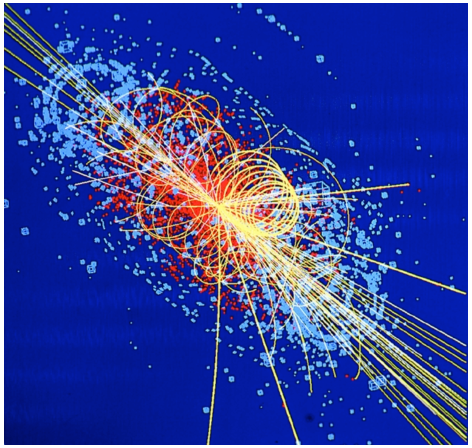
QCD at Colliders
Lecture 3



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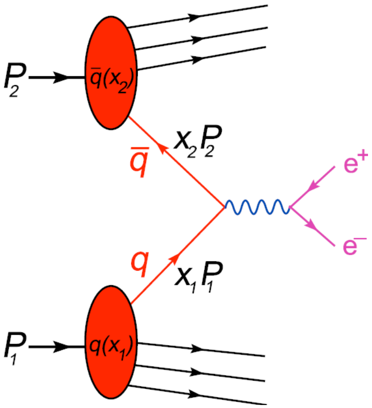
The Drell-Yan process



LO partonic cross section:

$$\hat{s} = x_1 x_2 s = M^2_{e^+e^-}$$

$$\hat{\sigma}(q\bar{q} \to e^+e^-) = \frac{1}{2\hat{s}} \frac{1}{4N_c^2} \sum_{h,c} |\mathcal{A}_4|^2$$

$$= \frac{4\pi\alpha^2}{3} \frac{1}{N_c} Q_q^2$$

$$\frac{d\hat{\sigma}}{dM^2} = \frac{\sigma_0}{N_c} Q_q^2 \delta(\hat{s} - M^2), \qquad \sigma_0 \equiv \frac{4\pi\alpha^2}{3M^2}$$

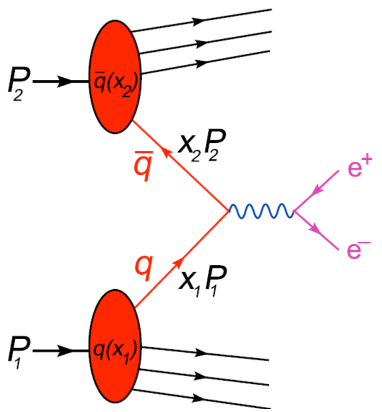
LO hadronic cross section:

$$\frac{d\sigma}{dM^2} = \int_0^1 dx_1 dx_2 \sum_q [q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)] \frac{d\hat{\sigma}}{dM^2}$$

$$= \frac{\sigma_0}{N_c} \int_0^1 dx_1 dx_2 \delta(x_1 x_2 s - M^2) \sum_q Q_q^2 [q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)]$$

$$= \frac{\sigma_0\, s}{N_c} \int_0^1 dx_1 dx_2 \delta(x_1 x_2 - \tau) \sum_q Q_q^2 [q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)], \qquad \tau \equiv \frac{M^2}{s}$$

Drell-Yan rapidity distribution



rapidity $$Y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$

$$\exp(2Y) = \frac{E+p_z}{E-p_z} = \frac{P_2\cdot P_Z}{P_1\cdot P_Z} = \frac{\frac{1}{x_2}p_{\bar{q}}\cdot P_Z}{\frac{1}{x_1}p_q\cdot P_Z} = \frac{x_1}{x_2}$$

combined with mass measurement,

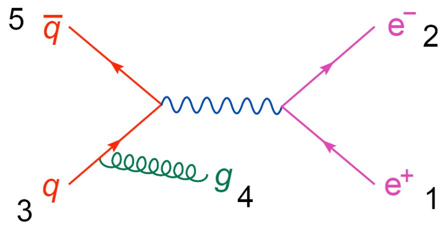
$$x_1 x_2 = \tau = \frac{M^2}{s}$$

double distribution $$\frac{d^2\sigma}{dM^2dY} = \frac{\sigma_0}{N_c\, s}\sum_q Q_q^2[q(x_1)\bar{q}(x_2)+\bar{q}(x_1)q(x_2)]$$

measures product of quark and antiquark distributions at

$$x_1 = \sqrt{\tau}e^{Y} \qquad x_2 = \sqrt{\tau}e^{-Y}$$

NLO QCD corrections to Drell-Yan production



$$|A_5|^2 = \frac{s_{13}^2 + s_{15}^2 + s_{23}^2 + s_{25}^2}{s_{12}s_{34}s_{45}}$$

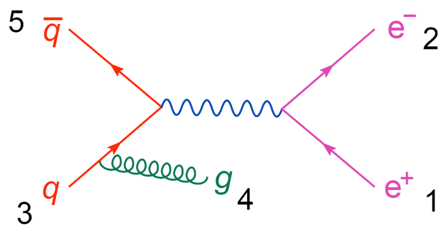
As at LO, average over decay direction of e^+ and e^-:

$$\langle k_1^\mu k_1^\nu \rangle_\Omega \equiv \int \frac{d\Omega_{e^+e^-}}{4\pi} k_1^\mu k_1^\nu = -\frac{s_{12}}{12}\eta^{\mu\nu} + \frac{1}{3}(k_1+k_2)^\mu (k_1+k_2)^\nu = \langle k_2^\mu k_2^\nu \rangle_\Omega$$

$$\langle s_{13}^2 \rangle_\Omega = \langle s_{23}^2 \rangle_\Omega = \frac{1}{3}(s_{13}+s_{23})^2 = \frac{1}{3}(s_{34}+s_{35})^2$$

$$\Rightarrow \quad \langle |A_5|^2 \rangle_\Omega = \frac{2}{3}\frac{(s_{34}+s_{35})^2 + (s_{35}+s_{45})^2}{s_{12}s_{34}s_{45}}$$

Phase space for DY @ NLO



Could use gluon energy, angle in CM frame, E_4, θ

Trade for $z, y \in [0,1]$ defined by:

$$z = \frac{s_{12}}{s_{35}}$$
$$y = \frac{1-\cos\theta}{2}$$

$$E_4 = -\frac{s_{34}+s_{45}}{2\sqrt{s_{35}}} = -\frac{s_{12}-s_{35}}{2\sqrt{s_{35}}} = \frac{1-z}{2}\sqrt{s_{35}}$$

$$s_{34} = -\sqrt{s_{35}}E_4(1-\cos\theta) = -y(1-z)s_{35}$$

$$\Rightarrow \quad s_{45} = -\sqrt{s_{35}}E_4(1-\cos\theta) = -(1-y)(1-z)s_{35}$$

$$s_{12} = M^2 = z s_{35}$$

cross section:

$$\langle |A_5|^2 \rangle_\Omega = \frac{2}{3\,M^2}\frac{(1-y(1-z))^2+(1-(1-y)(1-z))^2}{y(1-y)\,(1-z)^2}$$

P.S. measure in D=4-2ε:

$$\propto \left(\frac{\mu^2}{s_{35}}\right)^\epsilon \frac{d^{3-2\epsilon}p_4}{2E_4} \propto \left(\frac{\mu^2\,z}{M^2}\right)^\epsilon dE_4\,E_4^{1-2\epsilon}\,d\cos\theta(\sin^2\theta)^{-\epsilon}d\Omega^{1-2\epsilon}$$

$$\propto \left(\frac{\mu^2}{M^2}\right)^\epsilon dy\,dz\,[y(1-y)]^{-\epsilon}\,z^\epsilon\,(1-z)^{1-2\epsilon}$$

QCD corrections to DY (cont.)

Integral to do:

$$I = \left(\frac{\mu^2}{M^2}\right)^\epsilon z^\epsilon (1-z)^{-1-2\epsilon} \times \int_0^1 dy\, [y(1-y)]^{-\epsilon} \frac{(1-y(1-z))^2 + (1-(1-y)(1-z))^2}{y(1-y)}$$

Hard collinear divergences are at *y = 0,1*

Separate using

$$\frac{1}{y(1-y)} = \frac{1}{y} + \frac{1}{1-y}$$

related by symmetry

Expand *1/y* term in cross section about *y=0*

$$\begin{aligned} I &= 2\left(\frac{\mu^2}{M^2}\right)^\epsilon z^\epsilon (1-z)^{-1-2\epsilon} \int_0^1 dy\, y^{-1-\epsilon} \left[1+z^2-2y(1-y)(1-z)^2\right] \\ &\qquad \times (1-\epsilon \ln(1-y)) \\ &= 2\left(\frac{\mu^2}{M^2}\right)^\epsilon z^\epsilon (1-z)^{-1-2\epsilon} \left[-\frac{1+z^2}{\epsilon} - (1-z)^2 + \mathcal{O}(\epsilon)\right] \end{aligned}$$

QCD corrections to DY (cont.)

Including a few other omitted prefactors:

divergence absorbed into $q(x)$ in $\overline{\text{MS}}$ factorization scheme

$$\frac{d\hat{\sigma}^{\text{NLO, real}}}{dM^2} = \frac{\sigma_0}{N_c\, s} Q_q^2 \frac{\alpha_s}{2\pi} C_F \left[2\left(-\frac{1}{\epsilon} - \ln(4\pi) + \gamma\right) \frac{1+z^2}{1-z} - 2\frac{1+z^2}{1-z}\left(-2\ln(1-z) + \ln z - \ln\frac{M^2}{\mu^2}\right) - 2(1-z)^2 \right]$$

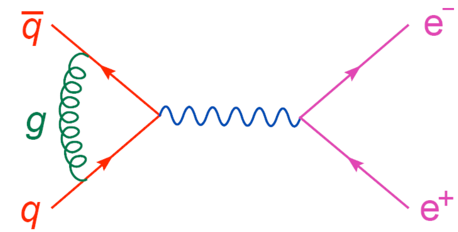
correction to cross section

artifact of my using unconventional FDH scheme with 2 gluon helicities, vs. standard 2-2ε of CDR – drop!

$$\begin{aligned} q(x,\mu) &= q_0(x) + \frac{\alpha_s(\mu)}{2\pi}\left(-\frac{1}{\epsilon} - \ln(4\pi) + \gamma_E\right) \int_x^1 \frac{d\xi}{\xi} \left[P_{qq}^{(0)}(x/\xi) q_0(\xi) + P_{qg}^{(0)}(x/\xi) g_0(\xi) \right] \\ &= q_0(x) + \frac{\alpha_s(\mu)}{2\pi}\left(-\frac{1}{\epsilon} - \ln(4\pi) + \gamma_E\right) \int_x^1 \frac{dz}{z} \left[C_F \frac{1+z^2}{1-z} q_0(x/z) + P_{qg}^{(0)}(z) g_0(x/z) \right] \end{aligned}$$

QCD corrections to DY (cont.)

Finally, virtual graph has support only at *z=1*.
-- kinematics same as at LO. Regulates *1/(1-z)* into plus distribution. Final result:



$$\frac{d\sigma^{\rm NLO}}{dM^2} = \frac{\sigma_0}{N_c\, s}\int_0^1 dx_1 dx_2 dz\, \delta(x_1 x_2 z - \tau)\sum_q Q_q^2 \Big[$$

$$q(x_1,\mu_F)\bar{q}(x_2,\mu_F)\Big(\delta(1-z) + \frac{\alpha_s(\mu_R)}{2\pi} C_F D_q(z,\mu_F)\Big)$$

$$+g(x_1,\mu_F)(q(x_2,\mu_F)+\bar{q}(x_2,\mu_F))\frac{\alpha_s(\mu_R)}{2\pi} T_R D_g(z,\mu_F)$$

$$+(x_1 \leftrightarrow x_2)\Big]$$

where

$$D_q(z,\mu_F) = 4(1+z^2)\left(\frac{\ln(1-z)+\ln(M/\mu_F)}{1-z}\right)_+$$

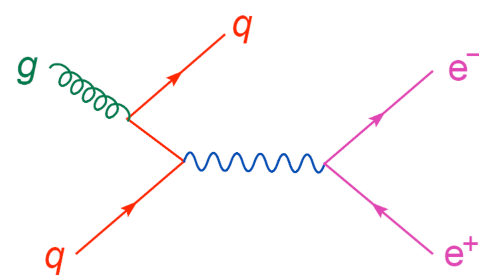
$$-2\frac{1+z^2}{1-z}\ln z + \delta(1-z)\left(\frac{2}{3}\pi^2 - 8\right)$$

singular distribution as $z \longrightarrow 1$

QCD corrections to DY (cont.)

and $$D_g(z,\mu_F) \;=\; (z^2+(1-z)^2)\left[\ln\frac{(1-z)^2}{z}+2\ln\frac{M}{\mu_F}\right]+\frac{1}{2}+3z-\frac{7}{2}z^2$$

comes from the *qg -> qγ** subprocess:

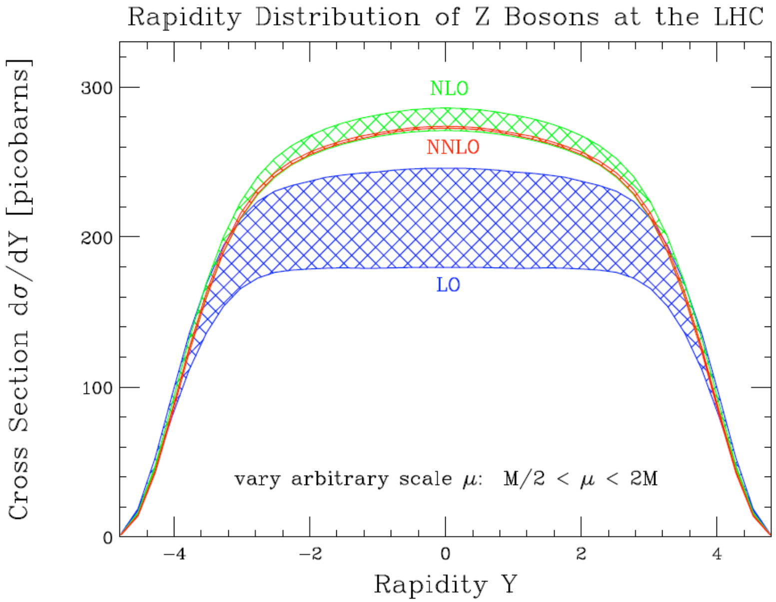


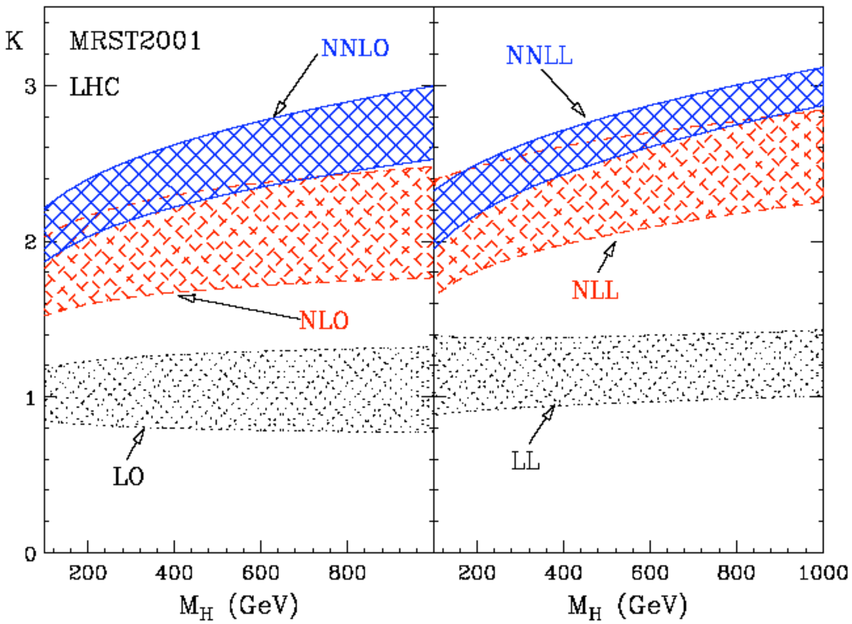
- Cross section related by crossing to *$\bar{q}q$ -> gγ**
- Remove *g -> $\bar{q}q$* collinear singularity in same way
- Note that there is no *1/(1-z)* (soft gluon) singularity in this term.

Why are NLO corrections large?

+ 30% typical for quark-initiated (W/Z, …)

+ 80-100% for some gluon-initiated ($gg \rightarrow$ Higgs $+ X$)

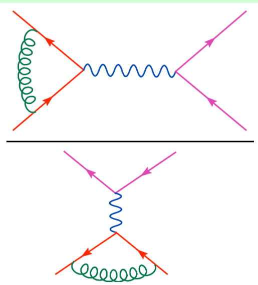




This is much bigger than $R_{e^+e^-} = 1 + \frac{\alpha_s}{\pi} \approx 1 + \frac{0.1}{\pi} \approx 1 + 0.03$!!

Some answers (not all for all processes)

1. LO parton distribution fits not very reliable due to large theory uncertainties
2. New processes can open up at NLO. In *W/Z* production at Tevatron or LHC, *qg -> γ*q* opens up, and *g(x)* is very large – but correction is negative!
3. Large π^2 from analytic continuation from space-like region where pdfs are measured (DIS) to time-like region (Drell-Yan/W/Z):

2 Re 

$$= 1 + \frac{\alpha_s}{\pi} C_F \left(-\frac{1}{\epsilon^2}\right) \mathrm{Re}\left[\left(\frac{\mu^2}{-Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{+Q^2}\right)^{\epsilon}\right]$$

$$= 1 + \frac{\alpha_s}{\pi} C_F \left(-\frac{1}{\epsilon^2}\right) \mathrm{Re}\left[\exp(i\pi\epsilon) - 1\right] = 1 + \frac{\alpha_s}{\pi} C_F \frac{\pi^2}{2}$$

4. Soft-gluon/Sudakov resummation

- A prevalent theme in QCD whenever one is at an edge of phase space.
- Infrared-safe but sensitive to a second, smaller scale
- Same physics as in (high-energy) QED: $e^+e^- = e^+e^-(\gamma)$
- What is prob. of no γ with $E > \Delta E,\ \theta > \Delta\theta$?

$$P = 1 - \frac{\alpha}{\pi}\int_{\Delta E}\frac{dE}{E}\int_{\Delta\theta}\frac{d\theta}{\theta} + \cdots = 1 - \frac{\alpha}{\pi}\ln\Delta E\ln\Delta\theta + \cdots$$

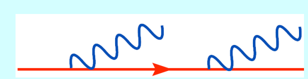
$$= \exp\left(-\frac{\alpha}{\pi}\ln\Delta E\ln\Delta\theta\right) + \cdots$$

soft

collinear

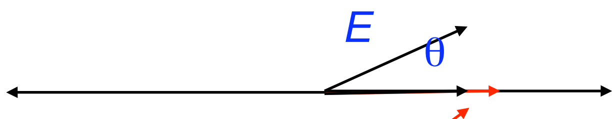
leading double logarithms -- in contrast to single logs of renormalization group, DGLAP equations.

exponentiation because soft emissions are independent

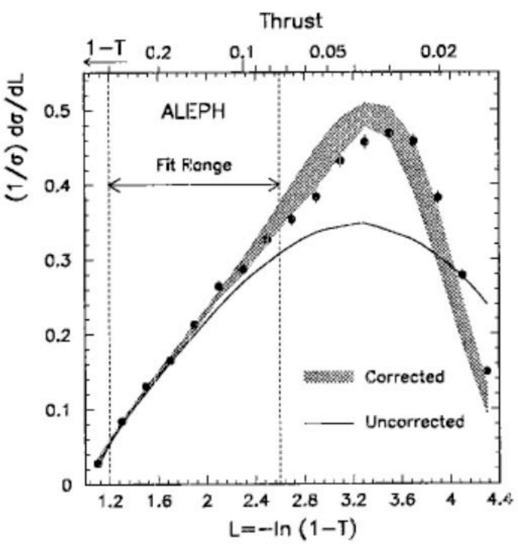


Example: e^+e^- Thrust $T \to 1$

$$T = \max_{\hat{n}} \frac{\sum_{j=1}^{N} |\hat{n} \cdot \vec{k}_j|}{\sum_{j=1}^{N} |\vec{k}_j|}$$



$$1 - T \approx \frac{E\theta^2}{\sqrt{s}}$$



Hard, wide angle radiation forbidden

$$E_{\min}(T) \propto (1-T)\sqrt{s} << \sqrt{s}$$

$$\frac{1}{\sigma}\frac{d\sigma}{dT} \approx \frac{4C_F\alpha_s}{\pi}\int \frac{dE}{E}\frac{d\theta}{\theta}\delta\left(1 - T - \frac{E\theta^2}{\sqrt{s}}\right)$$

$$= \frac{2C_F\alpha_s}{\pi}\frac{1}{1-T}\int_{E_{\min}(T)}^{E_{\max}}\frac{dE}{E}$$

$$= -\frac{2C_F\alpha_s}{\pi}\frac{\ln(1-T)}{1-T}$$

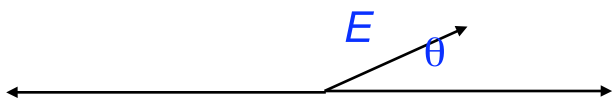
$$P(1 - T < \tau) = 1 - \frac{C_F\alpha_s}{\pi}\ln^2\tau + \cdots$$

$$\approx \exp\left(-\frac{C_F\alpha_s}{\pi}\ln^2\tau\right)$$

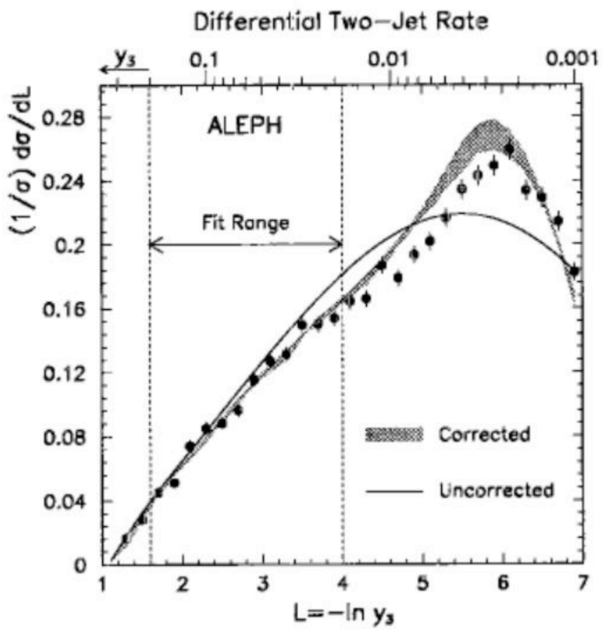
Known how to resum lnτ's in exponent to NLL (next-to-leading-log) accuracy for many variables, NNLL for some.

e$^+$e$^-$ jets with $y_{\mathrm{cut}} \to 0$

$$y_{\mathrm{cut}} = \frac{2E_iE_j(1-\cos\theta_{ij})}{s} \approx \frac{E\theta^2}{\sqrt{s}}$$



Hard, wide angle radiation again forbidden -- "pencil-thin jets"



Two-jet rate exponentiates like thrust.

Higher multijet rates can be resummed, but not exponentiated. $\sim \alpha_s{}^n L^{2n}$

Like thrust, dramatic effects at Z pole only start to happen when physical scale is getting close to Λ_{QCD}
-- large hadronization corrections

Hadron collider examples

$p_T(Z)$, important application to $p_T(W)$,
m_W measurement at Tevatron was discussed by Dieter

Another class of examples is provided by production of heavy states, like

- top quark at the Tevatron (W and Z production less so),
- even a light Higgs boson at the LHC, via $gg \rightarrow H$

Called threshold resummation or $x \rightarrow 1$ limit,
where $x = M^2/s$.
Can be important for $x \ll 1$ though.
For m_H = 120 GeV at 14 TeV LHC, $x = 10^{-4}$!
Radiation is being suppressed because you are running out of phase space – parton distributions are falling fast.

Threshold Resummation

We saw the first log of this type in the NLO corrections to Drell-Yan/*W*/*Z* production:

$$C_F D_q(z, \mu_F) = 4 C_F (1+z^2) \left(\frac{\ln(1-z) + \ln(M/\mu_F)}{1-z} \right)_+ - 2 \frac{1+z^2}{1-z} \ln z + \delta(1-z) \left(\frac{2}{3}\pi^2 - 8 \right)$$

Also a double-log expansion:

$$D_q^{(n)}(z, \mu_F) \propto (C_F \alpha_s)^n \left[\left(\frac{\ln^{2n+1}(1-z)}{1-z} \right)_+ + \cdots \right]$$

For *gg* -> *H*, same leading behavior at large *z*.
Except color factor is much bigger: $C_A = 3$, not $C_F = 4/3$

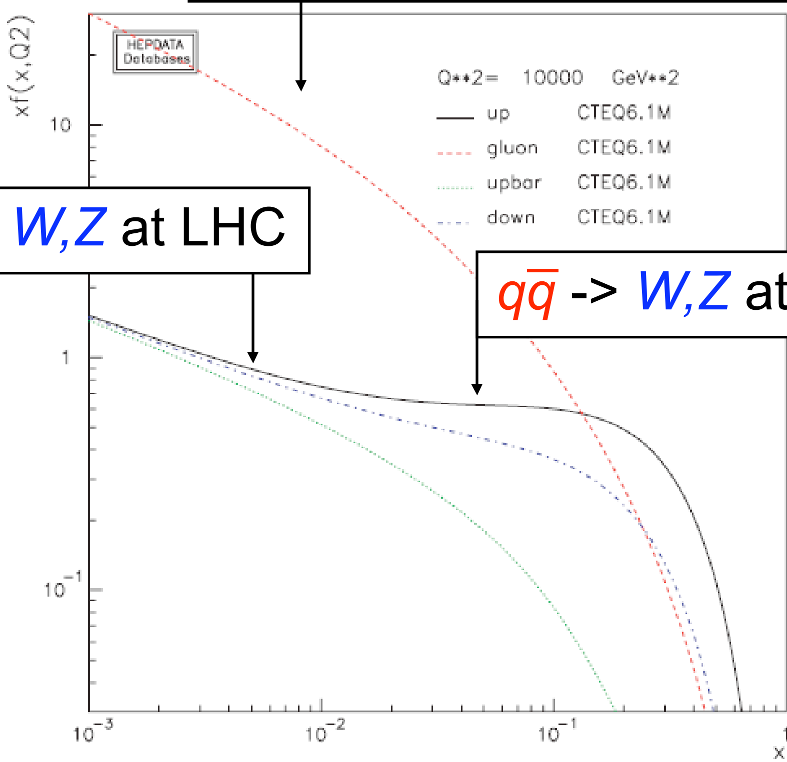
$$D_{gg \to H}^{(n)}(z, \mu_F) \propto (C_A \alpha_s)^n \left[\left(\frac{\ln^{2n+1}(1-z)}{1-z} \right)_+ + \cdots \right]$$

Fast falling pdfs -- worse for gluons

gg -> *light Higgs* at LHC

$q\bar{q}$ -> *W,Z* at LHC

$q\bar{q}$ -> *W,Z* at Tevatron



Q: If it is called Sudakov suppression, why does it increase the cross section?

A. Because the same suppression happens in the DIS process used to measure the pdfs. Both parton distributions "bigger than you thought": 2 – 1 > 0.

pdfs

partonic cross section

Conclusions

- QCD at colliders is an extremely rich field.
- I was only able to scratch the surface of it here.
- Indeed, at hadron colliders, the physics **is** QCD
– up to small, electroweak corrections!
- So, to uncover new physics of electroweak strength, we will need to understand QCD at colliders quite well.
- There is plenty of room for fresh, new ideas from young theorists and experimentalists (you!)