Pythagoras Theorem
Constructivist Lesson Plan

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Grade Level: 8

SCO: By the end of grade 8 students will be expected to demonstrate an understanding of the Pythagorean relationship, using models.

Pre-Discovery Phase:

- Historical Context
    - Pythagoras and Pythagoreans
    - Class Activity

Core-Discovery Phase:

- Meghan Trenholm's Manipulative Presentation
- Pythagorean Puzzle Worksheet

Post-Discovery Phase:

- Half Circles Worksheet
- Magic Triangle Worksheet

Enrichment Phase:

- Option 1: Pythagorean Tree
- Option 2: Golden Rectangle/Spiral

Pre-Discovery Phase: Historical Context

Objective:

The objective of the lesson is to introduce students to the personal history of Pythagoras and some elements of interest regarding the members of his school, the Pythagoreans. Students will use this information to assist in setting the context for later examination of Pythagoras' work with the lengths of the sides of a triangle.

Materials:
Teacher notes on Pythagoras and his school, PowerPoint presentation of notes, poster board, markers.

Advance Preparation:
Prepare PowerPoint slides, predetermine number and size of groups in the class, and obtain enough poster boards and markers for each group.

Instruction:

1. Inform students that over the next four classes we will be examining a mathematical concept developed by Pythagoras, an ancient Greek mathematician and thinker.
2. Inform students that in this class we examine the history of Pythagoras and the group of followers that surrounded him, as well as some of the more interesting characteristics of the group.
3. Introduce the life of Pythagoras PowerPoint slideshow. Elaborate on certain points of interest. (Appendix 1)
4. Introduce PowerPoint slideshow about characteristics of the Pythagorean school, touching on group philosophy, emblems, accomplishments and strictures. (Appendix 2)
5. Introduce group work and presentation. Form the class into 5 groups, giving each a poster board and a set of markers. Tell them that each group is a particular school that follows the teachings of a scholarly master, similar to the Pythagoreans, and must complete the following tasks:
    - Create a name for your group. The name may be based on an actual historical figure, or can be made up.
    - Develop a one-sentence group philosophy regarding the nature of the world and the relationships contained within it.
    - Create an emblem or symbol for the group, like the pentagram for the Pythagoreans. Explain the significance of the emblem.
    - Make a list of 5 peculiar guidelines or rules that each member of your group must follow, including possible punishments for breaking the rules.

- Choose a mathematical formula or concept that your group claims to have discovered. Examples include: area of a circle, area of a triangle, pi, volume of a cube etc.

Allow the students 20 minutes to complete their group work. The work must be recorded onto a poster board, and will then be presented to the rest of the class, along with an explanation of their material.

6. Conclude the class with a brief discussion about the importance of examining and understanding relationships in the world and explaining them using mathematics.

Core Discovery Phase

Refer to Meghan Trenholm's Manipulative Presentation

- http://people.stu.ca/~pheeney/5873ManipPythagoras.pdf

Objective:

- When students finish with the manipulative, there will likely still be some doubt as to the validity of the fact that the area of the small square plus the area of the medium square equals the area of the large square. This is due to answers that will be found that are *almost* correct, but a 90° angle was not formed therefore the results do not follow the correct pattern.
- This activity will eliminate experimental errors and confirms the pattern that was observed in the manipulative presentation.

Materials:

- Pythagorean Puzzle Worksheet (Appendix 3)
- Scissors
- Smart board worksheet for Pythagorean Puzzle 1

Advance Preparation:

- Photocopy Pythagorean Puzzle

Instruction:

- Have the students cut out the small square and the medium square.
- The medium square will need further cutting along the dotted lines.
- Have the students arrange the pieces they cut out to fit exactly into the larger square.
- Ask a student to demonstrate the answer on the Smart Board.
- This activity will eliminate any doubt that was formed during the manipulative activity.

Post Discovery (Reinforcement and Application):

Objective

- Reinforce idea of perfect squares.
- Reinforce idea of two smaller squares adding up to larger square.
- The idea of other shapes working for Pythagoras Theorem.
- Manipulation of numbers and squares to prove theorem.

Materials

- The "Half Circle" (Appendix 4) handout & overhead/ smart board.
- The "Magic Triangle" (Appendix 5) handout & overhead/ smart board.
- Pencils.
- Calculators if needed.

Advanced Preparation

- Photocopy enough "Half Circle" worksheets for everyone in class including a copy for overhead/ smart board.
- Photocopy enough "Magic Triangle" worksheets for everyone in class including a copy for overhead/ smart board.

Instructions:

- Begin class with a discovery video: http://www.youtube.com/watch?v=Z68uPQU2v3o

- Briefly review what you learned in previous class:
  - Question on whether this applies to other shapes beside squares (list possible shapes)
  - Can happen with many shapes but for purpose of class will use half circles.
  - Pass out worksheet, "Half Circle", put up copy on overhead/ smart board. Work through first question together; reviewing area of a circle $A = \pi r^2$ and area of a half circle $A = \pi r^2 / 2$.
  - Ask class what they found; how could you prove it to work with other shapes?

- Following talk about different shapes proving Pythagoras Theorem pass out "Magic Triangle" worksheet and place on overhead/ smart board.
  - Have class complete first question on their own.
  - Complete question 2(a) as a class mentioning that they should pretend the middle number is a square.
  - Have class complete the rest of the questions on their own.
  - Go over question 2(d) as a class. Reminder of the many amounts of answers the students may have come up with. If struggle ask class to add up all numbers in each box (refer to Appendix 5 for answer.)

Post Discovery (Enrichment Option 1): Pythagorean Tree

Objective:

- To reinforce the Pythagorean concept.
- To provide the students with an additional activity on Pythagoras. This also incorporates art into to the math lesson.

Materials:

- 'Creating your own Pythagorean Tree' handout (Appendix 6)
- Coloring tool.

Advance Preparation:

- Photocopy 'Creating your own Pythagorean Tree' handout for class.

Instruction:

- Begin the class with an online application showing how Pythagoras' concept can create different designs artistically.
- http://www.ies.co.jp/math/java/geo/pytree/pytree.html
- Inform students that they will be creating their own Pythagorean Tree, but using different angles then the example shown.
- Hand out the 'Creating your own Pythagorean Tree' handout.
- Collect trees for assessment to see if the students were able to confirm that the sum of the area of the smaller squares equals the area of the larger square in building the Pythagorean Tree.
- Return to online application and move the red dot so that it shows the tree constructed in class. You can now show the class what the tree would look like if they kept going.

Post Discovery (Enrichment Option 2): Golden Rectangle

Objective

- Reinforce the idea of the concrete formulas in math.
- Recognizing that math has been used for years for architecture.
- Pythagorean Theorem was not the only formula discovered around Pythagoras' time.

Materials

- The "The Golden iPod" (Appendix 7) handout & overhead/ smart board.
- The "The Golden Spiral" (Appendix 8) handout & overhead/ smart board.
- Pencils & Crayons, Colored Markers or Pencil Crayons
- Compass

Advanced Preparation

- Photocopy enough "The Golden Spiral" & "The Golden iPod" worksheets for everyone in class including a copy for overhead/ smart board.
- Supply the class with enough coloring utensils to complete the activity.

Instructions:

- Begin class with a discovery video: http://www.youtube.com/watch?v=2zWivbG0RIo

- Briefly review what you learned about Pythagoras at the beginning of the lesson and how his discovery helped with the construction of buildings in his time. Introduce the Golden Rectangle as another idea that the Pythagorean's used as a construction tool. The rectangle is introduced and the students are told that the formula for the ratio of the rectangle is $\frac{a+b}{a}$.
- To introduce the Golden Rectangle and how it is used in everyday objects, we'll use the example of the iPod touch. Students are given the "The Golden iPod" handout and are told to use the instructions which are given to them on the handout. The students then create the Golden Rectangle within the picture on the handout and can plainly see how the rectangle appears in the structure of this popular device. After discovering the Golden Rectangle the students then calculate the ratios between the two given lengths, which then equals the Golden Ratio of approximately 1.618.
- Following the discovery of the Golden Ratio, the students are given another activity of finding various examples of the Golden Rectangle where they can confirm that this ratio of approximately 1.618 holds.
- The class is given the "The Golden Spiral" handout, which has the first rectangle drawn out for them and are prompted on what to do next. They will continually add rectangles to the sheet, which will each create a new

Golden Rectangle. The students are encouraged to color each new rectangle they create a different color so they can easily decipher one rectangle form the other and see what rectangles are combined to create the new rectangle. When the students come to a point when they cannot fit anymore rectangles on their handout, they are then prompted to calculate the ratios within the rectangles they have created and will discover that the ratio holds for all the rectangles on their handout. The students will then create a spiral within their Golden Rectangles with their compasses and are asked to brainstorm what other objects around the world you have this Golden Ratio attributed to them.

Appendices

Appendix 1

The Father of Numbers:

Pythagoras and the Pythagoreans

Pythagoras

-Pythagoras was born in Samos Greece between 580-572 BC, and he died between 500-490 BC.

-He was considered one of the great mathematicians, philosophers, scientists and mystics of ancient Greece. He has been called "The Father of Numbers".

-As a young man he fled his home to escape a dictator. Pythagoras traveled to many places, including Egypt and Phoenicia, studying local knowledge and impressing people with his ideas.

-Later he settled in Calabria (present-day Italy) where he established a secret society that followed his teachings. His followers were called "Pythagoreans".

-Pythagoras demanded that his followers lead lives of virtue based on his teachings. He gave equal opportunity to males and females.

-Pythagoras and his school went on to make many discoveries about the nature of the world, and the relationships hidden in all things, not only in mathematics, but in science, music and religion.

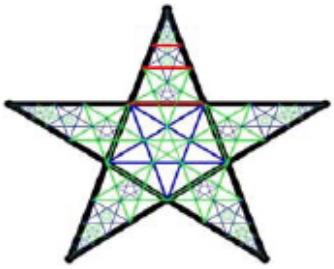
-None of Pythagoras' writings survive, so we don't know which discoveries were made by him personally.

-Over time, many interesting stories about him were told which became part of the legend of Pythagoras. Some of these include the belief that he had a golden thigh, an indication that he was related to the gods. Other believed that he could travel through time and space and communicate with plants and animals.

-Later in life, a noble family attempted to assassinate him. He escaped, and lived to the age of 90.

Appendix 2

The Pythagoreans



-The Pythagoreans were a very secretive group that followed the teachings of Pythagoras.

-They used the pentagram as their emblem, believing it to be an example of mathematical perfection.

-They believed that numbers were the foundation of the world, and that all relationships could be explained mathematically.

-Pythagoreans lived together, shared their belongings and followed a mainly vegetarian diet.

-They were divided into groups based on the work they chose. All discoveries were credited to Pythagoras as the leader of the school.

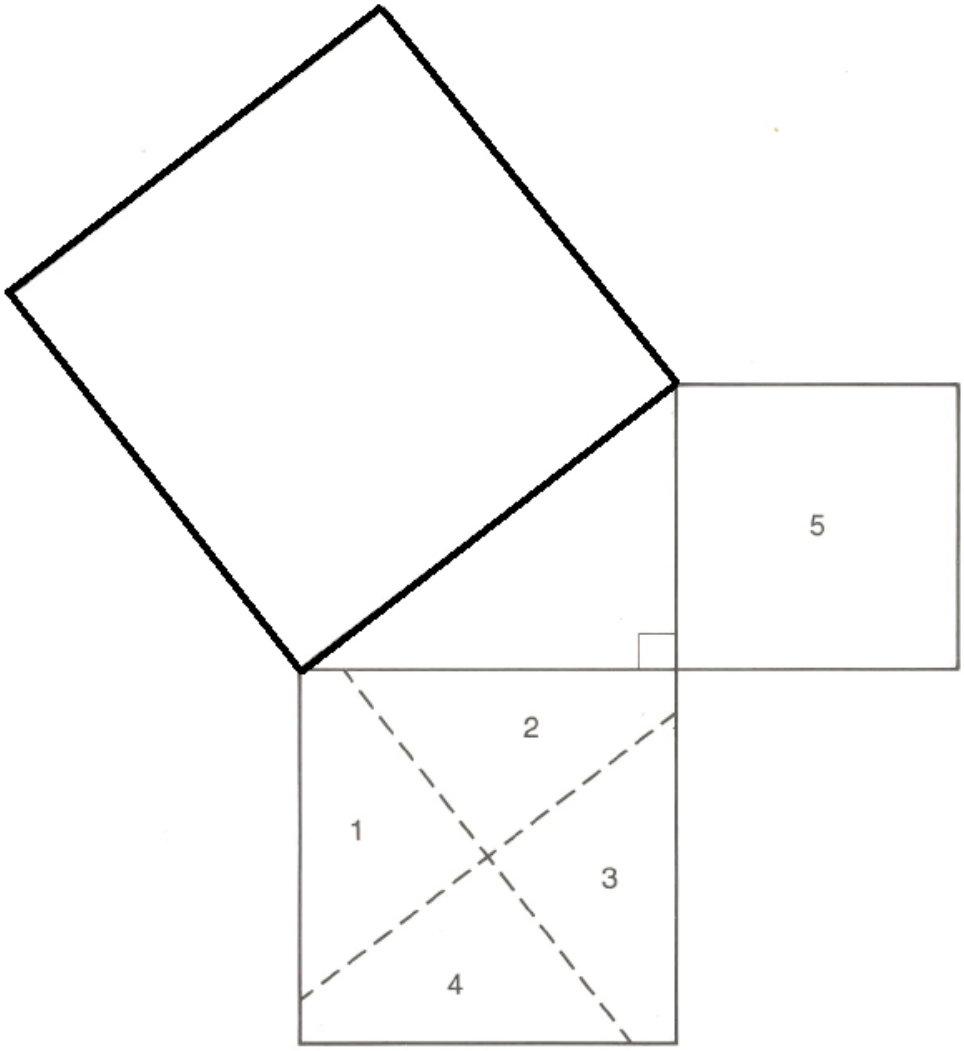
-The only people to actually see Pythagoras were his closest students.

-Pythagoreans followed a code of silence. They believed that if you didn't know what you were talking about, then you should say nothing. The punishment for breaking this rule was death.

-Pythagoreans also believed that their souls would return to Earth after they died. Their behaviour in life would dictate whether their souls would return in a better or worse position than before.

Appendix 3

Pythagorean Puzzle



Instructions:

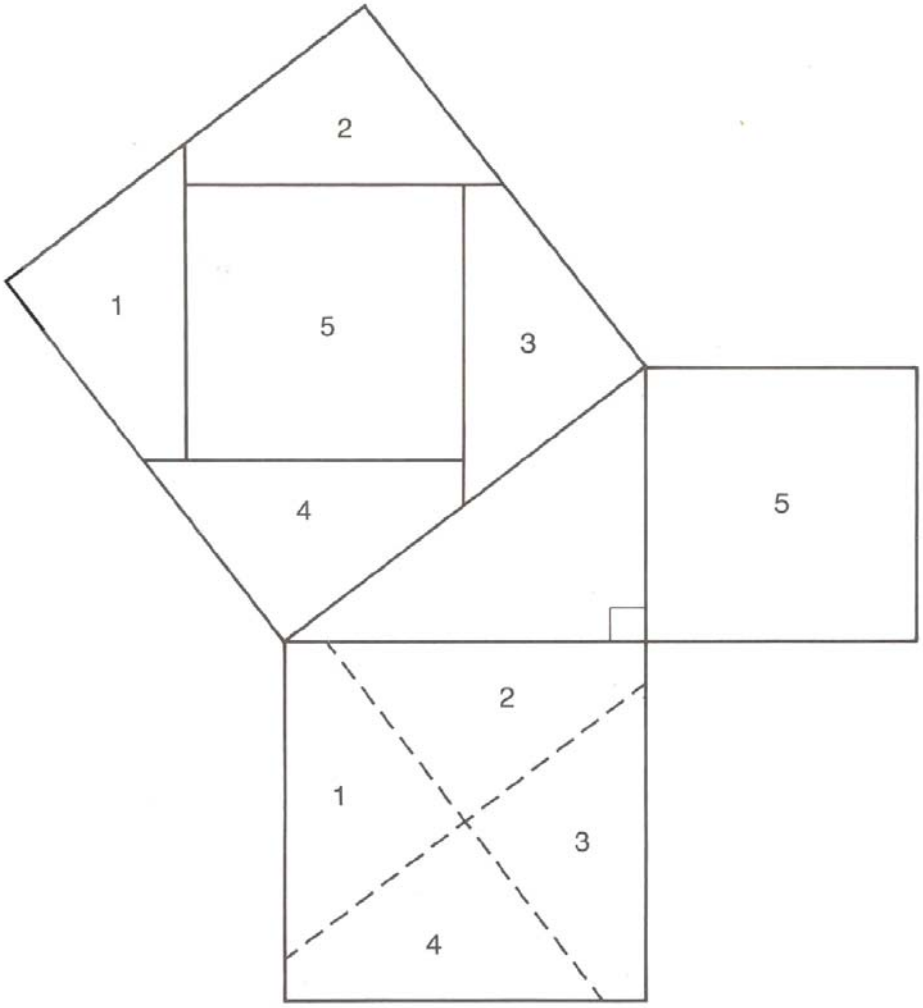
1) Cut off the small and medium squares. (Numbered 1-4 and 5)
2) Cut the medium square along the dotted lines.
3) Try to arrange the pieces (1-5) inside the larger, darker square.

What can you conclude from this exercise? _____

_____

_____.

Pythagorean Puzzle



Instructions:

1) Cut off the small and medium squares. (Numbered 1-4 and 5)
2) Cut the medium square along the dotted lines.
3) Try to arrange the pieces (1-5) inside the larger, darker square.
What can you conclude from this exercise? *I can conclude that the area of the small square and the area of the medium square is the same as the area of the large square in when arranged to form a right angle triangle.*

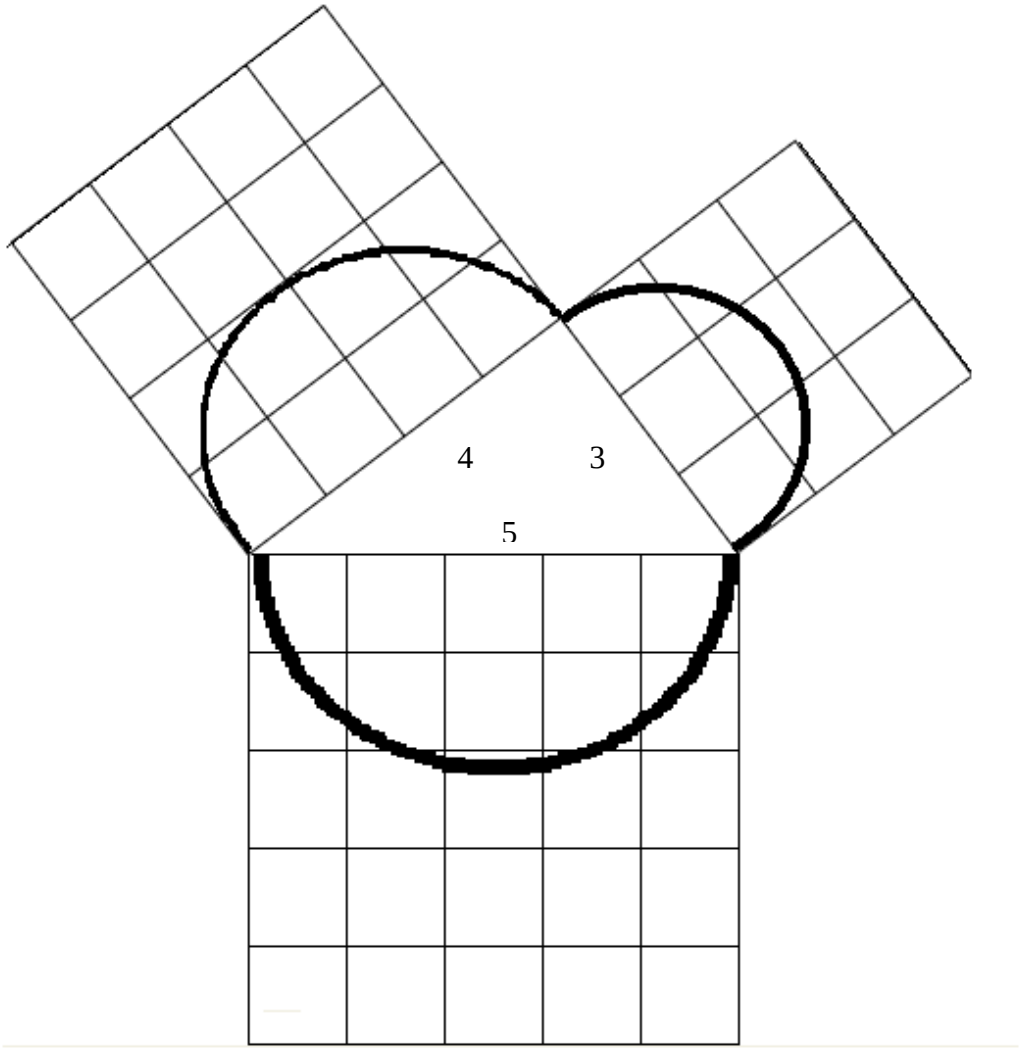
Appendix 4

Half Circles

1. Remember the formula for a circle. What formula would you use for a half circle?

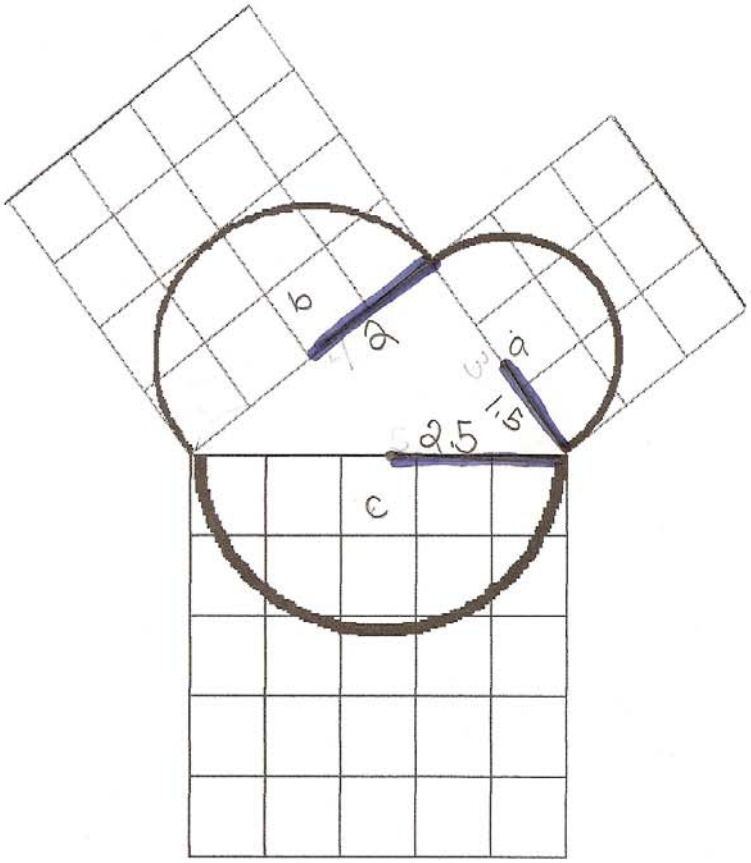
2. Find the area of all of the half circles.

3. When you add the two smaller half circles together what do you notice?



Solution:

Half Circles



1. Remember the formula for a circle. What formula would you use for a half circle?

2. Find the area of all of the half circles. (a,b,c)

3. When you add the two smaller half circles together what do you notice?

1) Area of a circle = πr^2 Area of a half circle = $\frac{\pi r^2}{2}$

2) $a = \frac{\pi(1.5)^2}{2} = 3.53 cm^2$ $b = \frac{\pi(2)^2}{2} = 6.28 cm^2$ $c = \frac{\pi(2.5)^2}{2} = 9.81 cm^2$

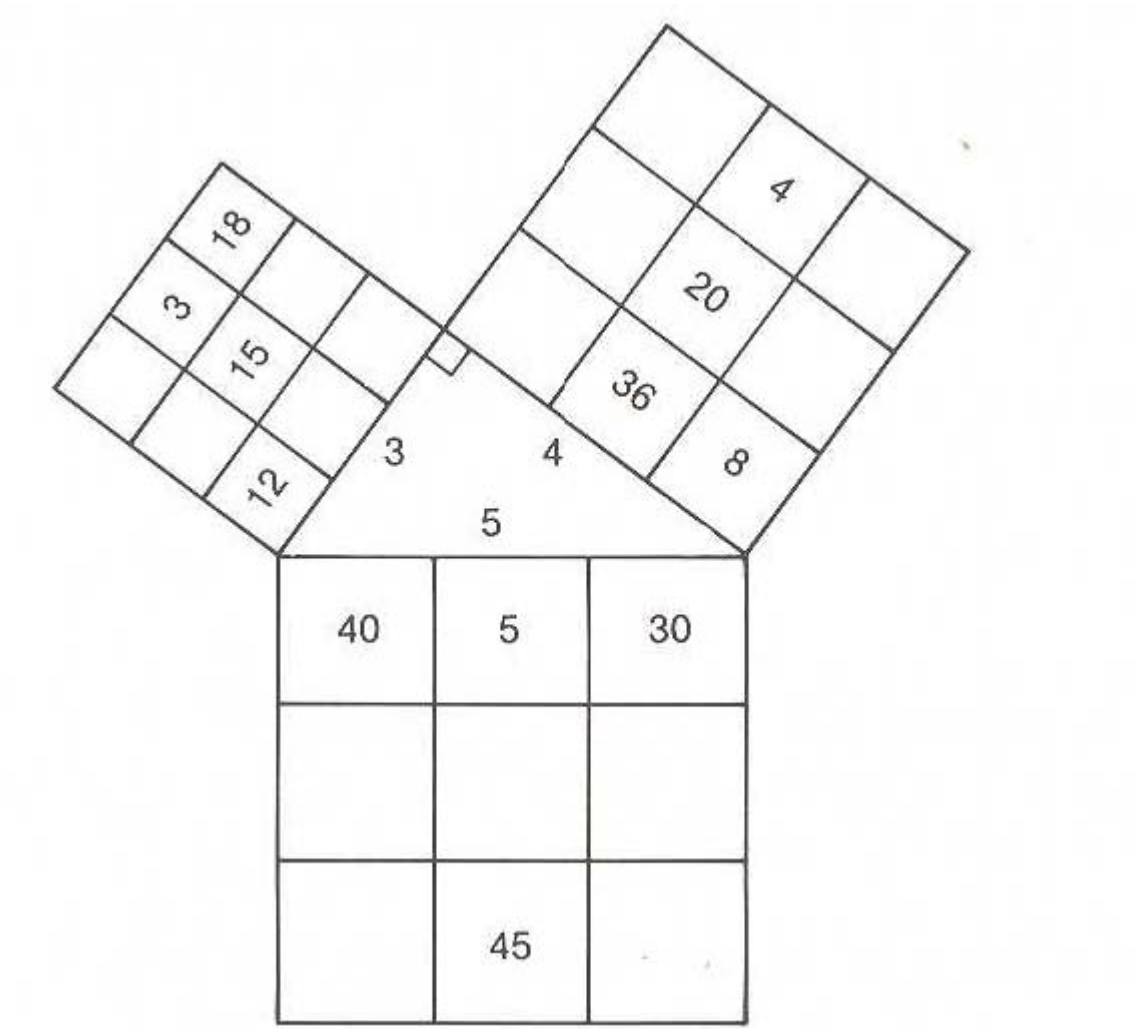
3) $3.53 + 6.28 = 9.81$

The 2 smaller half circles added together equals the larger circle therefore pythagoras theorem applies.

Appendix 5

Magic Triangle

1. Complete the 3 magic squares. (The sum of each column, each row and each diagonal must be the same or equal.)



2. Does Pythagoras Theorem apply to...

   a. The numbers written in the middle (central) boxes?
   b. The numbers written in the corresponding boxes?
   c. The sums of the numbers written in the 4 corners of each square?
   d. Any other combination of numbers written within the squares?

Magic Triangle

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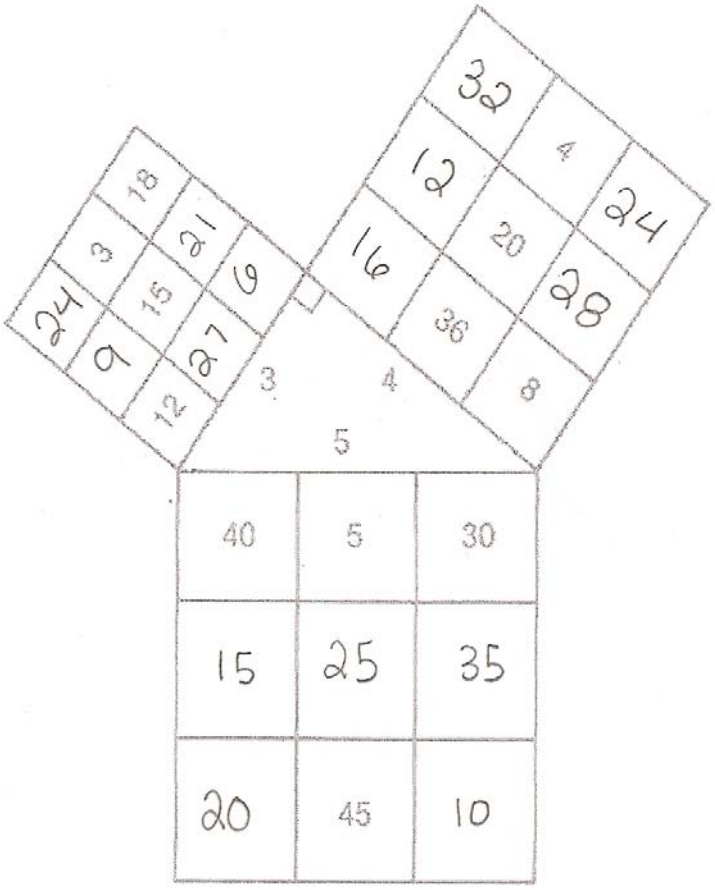

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   b. The numbers written in the corresponding boxes?
   c. The sums of the numbers written in the 4 corners of each square?
   d. Any other combination of numbers written within the squares?

2a) small - 15 (pretend numbers are
medium - 20 squares, so 15 × 15, 20 × 20,
large - 25 25 × 25)

The sum of the two smaller boxes equals the larger box.

b) The above applies to the corresponding numbers in each box.

c) small - 24 + 18 + 12 + 6 = 60
medium - 32 + 24 + 16 + 8 = 80
large - 40 + 30 + 20 + 10 = 100

(again apply 60 × 60, 80 × 80, 100 × 100)

2(a) again applies

d) Endless possibilities!
Example:
The sum of each seperate box (135, 180, 225) has the same result as 2(a)

Creating Your Own Pythagorean Tree:

Begin your tree with a 4x4 square at the bottom center of the page.
Form a 45°-45°-90° triangle on top of the square.
Shade in each square at each level a different color and clearly indicate its area.

Why do you think this is called a Pythagorean Tree?

_____

_____

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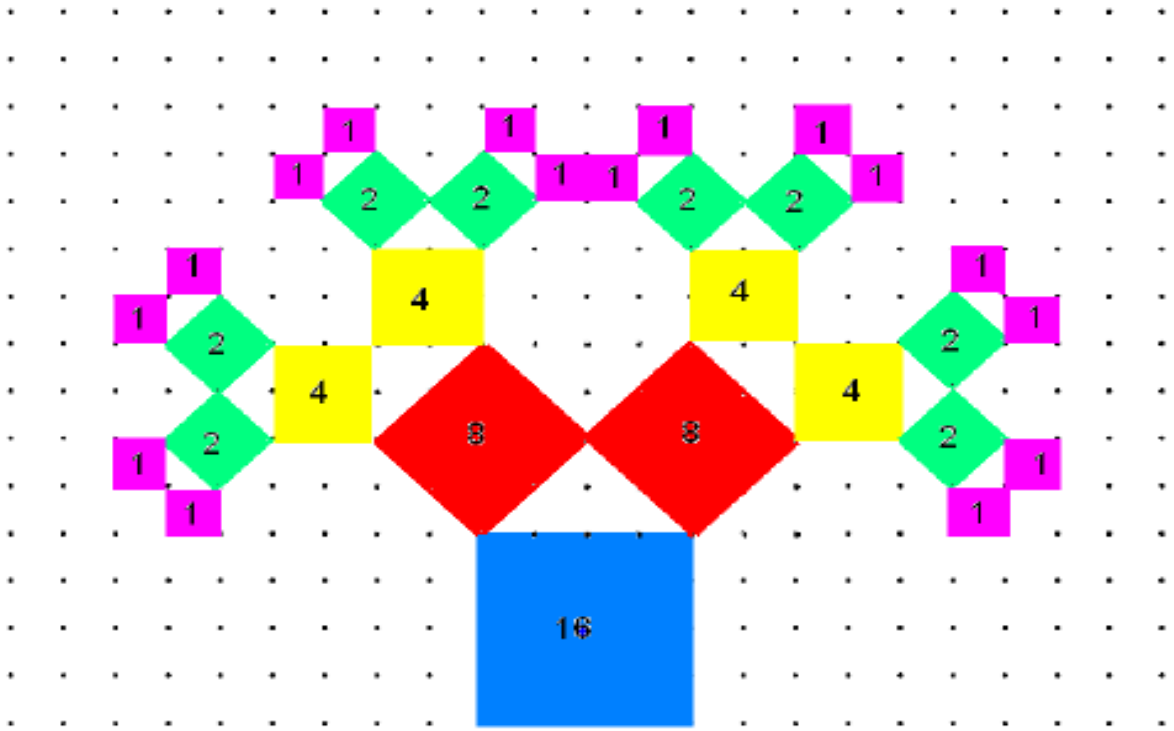


Why do you think this is called a Pythagorean Tree?

This is called a Pythagorean Tree because it demonstrates the Pythagorean pattern in which the sum of the smaller squares equals the sum of the large square when the squares are lined up to form a right angle triangle.

Appendix 7

Create the Golden Rectangle within this iPod Touch.
Measure the short side of the iPod, then use that measurement to create a square inside the perimeter of the iPod using the measured short side as one of the squares sides. Name the side of the square that is on the long side, a.
Mark the perimeter around the entire iPod. The line on the long side of the iPod that continues from a to the end of the iPod will be named b.

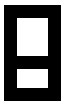
Calculate the ratio between a & b with the formula: $\frac{a+b}{a}$

Answer Sheet for “The Golden iPod”

This is what the students finished worksheet should look like and when they formulate the ratio of the rectangle, they will find that it equals the Golden Ratio of 1.618.

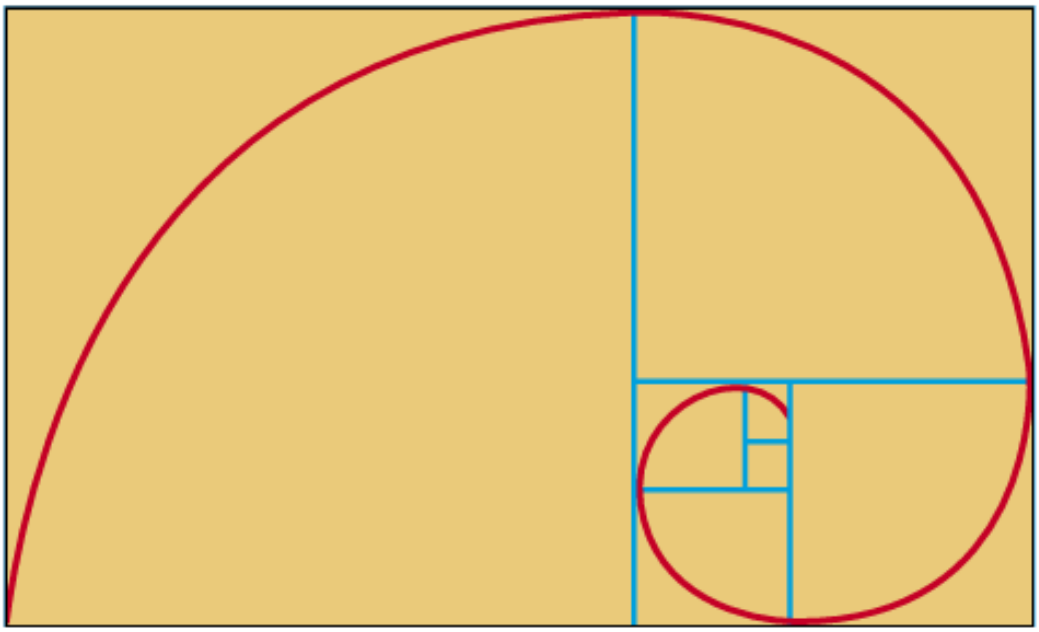
Appendix 8

The Golden Spiral



Use the long side of the rectangle that is composed of the smaller rectangle & square, to create a new square. The square will have all of its sides equal to the long side of the previous rectangle. Continue this pattern until you run out of room on the page.
After creating all of the rectangles, measures the sides of them and calculate the ratio of the rectangles using the formula: $\frac{a+b}{a}$
After calculating the ratios in your rectangles, use your compass to create half circles in all of the squares to added starting from the square that was provided at the beginning, creating a spiral. This is called The Golden Spiral. Can you think of any natural objects that have this form?

Answer Sheet for The Golden Spiral



This is what the students' finished project should look like. After calculating the ratios they should find that each rectangle has the Golden Ratio of approximately 1.618. They may respond to the finishing question with: snail shells, sunflower heads, hurricanes, tropical storms, etc.