QCD resummation for collider observables

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Quest for precision

- LHC's Run II has just started operations after the success of the Run I programme:
  - Discovery of the Higgs boson
  - No BSM effects observed yet, new physics constrained at high scales (< TeV ?)
  - Precision measurements of the SM Lagrangian
- The Run II will focus on:
  - measurements of the Higgs properties with higher precision
  - keep searching for signals of new physics beyond the SM

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>   - new search strategies/techniques to exploit data and enhance tiny signals
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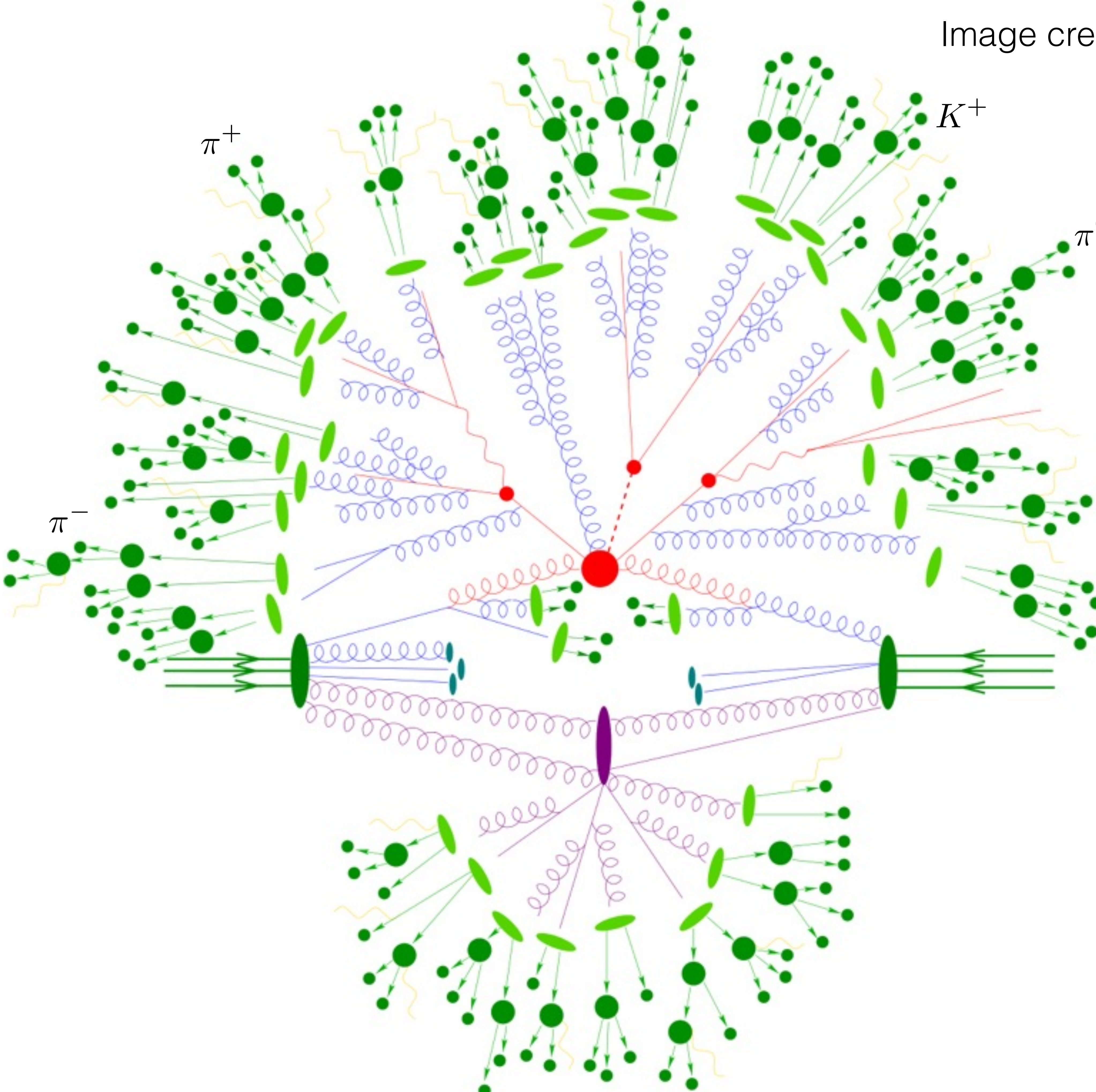
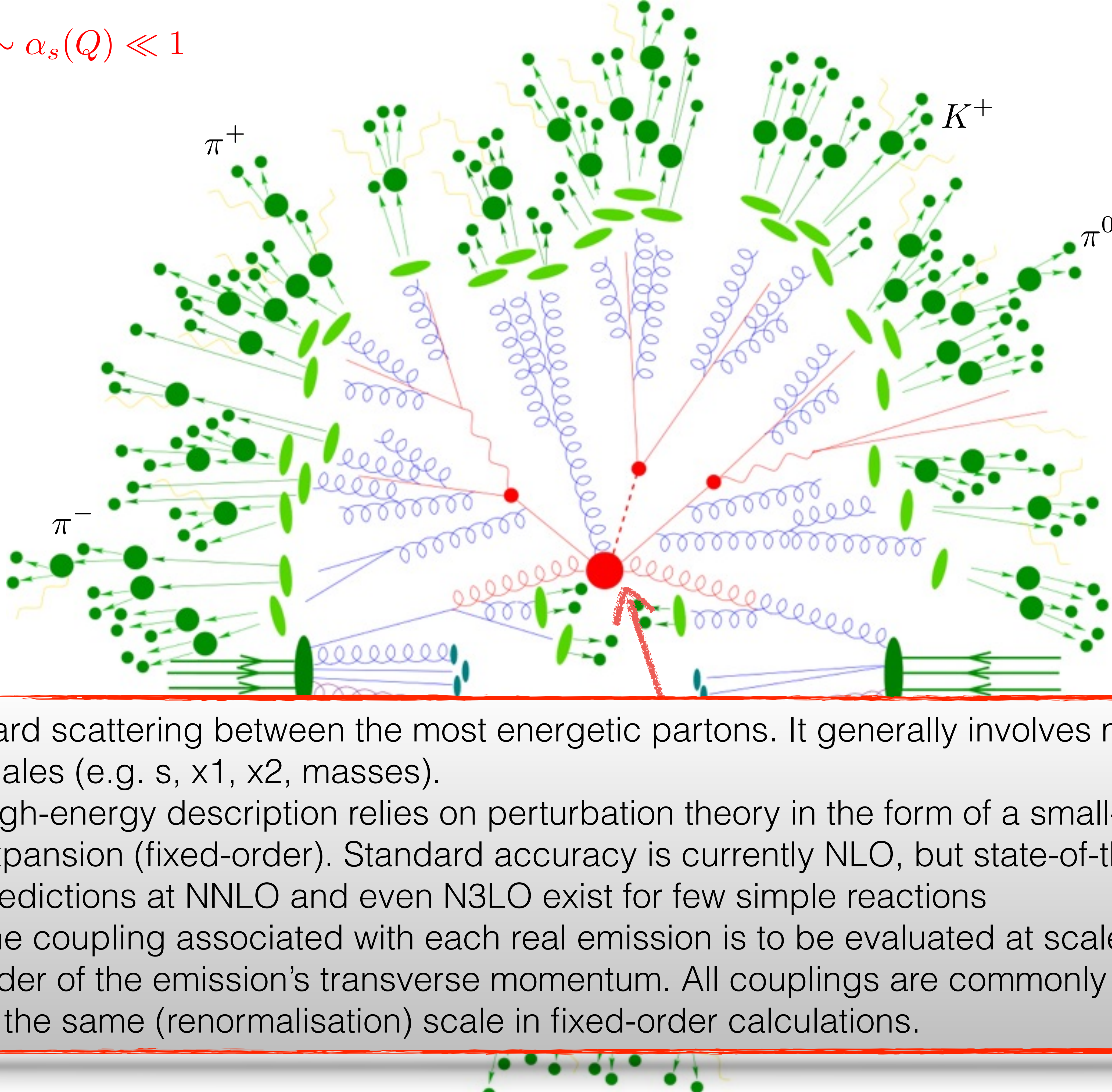
Image credits: F. Krauss



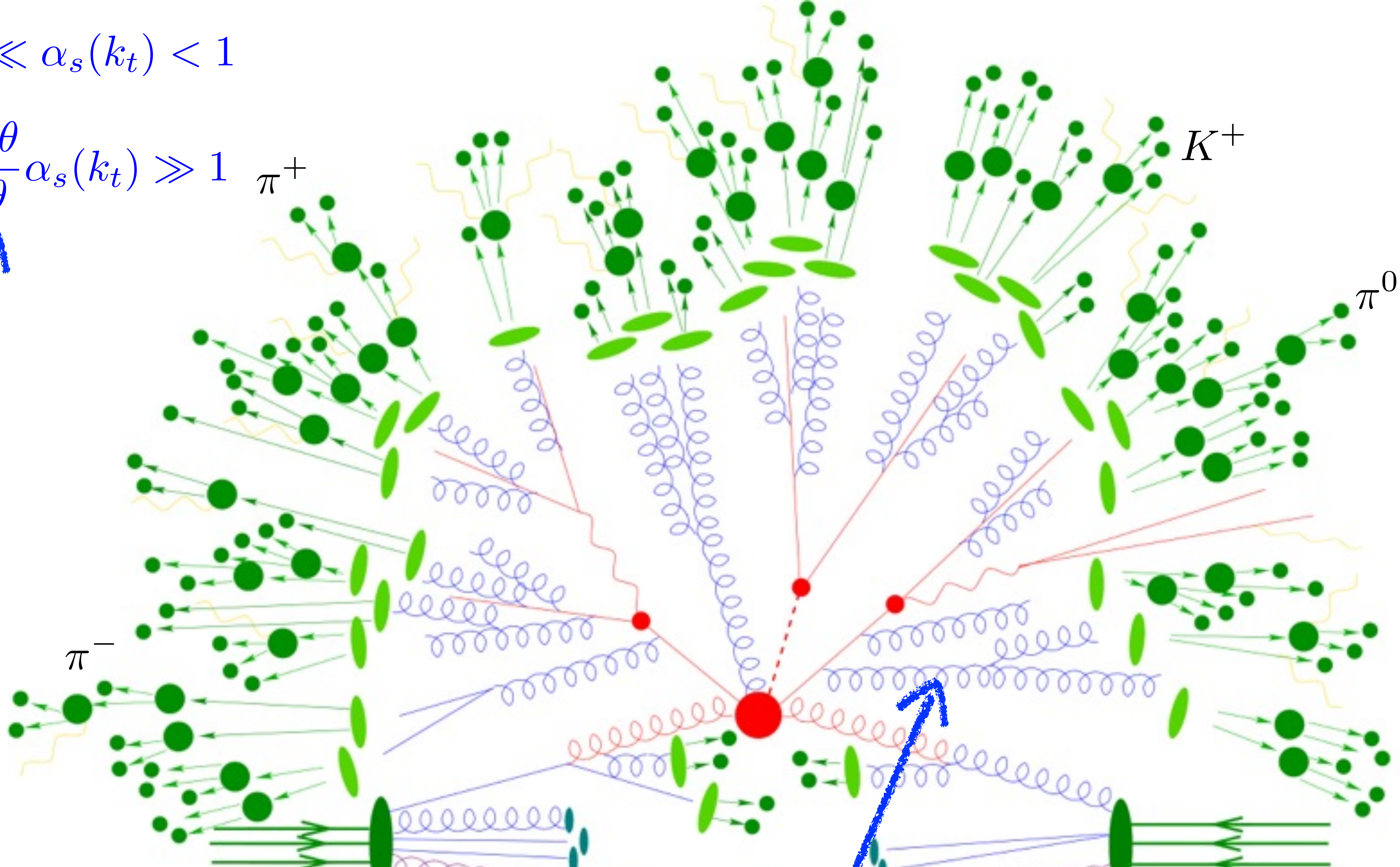
$\alpha_s(k_t) \sim \alpha_s(Q) \ll 1$



- Hard scattering between the most energetic partons. It generally involves multiple scales (e.g. s, x1, x2, masses).
- High-energy description relies on perturbation theory in the form of a small-coupling expansion (fixed-order). Standard accuracy is currently NLO, but state-of-the-art predictions at NNLO and even N3LO exist for few simple reactions
- The coupling associated with each real emission is to be evaluated at scales of the order of the emission's transverse momentum. All couplings are commonly evaluated at the same (renormalisation) scale in fixed-order calculations.

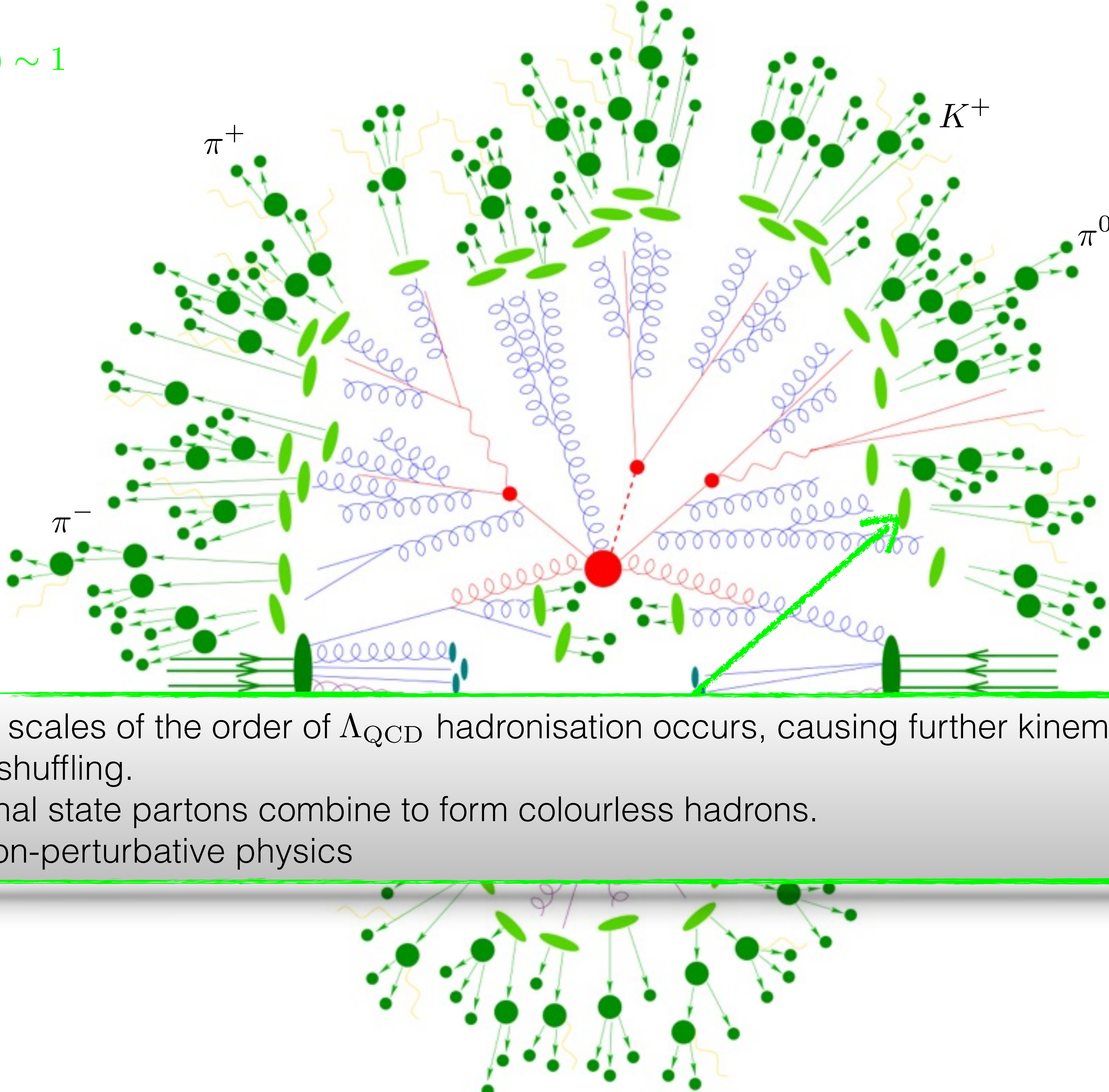
$$\alpha_s(Q) \ll \alpha_s(k_t) < 1$$

$$\int \frac{dE}{E}\frac{d\theta}{\theta}\alpha_s(k_t) \gg 1$$



- As the coupling grows large, coloured particles are very likely to emit soft and/or collinear radiation (i.e. small kt) all the way down to hadronisation scales.
- This radiation causes a kinematical reshuffling and it *normally* does not affect much the total production rates -> QCD "shower" is (nearly) unitary.
- Physical observables are insensitive to very soft/collinear radiation - otherwise they invalidate perturbation theory (*Infrared and Collinear Safety*)
- However, the sensitivity to these effects can become significant if one applies exclusive constraints on the real radiation

$\alpha_s(k_t) \sim 1$



- At scales of the order of Λ_{QCD} hadronisation occurs, causing further kinematics reshuffling.
- Final state partons combine to form colourless hadrons.
- Non-perturbative physics

Fixed order QCD and resummation

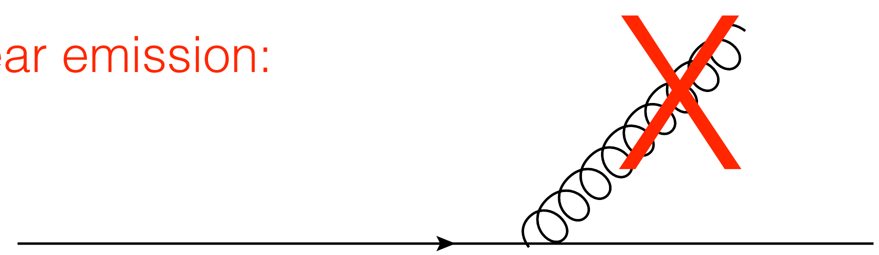
- Although the effect of soft/collinear radiation on total rates is very moderate, the sensitivity to these effects can grow dramatically if one constrains the QCD real radiation

  - real emission forced to be soft and/or collinear to the emitter

  - virtual corrections are unaffected

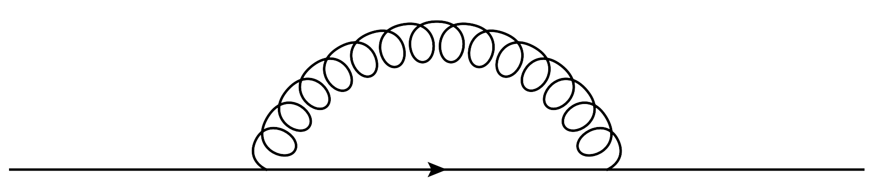
e.g. kt of a soft-collinear emission:

$$\frac{dk_t}{k_t} d\eta \alpha_s(k_t^2)$$



$$P(k_t < v) \sim 1 - \frac{\#\alpha_s C_F}{2\pi} \ln^2 v + \ldots$$

$$-\frac{dk_t}{k_t} d\eta \alpha_s(k_t^2)$$



  - single-logarithmic effects arise from less singular configurations

Fixed order QCD and resummation

- In the perturbative regime these logarithms can grow very large before the hadronisation takes over (breakdown of the PT)

$$L \sim \frac{1}{\alpha_s} \qquad\qquad L = \ln\frac{1}{v}$$

- This makes "higher order" corrections as large as leading order ones, i.e. $(\alpha_s L)^n L \sim \alpha_s L^2$

- The perturbative series breaks down and the probability of the reaction diverges logarithmically in the large L limit instead of being suppressed

- Need to reorganise PT in terms of all-order towers of logarithmic terms —> resummation.

- It is customary to define a new perturbative order at the level of the logarithm of the *cumulative* cross section

$$\Sigma(v) = \int_0^v \frac{1}{\sigma_{\text{Born}}}\frac{d\sigma}{dv'}dv' \sim e^{\overbrace{\alpha_s^n L^{n+1}}^{\text{LL}} + \overbrace{\alpha_s^n L^n}^{\text{NLL}} + \overbrace{\alpha_s^n L^{n-1}}^{\text{NNLL}} + \ldots}$$

Fixed-order vs. All-order Perturbative QCD

- Fixed-order calculations of radiative corrections are formulated in a well established way
  - i.e. recipe: compute amplitudes at a given order for a high-energy reaction and, provided an efficient subtraction of IR divergences, compute any IRC safe observable
  - technically extremely challenging, well-posed problem

- All-order calculations are still at an earlier stage of "evolution"
  - LL and NLL predictions for a wide class of observables can be obtained in a quite general (although not fully) way
  - No general recipe to tackle the problem beyond this order:
    - within a given reaction, each observable has its own IRC structure when radiation is considered
    - higher-order (> NLL) resummations commonly obtained in an observable-dependent way, for few collider observables
    - Single-observable resummations can be automated for classes of processes (e.g. production of colour singlets)

e.g. [Becher, Frederix, Neubert, Rothen '15]
[Grazzini, Kallweit, Rathlev, Wiesemann '15]

Monte Carlo Parton Shower

- The dominant (meaning LL / sometimes NLL) logarithmic towers can be predicted using modern parton shower generators, i.e. which shower the hard event with an ensemble of collinear partons (e.g. `Herwig++, Pythia, Sherpa`)

- Parton-shower (PS) simulations can be applied on top of NLO (e.g. `POWHEG, MC@NLO, MiNLO`) and in few cases NNLO (`MiNLO, UNNLOPS, Geneva`) calculations for the hard underlying reaction

- PS generators give a complete description of the event (i.e. fully exclusive in final state, non-perturbative effects modelled)

- Given the accuracy required by current experiments, and in order to match the high perturbative precision currently achieved in the computation of hard processes, the current PS simulations may be not enough

Why higher-order resummation

- In order to improve on that, methods to perform higher-order resummations are necessary — not an easy task (requires all-order treatment of the radiation in the relevant approximation). Despite being generally less flexible than PS simulations, higher-order resummations are important for a number of reasons:

- phenomenological interests:
    - precision physics
    - tuning/developing Monte Carlo event generators
    - matching of PS to fixed order
    - design of better-behaved observables (e.g. substructure)

- theoretical interests:
    - properties of the QCD radiation to all-orders
    - understanding of IRC singular structure (subtraction)
    - unveiling perturbative scalings in the deep IRC region
    - probing the boundary with the non-perturbative regime, and study of non-perturbative dynamics

Amplitude's properties to all orders

- We consider an *Infrared and Collinear (IRC) safe* observable normalised as $V = V(\{\tilde{p}\}, k_1, ..., k_n) \leq 1$, in the limit $V \to 0$

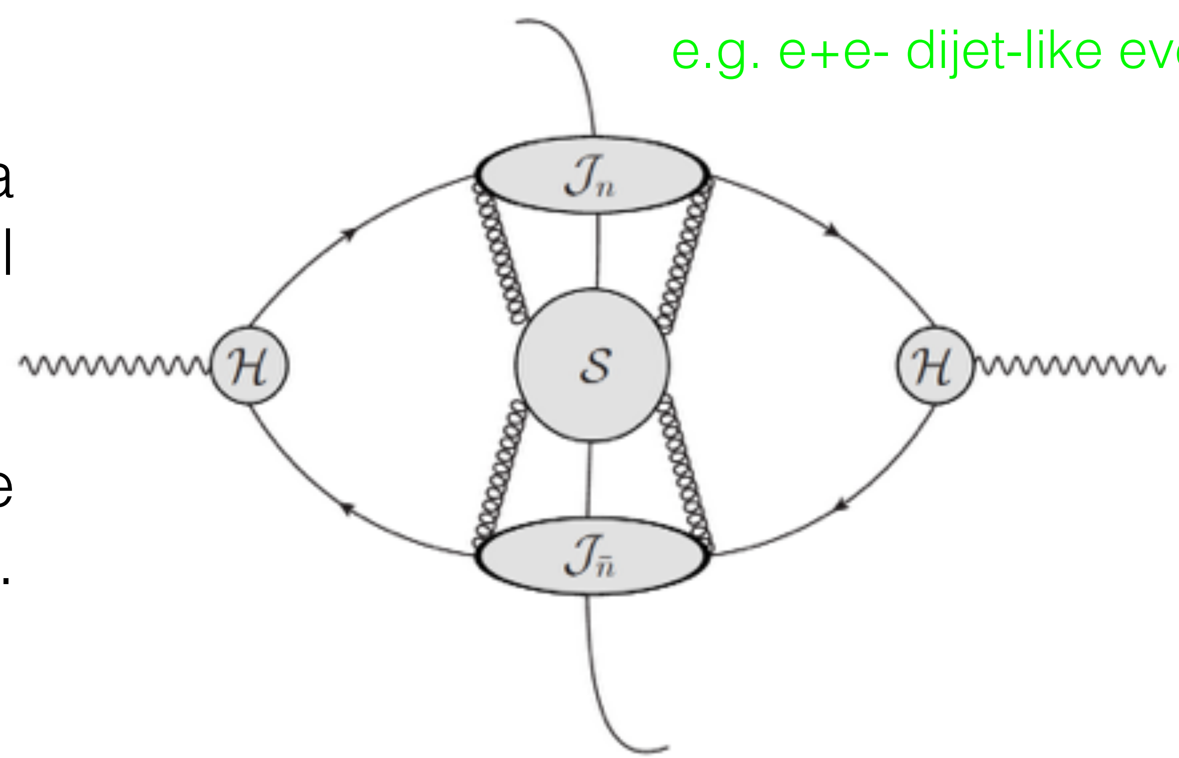
- In this limit radiative corrections are described exclusively by virtual corrections, and collinear and/or soft real emissions (logarithmic behaviour) — QCD amplitudes **factorise** in these regimes w.r.t. the Born up to regular (giving rise to non-logarithmic corrections) terms

$$|\mathcal{M}(\{\tilde{p}\}, k_1, ..., k_n)|^2 \simeq |M_{\rm Born}(\{\tilde{p}\})|^2 |M(k_1, ..., k_n)|^2 + \dots$$

e.g. e+e- dijet-like event

- Squared amplitude can be decomposed as a product of leading (singular) kinematical subprocesses

- Each of the subprocesses corresponds to the contribution of different singular modes (e.g. virtual, soft, collinear,…)



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- All-order treatment is possible only if factorisation of the QCD amplitudes holds true at all perturbative orders (often assumed)

- Cases of collinear factorisation breaking due to exchange of Glauber modes (i.e. Coulomb phases) found at high orders in multijet squared amplitudes

[Forshaw, Kyrieleis, and Seymour '06-'09]
[Catani, de Florian, and Rodrigo '12]
[Forshaw, Seymour, and Siodmok '12]
[Angeles-Martinez, Forshaw, and Seymour '15]

Factorisation theorems

- Different approaches exist (e.g. branching algorithm, `CAESAR/ARES`, SCET, CSS)

- In most of them, resummation is achieved through factorisation theorems for the studied observable (i.e. express the observable - if possible - as product of separate modes coming from different kinematical regions). Resummation is performed in a smartly-defined/observable-dependent conjugate space (e.g. Mellin - Laplace, Fourier) via RGE evolution of each of the involved subprocesses

  - OK for simple semi-inclusive cases: e.g. thrust in e+e-

$$\Theta(Q^2\tau - \bar{k}^2 - k^2 - wQ) = \frac{1}{2\pi i}\int_C \frac{d\nu}{\nu}\, \mathrm{e}^{\nu\tau Q^2}\mathrm{e}^{-\nu k^2}\mathrm{e}^{-\nu\bar{k}^2}\mathrm{e}^{-\nu wQ}$$

  - more difficult for involved observables: e.g. jet broadening in e+e-
  - tough/impossible for observables which mix various kinematic modes or require iterative optimisations: e.g. jet rates, thrust major

- Q: Is the factorisation of the observable a necessary requirement for (higher-order) resummation ?
- A: No, all one needs is specific scaling properties in the presence of multiple emissions.

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This talk

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Recursive IRC safety

- The standard requirement of IRC safety implies that the value of the observable does not change in the presence of one or more unresolved emissions (i.e. very soft and/or collinear)

- In addition, one requires recursive IRC (rIRC) safety (for a precise definition see backup slides), i.e. [Banfi, Salam, Zanderighi '04]

  - for sufficiently small $\bar{v}$ there exists some ϵ that can be chosen *independently* of $\bar{v}$ such that we can neglect any emissions at scales $\sim \epsilon\bar{v}$

$$V(\{\tilde{p}\}, k_1, \ldots, k_n, \ldots, k_m) \simeq V(\{\tilde{p}\}, k_1, \ldots, k_n) + \epsilon^p \bar{v}$$

  - that in the presence of multiple emissions the observable scales in the same fashion as for a single emission (IRC divergences have an exponential form)

$$\text{if } V(\{\tilde{p}\}, k_i) = v_i = \zeta_i \bar{v} \ \forall\ i \to V(\{\tilde{p}\}, k_1, k_2, \cdots, k_n) \sim \bar{v} \text{ as } \bar{v} \to 0$$

- We limit ourselves to *continuously global* observables* here, i.e. constrain the radiation equally everywhere in the phase space (it ensures the absence of non-global logarithms) [Dasgupta, Salam '01; Banfi, Marchesini, Smye '02]

*Not a real limitation, however no solution for the full NNLL structure of non-global logarithms currently known. Some activity recently [Caron-Huot; Larkoski et al.; Becher et al. '15/'16]

A glimpse of the method

- A generic cumulative cross section can be parametrised as

$$\Sigma(v) = \sigma_0 \int \frac{dv_1}{v_1} D(v_1) P(v|v1), \qquad D(v_1) = e^{-R(v_1)} R'(v_1)$$

Probability of emitting the hardest parton v1 = v(k1)

Probability of secondary radiation given the first emission, and the observable's value v

- rIRC safety guarantees:
  - the cancellation of IRC singularities at all orders in the probability $P(v|v1)$
  - all leading logarithms $(\alpha_s^n \ln^{n+1}(1/v))$ exponentiate $\rightarrow e^{-R(v)}$
    - the multiple-emission effects in $P(v|v1)$ start at most at NLL
  - a logarithmic hierarchy in the real emission probability (e.g. see backup) —> At NLL only independent emissions contribute !

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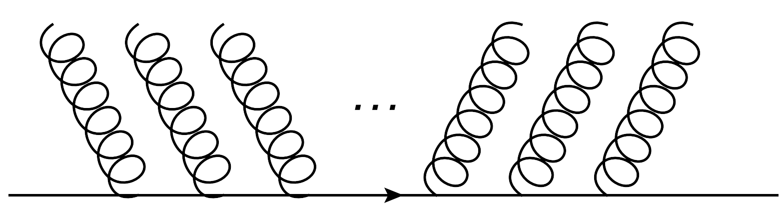
Probability of secondary radiation given the first emission, and the observable's value v

- NLL answer remarkably simple: grand-canonical ensemble of independent emissions widely separated in rapidity

[Banfi, Salam, Zanderighi '01-'04]

- The conditional probability $P(v|v1)$ is defined as the mean value of the observable's measurement function in the soft-collinear bath, at fixed v1

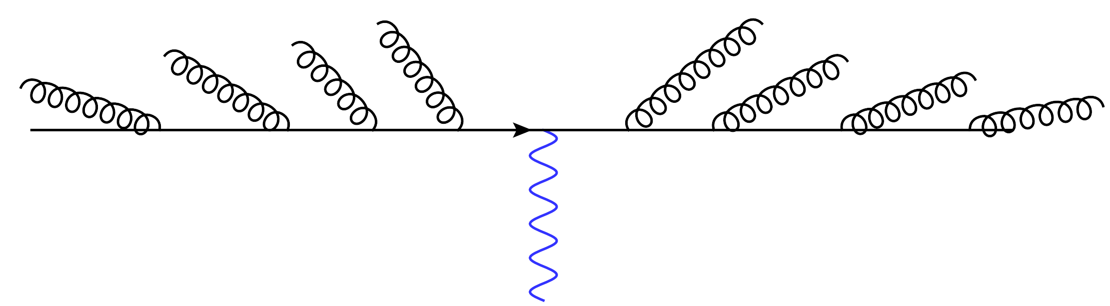


$$= \int d\mathcal{Z}[\{R'_{\mathrm{NLL},\ell_i}, k_i\}]\, \Theta\left(1 - \lim_{v\to 0} \frac{V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\})}{v}\right)$$

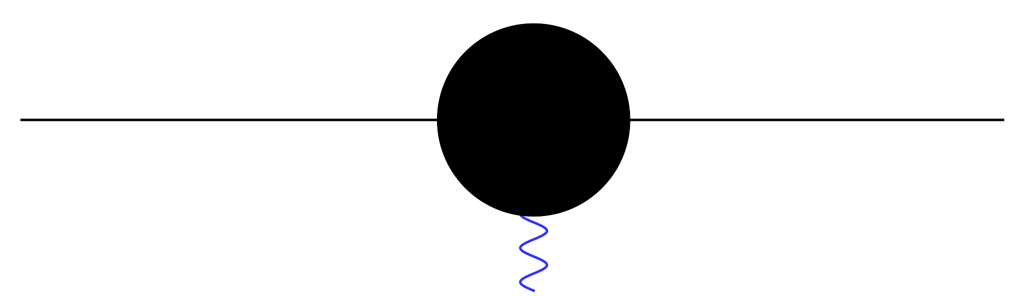
General structure of NNLL (ARES)

[Banfi, McAslan, PM, Zanderighi 1412.2126]

- Beyond NLL, a number of new corrections arise:
  - The structure of the anomalous dimensions which define the Sudakov radiator is more involved - requires RGE evolution (radiator is universal for certain classes of observables)
  - New configurations for the resolved real radiation contribute (O(as) corrections to the NLL configurations at all orders)
- Analogy with FO calculation: the NLL ensemble defines the Born level result



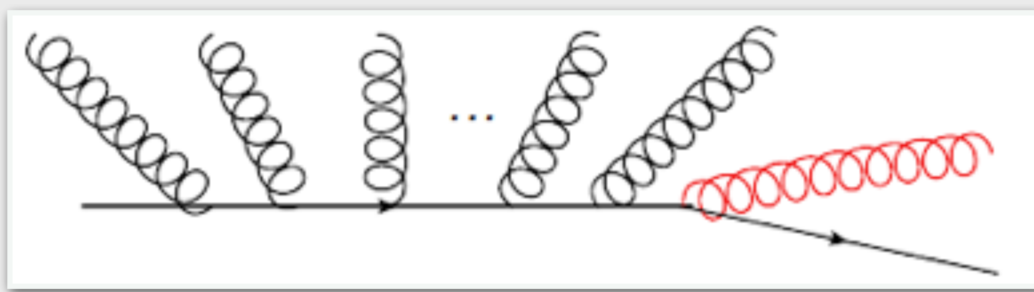
Sufficient to consider one correction at a time, for a single emission of the ensemble



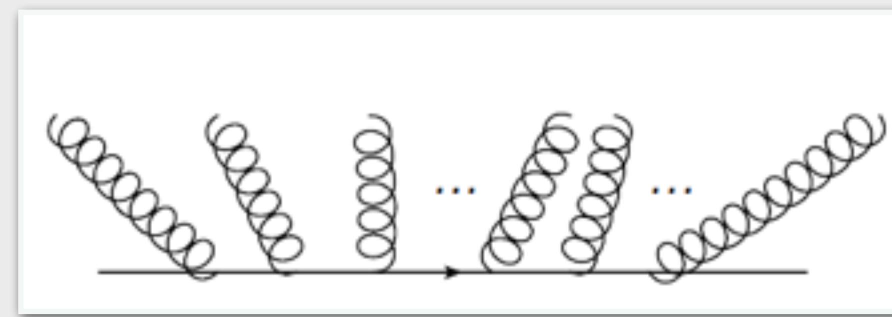
General structure of NNLL (ARES)

[Banfi, McAslan, PM, Zanderighi 1412.2126 and ongoing work]

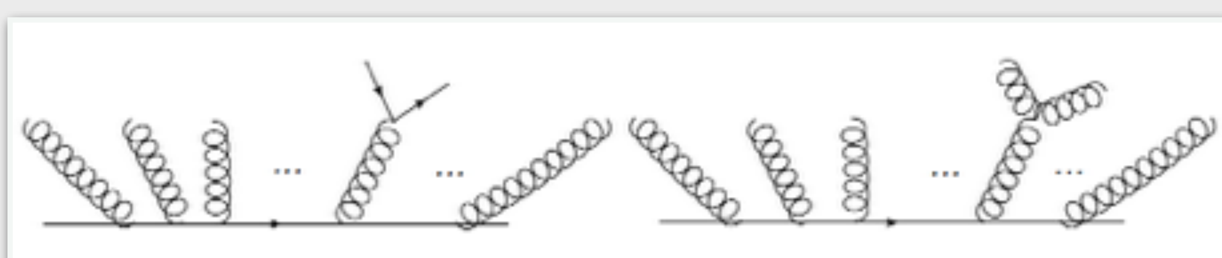
- (at most) one collinear emission can carry a significant fraction of the energy of the hard emitter (which recoils against it)
    - correction to the amplitude: hard-collinear corrections
    - correction to the observable: recoil corrections



- (at most) one soft-collinear emission is allowed to get arbitrarily close in rapidity to any other of the ensemble (relax strong angular ordering)
    - sensitive to the exact rapidity bounds: rapidity corrections
    - different clustering history if a jet algorithm is used: clustering corrections



- (at most) one soft-collinear gluon is allowed to branch in the real radiation, and the branching is resolved
    - correlated corrections



- (at most) one soft emission is allowed to propagate at small rapidities
    - soft-wide-angle corrections

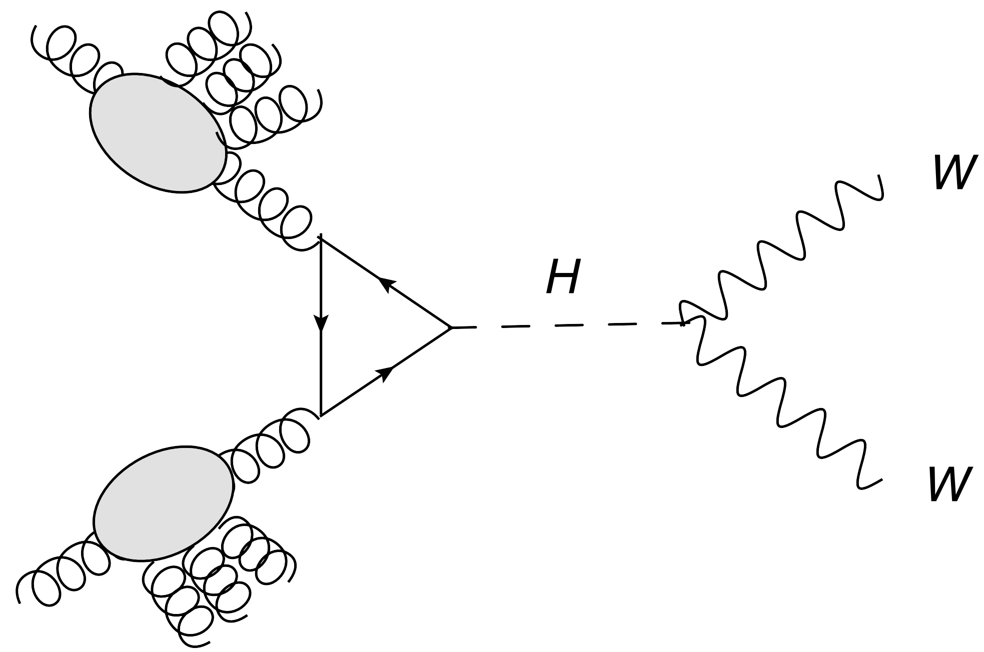
- Non-trivial abelian correction (~Cf^n, Ca^n) for processes with two emitting legs at the Born level - non-abelian contribution entirely absorbed into running coupling (CMW scheme)

- Non-abelian structure more involved in the multi leg case due to quantum interference between hard emitters (general formulation at NLL, still unknown at NNLL)

Applications at LEP and LHC

Higgs production with a jet veto

- pp-> H production with a jet veto

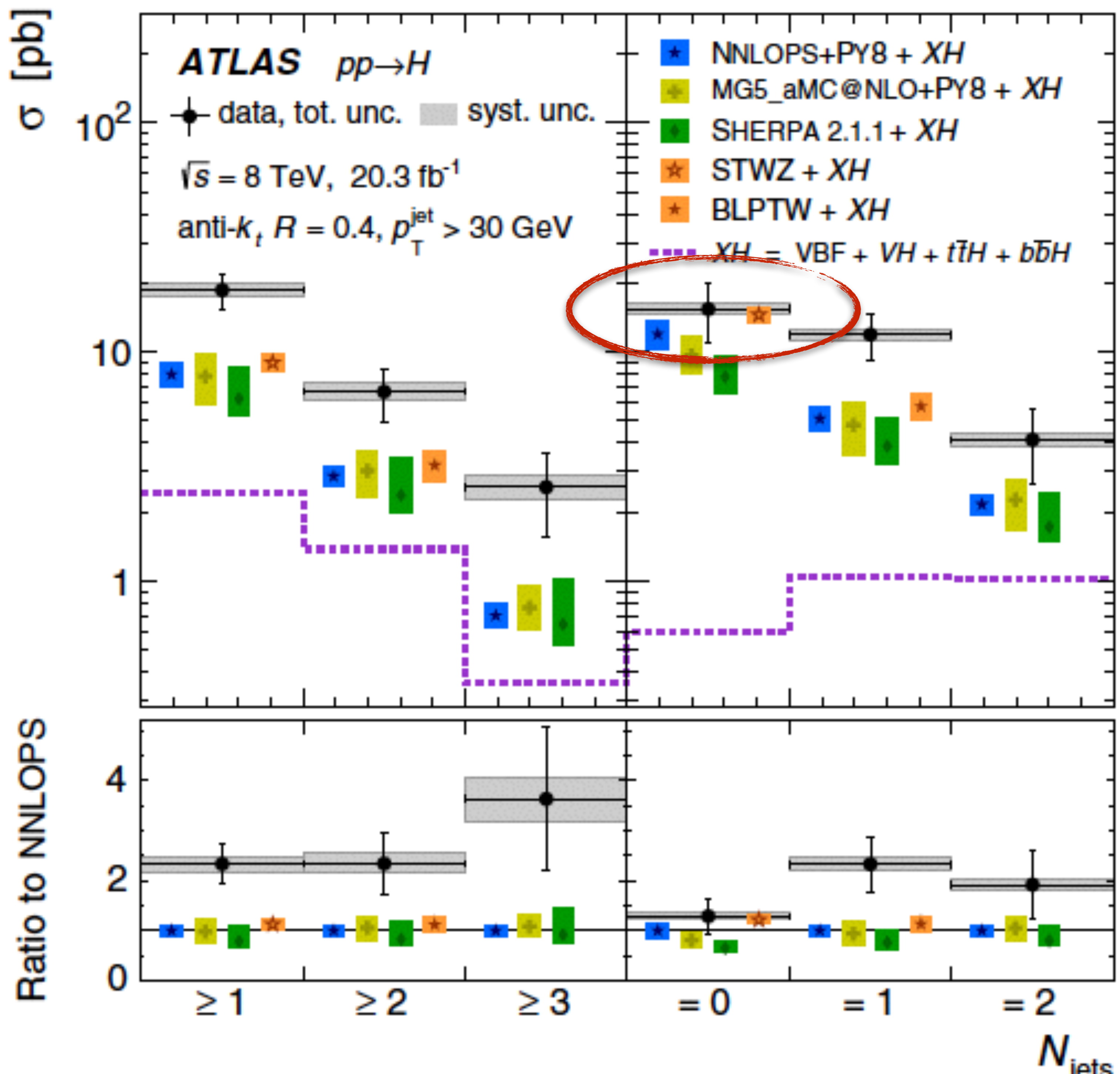


- Need to suppress massive background due to $t\bar{t} \to W^+W^-b\bar{b}$
- Veto all jets with a transverse momentum larger than $p_{\mathrm{t,veto}} \simeq 25 - 30\,\mathrm{GeV}$
- Relevant for HWW coupling/fiducial cross section measurements
- Same results apply to any colour-singlet production (e.g. Z, WW)

Higgs production with a jet veto

- Run I measurements of the 0-jet cross section

[PRL 115,091801 (2015)]

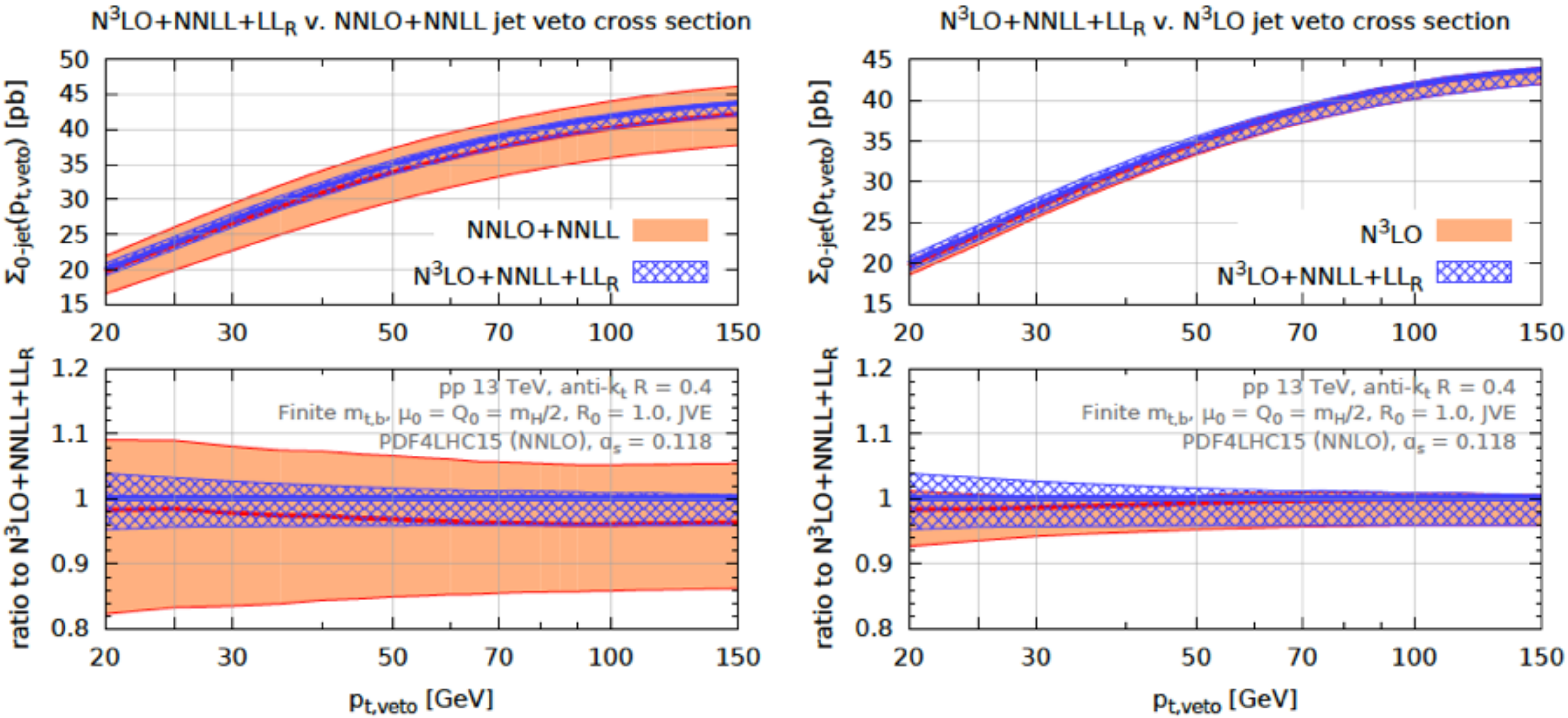


NNLL+NNLO: [Banfi, PM, Salam, Zanderighi 1206.4998; Becher, Neubert, Rothen 1307.0025; Stewart, Tackmann, Walsh, Zuberi 1307.1808]
Quark-mass effects: [Banfi, PM, Zanderighi 1308.4634]

Higgs production with a jet veto

[Banfi, Caola, Dreyer, PM, Salam, Zanderighi, Dulat 1511.02886]

- pp -> H + 0 jets at N3LO+NNLL+LL (small R) with quark-mass corrections



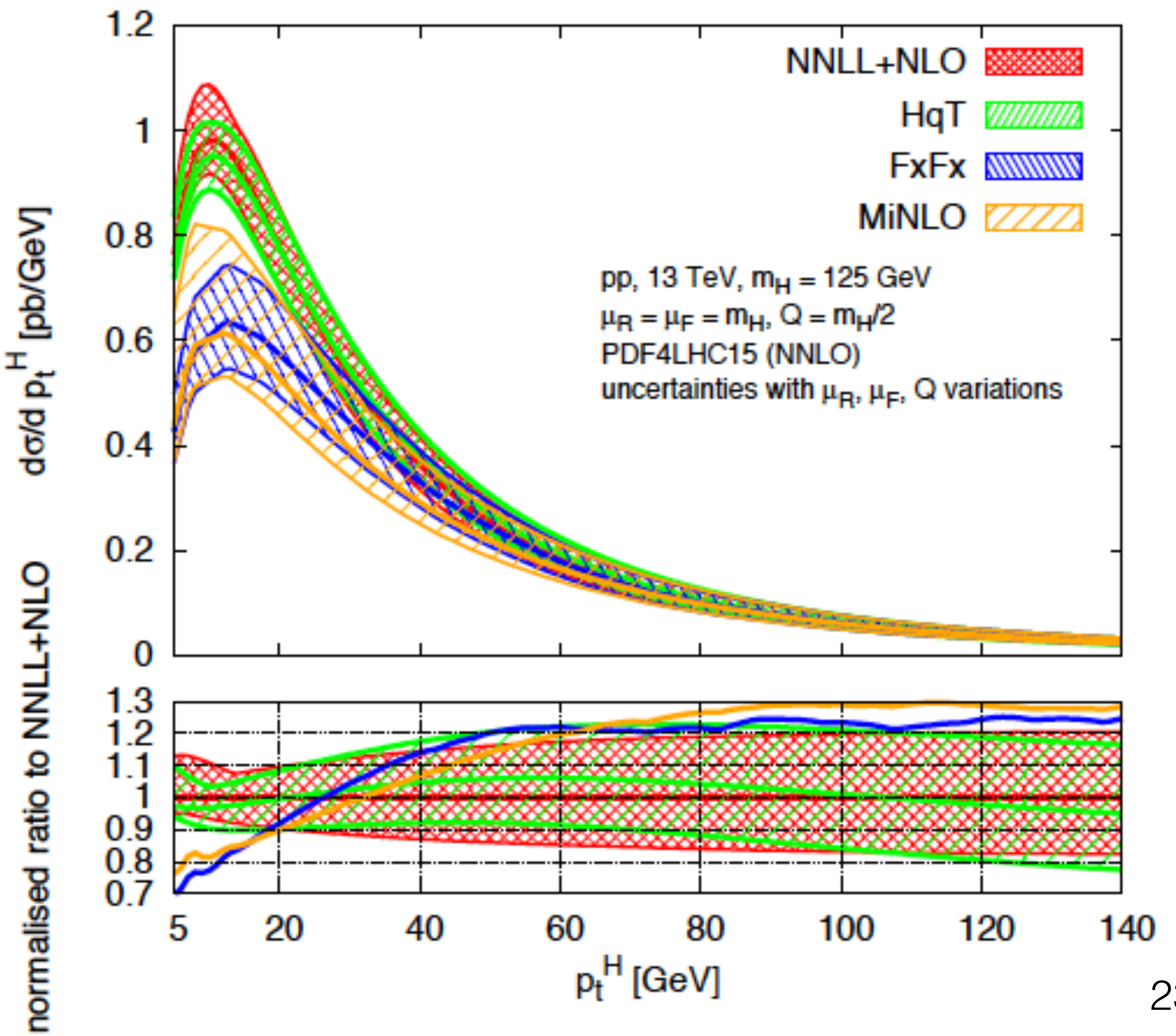
- Residual resummation effects of the same size as N3LO (~2%)
- Robust uncertainty assessment - residual theory error~3-4% (precision measurements, QCD effects under control)

Resummation performed in impact-parameter space up to NNLL+NLO:
[Bozzi, Catani, de Florian, Grazzini 0302104; Becher, Neubert 1007.4005]

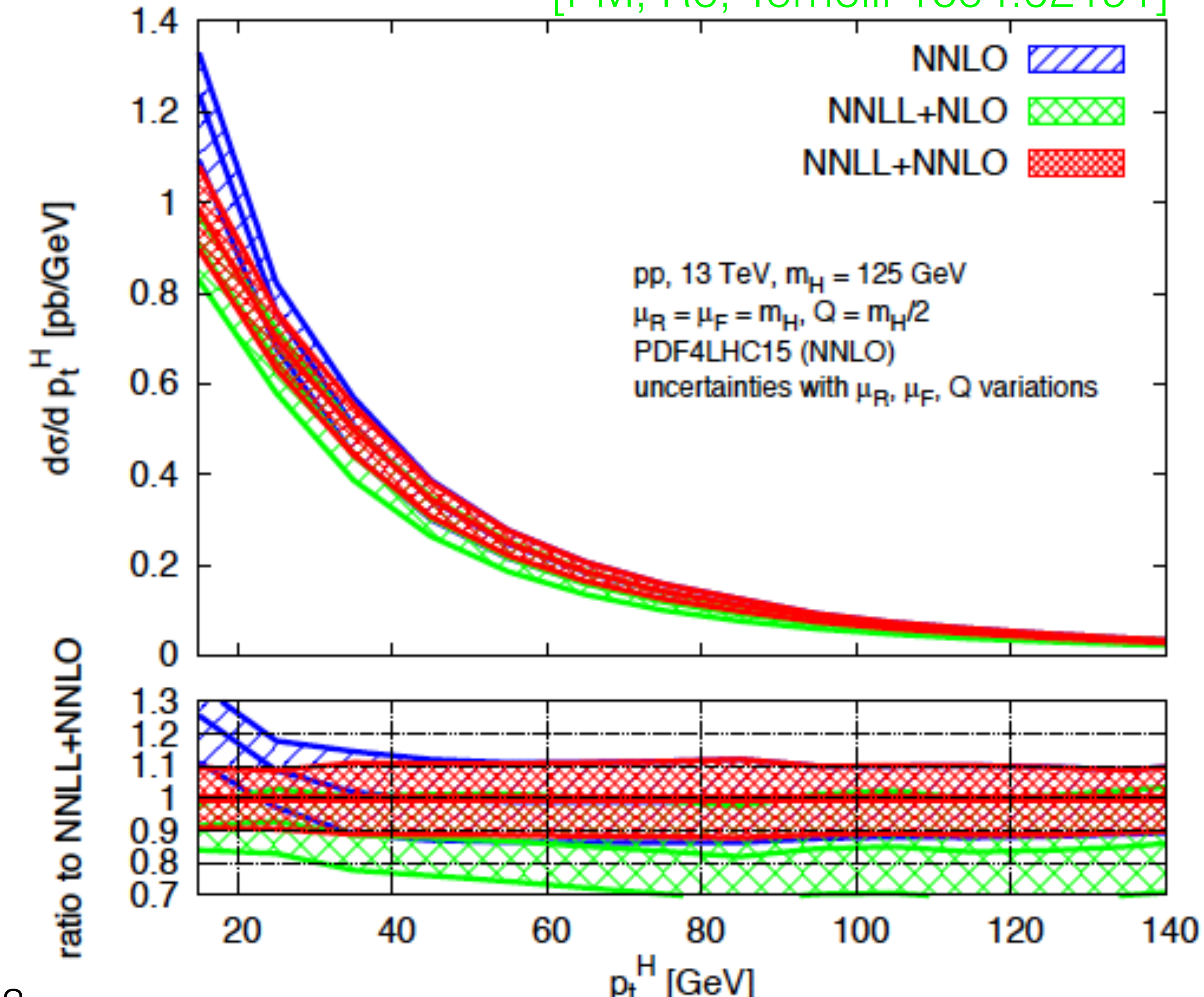
Higgs pT distribution at NNLL+NNLO

Fixed-order obtained combining N3LO cross section and H+1 jet @ NNLO
[Anastasiou et al. 1503.06056, 1602.00695] [Caola et al. 1508.02684; Boughezal et al. 1504.07922, 1505.03893; Chen et al. 1604.04085]

- Formulation in momentum space: no integral transforms required (luminosity in momentum space)
- Sizeable effects of NNLL resummation at small pt (~20% at 20 GeV), uncertainty reduced from 15-20% to10%
- Result in HEFT: beyond this accuracy heavy-quark (top, bottom) effects matter

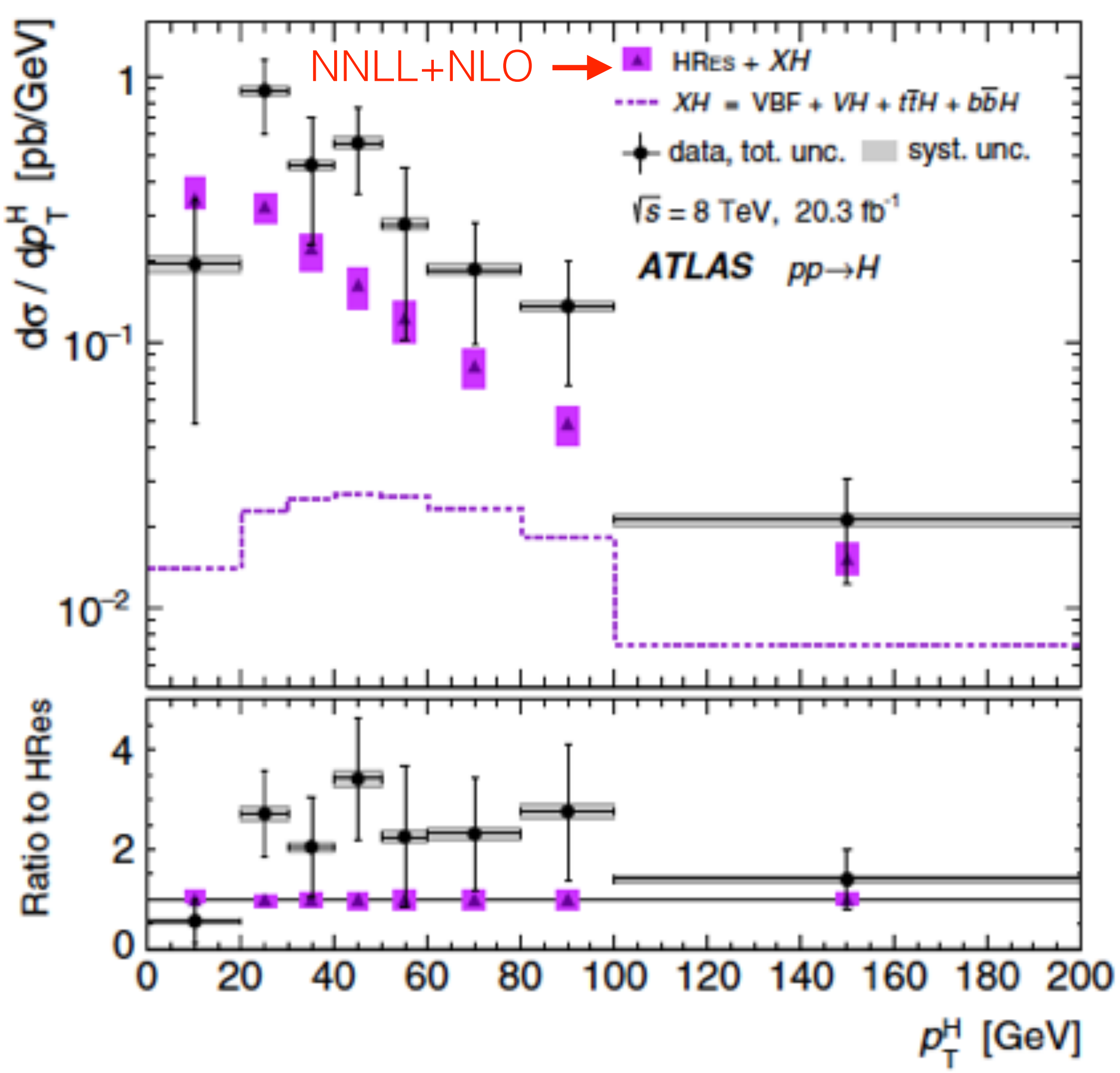


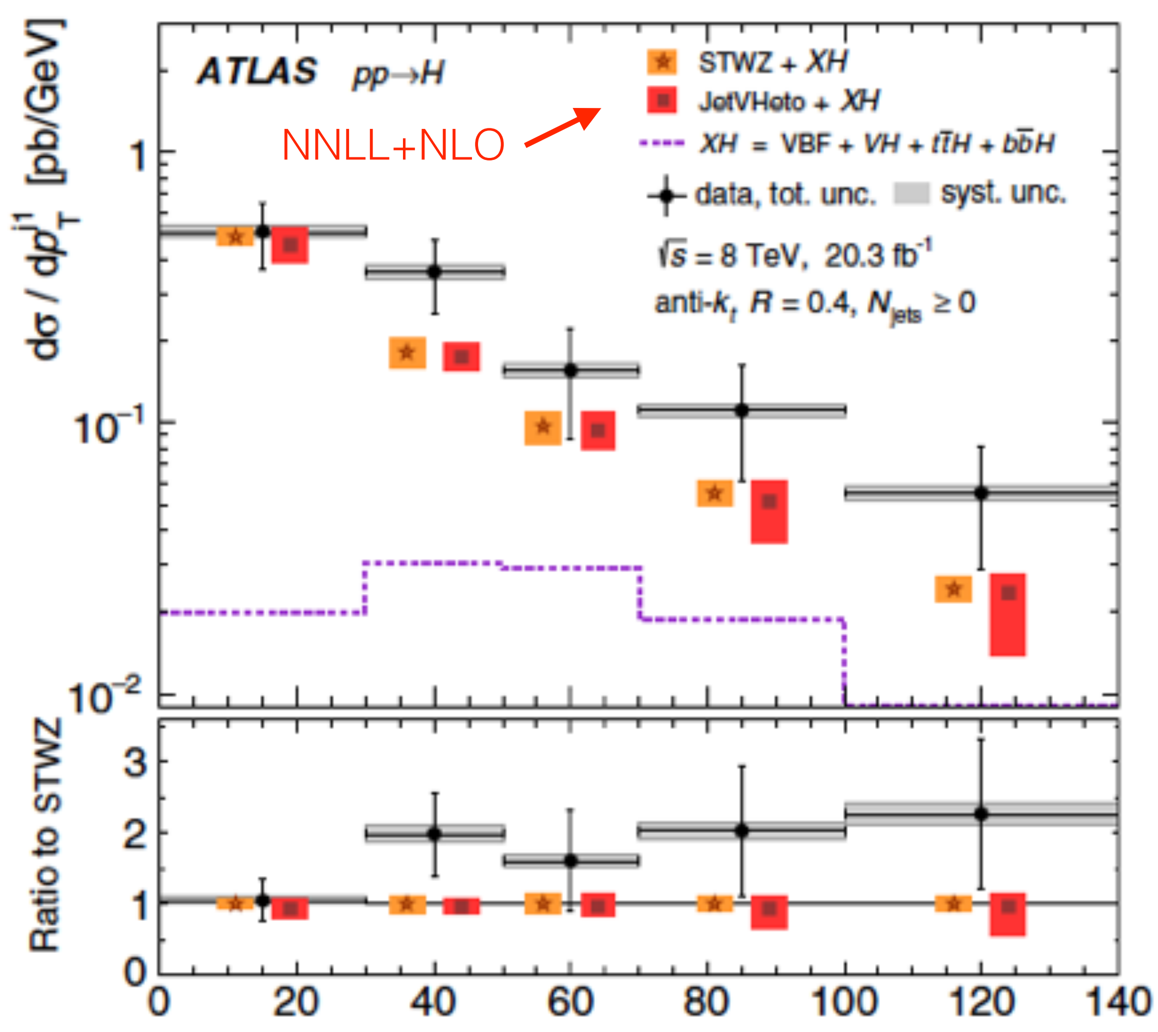
[PM, Re, Torrielli 1604.02191]



Differential distributions: Run I data

- Differential distributions sensitive to heavy-particle content in the production loop





[PRL 115,091801 (2015)]

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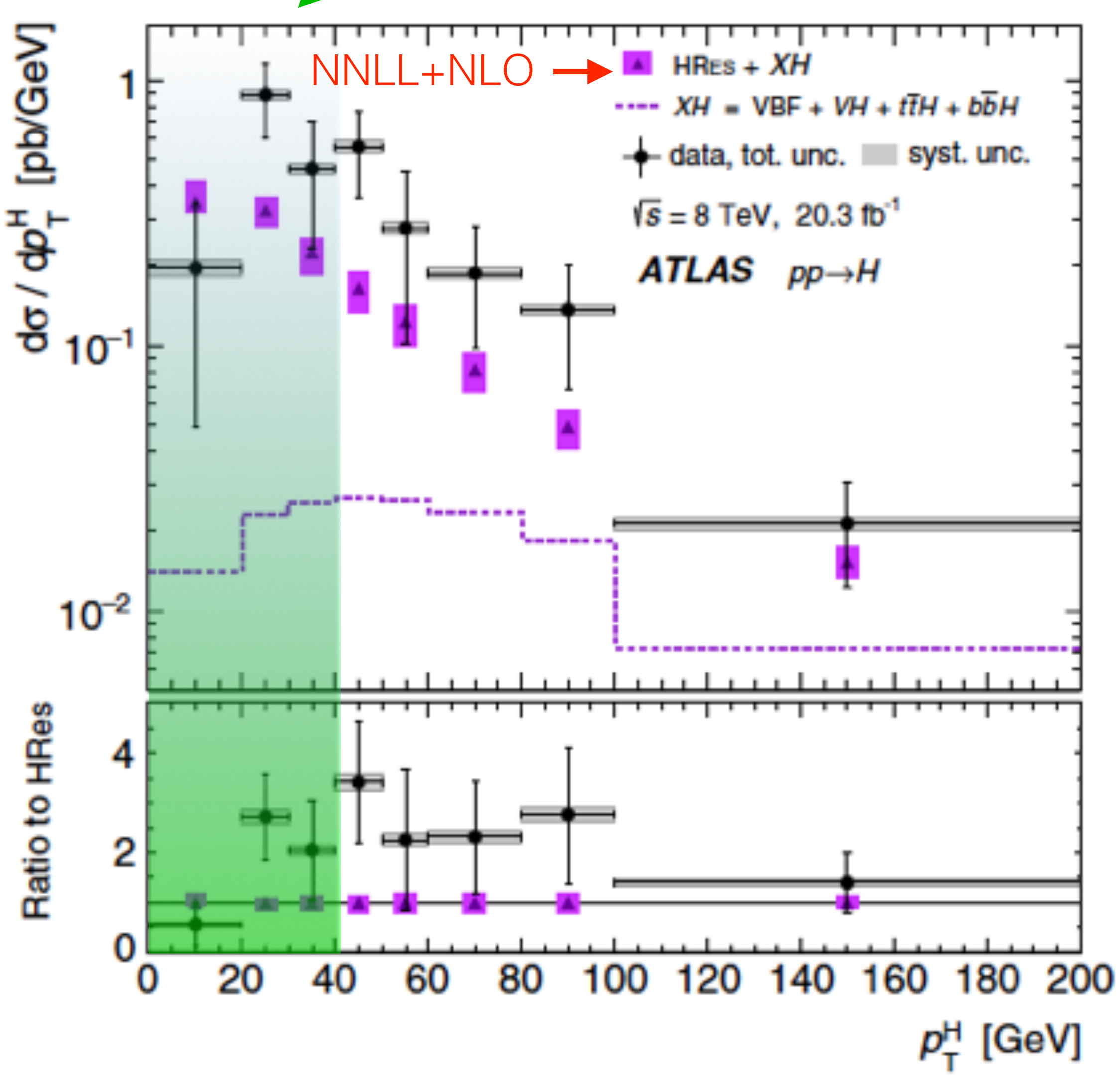
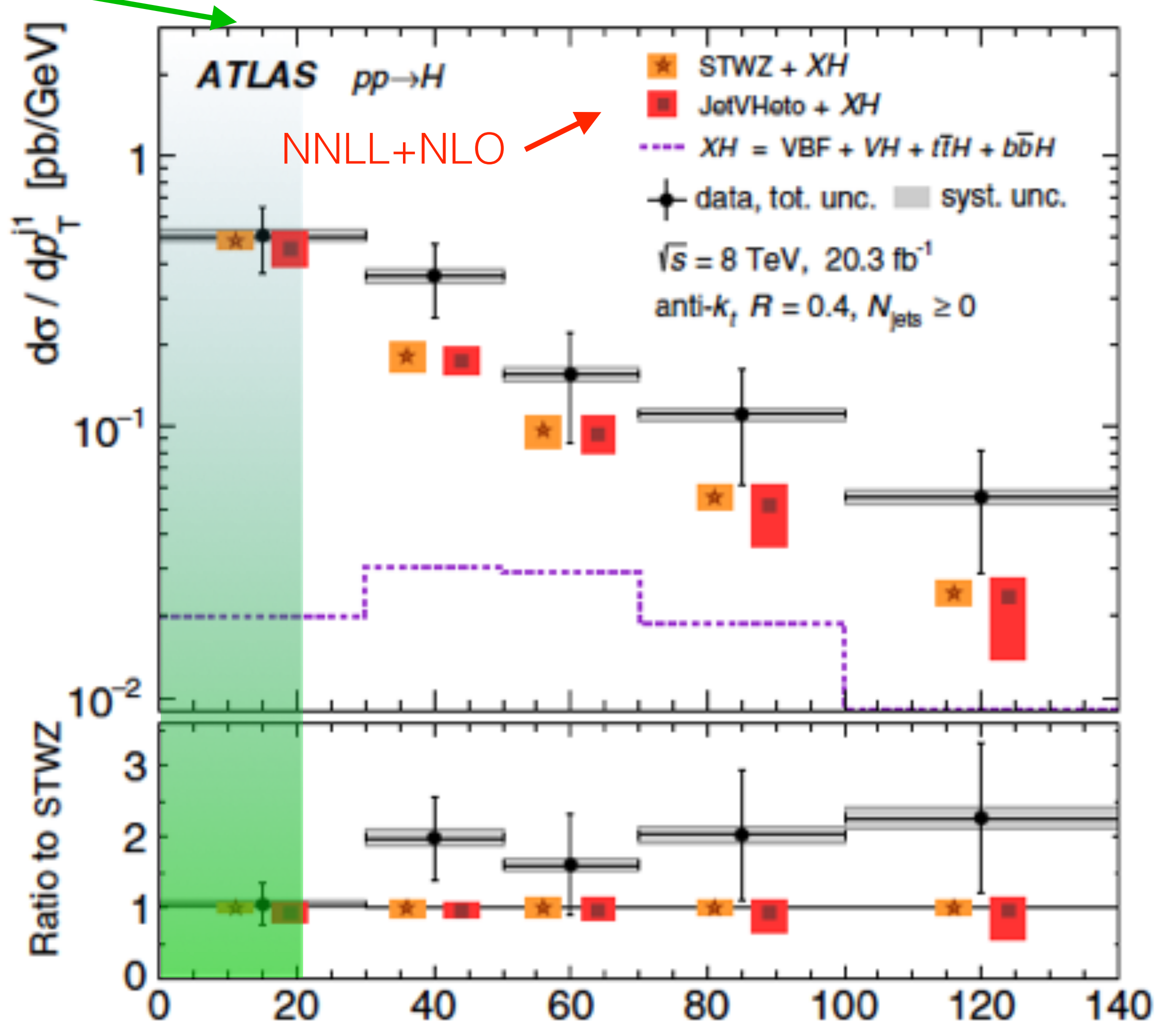
Areas sensitive to
Sudakov resummation:
NNLL effects very important





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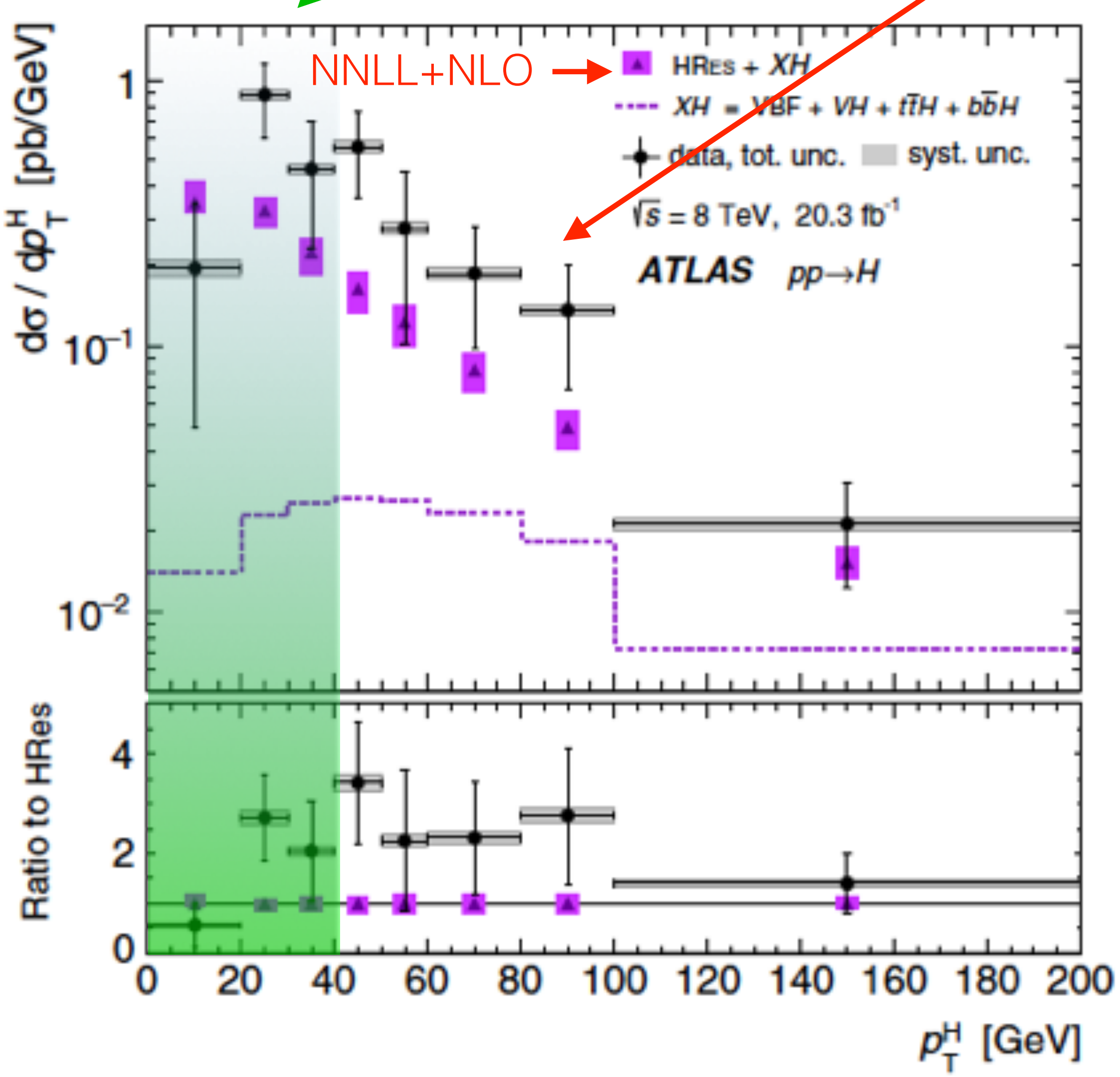
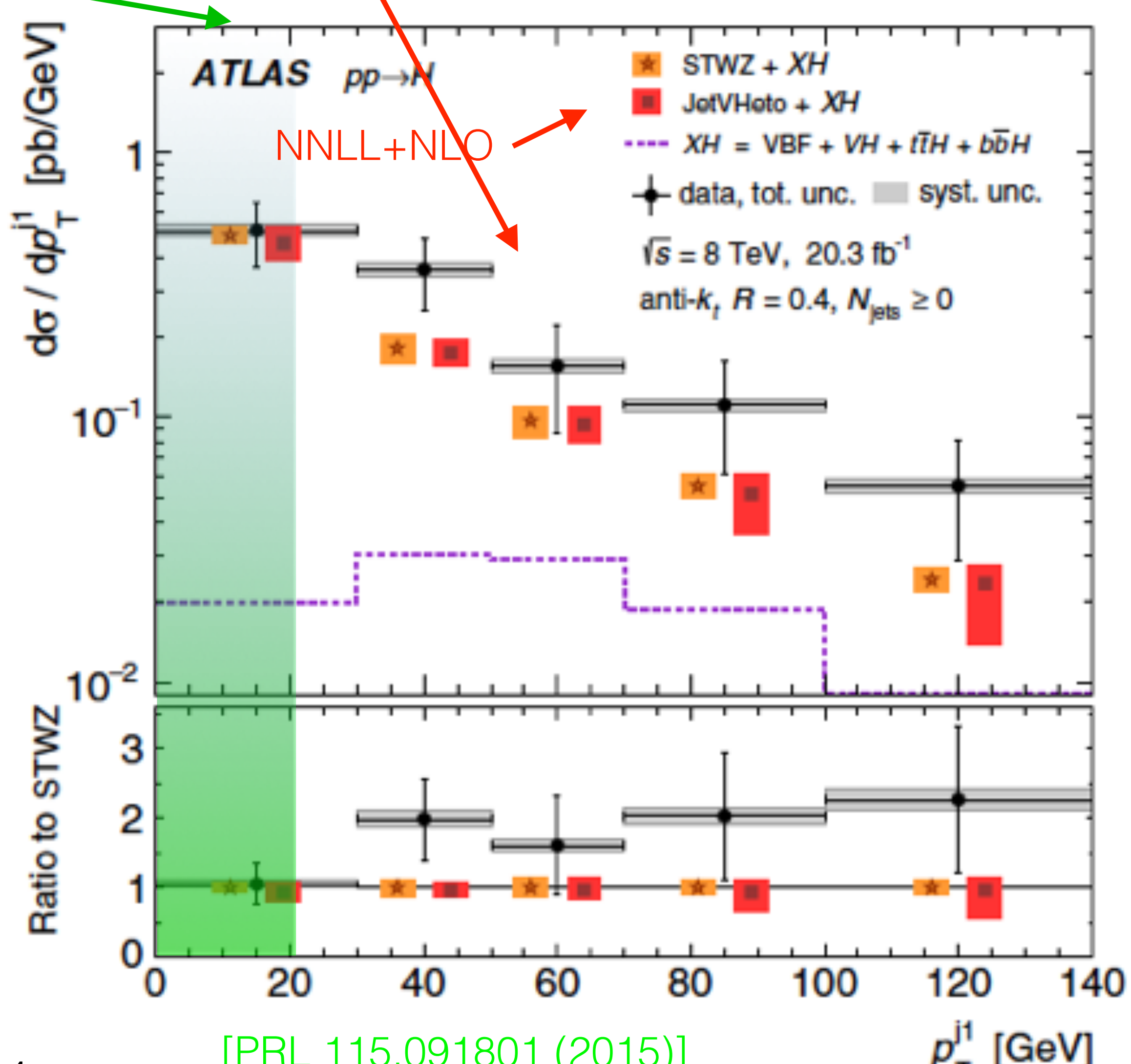
NNLO corrections important in the tails

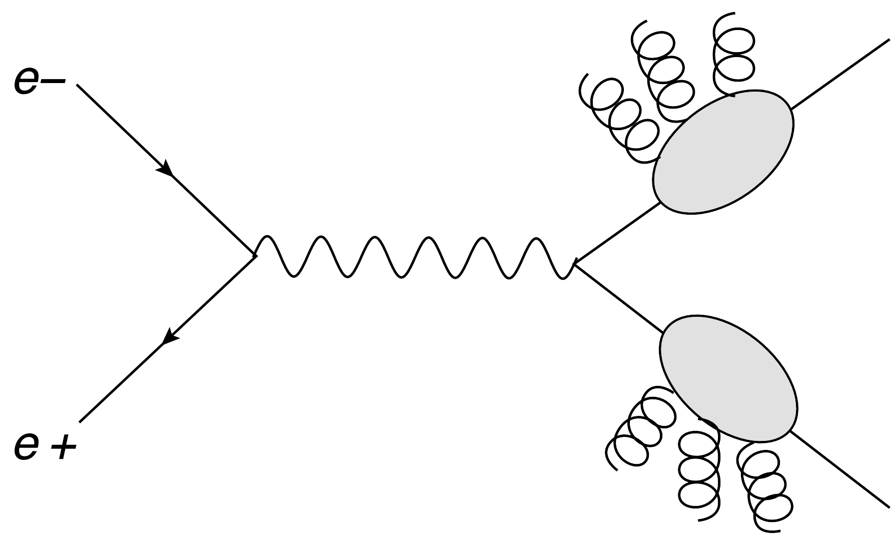




[PRL 115,091801 (2015)]

Event Shapes and Jet Rates in e+e-

- event shapes in e+e- -> 2 jets production



- Relevant (e.g.) for precise determination of the strong coupling - tension between recent extractions from thrust and C-parameter and world average (need for observables with different sensitivity to NP corrections)

- Toy model for final-state radiation (conceptually complete)

- Clean theory/exp laboratory to study non-perturbative corrections

- Development/tuning of MC generators

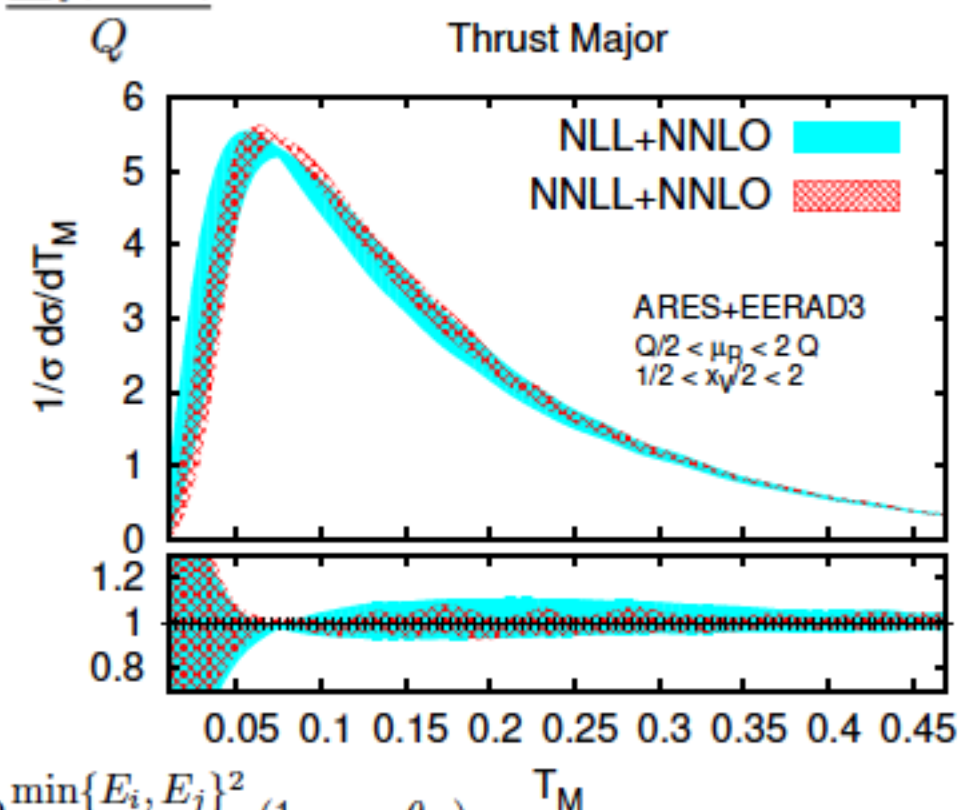
*NNLO fixed order from EERAD3
[Gehrmann-De Ridder et al.]

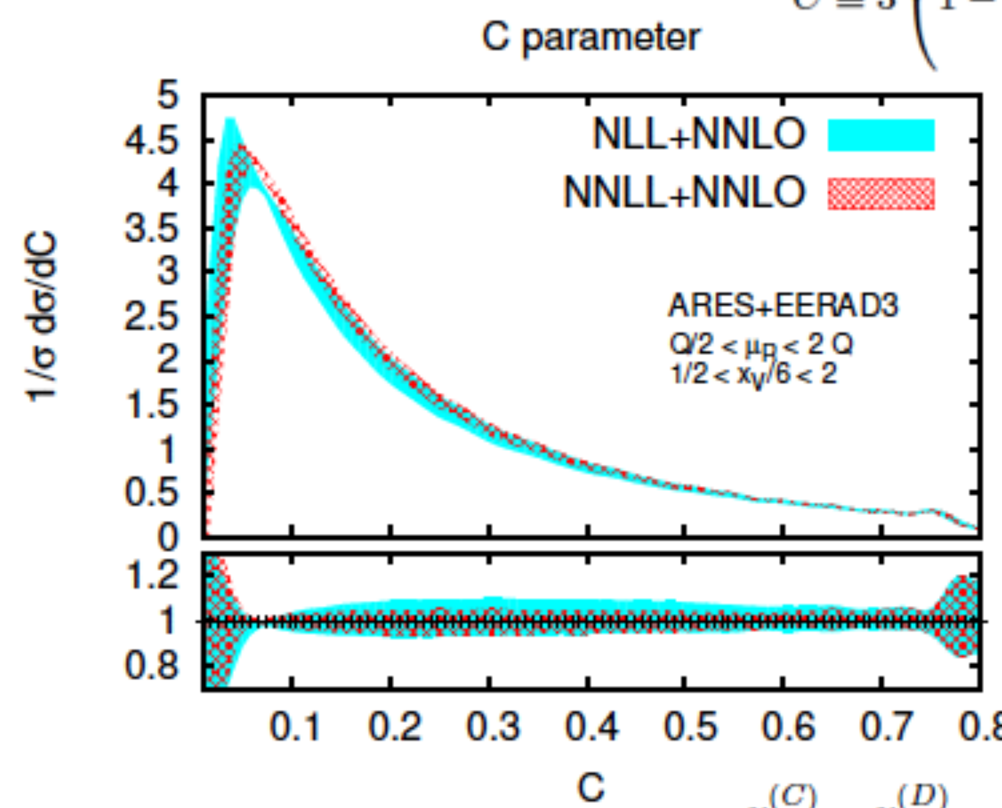
[Banfi, McAslan, PM, Zanderighi 1412.2126 and ongoing work]

e+e- Event Shapes and Jet Rates*

$$T_M \equiv \max_{\vec{n}\cdot\vec{n_T}=0} \frac{\sum_i |\vec{p_i}\cdot\vec{n}|}{Q}$$

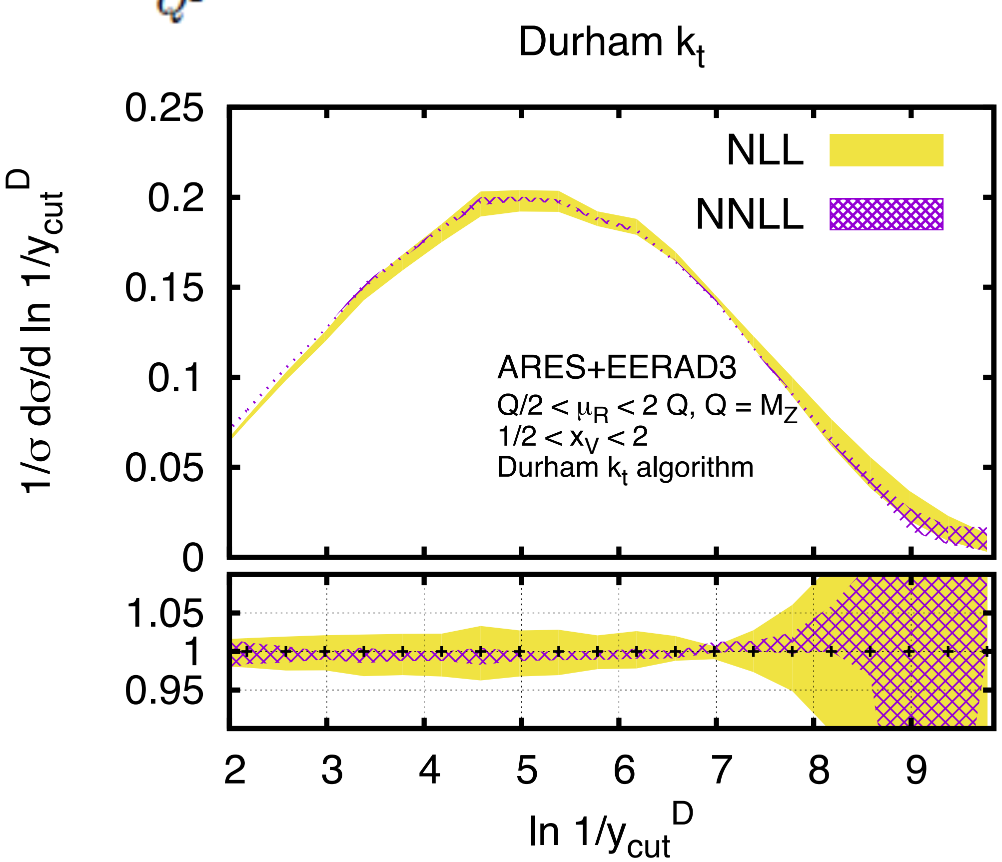
$$C \equiv 3\left(1 - \frac{1}{2}\sum_{i,j}\frac{(p_i\cdot p_j)^2}{(p_i\cdot Q)(p_j\cdot Q)}\right)$$

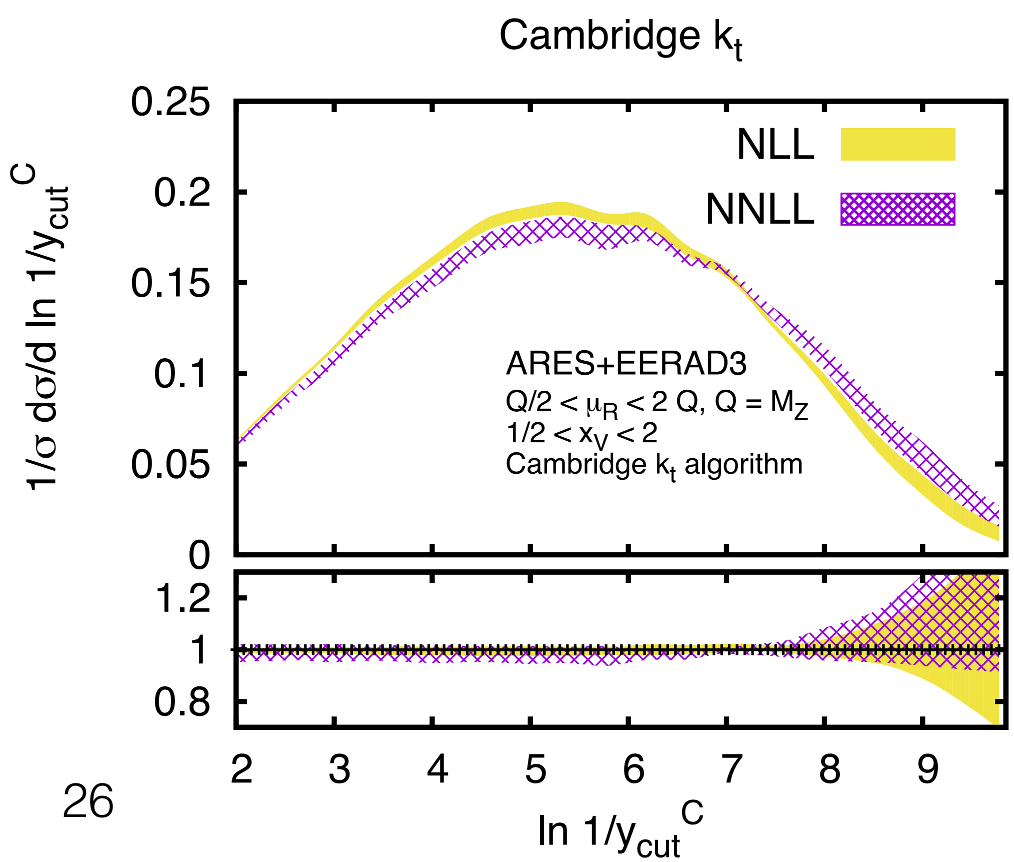




$$y_{ij}^{(D)} = v_{ij}^{(D)} = 2\frac{\min\{E_i, E_j\}^2}{Q^2}(1-\cos\theta_{ij})$$

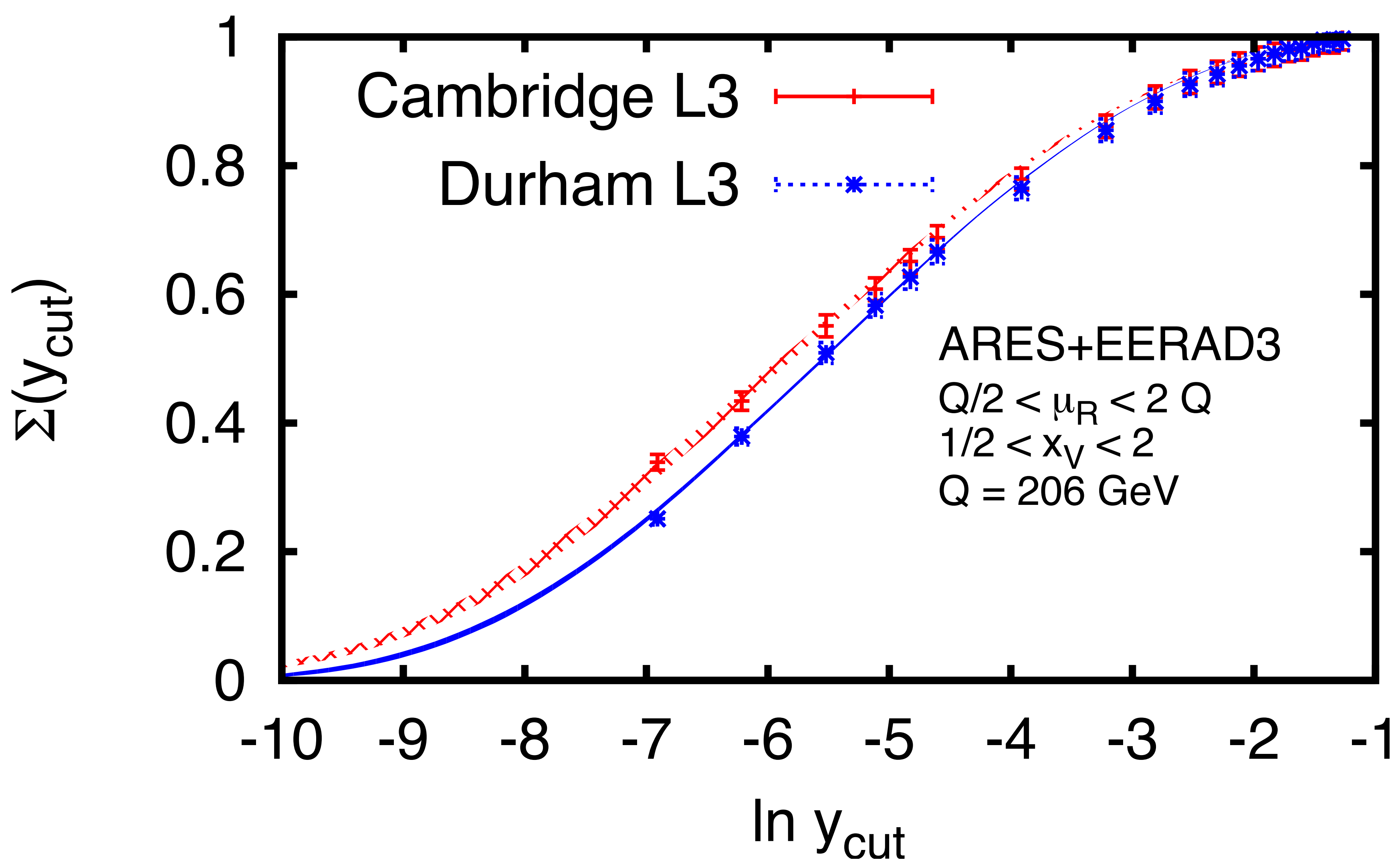
$$y_{ij}^{(C)} = y_{ij}^{(D)}, \qquad v_{ij}^{(C)} = 2(1-\cos\theta_{ij})$$





Jet Rates at LEP

- Jet rates less sensitive to hadronization corrections - helpful for accurate fits of the QCD coupling
- Improved agreement with LEP data at high energy



Reactions with 2 Born emitters

- General formulation now fully worked out for 2 hard emitters
  - Observables with very different logarithmic structures can be modelled with the same method (NNLL corrections generally sizeable)

**

correction type	$p_{t,\mathrm{veto}}$	$1-T$	B_T	B_W	C	ρ_H	T_M	O	$y_3^{\mathrm{Dur.}}$	$y_3^{\mathrm{Cam.}}$	p_t
$\mathcal{F}_{\mathrm{NLL}}$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$\delta\mathcal{F}_{\mathrm{rap}}$	x	✓	✓	✓	✓	✓	✓	✓	✓	x	✓
$\delta\mathcal{F}_{\mathrm{wa}}$	x	x	x	x	✓	x	x	x	✓	✓	x
$\delta\mathcal{F}_{\mathrm{hc}}$	x	✓	✓	✓	✓	✓	✓	✓	✓	x	✓
$\delta\mathcal{F}_{\mathrm{rec}}$	x	✓	✓	✓	✓	✓	✓	✓	✓	x	x
$\delta\mathcal{F}_{\mathrm{clust}}$	✓	x	x	x	x	x	x	x	✓	✓	x
$\delta\mathcal{F}_{\mathrm{correl}}$	✓	x	✓	✓	x	x	✓	✓	✓	✓	x

** no factorisation theorem available

- Extension to the most general case still requires:
  - non-global case
  - multileg case (e.g. pp -> Z+jet, pp -> 2 jets, e+e- -> 3 jet,…)
  - multiscale problems (more than one logarithmic family): joint resummations

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$\mathcal{F}_{\text{NLL}}$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$\delta\mathcal{F}_{\text{rap}}$	x	✓	✓	✓	✓	✓	✓	✓	✓	x	✓
$\delta\mathcal{F}_{\text{wa}}$	x	x	x	x	✓	x	x	x	✓	✓	x
$\delta\mathcal{F}_{\text{hc}}$	x	✓	✓	✓	✓	✓	✓	✓	✓	x	✓
$\delta\mathcal{F}_{\text{rec}}$	x	✓	✓	✓	✓	✓	✓	✓	✓	x	x
$\delta\mathcal{F}_{\text{clust}}$	✓	x	x	x	x	x	x	x	✓	✓	x
$\delta\mathcal{F}_{\text{correl}}$	✓	x	✓	✓	x	x	✓	✓	✓	✓	x

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Conclusions

- I presented a general formulation of the NNLL resummation for global rIRC observables: it is complete for two-scale problems in reactions with 2 hard Born emitters (i.e. given a process compute any global rIRC safe observable)
  - Easily implementable in a automated computer code
  - It can handle complex non-factorising observables - in principle extendable to higher orders
  - General formulation for multi leg processes and for non-global observables hasn't been worked out yet at this order
  - Entirely formulated in direct space (fast evaluation)
- Observables with non-Born-like cancellations now also treatable (e.g. ptH)
- First hints on how to tackle new problems with multiple scales at NNLL order - until now out of reach

Thank you for your attention

Requirements on the observable

- Parametrisation for single emission and collinear splitting

$$V(\{\tilde{p}\},\kappa_i(\zeta_i)) = \zeta_i\,; \qquad \kappa_i(\zeta) \to \{\kappa_{ia},\kappa_{ib}\}(\zeta,\mu)\,,\; \mu^2 = (\kappa_{ia}+\kappa_{ib})^2/\kappa_{ti}^2$$

- The standard requirement of IRC safety implies that

$$\lim_{\zeta_{m+1}\to 0} V(\{\tilde{p}\},\kappa_1(\bar{v}\zeta_1),\dots,\kappa_m(\bar{v}\zeta_m),\kappa_{m+1}(\bar{v}\zeta_{m+1})) = V(\{\tilde{p}\},\kappa_1(\bar{v}\zeta_1),\dots,\kappa_m(\bar{v}\zeta_m))$$

$$\lim_{\mu\to 0} V(\{\tilde{p}\},\kappa_1(\bar{v}\zeta_1),\dots,\{\kappa_{ia},\kappa_{ib}\}(\bar{v}\zeta_i,\mu),\dots\kappa_m(\bar{v}\zeta_m)) = V(\{\tilde{p}\},\kappa_1(\bar{v}\zeta_1),\dots,\kappa_i(\bar{v}\zeta_i),\dots\kappa_m(\bar{v}\zeta_m))$$

- We limit ourselves to *continuously* global observables*, i.e. the transverse momentum dependence is the same everywhere (it ensures the absence of non-global logarithms)

*Not a real limitation, although currently NNLL structure of non-global logarithms unknown

Requirements on the observable

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$$V(\{\tilde{p}\}, \kappa_i(\zeta_i)) = \zeta_i \,; \qquad \kappa_i(\zeta) \to \{\kappa_{ia}, \kappa_{ib}\}(\zeta, \mu)\,,\; \mu^2 = (\kappa_{ia} + \kappa_{ib})^2/\kappa_{ti}^2$$

- Impose the following conditions, known as recursive IRC (rIRC) safety

$$\lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_m(\bar{v}\zeta_m)) \qquad (1)$$

  - The above limit must be well defined and non-zero (except possibly in a phase space region of zero measure)
  - Condition (1) simply requires the observable to scale in the same fashion for multiple emissions as for a single emission (IRC divergences have an exponential form)
  - It is enough to ensure the exponentiation of double logarithms to all orders

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$$\lim_{\zeta_{m+1}\to 0} \lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_m(\bar{v}\zeta_m), \kappa_{m+1}(\bar{v}\zeta_{m+1}))$$

$$= \lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_m(\bar{v}\zeta_m)) \qquad (2.a)$$

$$\lim_{\mu\to 0} \lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \{\kappa_{ia}, \kappa_{ib}\}(\bar{v}\zeta_i, \mu), \ldots \kappa_m(\bar{v}\zeta_m))$$

$$= \lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_i(\bar{v}\zeta_i), \ldots \kappa_m(\bar{v}\zeta_m)) \qquad (2.b)$$

Requirements on the observable

- Parametrisation for single emission and collinear splitting

$$V(\{\tilde{p}\}, \kappa_i(\zeta_i)) = \zeta_i \,; \qquad \kappa_i(\zeta) \to \{\kappa_{ia}, \kappa_{ib}\}(\zeta, \mu)\,,\ \mu^2 = (\kappa_{ia} + \kappa_{ib})^2/\kappa_{ti}^2$$

- Impose the following conditions, known as recursive IRC (rIRC) safety

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$$\lim_{\zeta_{m+1}\to 0} \lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_m(\bar{v}\zeta_m), \kappa_{m+1}(\bar{v}\zeta_{m+1}))$$

$$= \lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_m(\bar{v}\zeta_m)) \qquad (2.\mathrm{a})$$

$$\lim_{\mu\to 0} \lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \{\kappa_{ia}, \kappa_{ib}\}(\bar{v}\zeta_i, \mu), \ldots \kappa_m(\bar{v}\zeta_m))$$

$$= \lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_i(\bar{v}\zeta_i), \ldots \kappa_m(\bar{v}\zeta_m)) \qquad (2.\mathrm{b})$$

Requirements on the observable

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$$V(\{\tilde{p}\}, \kappa_i(\zeta_i)) = \zeta_i\,; \qquad \kappa_i(\zeta) \to \{\kappa_{ia}, \kappa_{ib}\}(\zeta, \mu)\,,\ \mu^2 = (\kappa_{ia} + \kappa_{ib})^2/\kappa_{ti}^2$$

- Impose the following conditions, known as recursive IRC (rIRC) safety

- Conditions (2.a) and (2.b), in addition to plain IRC safety, require that for sufficiently small $\bar{v}$ there exists some ϵ that can be chosen *independently* of $\bar{v}$ such that we can neglect any emissions at scales $\sim \epsilon\bar{v}$

- The order with which one takes the limit is different in fixed-order and resummed calculations, and the final result must not change

$$\lim_{\mu\to 0}\lim_{\bar{v}\to 0}\frac{1}{\bar{v}}V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \{\kappa_{ia}, \kappa_{ib}\}(\bar{v}\zeta_i, \mu), \ldots \kappa_m(\bar{v}\zeta_m))$$
$$= \lim_{\bar{v}\to 0}\frac{1}{\bar{v}}V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_i(\bar{v}\zeta_i), \ldots \kappa_m(\bar{v}\zeta_m)) \qquad (2.\mathrm{b})$$