\, dx$

(A) 0 (B) $\dfrac{\pi}{2}$ (C) $\dfrac{\pi a^2}{2}$ (D) πa^2 (E) can't be evaluated

10. Find the area bounded by $y = x + 1/x$, and the lines $y = 2$ and $x = 3$ (see picture below):



(A) $\ln 3 + 4$ (B) $\ln 4 + 3$ (C) $\ln 3 - 4$ (D) $\ln 3$ (E) 4

11. The C&S law firm charges a variable rate for its consulting fees: after t hours, the consulting rate is $r(t) = 150(1 + e^{-t})$ dollars/hour. The average consulting rate for the first 10 hours is approximately

(A) $\$150$/hour
(B) $\$155$/hour
(C) $\$165$/hour
(D) $\$300$/hour
(E) $\$250$/hour

12. The function $f(x) = \dfrac{x^2 + 5x - 6}{\sqrt{x - 1}}$

(A) is continuous on the interval $[1, \infty)$.
(B) is continuous on the whole real line.
(C) is continuous on $[1, \infty)$ **provided that** we define $f(1) = 0$
(D) is continuous on $[1, \infty)$ **provided that** we define $f(1) = -5$
(E) cannot be extended to a continuous function on $[1, \infty)$

13. The line normal to the graph of $y = \sqrt{8 - x^2}$ at the point $(2, 2)$ has equation

(A) $y = x$
(B) $y = -2x + 6$
(C) $y = -x$
(D) $y = \frac{1}{2}x + 1$
(E) $y = x - 4$

14. Suppose that f is a function such that $f''(x) = 0$. Then f must be of the form

(A) f is a constant-valued function
(B) $f(x) = ax + b$, for suitable constants a, b
(C) $f(x) = ae^x + b$, for suitable constants a, b
(D) $f(x) = \dfrac{1}{a + bx}$, for suitable constants a, b
(E) $f(x) = ae^{bx}$, for suitable constants a, b

15. $\displaystyle\int_{-\pi}^{\pi} \frac{\cos x \, dx}{\sqrt{4 + 3\sin x}} =$

(A) $\frac{4}{3}$ (B) $-\frac{4}{3}$ (C) 0 (D) $\frac{4}{3}\sqrt{7}$ (E) $-\frac{4}{3}\sqrt{7}$

16. A circular conical reservoir, vertex down, has depth 20 ft and radius of the top 10 ft. Water is leaking out so that the level is falling at the rate of $\frac{1}{2}$ ft/hr. The rate, in cubic feet per hour, at which the water is leaving the reservoir when the water is 8 ft deep is

(A) 4π (B) 8π (C) 16π (D) $\frac{1}{4\pi}$ (E) $\frac{1}{8\pi}$

(You are given that the volume of a circular cone with base radius r and height h is Vol $= \frac{1}{3}\pi r^2 h$.)

17. Define the function $f(x) = \dfrac{L}{1 + Ce^{-kx}}$, where L, C and k are positive constants. Then $\lim_{x \to +\infty} f(x) =$

(A) 0 (B) L (C) does not exist (D) L/C (E) $L/(1+C)$

18. If $f(x) = \sqrt{1 + \sqrt{x}}$, then $f'(x) =$

(A) $\dfrac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$

(B) $\dfrac{1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$

(C) $\dfrac{1}{4\sqrt{1+\sqrt{x}}}$

(D) $\dfrac{-1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$

(E) $\dfrac{-1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$

19. $\displaystyle\int_0^{\sqrt{3\pi/2}} x \cos x^2 \, dx =$

(A) 1 (B) -1 (C) $\dfrac{1}{2}$ (D) $-\dfrac{1}{2}$ (E) 0

20. Given that $x^3 y + xy^3 = -10$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{3x^2y + y^3}{3xy^2 + x^3}$

(B) $-\dfrac{3x^2y + y^3}{3xy^2 + x^3}$

(C) $-\dfrac{x^2y + y^3}{xy^2 + x^3}$

(D) $3x^2 + 3xy^2$

(E) $-(3x^2 + 3xy^2)$

21. A 26-ft ladder leans against a building so that its foot moves away from the building at the rate of 3 ft/sec. When the foot of the ladder is 10 ft from the building, the top is moving down at the rate of r ft/sec, where r is

(A) $\frac{46}{3}$ (B) $\frac{3}{4}$ (C) $\frac{5}{4}$ (D) $\frac{5}{2}$ (E) $\frac{4}{5}$

22. If the function $f(x) = \begin{cases} 3ax^2 + 2bx + 1 & \text{if } x \leq 1 \\ ax^4 - 4bx^2 - 3x & \text{if } x > 1 \end{cases}$ is differentiable for all real values of x, then $b =$

(A) $-\frac{11}{4}$ (B) $\frac{1}{4}$ (C) $-\frac{7}{16}$ (D) 0 (E) $-\frac{1}{4}$

23. $\int_0^{\pi/4} x \sin 4x \, dx =$

(A) 0 (B) $\frac{\pi}{16}$ (C) $-\frac{\pi}{16}$ (D) $\frac{\pi}{4}$ (E) $-\frac{\pi}{4}$

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Semester 1, 2009 – 2010

AP Calculus (BC)

Part 2

Wednesday, December 16th, 2009
12:45 pm – 3:15 pm

Time: 40 minutes

Teacher: Mr. Surowski

Testing Site: HS Gymnasium

Student Name:____________________________________

Instructions to the Candidate

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Special Instructions:

- Graphing Calculators are allowed.
- Be sure to answer ALL questions.
- Part 2 has **7 pages** including the cover page and the Student Bubble Sheet.

There are 13 questions in this part (1 point each). A .25 point penalty is in force for incorrect answers.

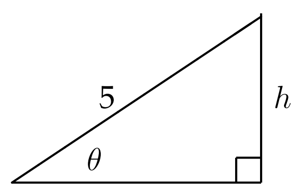
1. $\dfrac{d}{dx}\left(\displaystyle\int_{3x}^{2} f'(t)\,dt\right) =$

(A) $f(3)$ (B) $f'(3)$ (C) $-f'(3)$ (D) $-3f'(x)$ (E) $-3f'(3x)$

2. Which of the following is an equation of the line tangent to the graph of $f(x) = e^{2x}$ when $f'(x) = 10$

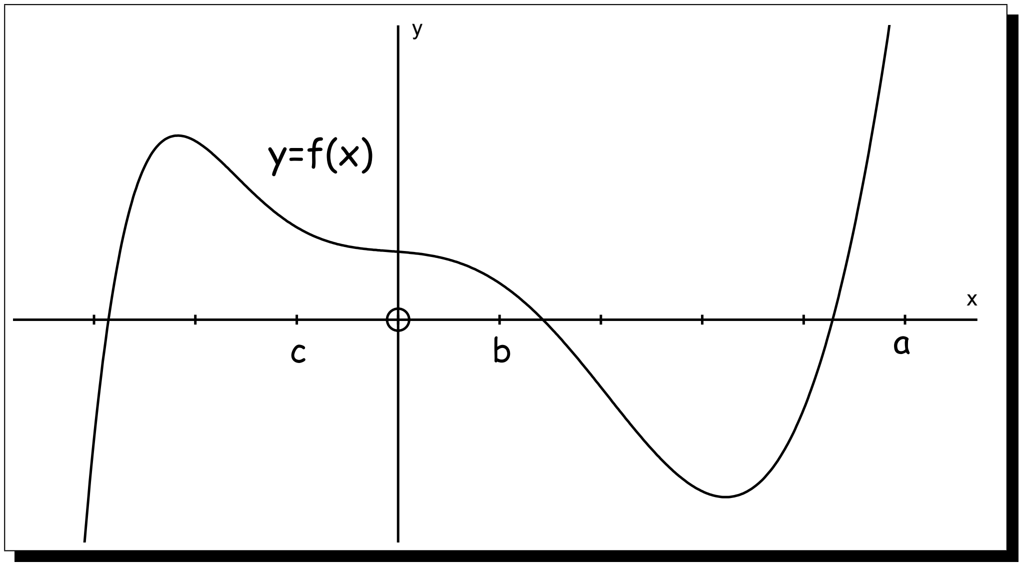
(A) $y = 10x - 8.05$
(B) $y = 10x - 3.05$
(C) $y = x - 3.05$
(D) $y = 10x - 11.5$
(E) $y = x - 8.05$

3. In the triangle below, the hypotenuse has fixed length 5, and θ is increasing at a constant rate of $\dfrac{2}{7}$ radians per minute. At what rate is the area of the triangle increasing, in units^2 per minute, when h is 3 units?



(A) 1 unit^2/min
(B) 2 unit^2/min
(C) 3 unit^2/min
(D) 4 unit^2/min
(E) 5 unit^2/min

4. The graph of $y = f(x)$ is depicted below.



Now set $A(x) = \int_0^x f(t)dt$. Then

(I) $A'(x) = 0$ for exactly two values of x in the interval $(0, a)$

(II) A is increasing on the interval $(0, b)$.

(III) A is concave up on the interval $(c, 0)$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only

5. Let $f(x) = (x^2 + 1)\ln x$. For which values of x is the slope of the line tangent to the curve $y = f(x)$ equal to 6.43?

I. 0.15

II. 2.27

III. 2.71

(A) I only (B) II only (C) III only (D) I and II (E) I and III

6. Values of a temperature function have been tabulated to the right:

x	0	1	5	6	8
$T(x)$	100	90	75	60	40

Using a trapezoidal approximation, estimate the average value of T over the interval $[0, 6]$

(A) 88.75 (B) 82.08 (C) 79.06 (D) 57.34 (E) 42.09

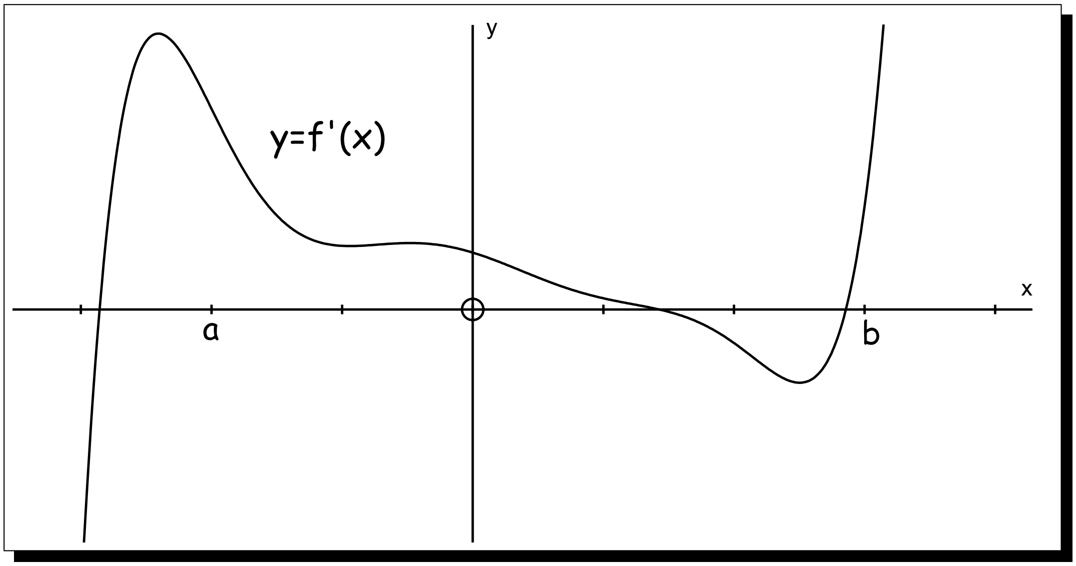
7. Let $f(t) = \dfrac{1}{1 + 2e^{-t}}$, $t > 0$. Then f has a point of inflection where $t =$

(A) 0.693
(B) 0
(C) 1
(D) 0.307
(E) There are no points of inflection for f on the interval $(0, \infty)$

8. Let $f(x) = e^{-x} \sin x$. How many relative maxima does f have on the open interval $(0, 4\pi)$

(A) none (B) one (C) two (D) three (E) four

9. The graph of the derivative f' of the function f is depicted below:



Which of the following describes all relative extrema of f on the interval (a, b)?

(A) One relative maximum and relative minimum
(B) Two relative maxima and one relative minimum
(C) One relative maximum and no relative minima
(D) No relative maxima and two relative minima
(E) One relative maximum and two relative minima

10. Define the function $F(x) = \displaystyle\int_2^x \frac{dt}{1+t^2}$. Then the value(s) of x at which the slope of the line tangent to the graph of $y = F(x)$ is 0.495 is

(A) 1.010
(B) -1.010
(C) -1.010 and -0.291
(D) ± 1.010
(E) There are no such values of x

11. Use differentials to approximate the change in the volume of a sphere when the radius is increased from 10 to 10.02 cm.

(A) 25.133 (B) 25.233 (C) 1256.637 (D) 1261.669 (E) 4213.973

12. If $y = 3x - 7$ and $x \geq 0$, what is the minimum product of x^2y?

(A) -5.646 (B) 0 (C) 1.555 (D) 2.813 (E) 3.841

13. The minimum distance from the point $(4, 0)$ to the parabola with equation $y = x^2$ is

(A) 2.678 (B) 10.212 (C) 3.141 (D) 0.841 (E) 0.917

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AP Calculus (BC)

Part 3

Wednesday, December 16th, 2009
12:45 pm – 3:15 pm

Time: 30 minutes

Teacher: Mr. Surowski

Testing Site: HS Gymnasium

Student Name:___________________________________

Instructions to the Candidate

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Special Instructions:

- Graphing Calculators are allowed.
- Be sure to answer ALL questions.
- Part 3 has **5 pages** including this cover page.

There are 2 free-response questions in this part (9 points each).

Part 3: Free-Response Questions—**30 minutes**, calculators allowed. There are **two problems**; each problem is worth **9 points**.

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

1. A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

   (a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.

   (b) Find a trapezoidal approximation of your integral in part (a).

   (c) Using correct units, explain the meaning of $\pi \int_{125}^{275} \left(\frac{B(x)}{2}\right)^2 dx$ in terms of the blood vessel.

   (d) Explain why there must be at least one value x, for $0 < x < 360$, such that $B''(x) = 0$.

(a) Avg. radius $= \frac{1}{720}\int_0^{360} B(x)\,dx$ (mm). (Note that $B(x)$ measures diameter, and so $\frac{1}{2}B(x)$ measures radius.) $\boxed{\text{2 points}}$

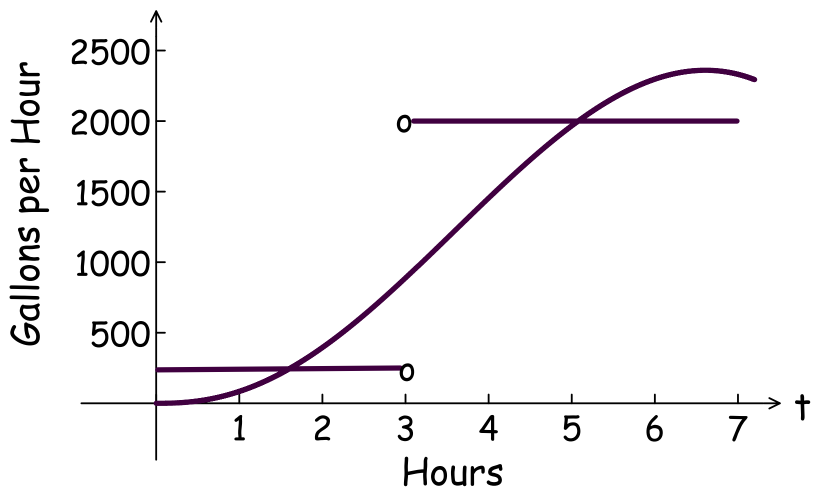
(b) We use the trapezoidal approximation

$$\begin{aligned}\int_0^{360} B(x)\,dx &\approx \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n)\\ &= 9780\end{aligned}$$

Therefore, Avg. radius $= \frac{1}{720}\int_0^{360} B(x)\,dx \approx \frac{1}{720} \times 9780 \approx 13.58$ mm. $\boxed{\text{2 points}}$

(c) The integral $\pi \int_{125}^{275} \left(\frac{B(x)}{2}\right) dx$ represents the total volume, in mm^3 of the 150 mm of blood vessel from $x = 125$ to $x = 275$. $\boxed{\text{2 points}}$

(d) By the Mean Value Theorem, there must exist a value c, where $60 < c < 120$ with $B'(c) = \frac{28-30}{120-60} = -\frac{1}{30}$. Similarly, there must exist a value d, $240 < d < 300$ satisfying $B'(d) = \frac{24-26}{300-240} = -\frac{1}{30}$. Applying the Mean Value Theorem to B' now gives a value x, $0 < 240 < c < x < d < 300 < 360$ with $B''(x) = \frac{B'(d)-B'(c)}{d-c} = 0$. $\boxed{\text{3 points}}$



2. The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

   (i) The rate at which water enters the tank is $f(t) = 100t^2 \sin\left(\sqrt{t}\right)$ gallons per hour for $0 \le t \le 7$.

   (ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases} \text{ gallons per hour.}$$

The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$ are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

(a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

(c) For $0 \le t \le 7$, at which time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

(a) During the time interval $0 \le t \le 7$ (hours) the amount of water entering the tank is given by the integral $\int_0^7 f(t)\,dt \approx 8263.8 \approx 8264$ gallons. $\boxed{\text{2 points}}$

(b) The water in the tank is decreasing when $g(t) > f(t)$. This happens on the interval $0 \le t < 1.62$ (hours) and on the interval $3 < t < 5.08$ (hours). $\boxed{\text{3 points}}$

(c) From (b) we conclude that the water in the tank is increasing on the interval $1.62 < t < 3$ and on the interval $5.08 < t < 7$ (hours). Therefore, we conclude that the water in the tank will be at a maximum either at $t = 0,\ t = 3$, or at $t = 7$.

At $t = 0$ hours there is 5000 gallons;

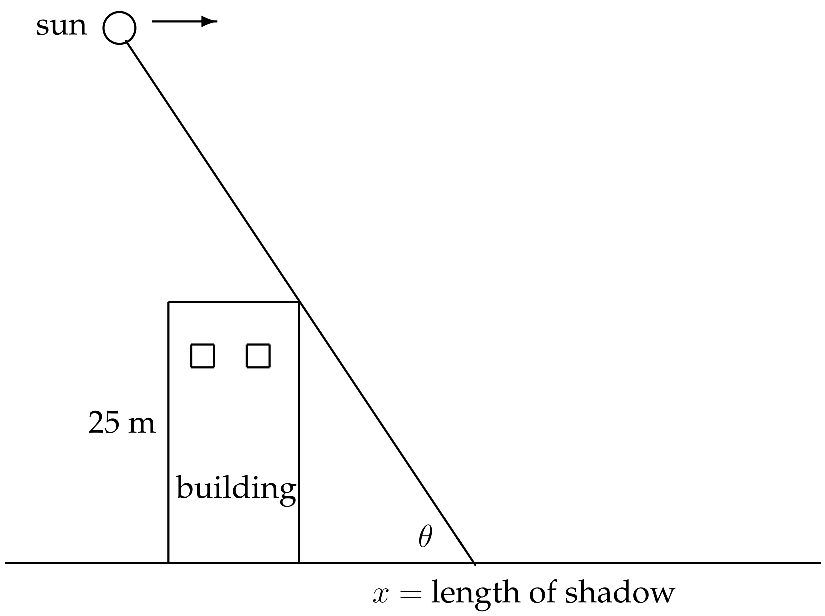
at $t = 3$ hours, there is $5000 + \int_0^3 (f(t) - g(t))\,dt \approx 5000 + 876.6 - 750 = 5126.6$ (gallons);

at $t=7$ gallons there is $5126.6+\int_3^7 (f(t)-g(t))\,dt \approx 5126.6+7387.2-2000\times 4 = 4513.8$ (gallons).

Therefore the maximum amount of water in the tank is $5126.6 \approx 5127$ gallons, which happens at time $t=3$ hours. $\boxed{\text{4 points}}$

Part 4: Free-Response Questions—**30 minutes**, no calculators allowed. There are **two problems**; each problem is worth **9 points**.

1. The picture below depicts the morning sun shining over a 25 m tall building, casting a shadow on the ground, as indicated. The indicated variables, θ and x, are therefore both functions of time t.



(a) Write an equation relating the variables θ and x.

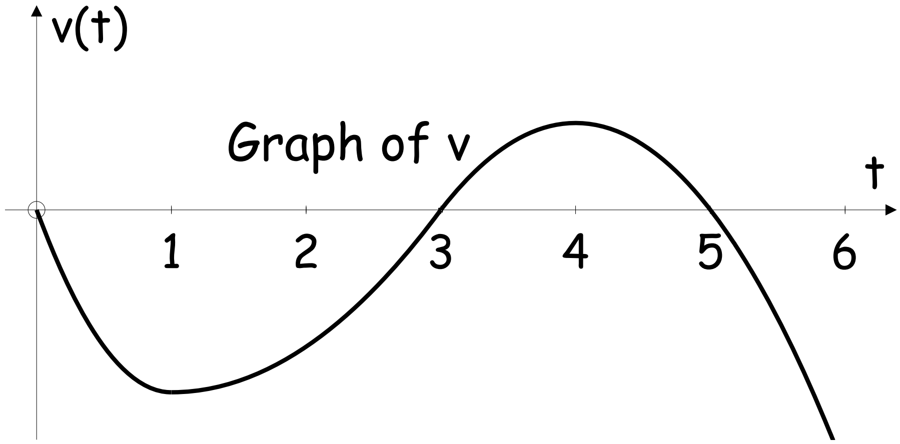
(b) If the position of the sun relative to the building is as indicated, determine the signs of $\frac{d\theta}{dt}$ and $\frac{dx}{dt}$ (i.e., whether they are positive or negative) and explain why.

(c) Assume that when $\theta = 60^\circ$, then θ is increasing at a rate of $.005$ radians/min. Compute the rate (measured in centimeters/min) at which the shadow is decreasing.

(a) $\boxed{\frac{x}{25} = \cot\theta}$ (note that x is measured in meters.) . $\boxed{\text{2 points}}$

(b) As the sun is approaching a point directly over the building, it is clear that θ is increasing, which implies that $\boxed{\frac{d\theta}{dt} > 0}$. At the same time, it's clear that x is decreasing, forcing $\boxed{\frac{dx}{dt} < 0}$. $\boxed{\text{3 points: answer, answer, reason}}$

(c) From $\frac{x}{25} = \cot\theta$ (part (a)), we have that $\frac{dx}{dt} = -25\csc^2\theta \cdot \frac{d\theta}{dt} \overset{\theta=\pi/3}{=} -25\csc^2(\pi/3)(.005) \approx$ 0.167 m/min. Therefore, $\boxed{\frac{dx}{dt} = 16.7 \text{ cm/min}}$ $\boxed{\text{4 points}}$



2. A particle moves along the x-axis so that its velocity at time t, for $0 \leq t \leq 6$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$ and $t = 5$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2. respectively. At time $t = 0$, the particle is at $x = -2$.

   (a) For $0 \leq t \leq 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

   (b) For how many values of t, where $0 \leq t \leq 6$, is the particle at $x = -8$? Explain your reasoning.

   (c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(a) The particle is moving to the left on the interval $0 < t < 3$ and on $5 < t < 6$. Therefore the particle will be farthest to the left either at $t = 3$ or at $t = 6$. At $t = 3$, the particle is at position $x(3) = -2 + \displaystyle\int_0^3 f(t)\,dt = -2 - 8 = -10$. At time $t = 6$, the particle is at position $x(6) = -2 + \displaystyle\int_0^6 v(t)\,dt = -2 - 8 + 3 - 2 = -9$. Therefore, the particle is furthest to the left at time $\boxed{t = 3}$. $\boxed{\text{3 points}}$

(b) We are trying to solve the equation $-8 = -2 + \displaystyle\int_0^t v(s)\,ds$. We note that x deceases from an initial value of $x = -2$ to the value $x(3) = -2 - 8 = -10$. On the interval $3 < t < 5$, x increases from -10 to $-10 + 3 = -7$. Thus, during the interval $0 < t < 5$, x has assumed the value -8 twice. Finally, on the interval $5 < t < 6$, x decreases from -7 to $-7 - 2 = -9$, and so x will assume the value -8 a $\boxed{\text{third}}$ time. $\boxed{\text{4 points}}$

(c) On the interval $2 < t < 3$ the velocity of the particle is negative, but with a positive acceleration. This implies that the speed is $\boxed{\text{decreasing}}$. $\boxed{\text{2 points}}$

Shanghai American School
An International Community

Puxi High School Examinations
Semester 1, 2009 – 2010

AP Calculus (BC)

Part 4

Wednesday, December 16th, 2009
12:45 pm – 3:15 pm

Time: 30 minutes

Teacher: Mr. Surowski

Testing Site: HS Gymnasium

Student Name:______________________________________

Instructions to the Candidate
- No food or drink to be brought into examination room.
- No cell phones/iPods are allowed during the examination at any time. You will be dismissed from the testing site if you are seen with one out. Please do not talk during the examination.
- If you have a problem please raise your hand and wait quietly for a teacher.
- Please do not open the examination booklet until directed to do so.
- Please ensure that you have the correct examination in front of you.
- No pencil cases allowed; bring only the writing materials you need into the examination room.
- Write your name clearly in the space above when directed to do so.
- At the conclusion of your examination please refrain from speaking until you are outside the exam room as there may still be other examinations still in progress.
- Students are reminded that they are not permitted to leave the examination room early.

Special Instructions:
- Graphing Calculators are NOT allowed.
- Be sure to answer ALL questions.
- Part 4 has **5 pages** including this cover page.

There are 2 free-response questions in this part (9 points each).