# Quadrilaterals and Their Properties
## A 4-gon Hypothesis

**SUGGESTED LEARNING STRATEGIES:** Activating Prior Knowledge, Group Discussion, Shared Reading, Create Representations, Think/Pair/Share

Mr. Minnow's art class is beginning a unit on mosaic tiling. He wants to introduce this unit with a 3-hour exploration involving pattern blocks of varying shapes and colors. Mr. Minnow directed Gilligan and Mary Ann to get the containers of pattern blocks and distribute them to the rest of the class as they split into groups.

Ginger batted her eyes and pleaded with Gilligan to give her only quadrilaterals, so he gave her all of the shapes that were not triangles or hexagons. Ginger really only wanted "the blue shapes because they look like diamonds." Ginger's partner, Roy, informed her that she should have asked for the rhombi that were not squares, and everyone looked at Roy as if he had two heads. (Roy was so smart most people called him Professor.)

Later that day, Mr. Minnow shared this episode with Mrs. Howell, who taught many of these students in her geometry class. Mrs. Howell decided that this was a "teachable moment." So she began her unit on quadrilaterals and tapped into her students' experiences with the pattern blocks. Who knew that Mr. Minnow's 3-hour exploration could become such an adventure for Mrs. Howell and her students!

In this activity, you will explore convex quadrilaterals. The term *quadrilateral* can be abbreviated "quad."
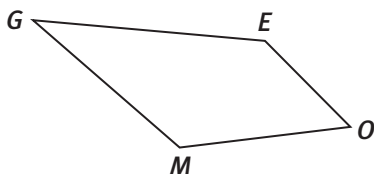


**1.** Given Quad *GEOM*

    **a.** List all pairs of opposite sides.

    **b.** List all pairs of consecutive sides.

    **c.** List all pairs of opposite angles.

    **d.** List all pairs of consecutive angles.

    **e.** Draw the diagonals, and list them.

My Notes

**My Notes**

**SUGGESTED LEARNING STRATEGIES:** Activating Prior Knowledge, Think/Pair/Share, Group Discussion, Interactive Word Wall, Group Presentations

A **kite** is a quadrilateral with exactly two distinct pairs of congruent consecutive sides.
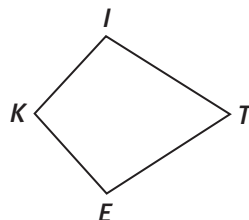


**2.** Given Quad *KITE* with $\overline{KI} \cong \overline{KE}$ and $\overline{IT} \cong \overline{ET}$

**a.** One of the diagonals divides the kite into two congruent triangles. Draw that diagonal and list the two congruent triangles. Explain how you know the triangles are congruent.

**b.** Draw the other diagonal. Explain how you know the diagonals are perpendicular.

**c.** Complete the following list of properties of a kite. Think about the angles of a kite as well as the segments.

1.  Exactly two pairs of consecutive sides are congruent.

2.  One diagonal divides a kite into two congruent triangles.

3.  The diagonals of a kite are perpendicular.

4.

5.

6.

**SUGGESTED LEARNING STRATEGIES:** Think/Pair/Share, Interactive Word Wall, Create Representations, Group Discussion

The segment whose endpoints are the midpoint of two sides of a triangle is called a **midsegment**.

**Triangle Midsegment Theorem** The midsegment of a triangle is parallel to the third side, and its length is one half the length of the third side.

**3.** Use the figure and coordinates below to complete the **coordinate proof** for the Triangle Midsegment Theorem.
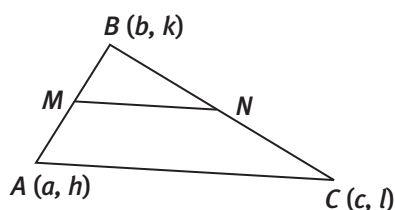


**a.** Complete the hypothesis and conclusion for the Triangle Midsegment Theorem.

> **Hypothesis**  *M* is the midpoint of _____
>
>  *N* is the midpoint of _____
>
> **Conclusion**  $\overline{MN} \parallel$ _____
>
>  $MN =$ _____

**b.** Find the coordinates of midpoints *M* and *N* in terms of *a*, *b*, *c*, *h*, *k* and *l*.

**c.** Find the slope of $\overline{AC}$ and $\overline{MN}$.

**d.** Simplify your response to Part c and explain how your answers to Part c show $\overline{MN} \parallel \overline{AC}$.

**My Notes**

**MATH TIP**

For this proof, you will want to use the following formulas:

Given $A(x_1, y_1)$ and $B(x_2, y_2)$

**Midpoint Formula:**
$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Slope of $\overline{AB}$: $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

**Distance Formula**
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**My Notes**

**e.** Find *AC* and *MN*.

**f.** Simplify your response to Part e and explain how your answers to Part e show that $MN = \frac{1}{2} AC$.

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**MATH TERMS**
**trapezoid**

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A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called **bases**, and the non-parallel sides are called **legs**. The pairs of consecutive angles that include each of the bases are called **base angles**.

**4.** Sketch a trapezoid and label the vertices *T, R, A,* and *P*. Identify the bases, legs and both pairs of base angles.

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**MATH TERMS**
A **trapezium** is a quadrilateral with no parallel sides.

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**CONNECT TO LANGUAGE**

The British use the term **trapezium** for a quadrilateral with exactly one pair of parallel sides and the term **trapezoid** for a quadrilateral with no parallel sides. They drive on a different side of the road, too.

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**The median of a trapezoid** is the segment each of whose endpoints is the midpoint of a leg of the trapezoid.
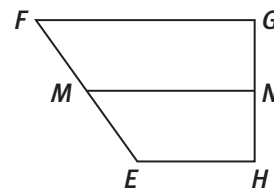
**Trapezoid Median Theorem** The median of a trapezoid is parallel to the bases and its length is the average of the lengths of the bases.

**Given:** Trapezoid *EFGH*
$\overline{MN}$ is a median

**Prove:** $\overline{MN} \parallel \overline{FG}$ and $\overline{MN} \parallel \overline{EH}$
$MN = \frac{1}{2} (FG + EH)$



**5.** Draw one diagonal in trapezoid *EFGH*. Label the intersection of the diagonal with $\overline{MN}$ as *X* and explain below how the Triangle Midsegment Theorem can be used to justify the Trapezoid Median Theorem.
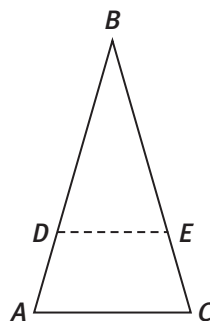
My Notes

**6.** Given Trapezoid *EFGH* and $\overline{MN}$ is a median. Use the figure, properties of trapezoids and/or the Trapezoid Median Theorem for each of the following.

    **a.** If $m\angle GFE = 42°$, then $m\angle NME =$ _____ and $m\angle MEH =$ _____.

    **b.** Write an equation and solve for $x$ if $FG = 4x + 4$, $EH = x + 5$ and $MN = 22$.

    **c.** Find $FG$ if $MN = 19$ and $EH = 12$.

An **isosceles trapezoid** is a trapezoid with congruent legs.

**7.** Given $\triangle ABC$ is isosceles with $AB = CB$ and $AD = CE$.

    **a.** $\angle A \cong$ _____ Explain.

    **b.** Explain why $\triangle BDE$ is isosceles.

    **c.** $\overline{AC} \parallel$ _____ Explain.

    **d.** Explain why Quad *ADEC* is an isosceles trapezoid.

    **e.** $\angle ADE \cong$ _____ Explain.

    **f.** Complete the theorem.

    The base angles of an isosceles trapezoid are _____.
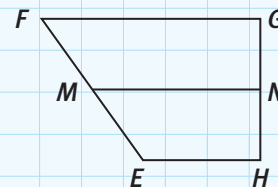
**My Notes**

**8.** On grid paper, plot Quad *COLD* with coordinates $C(1, 0)$, $O(2, 2)$, $L(5, 3)$ and $D(7, 2)$.

  **a.** Show that Quad *COLD* is a trapezoid.

  **b.** Show that Quad *COLD* is isosceles.

  **c.** Identify and find the length of each diagonal.

  **d.** Based on the results in Part c, complete the theorem.
  The diagonals of an isosceles trapezoid are _____.

**9.** At this point, the theorem in Item 8 is simply a conjecture based on one example. Given the figure below, write the key steps for a proof of the theorem. Hint: You may want to use a pair of overlapping triangles and the theorem from Item 8 as part of your argument.

**Hypothesis:** *CORE* is a trapezoid
$\overline{CO} \cong \overline{ER}$
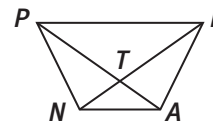**Conclusion:** $\overline{CR} \cong \overline{EO}$
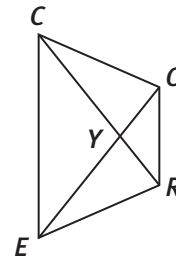


**10.** Given Quad *PLAN* is an isosceles trapezoid, use the diagram below and the properties of isosceles trapezoids to find each of the following.

  **a.** $\angle LPN \cong$ _____

  **b.** If $m\angle PLA = 70°$, then
  $m\angle LPN =$ _____ and
  $m\angle PNA$ _____.

  **c.** Write an equation and solve for
  $x$ if $AP = x$ and $NL = 3x - 8$.

My Notes

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. For the sake of brevity, the symbol ▱ can be used for *parallelogram*.

**11.** Given ▱*KATY* as shown.

> **a.** Which angles are consecutive to ∠*K*?
>
> **b.** Use what you know about parallel lines to complete the theorem.
>
> Consecutive angles of a parallelogram are ——————.
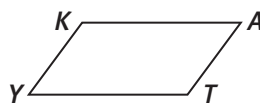
**MATH TERMS**
**parallelogram**

**12.** Fold an index card in half, and draw a scalene triangle on one half. As you cut out your triangle, keep the card folded so you cut out two identical (congruent) triangles. By putting the two triangular pieces together, how many different parallelograms can be formed? Sketch each parallelogram along with the side that is common to the two triangles.

**13.** Based upon the exploration in Item 12, complete the theorem.

Each diagonal of a parallelogram divides that parallelogram into

——————————————.
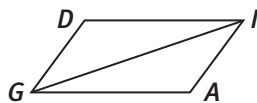
**14.** Given parallelogram *DIAG* as shown above. Complete the theorems.

> **a.** Opposite sides of a parallelogram are ————.
>
> **b.** Opposite angles of a parallelogram are ————.
>
> **c.** Prove the theorem you completed in part a. Use the figure in Item 13.
>
> **d.** Prove the theorem you completed in part b. Use the figure in Item 13.

**My Notes**

**MATH TERMS**
A **corollary** is a statement that results directly from a theorem.

**CONNECT TO AP**
Theorems are key to the development of many branches of mathematics. In calculus, two theorems that are frequently used are the Mean Value Theorem and the Fundamental Theorem of Calculus.
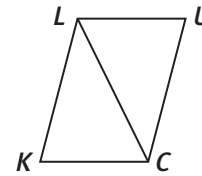
**15.** Explain why the theorems in Item 14 can be considered as corollaries to the theorem in Item 13.

**16.** Given $\square LUCK$, use the figure and the theorems in Items 11, 13, and 14 to find the following.

   **a.** $\triangle KCL \cong$ _____

   **b.** Solve for $x$ if
$m\angle KCU = 10x - 15$ and
$m\angle K = 6x + 3$.

   **c.** Solve for $x$ and $y$ if
$KL = 2x + y$, $LU = 7$,
$UC = 14$ and $KC = 5y - 4x$.

**Theorem:** The diagonals of a parallelogram bisect each other.

**17 a.** Rewrite the above theorem in "if-then" form.

   **b.** Draw a figure for the theorem, including the diagonals. Label the vertices and the point of intersection for the diagonals. Identify the information that is "Given" and what is to be proved.

Given:

Prove:

   **c.** Write a 2-column proof for the theorem.

| Statements | Reasons |
|---|---|
|  |  |

**SUGGESTED LEARNING STRATEGIES:** Activating Prior Knowledge, Think/Pair/Share, Interactive Word Wall, Group Discussion, Create Representations

A **rectangle** is a parallelogram with four right angles.

**18.** Given quad *RECT* is a rectangle. List all right triangles in the figure to the right. Explain how you know the triangles are congruent.
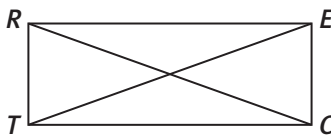


**19.** Complete the theorem.

The diagonals of a rectangle are _____.

**20.** Explain how you know the theorem in Item 19 is true.

**21.** List all of the properties of a rectangle. Begin with the properties of a parallelogram.

**22.** Given Quad *PINK* is a rectangle with coordinates $P(3,0)$, $I(0,6)$, and $N(8,10)$. Find the coordinates of point *K*.

**23.** Given Quad *TGIF* is a rectangle. Use the properties of a rectangle and the figure below to find the following.

**a.** If $TX = 13$, then $TI =$ _____ and $FG =$ _____.

**b.** Solve for $x$ if $TX = 4x + 4$ and $FX = 7x - 23$.

**c.** Solve for $x$ if $m\angle XFT = 6x - 4$ and $m\angle XTG = 10x - 2$.
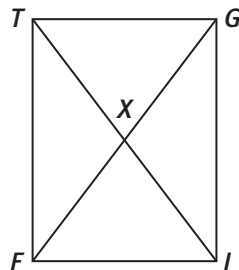


> **MATH TIP**
>
> If a figure is defined to be a parallelogram, then it has all of the properties of a parallelogram.

**My Notes**

**My Notes**

**ACADEMIC VOCABULARY**

An **Indirect proof** begins by assuming the opposite of the conclusion. The assumption is used as if it were given until a contradiction is reached. Once the assumption leads to a contradiction, the opposite of the assumption (the original conclusion) must be true.

**Indirect proofs** can be useful when the conclusion is a negative statement.

**Example of an Indirect Proof**

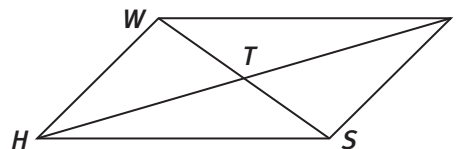> **Given** $m\angle SCR \neq m\angle CSI$
>
> **Prove** $\square RISC$ is not a rectangle.
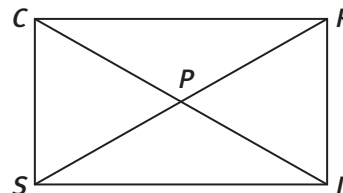
| Statements | Reasons |
|---|---|
| 1. $\square RISC$ is a rectangle | 1. Assumption |
| 2. $m\angle SCR = m\angle CSI = 90°$ | 2. Def. of a rectangle |
| 3. $m\angle SCR \neq m\angle CSI$ | 3. Given |
| 4. $\square RISC$ is not a rectangle | 4. The assumption led to a contradiction between statements 2 and 3. |

**24.** Complete the missing reasons in this indirect proof.

> **Given** $WT \neq TS$
>
> **Prove** Quad $WISH$ is not a $\square$

| Statements | Reasons |
|---|---|
| 1. $\square WISH$ | 1. |
| 2. $\overline{WS}$ and $\overline{HI}$ bisect each other | 2. |
| 3. $WT = TS$ and $HT = TI$ | 3. |
| 4. $WT \neq TS$ | 4. |
| 5. Quad $WISH$ is not a $\square$ | 5. |

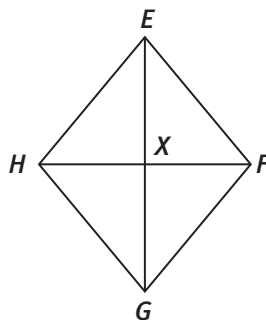**SUGGESTED LEARNING STRATEGIES:** Activating Prior Knowledge, Think/Pair/Share, Interactive Word Wall, Group Discussion

A **rhombus** is a parallelogram with four congruent sides.

25. Graph quad *USMC* with coordinates *U*(1, 1), *S*(4, 5), *M*(9, 5) and *C*(6, 1) on the grid to the right.

   **a.** Verify that Quad *USMC* is a parallelogram by finding the slope of each side.

   **b.** Verify that ▱*USMC* is a rhombus by finding the length of each side.

   **c.** Find the slopes of the diagonals, $\overline{MU}$ and $\overline{SC}$.

   **d.** Use the results in Part c to complete the theorem.

   The diagonals of a rhombus are _____.

26. Given Quad *EFGH* is a rhombus.

   **a.** List the three triangles that are congruent to △*HXE*.

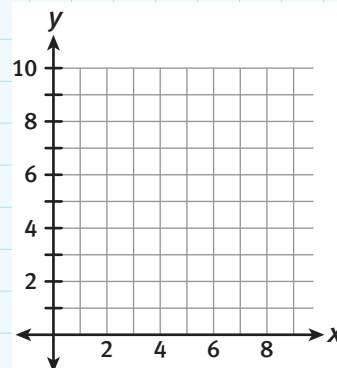   **b.** Explain why ∠*EFX* ≅ ∠*GFX* and ∠*HGX* ≅ ∠*FGX*.

   **c.** Complete the theorem.

   Each diagonal of a rhombus _____.

27. List all of the properties of a rhombus. Begin with the properties of a parallelogram.

## My Notes

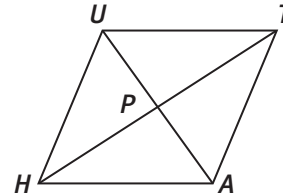A formal proof for the theorem in Item 25 is left as a practice exercise.

*My Notes*

**28.** Given Quad *UTAH* is a rhombus. Use the properties of a rhombus and the figure below to find each of the following.

**a.** Solve for *x* if
$m\angle UPT = 4x + 18$.

**b.** Solve for *x* and *y* if
$UT = 5x + 4$,
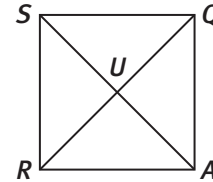$TA = 2x + y$,
$HA = 2y - 8$ and
$UH = 24$.

**c.** Solve for *x* if $m\angle PAH = 8x + 2$ and $m\angle PAT = 10x - 10$.

A **square** is a parallelogram with four right angles and four congruent sides.

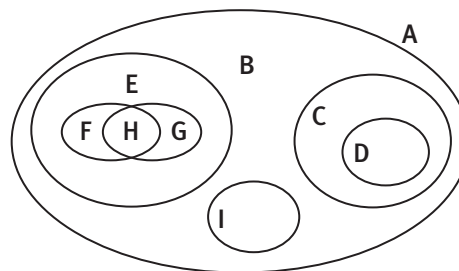**29.** Alternate definitions for a square.

**a.** A square is a rectangle with

**b.** A square is a rhombus with

**30.** List all of the properties of a square.

kites
isosceles trapezoids
parallelograms
polygons
quadrilaterals
rectangles
rhombi
squares
trapezoids

**31.** Match each region in the Venn Diagram below with the correct term in the table to the left.

## CHECK YOUR UNDERSTANDING

**Write your answers on notebook paper. Show your work.**

1. Make a true statement by filling in each blank with *always*, *sometimes*, or *never*.

   **a.** A trapezoid is _____ isosceles.

   **b.** A trapezoid is _____ a quadrilateral.

   **c.** The length of the median of a trapezoid is _____ equal to the sum of the lengths of the bases.

   **d.** Trapezoids _____ have a pair of parallel sides.

   **e.** Trapezoids _____ have two pairs of supplementary consecutive angles.

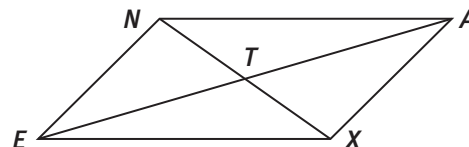2. Given Quad *GHJK* is a trapezoid. $\overline{PQ}$ is the median.

   

   **a.** If $HJ = 40$ and $PQ = 28$, find $GK$.

   **b.** If $HJ = 5x$, $PQ = 5x - 9$ and $GK = 3x + 2$, then solve for $x$.

3. Given Quad *JONE* is a trapezoid.

   

   **a.** $\angle ONJ \cong$ _____
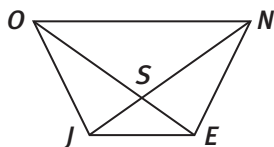
   **b.** If $\overline{OJ} \cong \overline{NE}$, then $\overline{OE} \cong$ _____.

   **c.** If $\overline{OJ} \cong \overline{NE}$, then $\angle NEJ \cong$ _____.

4. Quadrilateral *XENA* is a parallelogram. *T* is the point of intersection of the diagonals. For each situation, write an equation and solve for *y*.

   

   **a.** $EN = 5y + 1$ and $AX = 8y - 5$

   **b.** $m\angle ANX = 3y - 1$ and $m\angle NXE = 2y + 1$

   **c.** $ET = y - 1$ and $EA = 3y - 10$

   **d.** $m\angle ANE = 7y - 5$ and $m\angle NEX = 3y + 5$

5. *M* is the fourth vertex of a parallelogram. The coordinates of the other vertices are: (6,4), (8,1) and (2,0). *M* can have any of the following coordinates *except*:

   **a.** $(6, -2)$     **b.** $(12, 5)$

   **c.** $(4, -3)$     **d.** $(0, 3)$

6. Given Quad *QRST* with coordinates $Q(0, 0)$, $R(2, 6)$, $S(12, 6)$ and $T(12, 0)$.

   **a.** What is the best name for Quad *QRST*? Explain.

   **b.** Find the coordinates of the midpoint for each side of Quad *QRST* and label them *M*, *N*, *O*, and *P*. What is the best name for Quad *MNOP*? Explain.

7. Given Quad *WHAT* with vertices $W(2, 4)$, $H(5, 8)$, $A(9, 5)$ and $T(6, 1)$. What is the best name for this quadrilateral?

   **a.** parallelogram     **b.** rhombus

   **c.** rectangle     **d.** square

## CHECK YOUR UNDERSTANDING (continued)

**8.** Given Quad *ABCD* is a rhombus and $m\angle ABD = 32°$. Find the measure of each numbered angle.
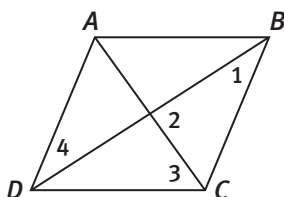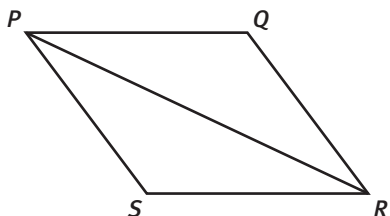


**9.** Given Quad *RIGH* is a rectangle.
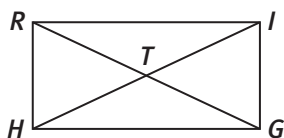


**a.** If $RT = 18$, then $RG =$ _____.

**b.** If $RG = 4x + 12$ and $HI = 10x - 15$, then $x =$ _____.
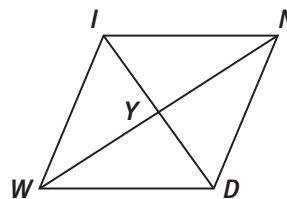
**10. Given:** parallelogram *PQRS* with diagonal *PR*

**Prove:** $\triangle PQR \cong \triangle RSP$



**11.** Write an indirect proof.

**Given:** $\triangle WIN$ is not isosceles

**Prove:** Quad *WIND* is not a rhombus



**12.** **MATHEMATICAL REFLECTION** Ginger noticed that no matter the height of the adjustable stand for her electric piano, the keyboard remains level and centered over the stand. What has to be true about the legs of the stand? Explain.