Contemporary Engineering Sciences, Vol. 11, 2018, no. 95, 4725 - 4732
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/ces.2018.89518

Qualitative Analysis of Bifurcations and Its Application in the Diode-Tunnel Circuit

Fernando Mesa

Department of Mathematics and GIMAE
Universidad Tecnológica de Pereira
Pereira, Colombia

Germán Correa Vélez

Department of Mathematics and GIMAE
Universidad Tecnológica de Pereira
Pereira, Colombia

Pedro Pablo Cárdenas Alzate

Department of Mathematics and GEDNOL
Universidad Tecnológica de Pereira
Pereira, Colombia

Abstract

This paper is about the analysis of a type bifurcation presents in the nonlinear typical circuit, known as diode-tunnel circuit. Furthermore, its presents a brief summary about the qualitative analysis to nonlinear dynamics systems, this allow to get a great variability of conclusions about dynamics of system when it varies one of its parameters.

Keywords: Bifurcation, diode-tunnel circuit, fixed points

1 Introduction

The objective of the analysis of the temporal evolution of natural or artificial phenomena is to be able to predict such evolution, and as much as possible explain it. To perform this task it is necessary to use mathematical models through the use of differential equations, which allow to describe analytically and qualitatively the behavior of dynamic systems from different areas of science such as mechanics, medicine, economics, electromagnetism, etc. This is why it is said that "differential equations are the cornerstone of applied mathematics" [1].

The qualitative analysis of differential equations arose at the end of the 19th century due to questions about celestial mechanics such as the problem of the 3 bodies, that the quantitative analysis was not able to give a clear answer. The pioneering works were carried out by Poincare, which introduces a new point of view that today is the basis of topology and differential geometry [2]

The concept of bifurcation of dynamic systems makes reference to the changes that these present in their qualitative structure when modifying one or several of their input parameters. Thus, the study of the typical properties of dynamic systems and their bifurcations provide a frame of reference for the possible results that are expected from a non-linear problem. The qualitative analysis is carried out according to the behavior of the trajectories in the phase space, which form a family of solutions depending on each of the initial conditions given [3].

2 Qualitative theory and Bifurcations in Dynamical Systems

The mathematical model of a dynamic system can be represented in the space of states or a set of first order differential equations in the following way:

$$\dot{x} = f(x, \mu), \tag{1}$$

where, $x \in \mathbb{R}^n$ and x is the vector of system states and $\mu \in \mathbb{R}^k$ with μ as the state parameter. The solution or trajectory of (1) $x(t)$ will depend on the initial conditions defined. For nonlinear dynamic systems, these are mostly calculated by computation integration techniques. [4]

The qualitative analysis is strictly related to the projection of the trajectories $x(t)$ in the phase space, called phase portrait. This diagram shows all the qualitative characteristics of the behavior of the system, one of the most relevant is the analysis of isolated equilibrium points, since there are enough mathematical backgrounds that allow a clear classification of these points, on the other hand, It should be noted that for non-linear dynamic systems the equilibrium points are not unique and depending on the initial conditions the system reaches different equilibria.

3 Application to the diode - tunel circuit

Considering the circuit of figure 1, known as diode-tunnel circuit, where its main characteristic is the current function through diode D in terms of its voltage, that is, $i_D = h(v_D)$.

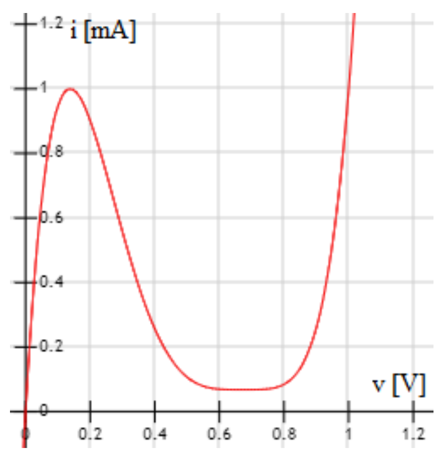

Figure 1: Property $i_D - v_D$.

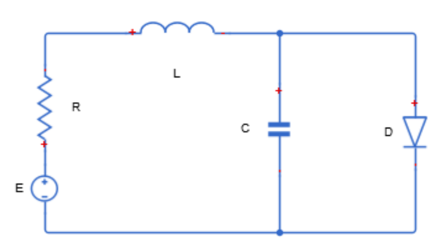

Figure 2: Diode-tunel circuit.

The previous circuit has two energy storage elements, whose behavior is considered linear and invariant with time, which are described by the equations:

$$i_C = C\frac{dv_C}{dt} \tag{2}$$

$$v_L = L\frac{di_L}{dt}, \tag{3}$$

where i and v are the current and the voltage through each element.

To represent the previous circuit in its corresponding state space, the most appropriate procedure is to take as state variables $x_1 = v_C$ and $x_2 = i_L$. Since we need to express all the variables present in the system according to the state variables, the Kirchhoff laws are applied, from which we obtain the following equations:

$$i_C + i_D - i_L = 0 \tag{4}$$

and

$$v_C - E + Ri_L + v_L = 0. \tag{5}$$

Replacing by the state variables, already selected, we obtain the following system:

$$\begin{aligned} \dot{x}_1 &= \tfrac{1}{C}\left(-h\left(x_1\right) + x_2\right) \\ \dot{x}_2 &= \tfrac{1}{L}\left(-x_1 - Rx_2 + E\right) \end{aligned} \tag{6}$$

The equilibrium points for the system are given when $\dot{x}_1 = \dot{x}_2 = 0$, that is:

$$\begin{aligned} 0 &= \tfrac{1}{C}\left(-h\left(x_1\right) + x_2\right) \\ 0 &= \tfrac{1}{L}\left(-x_1 - Rx_2 + E\right) \end{aligned}$$

Solving the above system of equations, we have that the equilibrium points for the tunnel-diode circuit are given by:

$$h\left(x_1\right) = \frac{E}{R} - \frac{x_1}{R} \tag{7}$$

Graphically you can see that the equilibrium points will be the intercepts between the function $h(x_1)$ shown in figure 1 with the line of Eq.(7), as indicated in figure 3.

From the figure 3, it can be noted that the equilibrium points can vary depending on how the line is modified, which depends on the input parameters E and R. That is, when moving the line from the bottom to the top, it is first that there is only a point of equilibrium, then two more points appear and finally a single point of equilibrium is maintained. This variation in the number of equilibrium points for a system is known as bifurcation, which we will analyze previously [5].

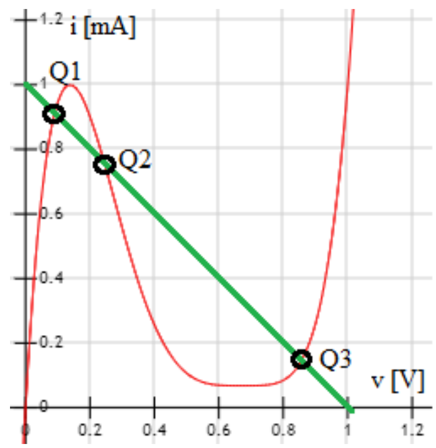

Figure 3: Equilibrium points diode-tunnel circuit.

3.1 Analysis of equilibrium points

Once we know how to calculate the equilibrium points of a system, it is important to make a characterization of its stability for each one of them. This task is achieved, obtaining an approximate linear system around said points by means of the Jacobian system.

The values of the system parameters (6) are shown in [3]: $E = 1.2\ V,\ R = 1.5\ k\Omega,\ C = 2\ pF,\ L = 5\ \mu H$. Therefore, the system will be described by:

$$\begin{aligned} \dot{x}_1 &= 0.5\left(-h\left(x_1\right) + x_2\right) \\ \dot{x}_2 &= 0.2\left(-x_1 - 1.5x_2 + 1.2\right), \end{aligned} \tag{8}$$

where, $h\left(x_1\right) = 17.76x_1 - 103.79x_1^2 + 229.62x_1^3 - 226.31x_1^4 + 83.72x_1^5$.

The phase diagram for this system can be visualized in the following figure, where three equilibrium points $Q_1 = (0.063, 0.758)$, $Q_2 = (0.285, 0.61)$ and $Q_3 = (0.884, 0.21)$ can be observed [9,10]. The nature of each of the previous points is given by the determination of the Jacobian values around the points of equilibrium [11,12].

$$J\left(x_1, x_2\right) = \begin{bmatrix} -0.5h\prime(x_1) & 0.5 \\ -0.2 & -0.3 \end{bmatrix},$$

where

$$h'\left(x_1\right) = 17.76 - 207.58x_1 + 688.86x_1^2 - 905.24x_1^3 + 418.6x_1^4$$

Evaluating the Jacobian in each of the equilibrium points we have:

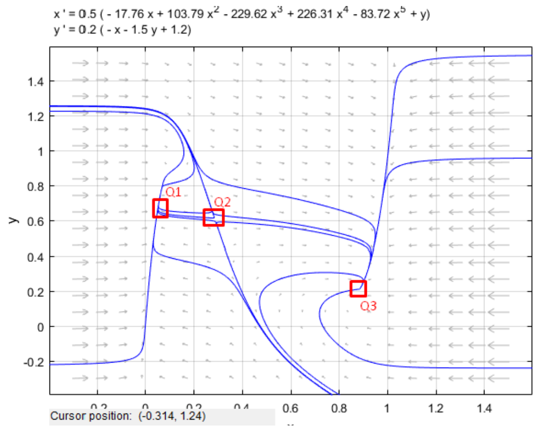

Figure 4: Phase space diode-tunnel circuit.

- For $Q_1 = (0.063, 0.758)$:

$$A_1 = \begin{bmatrix} -3.598 & 0.5 \\ -0.2 & -0.3 \end{bmatrix} \text{ with eigen-values } \begin{matrix} \lambda_1 = -3.57 \\ \lambda_2 = -0.33 \end{matrix}$$

- For $Q_2 = (0.285, 0.61)$

$$A_2 = \begin{bmatrix} 1.82 & 0.5 \\ -0.2 & -0.3 \end{bmatrix} \text{ with eigen-values } \begin{matrix} \lambda_1 = -0.25 \\ \lambda_2 = 1.770 \end{matrix}$$

- For $Q_3 = (0884, 0.21)$:

$$A_3 = \begin{bmatrix} -1.427 & 0.5 \\ -0.2 & -0.3 \end{bmatrix} \text{ with eigen-values } \begin{matrix} \lambda_1 = -1.33 \\ \lambda_2 = -0.40 \end{matrix}$$

Therefore, Q_1 and Q_3 are stable nodes and Q_2 is a saddle-point.

3.2 Bifurcation with the state variable

The study of the bifurcation for the diode-tunnel circuit is made from the Eq.(7) of a function is obtained in terms of the input parameter, that is, $x_1(E)$. Now, varying the input voltage, $E = \mu$, between $(0V - 4V)$, where you have two branch points one for $x_1^* = 0.149$ and the other for $x_1^* = 0.512$. This allows to divide the diagram into 4 intervals where the interval AB is observed that there is only one equilibrium solution that will be stable, in the interval BC two more solutions appear, one of which is unstable and the others stable, and finally two solutions disappear. Only one that will be stable from the value in C. This behavior can be seen in figure 5 [6,7,8].

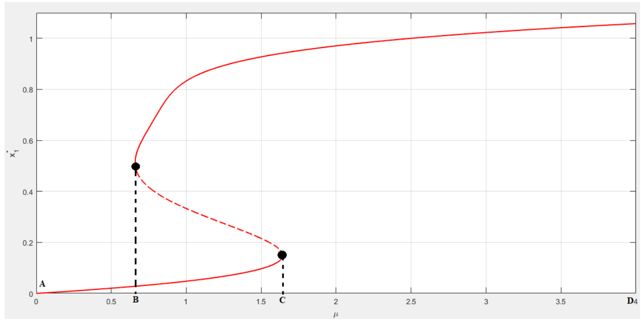

Figure 5: Diode-tunnel circuit bifurcation.

4 Conclusion

Qualitative theory is a very important tool when you want to analyze a non-linear dynamic system where finding the explicit solution is a complex task or in some cases it is not necessary, since you only need to know the asymptotic behavior of trajectories in space phase to conclude about the solution.

The chair-node bifurcation is the most common type of bifurcation in dynamic systems that present this phenomenon, its analysis is of great importance when designing systems that are stable. When performing linearization around a point of equilibrium, it must be taken into account that this point is hyperbolic to apply the stability criterion by means of the eigenvalues ??of the new system, since, if it is not, it can not be concluded that the stability of the linear system is identical to that of the non-linearized system.

Acknowledgements. We would like to thank the referee for his valuable suggestions that improved the presentation of this paper and our gratitude to the Department of Mathematics of the Universidad Tecnológica de Pereira (Colombia), the GEDNOL Research Group and GIMAE (Grupo de Investigación en Matemática Aplicada y Educación).

References

[1] E. Freire, *Análisis Cualitativo y de Bifurcaciones en Sistemas Dinámicos*, Diss., Universidad de Sevilla, 1982.

[2] C. Jung, Poincaré map for scattering states, *Journal of Physics A: Mathematical and General*, **19** (1986), no. 8, 1345-1353. https://doi.org/10.1088/0305-4470/19/8/016

[3] O. Bogoiavlenskii, *Methods of the Qualitative Theory of Dynamic Systems in Astrophysics and Gas Dynamics*, Moscow Izdatel Nauka, 1980.

[4] H. Baruh, *Analytical Dynamics*, Boston:WCB/McGraw-Hill, 1999.

[5] M. Khan, M. Swaleh, S. Sessa, Fixed point theorems by altering distances between the points, *Bulletin of the Australian Mathematical Society*, **30** (1984), no. 1. https://doi.org/10.1017/s0004972700001659

[6] D. Sullivan, Quasiconformal homeomorphisms and dynamics II: Structural stability implies hyperbolicity for Kleinian groups, *Acta Mathematica*, **155** (1985), 243-260. https://doi.org/10.1007/bf02392543

[7] Kening Lu, A Hartman-Grobman theorem for scalar reaction-diffusion equations, *Journal of Differential Equations,* **93** (1991), no. 2, 364-394. https://doi.org/10.1016/0022-0396(91)90017-4

[8] R. Landauer, Fluctuations in bistable tunnel diode circuits, *Journal of Applied Physics,* **33** (1962), no. 7, 2209-2216. https://doi.org/10.1063/1.1728929

[9] G. Layek, Theory of Bifurcations, *An Introduction to Dynamical Systems and Chaos, Springer, New Delhi,* 2015, 203-254. https://doi.org/10.1007/978-81-322-2556-0_6

[10] T. Monteiro, D. Delande, A. J. Fisher, G. S. Boebinger, Bifurcations and the transition to chaos in the resonant-tunneling diode, *Physical Review*, **56** (1997), no. 7, 3913-3921. https://doi.org/10.1103/physrevb.56.3913

[11] L. Chua, C. Desoer, E. Kuh, *Linear and Nonlinear Circuits*, McGraw-Hill, New York, 1987.

[12] P. Hartman, On the local linearization of differential equations, *Proceedings of the American Mathematical Society*, **14** (1963), no. 4, 568. https://doi.org/10.2307/2034276

Received: October 7, 2018; Published: November 12, 2018